Title: Internal Consistency in Models of Optimal Resource Use Under Uncertainty

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Abstract: For several decades, economists have been concerned with the problem of optimal resource use under uncertainty. In many studies, researchers assume that prices evolve according to an exogenous stochastic process and solve the corresponding dynamic optimization problem to yield an optimal decision rule for exploitation of the resource. This study is motivated by our attempt to understand the relationship between efficiency in resource markets and optimal harvest decisions in which price is an exogenous state variable. The literature on optimal commodity storage finds that in a rational expectations equilibrium commodity prices are stationary and serially correlated. Yet recent papers on optimal timber harvesting that assume exogenous stationary prices generate harvest rules inconsistent with the price processes on which they are based.

In this study, we investigate the appropriate form of the stochastic process governing prices of renewable resources. We develop a model in which timber is supplied by profit-maximizing managers with rational expectations and aggregate timber demand is subject to independent exogenous shocks. In contrast to earlier studies, prices are endogenously determined. Managers know the structure of the timber market and form expectations of future market equilibria in making optimal harvesting decisions. We show under general conditions that efficient timber prices are stationary and serially correlated. Stationarity and serial correlation are shown to arise from two sources: the occurrence of stock-outs (i.e., depletion of the inventory) and stock-dependent growth of the resource. Further, we show that prices retain these properties even in the absence of stock-outs. Simulations are used to further illustrate the analytical results.

Our findings have implications for a large number of economic analyses of optimal resource use. First, our results reveal why extraction rules for renewable resources based on exogenous price specifications are internally inconsistent, even when the specification conforms to the stochastic behavior of prices generated by an efficient market. These prices arise in a particular structural environment, and if large numbers of resource managers adopt the harvesting rule, the underlying structural environment would change, and the price process would deviate from that used to derive the harvesting rule. Second, we show that there can be no gains from exploiting the stochasticity of resource prices in a rational expectations world, a finding that challenges the prescriptive policies for resource use found in many studies, including those on option values.   Third, our results show that time-series analyses designed to test for the efficiency of renewable resource markets cannot distinguish prices generated in an efficient market from those generated in an inefficient market. Finally, we extend the literature on optimal storage. Previous models of commodity storage models are shown to be a special case of our model involving age-independent depreciation of the inventory.

Internal Consistency in Models of Optimal Resource Use Under Uncertainty

"That's the news from Lake Wobegon, where all the women are strong, all the men are good looking, and all the children are above average." – Garrison Keiller.

I. Introduction

For several decades, economists have been concerned with the problem of optimal resource use under uncertainty. Studies of nonrenewable resources have explored the implications of stochastic demand (Dasgupta and Heal, 1974; Pindyck, 1980) and reserves (Pindyck, 1980; Swierzbinski and Mendelsohn, 1989) and analyses of renewable resources have considered stochastic growth rates of resource stocks (Pindyck, 1984; Morck *et al*., 1989; Clarke and Reed, 1989). A number of studies treat resource prices (or net benefits) as uncertain, and assume that prices evolve according to an exogenous stochastic process (Arrow and Fisher, 1974; Norstrom, 1975; Brennan and Schwartz, 1985; Brazee and Mendelsohn, 1988; Reed, 1993).[1] The corresponding dynamic optimization problem is solved to yield an optimal decision rule for exploitation of the resource. In many of these studies, authors derive an expression for the option value associated with exploiting the resource today and foregoing the opportunity to use forthcoming information on resource prices.

In this paper, we investigate the appropriate form of the stochastic process governing prices for renewable resources. The study is motivated by our attempt to understand the relationship between efficiency in resource markets and optimal harvest decisions in which price is an exogenous state variable. The literature on optimal

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[1] Additional references include Fisher and Hanemann (1986), Morck *et al*. (1989), Clarke and Reed (1989), Haight and Holmes (1991), Zinkham (1991), Lohmander (1992), Thomson (1992), Albers (1996), and

commodity storage finds, in both stylized analytical models and empirical investigations, that in a rational expectations equilibrium commodity prices are stationary and serially correlated (Williams and Wright, 1991; Deaton and Laroque, 1992, 1996). Yet recent papers on optimal timber harvesting that assume exogenous stationary prices generate harvest rules inconsistent with the price processes on which they are based (e.g., Brazee and Mendelsohn, 1988; Haight and Holmes, 1991; Plantinga, 1998). These studies assume prices evolve according to a stationary stochastic process and find that, on average, timber managers can improve upon a fixed-length Faustmann rotation by timing harvests to take advantage of high prices—that is, prices above the mean of the price distribution. But, just as every kid in Lake Wobegon cannot be above average (by local standards), not all timber managers can harvest at prices above the mean, on average. In the limit, the mean of observed prices—those at which timber sales are consummated— must equal the mean of the price distribution and, therefore, the price process fails the test of internal consistency.

A casual explanation of this result is that the authors chose badly when specifying the price process. For instance, there might exist autoregressive price processes for which the optimal harvest rule is not characterized by the principle to cut only when the observed price is above its long-run mean, and these are the processes generated by an efficient market. However, if an optimal harvest rule is to reproduce the mean of the process, then it does no better on average than a myopic rule involving fixed-length rotations computed at the mean price (i.e., the Faustmann rotation). This implies that stochastic price variations provide no useful information to the resource manager making

Plantinga (1998), and Clarke and Reed (1990) provide a survey of applications to natural resource problems. In Pindyck (1980, 1984), prices are uncertain, but are determined endogenously.

an irreversible harvesting decision, which contradicts the central result of real options theory (e.g., Dixit and Pindyck, 1994). Two related questions follow. Are markets for renewable resources somehow fundamentally different than other commodity markets, in the sense that, *necessarily,* resource prices are non-stationary and serially independent? And, if not—if, in fact, renewable resource prices can be stationary and serially correlated—how does one interpret the recent literature on optimal timber harvesting?

An answer to the first question is suggested by Washburn and Binkley (1990). In an analysis of stumpage markets in the U.S. South, the authors adapt a commodity storage model to timber. They assume that the store of timber is not exhausted, in which case there is full market adjustment in each period and prices follow a martingale process (Hultkrantz, 1993). In the commodity storage context, this is equivalent to there being no occurrence of stock-outs (i.e., depletion of the inventory). Stock-outs drive the result that commodity prices are stationary and serially correlated (Williams and Wright, 1991). Thus, if inexhaustibility of the current inventory is a reasonable assumption for renewable resources[2], we might conclude that there is a fundamental difference between commodity and renewable resource markets and that this difference gives rise to non-stationary and serially independent resource prices. This explanation has some appeal since it appears to reconcile the issues of price behavior and market efficiency. If markets are efficient, current prices should embody all relevant information contained in the past sequence of prices and evolve according to a martingale process. This is the justification given by some authors for specifying random walk prices (e.g., Paddock et al., 1988; Thomson, 1992).

