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Predicting Arrival Delays: An Application of Spatial Analysis

Analysts have many tools available to forecast delays. However, spatial analysis does not readily come to mind when predicting delays. Based on the case study of Newark Liberty International airport (EWR), this study proposes to illustrate the potential application of prevalent geostatistical techniques to delay forecast. Arguably, there is a high degree of dependence among delays in a space or neighborhood defined by hour of operation and by day. Local spatial autocorrelation statistics can help determine how delays in a space are autocorrelated to other ones. Among the other spatial analytical techniques, kriging enables the interpolation of delay estimates at unobserved spaces based on the values at observed spaces. Error estimates can be mapped to define spatial patterns (spaces where delays are likely to be more intense). Finally, spatial error regression provides a method for analysts to test the reliability of their findings when spatial dependency in errors is not taken into account.

1. INTRODUCTION

The airspace surrounding the New York City metropolitan area is among the busiest and most complex in the world. In 2008, the three largest New York City airports accounted for 1,273,145 takeoffs and landings[i]. These airports have also stood among the most delayed facilities in the United States: In 2008, 64.73% of the flights reported by the major domestic airlines arrived on time at EWR, 70.85% at JFK and 66.85% at LGA. During the same time period, the national average was 77.76%[ii]. These on-time statistics are compared with published airline schedules[iii].

This study serves two purposes. First, it provides an illustration of how statistical techniques used in geostatistics and spatial econometrics can be applied to the forecast of delays as an alternative to more prevalent methods such as regression analysis. Secondly, it is meant to provide an introduction to spatial analysis for government, airline and airport analysts who need to evaluate the reliability of their models and findings.

Spatial analysis has been used in the airline sector to evaluate origin and destination flows (Derudder et al. 2009; Lee et al. 1994), market attributes and concentration mapping (Daraban and Fournier 2008; Reynolds-Feighan 2007; Spiller 2006). However, spatial analysis has not been utilized as an alternative methodology to forecast delays.

This study assumes that delay is a spatially distributed variable based on time of operations and day of the week (the space or neighborhood of a delay event). It focuses on spatial dependency in total minutes of arrival delays at EWR (i.e., the centroid). After describing the data and their sources, the discussion will proceed with the definition of the model variables. The analysis of the data structure through semivariograms and their dependency through the identification of spatial autocorrelation will lead to the interpolation of delays between sample points using kriging as an optimal interpolator. Finally, the exposition of spatial analytical techniques will end with a comparison between the Ordinary Least Square regression and the spatial error models and identify differences in the model outcomes.

2. DATA ANALYSIS

1.1 Data Sample

The sample includes 558 instances of hours with arrival delays. The July 2009 data were organized by hour and by day. The choice of July was motivated by the high impact of convective weather on airport on-time performance and high volumes of traffic.

1.2 Data Sources

The operations, delay and capacity metrics are stored and compiled in the Aviation Systems Performance Metrics (ASPM) data warehouse based on the following sources:

- Out-Off-On-In (OOOI) data provided by ARINC,
- Runways configuration and airport rate information collected by the Air Traffic Control System Command Center (ATCSCC),
- Air carriers' flight schedules published by Innovata,
- Flights records assembled from the Enhanced Traffic Management System (ETMS), and
- On-time data compiled by the Bureau of Transportation Statistics (BTS) in their monthly survey called the Airline Service Performance Quality (ASQP).

1.3 Definitions of Variables

Below are the definitions of all the model variables including the sources of data:

- *Total Minutes of Arrival Delays* (DLASCHARR) represent the total delays of one minute of more compared with the airline's gate-in schedule (sources: ASPM, ARINC, Innovata).
- *Weather* (MC) stands for the approach conditions at a particular hour based on ceiling and visibility minima (Instrument Approach Conditions in case of low visibility and ceiling v. Visual Approach Conditions in case of clear meteorological conditions). The periods in 'Instrument Approach Conditions' (IAC) are the hours when an airport is not capable of vectoring for visual approaches based on minima for ceiling and visibility. The minimum values are 3,000-ft ceiling and four-mile visibility at EWR, 2,000 feet and four miles at JFK and 3,200 feet and four miles at LGA. The facilities provide the information to the ATCSCC.
- *Runway Configurations* (RWYCONFIG, JFK_RWY, LGA_RWY) are the set of arrival and departure runways in use at a particular hour (source: ATCSCC).
- *Wind Angle* (WINDANGLE) is the direction of the wind in degrees (source: National Oceanic and Atmospheric Administration).
- *Total Minutes of Taxi-In Delays* at EWR (DLATI) are the difference between the actual minutes of taxiing from wheels on to gate in and an impeded taxi in times computed in ASPM based on ASQP carriers and season (source: ASPM, ARINC, ASQP).
- *Total Minutes of Taxi-Out Delays* at EWR (DLATO) represent the additional minutes of taxiing from gate out to wheels off compared with unimpeded taxi-out times (source: ASPM, ARINC, ASQP).
- *Arrival Demand* (ARRDEMAND at EWR, JFK_ARRDEM, LGA_ARRDEM) is computed on an individual flight basis within 15-minute increments. Arrival demand

starts at the time when aircraft wheels are off and includes the estimated time enroute. It ends when wheels are on (sources: ASPM and ARINC).

