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Equipment Replacement Decision Making: Opportunities and Challenges

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ABSTRACT

The primary function of equipment managers is to replace the right equipment at the right time and at the lowest overall cost. To accomplish this task, a theoretically sound and practically feasible equipment replacement optimization methodology has been developed so that a significant amount of money can potentially be saved. In this paper, the opportunities and challenges associated with equipment replacement decision making are discussed in detail. First, a comprehensive review of the state-of-the art and state-of-the practice literature for the equipment replacement optimization (ERO) problem is conducted. Second, a dynamic programming (DP) based optimization solution methodology is presented to solve the ERO problem. The Bellman's formulation for the ERO deterministic (DDP) and stochastic dynamic programming (SDP) problems are discussed in detail. Finally, comprehensive ERO numerical results and implications are given.

KEYWORDS: Equipment replacement; decision making; optimization; dynamic programming.

INTRODUCTION

As assets age, they generally deteriorate, resulting in rising operating and maintenance (O&M) costs and decreasing salvage values. Furthermore, newer assets that are more efficient and better at retaining their value may exist in the marketplace and be available for replacement. The conditions of deterioration and technological changes motivate public and private agencies that maintain fleets of vehicles and/or specialized equipment to periodically replace vehicles composing their fleet. This decision is usually based upon a desire to minimize fleet costs, which typically include the acquisition, operating and maintenance cost, and salvage value over a definite or infinite horizon.

Much research has been undertaken in equipment replacement optimization (ERO) including the Texas Department of Transportation's (TxDOT) ongoing equipment replacement optimization efforts. A detailed review of the state-of-the art and state-of-the-practice literature of the ERO problem and commercial fleet management systems currently available worldwide can be seen and examined in a separate research paper (Fan et al. 2011). In summary, previous research efforts have been made to examine the ERO problem, which can be classified into and solved using three categories from the *solution approach perspectives*:

1) Minimum Equivalent Annual Cost (EAC) Approach

The most basic ERO problem is studied under the assumption of no technological change over an infinite horizon (i.e., the equipment is needed indefinitely). The “no technological change” is sometimes also referred to as “stationary cost” by some researchers in the sense that an asset is replaced with the purchase of a new, identical asset with the same cost. Under this assumption, the optimal solution to the infinite-horizon equipment replacement problem with stationary costs is to continually replace an asset at the end of its economic life. Once determined, the asset should be continuously replaced at this age under the assumption of repeatability and stationary costs (Fan et al. 2011).

2) Experience/Rule based Approach

Many state DOTs use the experience/rule based approach to make keep/replacement decisions for their equipment, particularly during the early stages of ERO implementation (Fan et al. 2011). For example, TxDOT uses threshold values for age, use of an equipment unit, and repair cost as inputs for replacement (Fan et al. 2011, TERM 2004). This experience/rule based approach to the ERO problem can work really well for the fleet manager under certain circumstances. However, this approach heavily depends upon the fleet manager's engineering judgment and experience with the ERO.

3) Dynamic Programming (DP) Approach

The solution of continuously replacing an asset at the end of its economic life based on the minimum EAC method is optimal only under the assumptions of an infinite horizon and stationary costs. However, many situations occur in practice in which an asset is required for a finite length of service (i.e., finite horizon). In particular, if the

costs (including O&M cost and salvage value) are age based, assuming constant or predetermined utilization over a finite horizon, the DP approach is commonly used to solve the ERO problem.

There has been an enormous amount of research on the ERO with finite time horizon using the Deterministic Dynamic Programming (DDP) approach (Hartman and Murphy 2006, Hartman and Rogers 2006, Hillier and Liberman 2005, Wolsey 1998, Nemhauser and Wolsey 1999). However, it should be noted that almost all the previous research efforts are devoted to the DDP solution formulation and its limited applications to extremely simplified case studies and/or toy examples. To the best of our knowledge, there have been no research efforts made so far (except Fan et al. 2012a, 2012b) to apply such DP approaches to solving the real-world ERO problem. In our research a comprehensive DP-based optimization solution methodology has been developed to solve the ERO problem. The developed ERO software consists of three main components: 1) A SAS Macro based Data Cleaner and Analyzer, which undertakes the tasks of raw data reading, cleaning and analyzing, as well as cost estimation & forecasting; 2) A DP-based optimization engine that minimizes the total cost over a defined time horizon; and 3) A Java-based Graphical User Interface (GUI) that takes parameters input by users and coordinates the Optimization Engine and SAS Macro Data Cleaner and Analyzer.

When using the DDP approach, both the vehicle usage and the annual O&M cost are assumed to be constant or predetermined. However, due to randomness in real operations, these expected equipment utilizations are not normally realized in practice, thus invalidating the replacement optimization decisions in some aspects.