Martingale prices have not been formally supported by a model that captures realistic features of renewable resources. In keeping with the commodity storage literature, which usually assumes a constant, often zero, rate of spoilage, Washburn and Binkley (1990) assume a constant growth rate for timber, thus eliminating a defining characteristic of renewable resources—growth depends on the state of the resource stock. In this study, we show that this assumption has important consequences. We develop a model in which timber is supplied by profit-maximizing managers with rational expectations and aggregate timber demand is subject to independent exogenous shocks. While we focus on timber markets, our results are applicable to the broader class of renewable resources with age-dependent growth (e.g., fish, wildlife). In contrast to earlier studies, prices are endogenously determined. Managers know the structure of the timber market and form expectations of future market equilibria in making optimal harvesting decisions. We show under general conditions that efficient timber prices are stationary and serially correlated even though demand shocks are uncorrelated and even when stock-outs do not occur. Key to this result is the age-dependent growth of the timber inventory. A demand shock induces a supply response that modifies the current timber inventory, and this affects future timber growth rates, optimal supply decisions, and equilibrium prices. The persistence of demand shocks implies that prices are correlated over time.

Having answered no to the first question posed above, we must confront the second question: if efficient prices are indeed stationary and serially correlated, why are harvesting rules based on such prices internally inconsistent? Our modeling results

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[2] To be more precise, we should say that the probability of depleting the entire stock of the resource in a given period is very small. If the resource were truly available in infinite quantity, its price would be

reveal the source of the inconsistency. These prices arise in a particular structural environment, and if large numbers of resource managers migrate to the harvesting rule, the underlying structural environment would change, and the endogenous price process would deviate from that used to derive the harvesting rule. Further, we show that there can be no gains from exploiting the stochasticity of resource prices in a rational expectations world. This point is quite subtle. A timber manager, for example, would indeed raise the expected net present value of his timber by taking advantage of price correlations, if the point of comparison is a Faustmann decision rule, or a decision rule based on an incorrect assumption that prices follow a random walk (Plantinga, 1998). But the manager could not outperform a neighbor whose harvest decisions are based on a complete understanding of the structure of the system generating the correlated prices.

These results have implications for the large number of analyses (referenced above) that model resource prices or net benefits using an exogenous stochastic process. They raise the issue of whether structural characteristics of the resource and of resource markets lead to internal consistency problems with the assumed exogenous price processes. In this case, the derived decision rules for exploitation of the resource are suboptimal and conclusions regarding incorporation of option values into the decision problem are potentially misleading. Beyond these studies, our results show that time-series analyses designed to test for market efficiency (e.g., Washburn and Binkley, 1990; Haight and Holmes, 1991; Yin and Newman, 1995) cannot distinguish prices generated in an efficient market from those generated in an inefficient market. In addition, we extend the literature on optimal storage mentioned above. Commodity storage models are shown to be a special case of our model involving age-independent depreciation of

driven to zero since managers would always be willing to sell more when prices are positive.

the inventory. Our more general model reveals that prices can serially correlated as the result of stock-outs, an age-dependent inventory, or both.

In the next section, we study the behavior of timber prices analytically and, then in Section III, through the use of simulations. In Section IV, we elaborate on the points made above in light of our results, and additional implications of the analysis. A final section present conclusions.

II. Theoretical Development

In this section, we present a partial equilibrium model of a competitive market for timber. We derive the basic result that prices are stationary and serially correlated in a rational expectations equilibrium, even when stock-outs do not occur, and show formally the role of nonconstant timber growth in determining equilibrium prices.

The Timber Manager's Harvest Decision

The manager of property j chooses the amount of timber to harvest, q_{jt}, to maximize discounted timber revenues, with the understanding that: (a) the stock of timber s_{jt} grows over time; (b) he cannot harvest more timber than he holds in stock, and (c) future timber prices are stochastic, and conditional on current market state variables. Some timber managers may use only the current price to forecast future prices. Others may understand that the current price is endogenous to the market, and thus may use other market state variables, such as the distribution of the aggregate timber stock across growth rates, to forecast future timber prices. In the timber manager's problem presented below the set of market-level state variables the timber manager uses to forecast timber

price is defined generally as m_t. The particular composition of this set depends on the

manager's information and rationality.

Let x_{jt} denote the difference between stock and harvest at time t, $x_{jt} = s_{jt} - q_{jt}$.

Formally, the manager solves,

(1) $\quad \max_{q_{jt}} E \left[\sum_{t=0}^{\infty} \beta^t p_t q_{jt} \right],$

subject to,

(2) $\quad \begin{aligned} s_{j,t+1} &= f_j\left(s_{jt}, q_{jt}\right) \\ &= x_{jt} + g_j\left(x_{jt}\right), \\ x_{jt} &\geq 0 \end{aligned}$

(3) $\quad \left\{ \pi_{j,t+1}\left(m_{t+1} \mid m_t\right) : M \Rightarrow (0,1) \right\},$

where $E[\cdot]$ is the expectation with respect to information available in period t; β is the

discount factor; $g(\cdot)$ is an increasing, concave growth function specific to manager j; and

(3) denotes the family of transition probabilities over the market state space M.[3]

Transition probabilities indicate the probability of moving into any feasible market state,

conditional on the market state at time t, m_t. These probabilities reflect the timber

manager's information and beliefs about the market forces generating timber prices and,

hence, are manager-specific. So, for instance, if a manager believes that timber prices

evolve according to a traditional first-order autoregressive process with normal iid

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[3] Note that we do not include a non-negativity constraint on q_{jt}; we assume $q_{jt} > 0$. Excluding the option for the manager to harvest no timber does not affect the basic nature of our results and allows us to focus our attention on the effect of the stock level on prices.

innovations, then the state space M is the set of all feasible prices, $m_t = p_t$, and the relevant parameters governing transition probabilities are the coefficients of the autoregressive process and the variance of the innovations. Other specifications are possible, of course, and in particular, for a timber manager who treats the price of timber as endogenously determined by market forces, transition probabilities do not depend on price. This point is revisited at the end of this section.

Two aspects of this specification of the timber manager's problem deserve emphasis. First, the state of forest property j is described by a single state variable s. In reality, this is typically not possible. Instead, the growth in the timber stock depends on the distribution of the stock across age classes. Here we simplify matters without affecting the points to be made in the paper. Second, market transition probabilities are not conditional on the timber manager's decision, because the manager understands the atomistic form of the market, and thus is a price-taker.

Efficient Market Equilibrium with Independent Demand Shocks

Because all benefits and costs of timber harvesting are private, a competitive market with fully-informed timber managers will maximize the expected discounted surplus, in which case the market equilibrium can be found from a social planner's problem (the Invisible Hand as planner). In light of the development above, it is clear that the planner must choose the amount of timber to cut from *each* of the J timber properties, as this determines the evolution of the stock on each property, which in turn determines the growth of the aggregate stock.