- *Arrivals and Departures* (EWR_ARR, EWR_DEP) are the volume of traffic handled by the EWR facility (source: ETMS).

- *Total Minutes of Airport Departure Delays* (DLASCHOFF) include the gate departure delays in minutes (actual gate out time compared with scheduled gate-out time) plus taxi-out delays as explained above. The sources are ETMS, ARINC and Innovata.

- *Total Minutes of Airborne Delays* (DLAAIR). Airborne delay is the actual airborne time minus a carrier's submitted estimated time enroute (sources: ASPM and ETMS).

- *Total Minutes of Taxi-In* (ACTTI) account for the total minutes of operations from wheels on to gate in (sources: ARINC and ETMS).

1.4 Data Processing

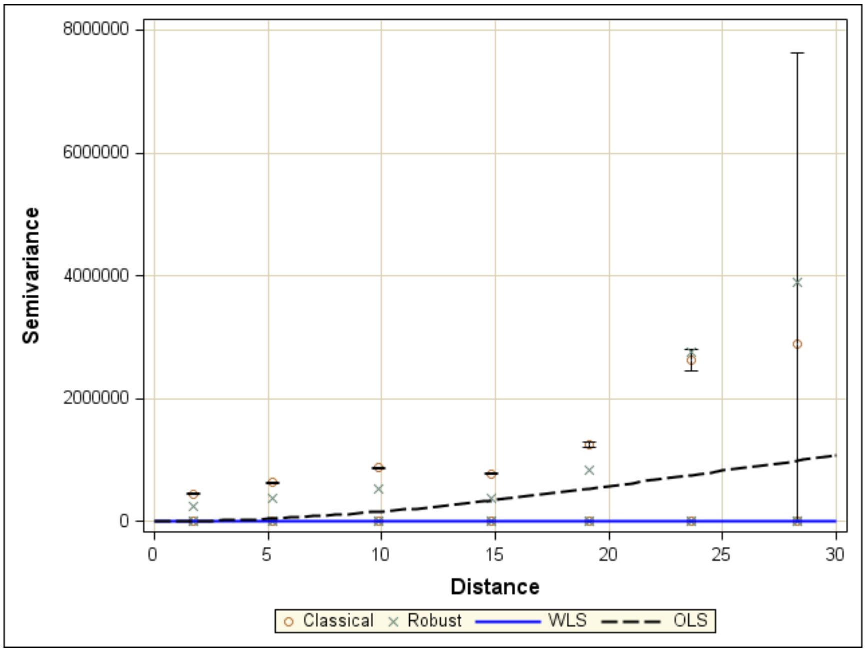
The data were input into SAS®. The MIXED procedure enabled to select the covariate structure based on the lowest value of the fit statistics (residual log likelihood, Akaike's and Schwarz's criteria). The VARIOGRAM procedure made it possible to generate the theoretical and empirical semivariograms. Kriging analysis was performed with the KRIEGE2D procedure. The REG procedure provided regression analysis.

1.5 Measurement of Spatial Dependency

The variogram[iv] is a tool for quantifying spatial correlation (see Appendix 1 for further details). It displays the variances within groups of observations plotted as a function of distance between observations. The variogram is also used to examine the degree of spatial continuity in data at various lags or distances of separation—assuming the data are stationary.

Semivariance represents half the average squared difference between pairs of arrival delays at a given distance apart. The semivariogram measures variance among sites as a function of distance. Figure 1 provides a comparison between the empirical and the theoretical semivariograms (see Cressie 1993). In geostatistics, semivariograms are more readily used than covariograms which express dependency between random variables in terms of covariances. The comparison of the fit statistics determined that the Gaussian covariance structure was preferable to others (spherical or exponential, for instance).

Figure 1: Empirical and Fitted Theoretical Semivariograms



The semivariograms suggest that points separated by short distances are correlated (Figure 1). The correlation decreases as the distance between points increases (up to a distance of 15 in the present case). Because the empirical variogram provides estimates for a definite set or classes of lags, it is necessary to fit a parametric semivariogram in order to derive large-scale trends and provide spatial predictions. Other forms of fit are also indicated in Figure 1: weighted least square (WLS) and ordinary least square (OLS).