Define $q_t = \sum_J q_{jt}$, $\mathbf{q}_t = \left(q_{1t},...q_{Jt}\right)$, and $\mathbf{s}_t = \left(s_{1t},...s_{Jt}\right)$. Also, let $p(q_t,\varepsilon_t)$

denote the inverse demand function, where ε_t is a demand shock drawn from an iid

distribution, and let $v(\mathbf{s}_t,\varepsilon_t)$ denote the aggregate value of forestland, given optimal

harvesting decisions are made in the current and all future periods. The planner chooses

$\mathbf{q}_t$ to solve

$$(4) \qquad v(\mathbf{s}_t,\varepsilon_t) = \max_{\mathbf{q}_t} \int_0^q p(q_t,\varepsilon_t)dq + \beta E[v(\mathbf{s}_{t+1})]$$

subject to (2) and (3) for all $j=1,...J$. Under mild regularity conditions, the value function

in (4) is stationary, as indicated by the lack of time subscripts. This follows from the

stationarity of demand, the growth function g, and the discount factor β (Rust, Theorem

2.3, part 1).

Let γ_{jt} denote the Lagrange multiplier on the inequality constraint in (2). For all

j, the solution to (4) satisfies the necessary conditions,

$$(5a) \qquad p(q_t,\varepsilon_t) - \beta\, E\left(\frac{\partial v_{t+1}}{\partial s_{j,t+1}}\right)\left(1+\frac{dg_j}{dx_{jt}}\right) - \gamma_{jt} = 0$$

$(5b) \qquad x_{jt} \geq 0$

$(5c) \qquad \gamma_{jt}x_{jt} = 0$

where the time subscript on the value function indicates the function is evaluated at the

argument values in the indexed period. The Lagrange multiplier γ_{jt} has an important

economic interpretation. It is the net value of harvesting the last unit of stock j in the

current period, rather than leaving it for the future. As long as the carryover on property j

is positive, this value is zero; the stock is allocated over time so that the value of the last unit harvested in the current period is equal to its discounted value in the future. A stock-out occurs when γ_{jt} is strictly positive for all J properties.

The solution to (4) gives the equilibrium harvest quantity $q_t^e = q^e(\mathbf{s}_t, \varepsilon_t)$, which is stationary under the same conditions ensuring the stationarity of the value function (Rust, Theorem 2.3, part 2). Substitute the equilibrium harvest quantity into (4) and take the derivative of both sides of (4) with respect to s_{jt} to obtain

$$(6) \qquad \frac{\partial v_t}{\partial s_{jt}} = \left[p(q_t^e, \varepsilon_t) - \beta\ E\left(\frac{\partial v_{t+1}}{\partial s_{j,t+1}} \right)\left(1 + \frac{dg_j}{dx_{jt}} \right) \right] \cdot \frac{\partial q_t^e}{\partial s_{jt}} + \beta\ E\left(\frac{\partial v_{t+1}}{\partial s_{j,t+1}} \right)\left(1 + \frac{dg_j}{dx_{jt}} \right).$$

Our purpose is to derive the equilibrium relationships between current and future prices, analogous to the arbitrage relationships for prices in the commodity storage literature (e.g., Equation 1 in Deaton and Laroque, 1992). To this end, we begin by evaluating (6) in the cases of positive and zero carry-over of the stock (i.e., $x_{jt} > 0$ and $x_{jt} = 0$, respectively).

When some, but not all, timber is cut from property j, the Lagrange multiplier γ_{jt} is zero, in which case from (5a) we have,

$$\beta\ E\left(\frac{\partial v_{t+1}}{\partial s_{j,t+1}} \right)\left(1 + \frac{dg_j}{dx_{jt}} \right) = p(q_t, \varepsilon_t),$$

and so (6) reduces to

(7) $\dfrac{\partial v_t}{\partial s_{jt}} = p\big(q^e(\mathbf{s}_t, \varepsilon_t), \varepsilon_t\big),$

which indicates that the value of one more unit of stock is the current market price. This relationship also holds even when *all* the timber is harvested from property j. This is shown formally by substituting q^e into (5c) and differentiating both sides with respect to s_{jt}:

(8) $\dfrac{\partial \gamma^e_{jt}}{\partial s_{jt}} \cdot \big[s_{jt} - q^e_{jt} \big] + \gamma^e_{jt} \cdot \left[1 - \dfrac{\partial q^e_t}{\partial s_{jt}} \right],$

where $q^e_{jt} = q^e_j(\mathbf{s}_t, \varepsilon_t)$ is the equilibrium quantity harvested from property j in period t. In the event of a stock-out, this relationship holds only if either (a) $\gamma_{jt} = 0$, the degenerate case, with (7) thus obtained via the same algebra used for an interior solution; or (b) $1 - \dfrac{\partial q^e_t}{\partial s_{jt}} = 0$. Substituting this equality into (6) gives the result in (7).

Using (7), we derive the equilibrium price relationships. Shift (7) forward by one period, multiply both sides by the discount factor, and taking the expectation of the result at time t to obtain,

(9)
$$
\begin{aligned}
\beta\, E\!\left(\dfrac{\partial v_{t+1}}{\partial s_{j,t+1}} \right) &= \beta\, E\big\{ p\big(q^e(\mathbf{s}_{t+1}, \varepsilon_{t+1}), \varepsilon_{t+1} \big) \big\} \\
&= \beta\, E\big\{ p\big(q^e(\mathbf{h}(\mathbf{s}_t, \varepsilon_t), \varepsilon_{t+1}), \varepsilon_{t+1} \big) \big\}
\end{aligned}
$$
,

where **h** expresses the vector of stocks in period $t+1$ as conditional on previous stocks $\mathbf{s}_t$ and equilibrium decisions.[4] Substituting (9) into (5a) at the solution and rearranging gives,

$$(10)\quad p\left(q^e(\mathbf{s}_t,\varepsilon_t),\varepsilon_t\right) = \beta E\left[p\left(q^e(\mathbf{h}(\mathbf{s}_t,\varepsilon_t),\varepsilon_{t+1}),\varepsilon_{t+1}\right)\cdot\left(1+\frac{dg_j}{dx_{jt}}\right)\right]+\gamma_j^e(\mathbf{s}_t,\varepsilon_t).$$

Analogous to the commodity storage literature, the equilibrium price relationships can be expressed as:

$$(11)\quad \begin{aligned} p\left(q^e(\mathbf{s}_t,\varepsilon_t),\varepsilon_t\right) &= \beta E\left[p\left(q^e(\mathbf{h}(\mathbf{s}_t,\varepsilon_t),\varepsilon_{t+1}),\varepsilon_{t+1}\right)\cdot\left(1+\frac{dg_j}{dx_{jt}}\right)\right] && if\ \ x_{jt}>0 \\[2ex] p\left(q^e(\mathbf{s}_t,\varepsilon_t),\varepsilon_t\right) &\geq \beta E\left[p\left(q^e(\mathbf{h}(\mathbf{s}_t,\varepsilon_t),\varepsilon_{t+1}),\varepsilon_{t+1}\right)\cdot\left(1+\frac{dg_j}{dx_{jt}}\right)\right] && if\ \ x_{jt}=0 \end{aligned},$$

for all j. According to (11), prices are stationary and serially correlated. Stationarity is assured by the stationarity of inverse demand and the equilibrium harvest rule $q^e(\mathbf{s}_t,\varepsilon_t)$. Serial correlation is established by the presence of the demand shock ε_t in the equilibrium prices at time t *and* time $t+1$. Recursive substitution of the state equation into both sides of (10) reveals that time t and $t+1$ prices are functions of the sequence of realized demand shocks from time 0 to time t and the initial inventory $\mathbf{s}_0$.

The stock vector $\mathbf{s}_t$ is a relevant indicator of the state of the timber market for two reasons. First, it indicates physical scarcity—the potential for a stock-out. Second, it indicates the rate of stock growth. The correlation in prices is induced by the state

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[4] From (2) we have in equilibrium, $s_{j,t+1} = s_{jt} - q_{jt}^e(\mathbf{s}_t,\varepsilon_t)+g_j(s_{jt}-q_{jt}^e(\mathbf{s}_t,\varepsilon_t)) = \mathbf{h}_j(\mathbf{s}_t,\varepsilon_t)$; the vector **h** is composed of the J elements h_j.

equation (2), and so it follows that *either* of these factors—physical scarcity or stock-dependent growth rates—is a source of correlated prices. In the traditional commodity storage model, in which stock growth rates are constant (usually zero), correlated prices arise only due to the positive probability of stock-outs. In these models, stock-outs are integral to the functioning of efficient markets. If prices are so high that the probability of a stock-out is zero, there is at least one unit of stock wasted, never to be used; this is inefficient.

The first equation in (11) confirms that with stock-dependent growth, stock-outs are not necessary for correlated prices. If there is a positive demand shock in period t, more stock will be harvested in period t, but at the cost of leaving to the future only relatively high-growth timber—timber that managers would prefer to hold rather than cut. This serves to raise the price of timber in period $t+1$ and, thus, the period t demand shock exhibits some degree of persistence. Note that the conditions in (11) can simultaneously hold for subsets of the J properties. Accordingly, one can think of this problem as involving a continuum of stock-outs, where timber managers progressively deplete low-growth timber to high-growth timber, possibly never depleting the entire stock.

We now return to the problem faced by timber managers. Expectations are represented by the transition probabilities in (3). For timber managers with rational expectations, the preceding development makes clear that transition probabilities are governed by the distribution of ε, and conditional on the stock vector $\mathbf{s}_t$. Of course, these variables are interesting to managers only because they hold information about future prices. The net revenue generated by a timber manager who instead conditions transition probabilities on the observed price of timber is *necessarily* lower. In effect, such a

manager is attempting to predict future prices with limited information. Nonetheless, *given* that the only state variable used by the manager is the current price, it clearly serves the manager to use the information embedded in correlated prices.

The timber manager may choose to forecast prices with limited information because full information is costly. Of course, as the number of managers using limited information grows large, the actual behavior of timber prices deviates from that predicted by the model; recent literature examines the behavior of commodity markets when information is costly (Brock and Hommes, 1997; Chavas, 1999). Yet the point remains that correlated, stationary prices may arise in efficient timber markets even in the absence of stock-outs. This point is demonstrated in the simulations below.

III. A Simulation to Illustrate the Results

In this section we illustrate using simulations that (a) when the growth of a renewable stock does not depend on the state of the stock, endogenous stockouts are the source of correlated prices, and (b) when the growth of the stock does depend on the state of the stock, stockouts may not arise, and yet prices are correlated due to state-dependent growth.

The simulation is based on the following model of the flow of timber from forest land. Just after harvesting, and in the absence of additional harvesting, timber stock (volume) per unit land increases for four consecutive periods across four "stock classes", and then stops growing, remaining in class four. Harvesting interrupts this growth process, causing the stock on the harvested land to revert to class one in the period following harvest. Examining a model where timber grows for only a few periods is

necessary because the state of the system is defined by the amount of timber in each stock class. As the number of classes increases, the size of the numerical simulation increases exponentially (this is Bellman's curse of dimensionality).

The amount of land in timber production is fixed; the model abstracts from the movement of land into and out of timber production. This being the case, three state variables fully define the state of the forest; namely, the amount of stock in classes 2, 3, and 4 (the amount of stock in class 1 can be deduced given the amount of stock in the other classes and the invariance of land in timber production). Denote by s_{jt}, j=1,2,3,4, the stock in class j in period t, and denote by q_{jt} the harvest in period t of stock in class j. Note that in this section, the subscript j refers to a stock class, rather than a property. Let g_j denote the additional volume per unit stock that accrues when a unit of stock grows from class j to class j+1: $s_{j+1,t+1} = \left(1 + g_j\right)\left(s_{jt} - q_{jt}\right)$. With stock per unit land normalized to one for stock in class one, and with total land in timber production also normalized to one, the state of the forest is governed by the state equations,

$$
(12) \quad
\begin{aligned}
s_{2,t+1} &= \left(1 + g_1\right)\left(s_{1t} - q_{1t}\right) \\
&= \left(1 + g_1\right)\left(1 - \left(\frac{s_{2t}}{1+g_1} + \frac{s_{3t}}{1+g_1+g_2} + \frac{s_{4t}}{1+g_1+g_2+g_3}\right) - q_{1t}\right) , \\
s_{3,t+1} &= \left(1 + g_2\right)\left(s_{2t} - q_{2t}\right) \\
s_{4,t+1} &= \left(1 + g_3\right)\left(s_{3t} - q_{3t}\right) + \left(s_{4t} - q_{4t}\right)
\end{aligned}
$$

and the inequality constraints,

$$q_{jt} \geq 0, \quad j = 1,2,3,4$$

(13) $\quad s_{jt} - q_{jt} \geq 0, \quad j = 2,3,4$

$$1 - \left[\frac{s_{2t}}{1+g_1} + \frac{s_{3t}}{1+g_1+g_2} + \frac{s_{4t}}{1+g_1+g_2+g_3} \right] - q_{1t} \geq 0$$

These inequality constraints state that the stock in class j harvested at time t is bounded

by zero[5] and the amount of stock actually in the class.

For the sake of simplicity, inverse demand for timber is linear, with a simple

additive disturbance shifting the intercept in period t:

(14) $\quad p_t = c_0 + \varepsilon_t - c_1 \cdot q_t.$

The disturbance is independently and identically distributed with open lower bound

greater than $c_1 s_4 - c_0$, assuring that timber is always scarce (positive price).

Because all benefits and costs are private, a competitive market with fully-

informed participants will generate as an equilibrium the solution to the problem of a

planner maximizing surplus. Let $\mathbf{s}_t$ denote the 3-dimensional vector of stock in classes 2

through 4, let $\mathbf{q}_t$ denote the 4-dimensional vector of harvest decisions, and let q_t denote

the sum of the elements of $\mathbf{q}_t$. The planner's problem can be stated,[6]

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[5] For the simulations, we must incorporate the non-negativity constraint on q_{jt} to ensure that none of the solutions involve negative harvest quantities.