Figure 2 shows the distribution of arrival delay minutes by day and by hour. Delays were more significant in the afternoon than in the morning. The surface plot also highlights the delay outliers and their day of occurrence. Figures 3 provides the duration, intensity and days when significant arrival delays happened.

Figure 2: Surface Plot of Arrival Delay Minutes

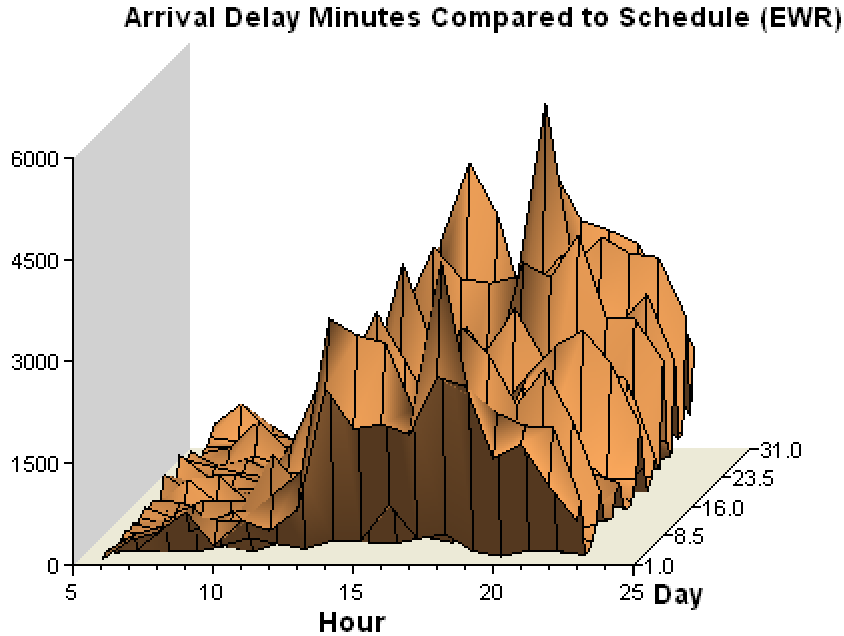
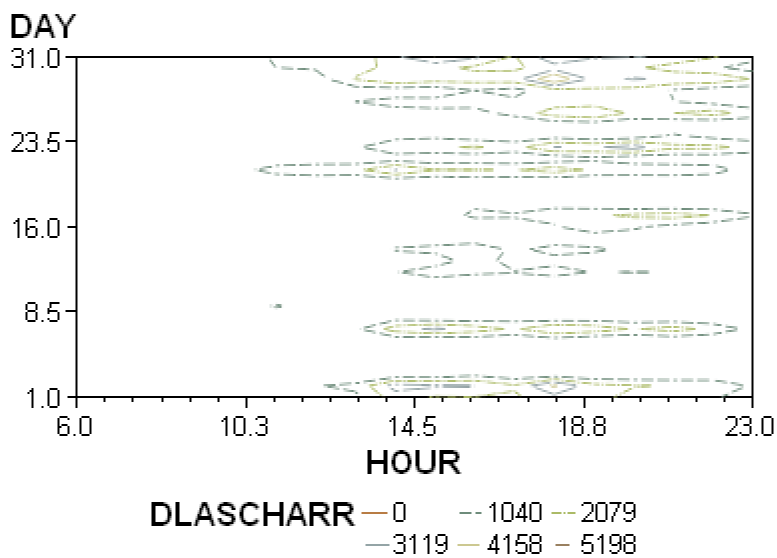


Figure 3: Contour Plot of Arrival Delay Minutes (By Hour and By Day)



3. SPATIAL ANALYSIS

Spatial analysis refers to a set of techniques applied to autocorrelation, interpolation, regression and interaction models. After calculating the empirical semivariogram and modeling the theoretical variogram, the next step consists in identifying spatial autocorrelation and then using ordinary kriging to identify the best linear unbiased predictors and errors. Finally, we will focus on one type of spatial regression based on the error as opposed to lagged dependent variable.

2.1 Spatial Autocorrelation

The pattern of spatial distribution can be measured with two statistics:

- Moran's *I* computed as $I(d) = \sum_{i}^{n} \sum_{j}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x}) \Big/ (S^2 \sum_{i}^{n} \sum_{j}^{n} w_{ij})$

With $S^2 = \frac{1}{n}\sum_{i}^{n}(x_i - \bar{x})^2$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

- Geary's ***c*** as $C(d) = (n-1) \Big/ (2\sum_{i}^{n}\sum_{j}^{n} w_{ij}) \{\sum_{i}^{n}\sum_{j}^{n} w_{ij}(x_i - x_j)^2 \Big/ \sum_{i}^{n}(x_i - \bar{x})^2\}$

Table 1: Autocorrelation Statistics for Selected Variables

Autocorrelation Statistics for Total Minutes of Arrival Delays

Assumption	Coefficient	Observed	Expected	Std Dev	Z	Pr > |Z|
Normality	Moran's I	0.0825	-0.0018	0.0018	46.9	<.0001
Normality	Geary's c	0.8649	1	0.00743	-18.2	<.0001

Autocorrelation Statistics for EWR Arrivals

Assumption	Coefficient	Observed	Expected	Std Dev	Z	Pr > |Z|
Normality	Moran's I	0.069	-0.0018	0.0018	39.4	<.0001
Normality	Geary's c	0.909	1	0.00743	-12.3	<.0001

Autocorrelation Statistics for Actual Taxi In Times

Assumption	Coefficient	Observed	Expected	Std Dev	Z	Pr > |Z|
Normality	Moran's I	0.0875	-0.0018	0.0018	49.6	<.0001
Normality	Geary's c	0.8873	1	0.00743	-15.2	<.0001

Autocorrelation Statistics for EWR Departure Demand

Assumption	Coefficient	Observed	Expected	Std Dev	Z	Pr > |Z|
Normality	Moran's I	0.0277	-0.0018	0.0018	16.41	<.0001
Normality	Geary's c	0.9624	1	0.00743	-5.06	<.0001

Autocorrelation Statistics for EWR Arrival Demand

Assumption	Coefficient	Observed	Expected	Std Dev	Z	Pr > |Z|
Normality	Moran's I	0.0855	-0.0018	0.0018	48.5	<.0001
Normality	Geary's c	0.8475	1	0.00743	-20.5	<.0001

Autocorrelation Statistics for JFK Arrival Demand

Assumption	Coefficient	Observed	Expected	Std Dev	Z	Pr > |Z|
Normality	Moran's I	0.0793	-0.0018	0.0018	45.1	<.0001
Normality	Geary's c	0.8798	1	0.00743	-16.2	<.0001

Autocorrelation Statistics for LGA Arrival Demand

Assumption	Coefficient	Observed	Expected	Std Dev	Z	Pr > |Z|
Normality	Moran's I	0.0503	-0.0018	0.0018	29	<.0001
Normality	Geary's c	0.8958	1	0.00743	-14	<.0001

Source: ASPM

A positive value of Moran's *I* suggests that observations tend to be similar contrary to a negative value. The observations are arranged randomly over space when Moran's *I* is close to zero. Based on the p values of the reported Moran's *I*, we can reject the null hypothesis that there is zero spatial autocorrelation in the values of the specified variables in Table 1. With Geary's ***c***, values greater than or equal to 1 imply that observations are likely to be dissimilar—as opposed to a ***c*** value less than 1. The distance matrix used in this study is 558 X 558 observations.

2.2 Spatial Interpolation: Ordinary Kriging

After identifying the spatial correlation structure of a variable, the data from the measured locations can be used to estimate a variable at locations where it had not been measured. This interpolation from measured locations or spaces to unmeasured locations is referred to as kriging[v].

There are three steps in kriging analysis. The first one consists in computing the empirical variogram of the sampled data. Secondly, a theoretical variogram that fits the sample variogram for spatial extrapolation is selected among different mathematical forms and parameters (exponential, Gaussian, spherical, and power). Third, the variogram is used to solve the kriging system at a specified set of spatial points.

In ordinary kriging, the variable values from the measured locations become covariates for the values at unmeasured locations. In the present ordinary kriging model, the coordinates used to determine the location of points in the grid are days and hours (Figure 2). There are 49 prediction grid points and the neighborhood is based on a radius of 30 days to fully reflect the spatial correlations in the data. The type of covariance model is spherical with a sill of 6.5 and a range of 30. The nugget effect is 0.

For each predicted location, the variance-covariance structure identified in the variogram is used to fit a linear function to these covariates. As a result, it is possible to derive an unbiased linear prediction (ESTIMATE) and a standard error of the value at the location detail (STDERR) as shown in Table 2. Predictions are derived as weighted averages of known values. The points closer to the one to interpolate have a greater weight. Clusters of points become a single point. Finally, the error of each prediction is based on the sample point locations.