[6] Although it is not immediately obvious, this model corresponds to the one investigated in the previous section, for the case where (a) the growth function, as defined by g_j, is the same for all properties, and (b) properties are points on the land surface, with the harvest decision on each property reduced to the binary decision of either cutting the entire stock on the property, or postponing harvest. This being the case, then under the assumption that all timber managers are rational, fully-informed profit maximizers, all managers behave the same way (employ the harvest policy favored by the surplus-maximizing planner), and so the relevant state variables are the total amount of stock in each class.

$$
(15) \quad v\left(\mathbf{s}_t, \varepsilon_t\right) = \max_{\mathbf{q}_t}\left[\left(c_0 + \varepsilon_t\right)q_t - \frac{c_1 q_t^2}{2} + \beta E\left\{v\left(\mathbf{s}_{t+1}, \varepsilon_{t+1}\right)\right\}\right]
$$

$$
s.t. \quad (12), \ (13)
$$

If at the solution the stock in class j is harvested but not completely eliminated, we have the first-order necessary condition,

$$
(16) \quad p_t - \beta E\left\{\frac{\partial v\left(\mathbf{s}_{t+1}, \varepsilon_{t+1}\right)}{\partial s_{j+1,t+1}}\right\}\left(1 + g_j\right) = 0 \ ,
$$

which corresponds to (5a), and makes clear that because we are interested in the evolution of equilibrium prices, we are more interested in accurate estimation of the gradient of the value function, $\dfrac{\partial v\left(\mathbf{s}_t, \varepsilon_t\right)}{\partial s_{jt}}$, $j = 2, 3, 4$, than of the value function itself.

With this in mind, the appropriate algorithm is one in which we solve iteratively the necessary conditions and adjoint equations associated with (15).

Disturbances are drawn from a discrete uniform distribution with ten values evenly spaced in the interval (-1,1). The value function gradient is approximated for every possible value of the disturbance by a 10th-order Chebychev polynomial in each of the three dimensions of the state space.[7] Details of the approximation algorithm are provided in the appendix. Table 1 provides parameter values used in the analysis.

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[7] Miranda and Fackler (2001) elucidate the advantages of Chebychev polynomial approximations in numerical analysis. In the simulation, higher order polynomial approximations had no discernible effect on results.

Simulation Results

Figure 1(a-b) presents carryover and timber price for a typical one-hundred year sequence, for the case where growth is constant ($g_j=1$, $j=2,3,4$). Equilibrium carryover is always less than 1.0 because the stock is never carried past stock class s_2 (recall that total available forestland is normalized to unity). As expected, stockouts (carryover =0) are frequent (55% of periods). Figure 1b exhibits the behavior typical of stationary prices with positive correlation; prices cross the mean value (8.0) frequently, and peaks are greater than troughs. Fitting the full price series (10^4 observations) to an AR(1) process,

$$p_t = d_0 + d_1 p_{t-1} + \eta_t,$$

yields parameter values $d_0=6.61$ and $d_1=.17$, with standard errors .0098 and .079, respectively. The Dickey-Fuller test rejects the hypothesis of a unit root.[8]

Consider now the case where growth is nonconstant, with $g_1=1.2$, $g_2=1.1$, and $g_3=1.0$; as a stand ages, it increases in volume at a decreasing rate. Figure 2(a-b) presents carryover and timber price for a typical 100-year sequence. As shown in Figure 2a, carryover never falls to zero (no stockouts), and in fact, over a sequence of 10^6 periods the carryover never falls to zero, suggesting that the probability of a stockout is zero. Nonetheless, Figure 2b clearly indicates that prices are stationary and positively correlated.[9] Fitting the full price series to an AR(1) process yields parameter estimates

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[8] The test statistic for H_0: $d_0=0$, $d_1=1$ is 42.25; the critical value at the 90% confidence level is 3.78.
[9] In this case, the mean value is 7.8, and so peaks are clearly greater than troughs. The mean value is lower than for the case where growth is constant (Figure 1) because the average harvest is greater: 1.10 compared to .94.

d_0=5.74 and d_1=.27, with standard errors .0096 and .075, respectively. The Dickey-Fuller test rejects the hypothesis of a unit root.[10]

Finally, consider the effect of the discount factor on price behavior when timber growth is nonconstant. We might expect that as the discount factor β falls from .95, stockouts begin to arise because the future becomes increasingly unimportant to current decisions. The increase in stockouts would serve to increase price correlation, but the increasing disengagement of current decisions from future outcomes would serve to reduce the price correlation; when β becomes sufficiently low, all available timber is harvested in the current period, in which case prices are serially uncorrelated because the demand shock is serially uncorrelated. These countervailing forces could lead to nonlinearities in the relationship between β and d_1. By comparison, as β increases from .95, we might expect the correlation of prices to increase, because the effect of current decisions on future states becomes more significant in the decision problem, and stockouts occur with zero probability. Table 2 presents selected results from a sensitivity analysis of β. As expected, stockouts increase as the discount factor falls. The correlation coefficient d_1 behaves in a nonlinear fashion. In all cases, nonstationarity is statistically rejected.

IV. Discussion

In this section, we discuss the implications of our results for analyses of optimal resource use under uncertainty, time-series analyses of resource prices, and models of optimal storage.

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[10] The test statistic for H$_0$: d_0=0, d_1=1 is 42.25; the critical value at the 90% confidence level is 12.66.

Optimal Resource Use

There are numerous studies in the economics literature concerned with the

optimal use of natural resources under uncertainty. Many of these assume that resource

prices (or net benefits) evolve according to an exogenous stochastic process and then use

stochastic dynamic optimization techniques to derive an optimal decision rule. Among

these studies is a large group of analyses of the optimal timber rotation decision (e.g.,

Norstrom, 1975; Brazee and Mendelsohn, 1988; Morck *et al*., 1989, Clarke and Reed,

1989; Haight and Holmes, 1991; Lohmander, 1992; Thomson, 1992; Reed, 1993;

Plantinga, 1998). The central finding of rotation analyses is that the optimal harvesting

rule involves a reservation price strategy: harvest if the current price is above the

reservation price and, otherwise, delay the harvest and reconsider the decision in the next

period. The reservation price strategy is shown to produce higher expected returns than

the Faustmann rotation computed at the expected value of future prices.[11]

A second group of studies consider the decision to develop a parcel of land when

the future net benefits from preservation are uncertain (e.g., Arrow and Fisher, 1974;

Fisher and Hanemann, 1986; Albers, 1996). In the current period, the future benefits of

preservation are assumed to be unknown but distributed according to a known

probability distribution. In a later period, the uncertainty is resolved and the benefits of

preservation are revealed. The key result from these studies is that the expected net

benefits from preservation are higher when uncertainty and the irreversibility of the

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[11] When prices are stationary, the reservation price strategy always does at least as well as the Faustmann rotation. When prices are non-stationary (and serially uncorrelated), there are potential gains from a reservation price strategy only when fixed costs are present, such as timber management costs or alternative land uses.

development decision are taken into account. The option value (in the early literature, the

quasi-option value) is computed as the difference between the *ex ante* value of the land

when uncertainty and irreversibility are incorporated into the decision calculus and the *ex

ante* value when uncertainty and irreversibility are ignored.[12]

The basic message of optimal timber harvesting and option value studies is that

resource managers can take advantage of information embodied in stochastic variations

in prices or net benefits. Our results make clear that no such opportunities exist in a

rational expectations world. In this case, market participants incorporate all relevant

information about the structure of the market into their decision calculus. The optimal

behavior of resource managers produces efficient prices and, by definition, there is no

room for improving on the solution. Williams and Wright (1991) reinforce this point in

the context of storage models:

> Lest there be some confusion, a finding that actual land prices, commodity
> prices, or the S&P stock index for that matter, do not behave as non-
> stationary random walks does not imply that opportunities exist to profit
> from predictable price movements. … The land prices in the storage
> model are rational and offer no opportunity for assured speculation.