Table 2: Kriging/Estimates and Standard Errors of Arrival Delay Minutes

Obs	GXC	GYC	NPOINTS	ESTIMATE	STDERR
1	0	0	452	-22.6430	1.7837
2	0	5	527	129.6150	1.6430
3	0	10	555	92.3830	1.6197
4	0	15	558	24.7340	1.6179
5	0	20	558	35.7970	1.6176
6	0	25	543	105.1210	1.6283
7	0	30	488	75.0180	1.7001
8	5	0	492	68.6300	0.9255
9	5	5	552	38.5180	0.6881
10	5	10	558	102.6250	0.6874

Figure 4 illustrates the estimated arrival delay minutes and Figure 5 shows the surface of the standard errors to identify stable estimates. Standard errors are higher off peak times—which can be explained by fewer points in the neighborhood defined by the radius—than points at the center.

Figure 4: Surface Plot of Estimated Arrival Delay Minutes

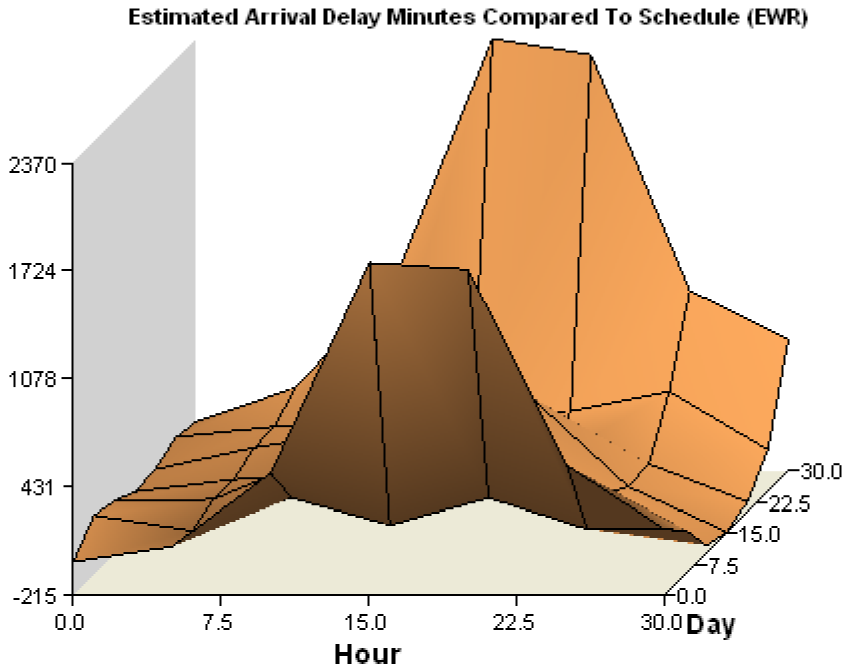
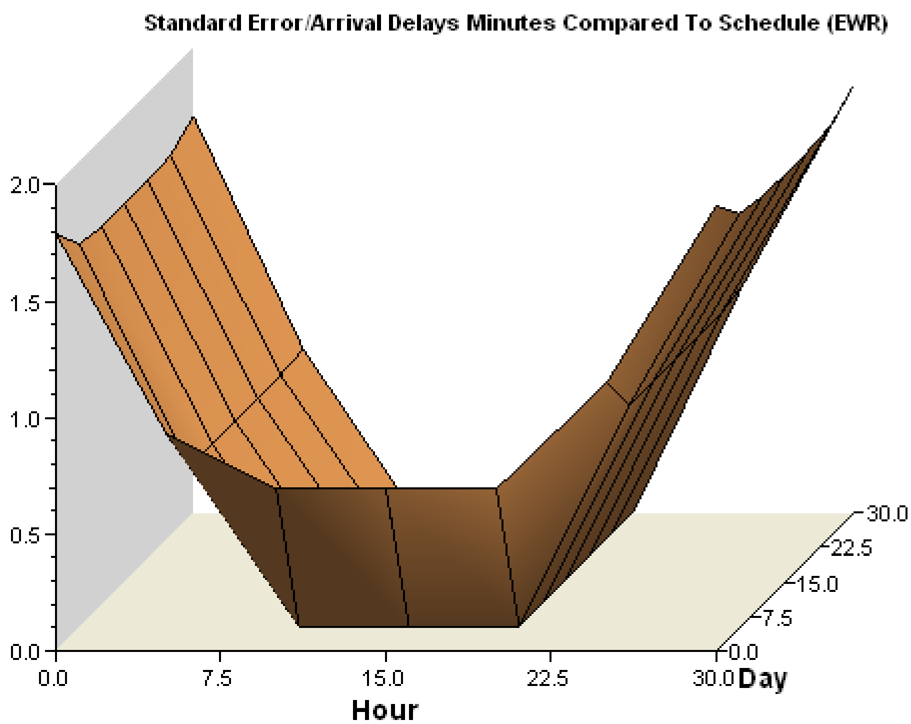


Figure 5: Standard Error Plot



2.3 <u>Spatial Error Model</u>

The MIXED procedure was used to fit a regression model in which the results and expected errors are spatially autocorrelated. The spatial error model can be written as

$$y_i = \mathbf{xi}^*\beta + \lambda^*\mathbf{w}_i^*\xi_i + \varepsilon_i$$

where $\mathbf{w}_i$ is a vector of **W** that denotes the proximity of units with each other, ξ_i the spatial component of the error term and ε_i a spatially uncorrelated error term.

The outcomes of three models were compared in order to assess the significance of the variables in predicting arrival delay minutes at EWR. The first spatial error model is characterized by spatial Gaussian covariance and an intercept subject effect (Exhibit 1). The outcomes indicate that the number of arrivals and arrival demand at EWR, as well as the arrival demand at JFK all have a significant effect on the arrival delay minutes at EWR (based on schedule). Despite the proximity of LGA and EWR in physical distance, the arrival demand at LGA does not have a significant fixed effect on the arrival delay minutes at EWR.

Exhibit 1: Spatial Error Model 1

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
SP(GAU)	Intercept	0.8073
Residual		132491

Fit Statistics

-2 Res Log Likelihood	8092.70
AIC	8096.70
AICC	8096.70
BIC	8105.30

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	60	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	278	63.9079	0	4	.
EWR_ARR	-23	2.0115	553	-11	<.0001
ARRDEMAND	16	0.7646	553	21	<.0001
JFK_ARRDEM	3	0.7818	553	4	<.0001
LGA_ARRDEM	1	0.7607	553	1	0.1947

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
EWR_ARR	1	553	126.24	<.0001
ARRDEMAND	1	553	451.89	<.0001
JFK_ARRDEM	1	553	16.99	<.0001
LGA_ARRDEM	1	553	1.69	0.1947

In the second spatial error model (Exhibit 2), the dependent variable is the total minutes of arrival delays minutes at EWR (DLASCHARR), while the explanatory variables can be classified in three main groups:

- *Class Level Information:* RWYCONFIG, MC, JFK_RWY, LGA_RWY, WINDSPEED, WINDANGLE.
- *Random Effects:* DLATI, DLATO, and ARRDEMAND.
- *Fixed Effects*: EWR_ARR, EWR_DEP, DLASCHOFF, DLAAIR, ACTTI, JFK_ARRDEM, LGA_ARRDEM, WINDSPEED, WINDANGLE, and MC.

Exhibit 2: Spatial Error Model 2

Covariance Parameter Estimates

Cov Parm	Estimate
DLATIA	697.48
DLATO	0.10
ARRDEMAND	196.41
SP(GAU)	1.00
Residual	79,128.00

Fit Statistics

-2 Log Likelihood	7893.5
AIC	8,025.50
AICC	8,043.50
BIC	7,893.50

Solution for Random Effects

Effect	Estimate	Std Err Pred	DF	t Value	Pr > |t|	Alpha	Lower	Upper
DLATIA	-24.94	13.105	493.00	-1.90	0.06	0.9	-26.58	-23.2893
DLATO	-0.32	0.05111	493.00	-6.29	<.0001	0.9	-0.33	-0.3149
ARRDEMAND	14.01	0.66	493.00	21.23	<.0001	0.9	13.93	14.0956

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
EWR_DEP	1	493	7.70	0.0057
EWR_ARR	1	493	124.40	<.0001
DLASCHOFF	1	493	59.33	<.0001
DLAAIR	1	493	1.76	0.1852
ACTTI	1	493	52.90	<.0001
JFK_ARRDEM	1	493	2.12	0.1458
LGA_ARRDEM	1	493	0.02	0.8874
WINDSPEED	16	493	4.54	<.0001
WINDANGLE	36	493	1.60	0.0169
MC	1	493	8.64	0.0034

The second spatial error model shows lower values for the fit statistics (Akaike Information Criterion, Akaike Information Corrected Criterion, Bayesian Information Criterion, and -2 Log Likelihood) than the first one—which is preferable in selecting a model selection. The second model also implies that the random effects of taxi-in/out delay and arrival demand at EWR are significant at a 95% confidence level. These variables are indicators of airport congestion, most likely to impact delays. However, the fixed effects of airborne delay, arrival demand *both* at JFK and LGA are not significant at 95% confidence level. This may be counterintuitive in an airspace environment where the Terminal Radar Approach Control (TRACON) affects each airport's capacity utilization and on-time performance by managing demand for arrivals and departures.

Exhibit 3 provides the estimates from an ordinary least square (OLS) regression model computed with the REG procedure in SAS.