Similarly, no opportunities exist to exploit serial correlation in resource prices if these

prices are generated in an efficient market.

Our analysis draws attention to the fact that the timber harvesting and option

value studies involve a comparison between two suboptimal decision rules. These

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[12] As discussed by Fisher and Hanemann (1990), the first value is derived from a closed-loop control policy whereas the second is from an open-loop control policy. Dixit and Pindyck (1994) analyze option values in the context of irreversible investment decisions. In their terminology, the option value is the difference between the expected value of the investment obtained as the solution to a stochastic dynamic programming problem and the expected value derived from a simple net present value rule. Lastly, Plantinga (1998) shows that the reservation price strategy in forestry studies is a mechanism for incorporating option values into the timber rotation decision.

studies show that resource managers can increase expected profits by recognizing uncertainty and irreversibility. However, the assumption that prices or net benefits evolve according to an exogenous stochastic process necessarily means that information about the structure of resource markets is ignored. Accordingly, the derived decision rule cannot be optimal. Our results suggest a general problem with applying decision rules based on exogenous price specifications to goods that involve physical stocks (e.g., renewable resources, agricultural commodities, land resources). Due to stock-outs and, in some cases, age-dependent growth, prices for these goods are likely to be stationary and serially correlated. A decision rule based on an assumption of exogenous stationary and serially correlated prices is internally inconsistent, and widespread adoption of the rule would alter the underlying structural environment and the form of the stochastic price process. Decision rules based on alternative prices specifications (e.g., geometric Brownian motion) are apt to lead to bad decisions.

In using exogenous prices to develop prescriptive policies for dynamic resource use, from harvesting trees to catching fish, the analyst embraces a tenuous proposition: that the particular price process used to develop the policy is "close enough" to the true, endogenous process to render the analysis meaningful. "Close enough" depends on the audience for the prescription. In this prescriptive literature, then, the analyst believes he has chosen an exogenous price process closer to the true process than that chosen by his audience. So, for instance, the option value literature is aimed at decision-makers who fail to consider in their decision-making the stochastic elements of the decision environment. But, as we have shown, the prescription is not necessarily the best medicine, and so the burden falls to the analyst to carefully rationalize a particular

exogenous specification of endogenous variables like price. Consider, for instance, studies that model timber prices as (nonstationary) random walks (e.g., Morck *et al*., 1989; Clarke and Reed, 1989; Thomson, 1992; Reed, 1993). This price specification is often justified by the claim that it is consistent with market efficiency. But the previous analysis disputes this claim: an efficient timber market may generate stationary prices. In the future, alternative rationales must accompany the use of random walk processes such as Brownian motion in stochastic models of resource use.

Time-Series Analysis

A number of studies have examined the relationship between reservation price strategies, market efficiency, and results of time-series analyses of timber prices. Washburn and Binkley (1990) argue that the reservation price strategies in timber harvesting studies exploit predictable departures from an equilibrium price level and, therefore, rely on an implicit assumption of market inefficiency. To investigate if timber markets are efficient, Washburn and Binkley (W&B) conduct tests of "weak-form" market efficiency (Fama 1970) using time-series data on southern pine timber prices.[13] W&B test whether *ex post* deviations of actual rates of price change from equilibrium rates are white noise. Some evidence of weak-form efficiency in timber markets is found: deviations are serially independent for annual and quarterly time series, but serially correlated for monthly time series. This finding suggests that the current price is the best predictor of price a quarter or more in the future. In the nearer term, additional information may be contained in the past sequence of prices.

A number of follow-up studies to W&B have produced different results. Haight and Holmes (1991) find evidence of stationarity in time-series of quarterly opening southern pine series. The annual and quarterly series analyzed by W&B are averaged prices, and Haight and Holmes demonstrate that a series of averaged prices tend to behave as a non-stationary random walk even when the underlying price series is stationary and autoregressive. Similarly, Yin and Newman (1995) find evidence of price stationarity in fourteen southern pine sawtimber markets. In a comment on W&B, Hultkrantz (1993) shows that, while the W&B results fail to reject the null hypothesis that quarterly and annual prices are non-stationary, they also fail to reject stationarity. To gain greater precision on the estimates, Hultkrantz re-estimates the model with pooled data and finds that non-stationarity can be rejected. However, in a response to Hultkrantz (1993), Washburn and Binkley (1993) question the appropriateness of the pooling strategy, citing evidence of distinct markets in different regions.

In our judgment, the weight of the evidence suggests that timber prices are stationary. What does this result indicate about market efficiency and the effectiveness of a reservation price strategy? Our results indicate that price stationarity has little bearing on these issues. We show that stationary and serially correlated prices can be generated in a rational expectations equilibrium in which all agents are optimizing. Of course, an informationally inefficient market can produce stationary prices as well. Thus, a finding of stationarity does not clarify the issue of market efficiency. Moreover, we demonstrate that stationarity is not a sufficient condition for the efficacy of a reservation price strategy. In our model, optimizing agents cannot use the stationarity of prices to their

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[13] As stated in W&B, a timber market is weak-form efficient if "the current price of sawtimber stumpage incorporates all of the information obtainable by studying past departures from equilibrium rates of price

further advantage—stationary prices are the endogenous result of optimizing agents.

Our results contradict views commonly expressed in the literature. Hultkrantz (1993) states that W&B's conclusion regarding reservation price strategies—"that 'there can be no can gain from using past price movements to play the market in timing timber harvests'"—"follows if timber prices are non-stationary." In other words, the claim is that price non-stationarity is a necessary condition for the ineffectiveness of a reservation price strategy. We show that a reservation price strategy can be ineffective when prices are stationary, in the sense that it represents a second-best decision rule. Timber managers using reservation price strategies are using past prices to forecast future prices; such adaptive price expectations are by definition inferior to rational expectations. As stated above, whether a reservation price strategy is "close enough" depends on both the alternative harvest policy (such as the Faustmann rule), and the true process generating prices. This is an empirical issue requiring additional research.

Optimal Storage

Our theoretical model generalizes earlier models of optimal commodity storage (Samuelson, 1971; Scheinkman and Schechtman, 1983; Williams and Wright 1991, Deaton and Laroque 1992, 1996). In Williams and Wright, for example, the condition for a competitive equilibrium with storage is,

$$(17) \quad \begin{aligned} p_t + k - \beta E[p_{t+1}](1-\varpi) &= 0 \qquad S_t > 0 \\ p_t + k - \beta E[p_{t+1}](1-\varpi) &\geq 0 \qquad S_t = 0 \end{aligned},$$

---

change."

26

where k is the unit cost of storage, ω is the rate of depreciation in the inventory, and S_t is aggregate storage (Williams and Wright, Equations 2.5 and 4.3). The price relations in (11) reduce to (17) when the growth rate of the inventory is constant and storage costs are assumed to be zero. Note that in the commodity storage case, "appropriately deflated" prices follow a martingale process if stock-outs do not occur (i.e., if S_t is always positive).

In our model of the timber market, we evaluate the case in which stock-outs do not take place. There are at least two reasons why the absence of stock-outs is plausible in the case of renewable resources. First, the growth rate of renewable resources increases as the inventory, or the age of constituent elements of the inventory, falls. High growth rates increase the incentive for managers to hold stocks for the future and, thus, act as a barrier against total depletion of the inventory. Second, in the case of some renewable resources (e.g., forests), landowners hold stocks for reasons other than commodity production (e.g., private recreational benefits). Under typical demand conditions, prices may not be high enough to induce supply. However, when prices are high enough to compensate these owners for foregone non-commodity benefits, they may supply to the market and, thus, stock-outs are averted. In this paper, we demonstrate that the absence of stock-outs does not eliminate serial price correlation, as in the commodity storage case, and that the key to this result is stock-dependent growth of the inventory.

V. Conclusions

For several decades, economists have worked to extend deterministic models of optimal resource use (e.g., Clark, 1976) to account for the uncertainty inherent in

biological and economic processes. In particular, there have many efforts to apply the theory on real options to the optimal management of renewable resources. Options theory is concerned with optimal investment decisions in a dynamic and uncertain environment and, thus, seems a natural fit to problems of optimal resource use. Moreover, elegant and tractable analytical models of investment have been developed for the case in which uncertainty can be represented as geometric Brownian motion (e.g., Dixit and Pindyck, 1994). The main conclusion of this paper is that researchers need to think carefully about the structural features of the market they are analyzing before choosing to model endogenous market variables as exogenous stochastic processes. In the case of renewable resources, modeling prices as an exogenous stochastic process leads to decision rules that are either inconsinent with market equilibrium or simply wrong.

The issues examined in this paper have many parallels to the finance literature. In particular, there is the enduring question of whether stock investors can profit from information gleaned from analyses of past prices. Traditionally, economists has dismissed such practices on theoretical grounds, arguing that in an efficient market all relevant information about a stock's price in the future will be embodied in the current price (e.g., Malkiel, 1981). However, the efficient market hypothesis has come under attack in recent years as the result of emerging empirical evidence of the predictability of stock returns from past returns (e.g., Brock *et al*., 1992). The analogue to the renewable resource case is the development of an appropriate test of market efficiency applied to historical data on renewable resource prices. Such a test would need to recognize that an efficient market for renewable resources can generate stationary and serially correlated

prices. Another avenue for future research is the examination of how well decision rules based on past prices perform in a rational expectations environment and how widespread adoption of such rules alter the underlying market structure.

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Table 1. Parameter values used in simulation

Parameter	Value
g_1	1.0, 1.2
g_2	1.0, 1.1
g_3	1.0
c_0	10.0
c_1	2.0
δ	.95[a]

[a] Other values of the discount factor were examined; see text and Table 2.

Table 2. The effect of the discount factor on stock-out frequency and price correlation[a]

Value of β	Frequency of stockouts (%)	Estimate of d_1 (standard error in parentheses)
.99	0	.51 (.0086)
.96	0	.32 (.0095)
.95	0	.28 (.0096)
.94	0	.25 (.0096)
.90	0	.23 (.0097)
.88	0	.26 (.0096)
.87	.017	.29 (.0096)
.86	.46	.37 (.0093)
.85	3.92	.44 (.0089)
.83	19.99	.51 (.0086)
.80	51.99	.27 (.0096)
.50	100	-0.02 (.010)

[a] In all cases save two, the simulation was run for 10^4 years. The exceptions were $\delta = .87, .88$, where the search for the possibility of stockouts at very low frequency required a longer time series.

Appendix. Simulation Algorithm.

The simulation proceeds in two steps. In the first, the equilibrium price function is approximated. In the second, this approximation is used with initial state vector $\mathbf{s}_t$, the state equations (12), and random draws from the disturbance function to generate an extended sequence of prices (10^4 to 10^6 periods). Here we discuss the derivation of the equilibrium price function.

First, note that the first order necessary conditions are,

(A1)
$$
\begin{aligned}
p_t - \beta E\left\{\frac{\partial v_{t+1}}{\partial s_{j+1,t+1}}\right\} - \gamma_{jt} + \mu_{jt} = 0 \quad j = 1,2,3 \\
p_t - \beta E\left\{\frac{\partial v_{t+1}}{\partial s_{4,t+1}}\right\} - \gamma_{4t} + \mu_{4t} = 0 \quad j = 4
\end{aligned}
\quad ,
$$

where γ_{jt} is the Lagrange multiplier for the nonnegativity constraint on class j, and μ_{jt} is the Lagrange multiplier on the nonnegativity of q_{jt}. The adjoint equations are,

(A2)
$$
\begin{aligned}
\frac{\partial v_t}{\partial s_{jt}} = \gamma_{jt} - \gamma_{1t} + \beta E\left\{\frac{\partial v_{t+1}}{\partial s_{j+1,t+1}} - \frac{\partial v_{t+1}}{\partial s_{2,t+1}}\right\} \quad j = 2,3 \\
\frac{\partial v_t}{\partial s_{4t}} = \gamma_{jt} - \gamma_{1t} + \beta E\left\{\frac{\partial v_{t+1}}{\partial s_{4,t+1}} - \frac{\partial v_{t+1}}{\partial s_{2,t+1}}\right\} \quad j = 4
\end{aligned}
\quad .
$$

With this in mind, the algorithm involves the following steps:

1. With the three functions, $\beta E\left\{\frac{\partial v_{t+1}}{\partial s_{j,t+1}}\right\}$ is approximated from the previous iteration (initially these functions are identically equal to zero) and for each grid point in the state space a search across harvest volumes is made to find the harvest

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decision $\mathbf{q}^e$ satisfying the four necessary conditions. This search exploits the structure of the problem; namely, that because the rate of growth of timber declines with age (time since last harvest), harvest must proceed monotonically from stock class four to class one. So long as some, but not all, timber is harvested, the solution is characterized by the result that

(A3) $\quad p\left(q^e\left(\mathbf{s}_t, \varepsilon_t\right), \varepsilon_t\right) - \beta E\left\{\dfrac{\partial v_{t+1}}{\partial s_{j+1,t+1}}\right\} = 0$

for that stock level j for which the acreage harvested is an interior solution or a degenerate corner solution. Otherwise the equilibrium price is found from the demand function with all stock consumed. The solution is thus quickly bracketed within a stock class, and quasi-Newton methods are then used to find the solution to (A3).