Exhibit 3: OLS Model

Number of Observations Read	558
Number of Observations Used	558

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	306859914	43837131	198.74	<.0001
Error	550	121318912	220580		
Corrected Total	557	428178826			

Root MSE	469.65928	R-Square	0.7167
Dependent Mean	718.41577	Adj R-Sq	0.7131
Coeff Var	65.3743		

Parameter

Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	Intercept	1	122.0854	86.9338	1.4000	0.1608
EWR_DEP	EWR_DEP	1	-14.5147	2.1412	-6.7800	<.0001
EWR_ARR	EWR_ARR	1	-8.8179	3.1691	-2.7800	0.0056
DLASCHOFF	DLASCHOFF	1	0.2506	0.0264	9.4900	<.0001
DLAAIR	DLAAIR	1	0.4516	0.1665	2.7100	0.0069
ACTTI	ACTTI	1	0.6807	0.3389	2.0100	0.0450
JFK_ARRDEM	JFK_ARRDEM	1	7.6878	0.8732	8.8000	<.0001
LGA_ARRDEM	LGA_ARRDEM	1	9.0392	0.7409	12.2000	<.0001

In Exhibit 3, only the intercept is not significant at 95% confidence level. Contrary to the spatial error models 1 and 2, the arrival demand at JFK and LGA are all significant at a 95% confidence level. As a result, a policy maker may focus on a solution that accommodates the three airports as a whole instead of focusing on the management of arrival demand where it is significant (as the spatial error models 1 and 2 suggested).

4. CONCLUDING REMARKS

Spatial analysis is a relatively new methodology in the airline industry, while it is a mainstay in geostatistics. Spatial analysis assumes that delays represent a location in a space determined by coordinates such as hour of operation and day in the present case.

This paper was designed to illustrate how spatial analytical methods can be applied to the study of delay and its forecast. Kriging can be instrumental in identifying clusters of delays and their mapping, to interpolate estimates and to determine errors. The mapping can be based on other coordinates such as runway configurations or sections of the airport.

Finally, the results show that analysts have to be careful when evaluating the factors likely to impact delays. Variables that are significant in an OLS model may not be so in spatial error models. In this study, the outcomes of the spatial error models point out to the significance of internal operational factors such as taxi delays and demand management in reducing delays at EWR.

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APPENDIX: The Variogram/Semivariogram

Figure 1 shows the key components of the empirical variogram/semivariograms and Figure 2 indicates the location of the different vectors. In Figure 1, the range is the point on the x-axis where the curve reaches a plateau. The sill is the height of the curve at the plateau. The nugget effect is the resulting discontinuity in the semivariograms at the origin. The relative nugget effect is the percentage of the overall variance attributable to measurement error. In Figure 2, the lag is the difference between two sites in a pair. The number of pairs available for computing a semivariograms depends on the lag distance.