2. Given the solution of the problem at the state grid points, the values of γ_{jt}^e are used in the adjoint equations to find new values of $\dfrac{\partial v\left(\mathbf{s}_t, \varepsilon_t\right)}{\partial s_{jt}}$. Taking the expectation of these values over the disturbance term yields new approximations of the functions $E\left\{\dfrac{\partial v_t}{\partial s_{jt}}\right\}$ at the grid points. To these values a 10$^{\text{th}}$-order Chebychev polynomial is fit (note, then, that the grid points are the Chebychev

nodes). This generates three-dimensional polynomial approximations to each of

the three functions $E\left\{\dfrac{\partial v_t}{\partial s_{jt}}\right\}$, j=2,3,4.

3. The algorithm returns to step 1 with these new approximations. The algorithm

terminates when the new approximations of equilibrium prices in (A3) are "close

enough" to the old approximations.[14]

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[14] At each grid point, prices are within .01 of their values in the previous iteration.

Figure 1. Carryover and Prices for the Constant Growth Scenario
_____

Figure 1a. Carryover under constant growth

Figure 1b. Timber prices
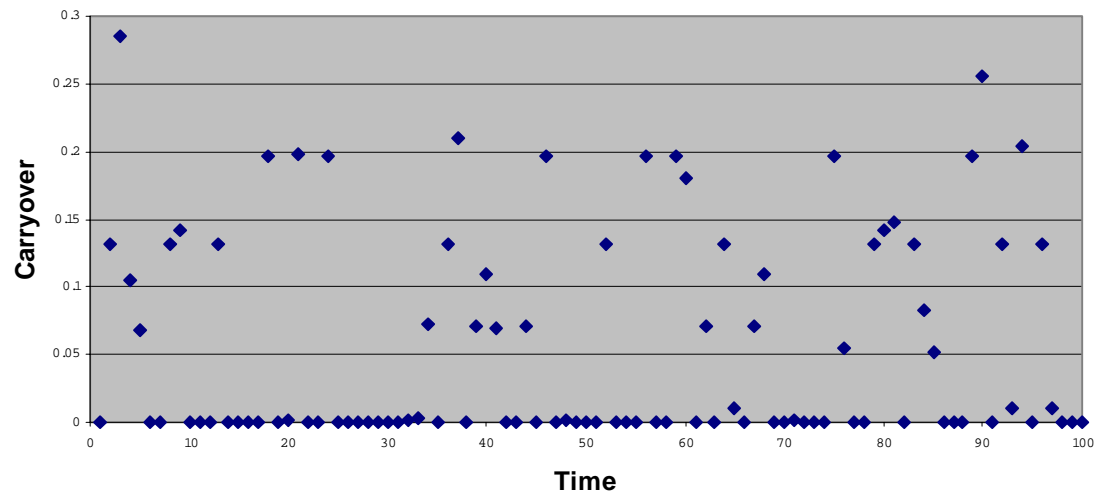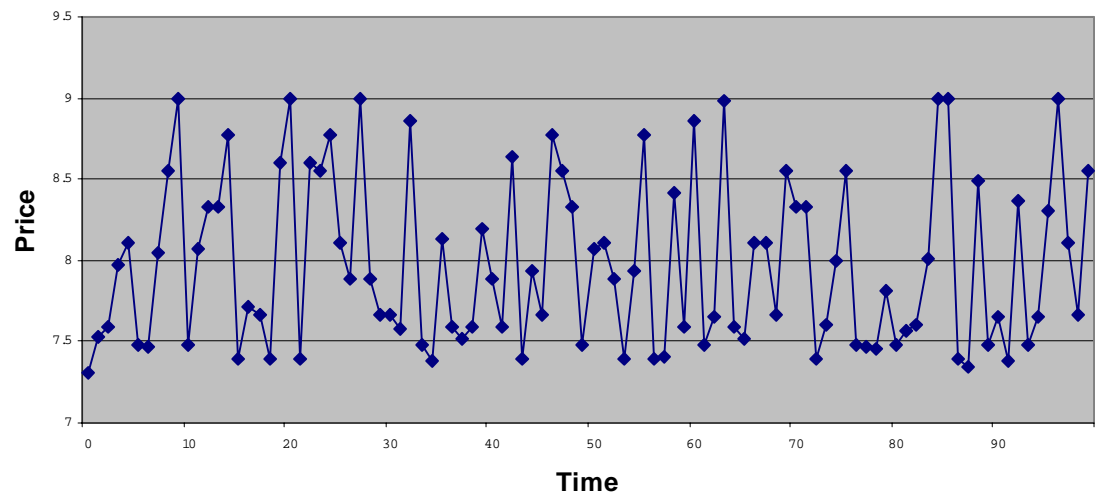
Figure 2. Carryover and Prices for the Non-Constant Growth Scenario
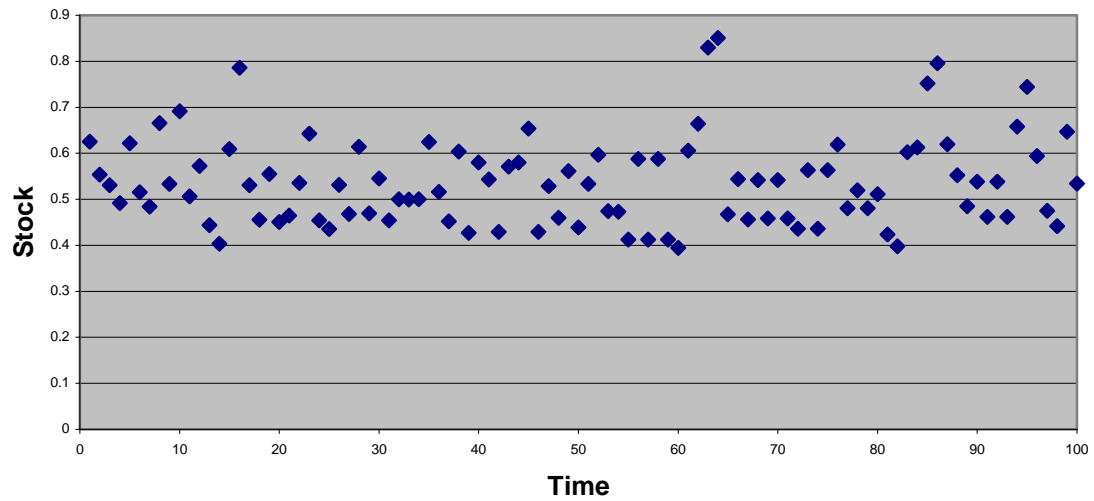
_____

Figure 2a. Carryover

Figure 2b. Timber Prices