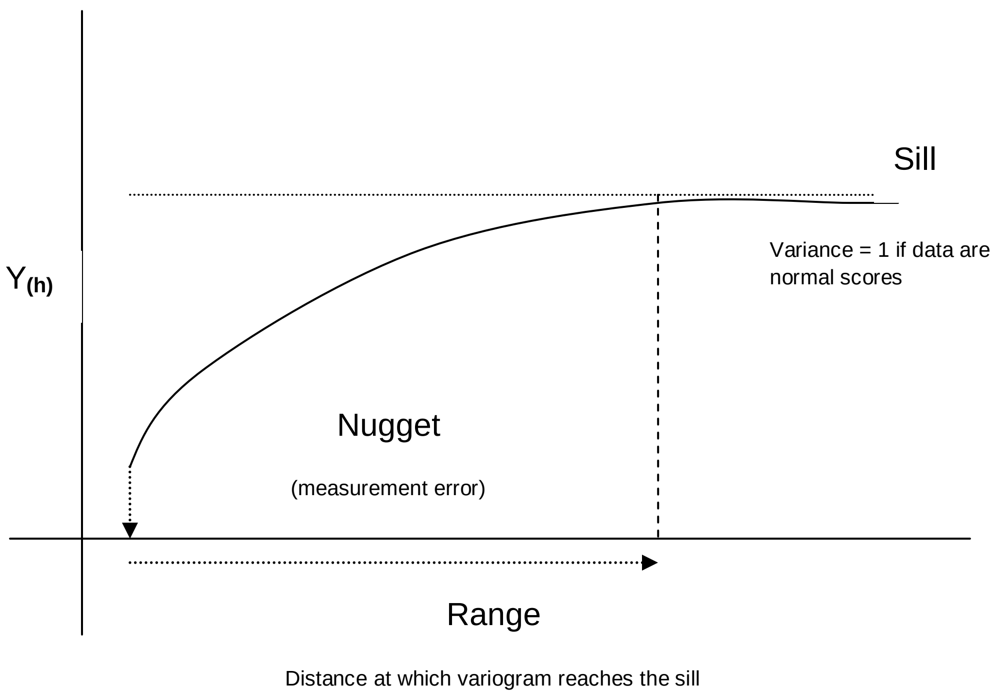

Figure A: Key Components of the Empirical Variogram

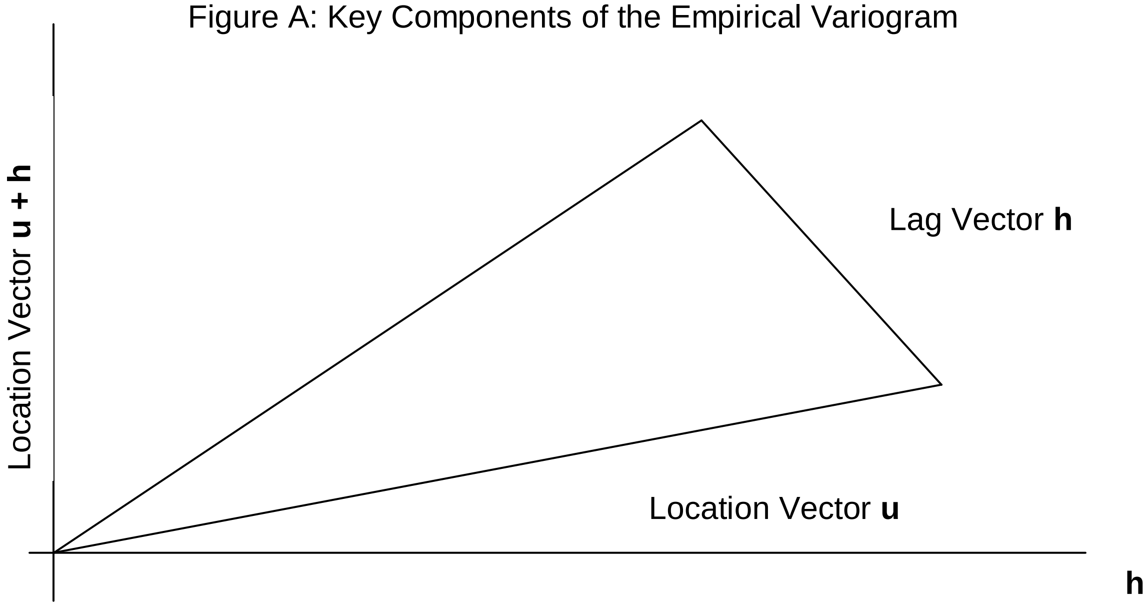

Figure B: The Location Vector

When spatial correlation is stronger in one direction than other, the spatial correlation pattern is *anisotropic*. Anisotropy can be geometric or zonal. Geometric anisotropy can usually be corrected by linear transformation. When the spatial correlation depends on the distance and not the direction of separation, the spatial correlation pattern is characterized as *isotropic*. The lag tolerance is chosen to be *half the smallest lag spacing*. As a result, no pair of points will be used for more than one lag distance.

The theoretical variogram in this study is Gaussian and can be characterized as

$$\gamma_z(h) = c_0 \left[1 - \exp\left(-\frac{h^2}{a_0^2}\right)\right]$$

[i] The data source is the Operations Network (OPSNET).

[ii] ASQP (Airline Service Performance Quality) provides data on on-time performance, tarmac delays and the causes of delays for a sample of 19 carriers at the time of this writing.

[iii] For the Bureau of Transportation Statistics (BTS), a flight is delayed if it arrives 15 minutes or more past its schedule gate-in time. In ASPM, delays can be measured as one minute or more compared with the published schedule or flight plan (delay for all arrivals) or as 15 minutes or more (delay for delayed arrivals) based on the same criteria.

[iv] The theoretical variogram can be defined as follows:

$$\hat{\gamma}(\boldsymbol{h}) = \frac{1}{2n(\boldsymbol{h})} \sum_{(i,j)|\boldsymbol{h_{ij}} = \boldsymbol{h}} (y_i - y_j)^2$$

where $s_i = (s_{1i}; s_{2i})$ = *ith* location; $h_{ij} = s_j - s_i$; the vector connecting points s_i and s_j ; y_i = the observed response at site s_i. It can be Gaussian, exponential, spherical, linear and power. The formula for the experimental variogram is $\hat{\gamma}(h_k) = \frac{1}{2|N(h_k)|}\sum_{i=1}^{N_k}[z(x_i) - z(x_i^{'})]^2$.

[v] The value of a point x* is estimated as a linear combination such that $\hat{f}(x^*) = \sum_{i=1}^{n} \lambda_i(x^*) f(x_i)$ with the weight λ_i as solutions of a system of linear equations.