The stochastic dynamic programming (SDP) approach will undoubtedly be the preferred approach to solving the ERO problem because it can explicitly consider the uncertainty in the vehicle utilization and the annual O&M cost accordingly. Meyer (Meyer 1971), perhaps due to computational constraints, is one among the very few to study the ERO problem under uncertainty. With the advances in computing technology, a lot of research effort has been put forth to examine the ERO problem under uncertainties during the past decade, as can be seen by much of Hartman's research work (Hartman and Rogers 2006). However, none of these previous research efforts made (except Fan et al. 2012b) uses real-world fleet cost/usage data, and all previous case studies are limited and based on small examples. As a result, many underlying characteristics of the ERO SDP problem have yet to be explored and identified. To our best knowledge, this is the first ERO SDP software that is targeted at a real-world application (using TxDOT's current fleet data) and can explicitly consider the uncertainty in the vehicle utilization and the annual O&M cost. It is believed that the pilot SDP-based work is very general and can be used to make only broad statements regarding the ERO problem. Nonetheless, it can be seen as an example demonstrating its promising feasibility for more large-scale applications. When enough cost/mileage data is collected, the SDP-based optimization solution can be of immediate use and will yield substantial cost savings for years to come in the fleet management industry worldwide.

ERO MODEL FORMULATION

General DP Characteristics

The basic features that characterize DP solution algorithms can be presented as follows (Bellman 1995): 1) The problem can be divided into stages with a policy decision required at each stage. The stages are usually related to time and are often solved by going backwards in time. 2) Each stage has a number of states associated with that stage. 3) The decision at each stage transforms the current state at this stage to a state associated with the beginning of the next stage (possibly with a probability distribution applied). 4) The solution procedure is designed to find an optimal policy for the overall problem, i.e., a prescription of the optimal policy decision at each stage for each of the possible states. 5) Given the current state, the optimal policy decision for the remaining stages is independent of decisions made in previous stages. 6) The solution procedure begins by finding the optimal policy for the last stage. 7) A recursive relationship is available to traverse between the value of the decision at a stage N and the value of the optimum decisions at previous stages $N+1$. 8) When using the recursive relationship, the solution procedure starts at the end and moves backward stage by stage – each time finding the optimal policy for that stage – until the optimal policy starting at the initial stage is found (Bellman 1995, Bellman 2003, Wagner 1975, Waddell 1983, Hartman 2005, Hartman and Murphy 2006).

DP can generally be classified into two categories: DDP and SDP. For DDP, the state at the next stage is completely determined by the state and policy decision at the current stage. In SDP the state at the next stage is not completely determined by the state and policy decision at the current stage. Rather, there is a probability distribution applied for what the next state will be. However, the probability distribution is still determined entirely by the state and policy decision at the current stage (Bellman 2003, Wagner 1975, Meyer 1971). In SDP, the decision maker's goal is usually to minimize *expected* (or expected discounted) cost incurred or to maximize *expected* (or expected discounted) reward earned over a given time horizon.

DP Model Formulation

The TxDOT fleet manager identifies equipment items as candidates for equipment replacement one year in advance due to the fact that generally one year is required to allow sufficient time for the procurement and delivery of a new unit of equipment. Since the TxDOT fleet manager makes decisions as to whether to keep or replace a piece of equipment at the beginning of each year, it is very natural to consider each year a stage. As a result, we refer to the year count (or index) as the *stage variable* and the age of the equipment in service at the beginning of each year as the *state variable*. In this project, the TxDOT fleet manager highly recommended that all the equipment must be salvaged at the end of a planning horizon of 20 years. In other words, it is assumed that an equipment unit will be kept no longer than 20 years. It is expected that the value of the planning horizon selected by the fleet manager may

have some impacts on the equipment optimal keep/replacement decisions. However, it is also believed that 20 years is a very reasonable value and is therefore highly recommended for the ERO problem for State DOTs.

The equipment purchase cost model is year-based, the annual operating & maintenance cost and the usage of the equipment unit are both age-based, and the salvage values are dependent upon both the model year and equipment age. All of this data comes from SAS as outputs of the SAS macro based Data Cleaner and Analyzer (Fan et al. 2011) and act as inputs to the DDP-based optimization engine. Moreover, we have realized that it is standard practice to allow for discounting of future costs in any DDP model and solution process. Put another way, solving the ERO problem using the dynamic programming approach requires all costs (such as annual O&M costs including all repairs, regular maintenance and down time penalty costs, and salvage values, as well as purchase costs of the new model year) at each stage to be converted from the equipment model year (for the equipment purchase cost) and/or calendar year (for annual O&M costs and salvage value) to a benchmark year using the inflation rate. Such calculations for the discounting of future costs have been successfully performed (Fan et al. 2011).

DP SOLUTION APPROACH

Bellman's Formulation for the ERO DDP Problem

Bellman (Bellman 1995, Bellman 2003) introduced the first DDP solution to the finite horizon equipment replacement problem where the age of the asset defines the state of the system with the decision to keep or replace the asset at the end of each period (stage). We have implemented the Bellman DDP approach so that the solution caters to TxDOT's needs in solving the ERO problem (Fan et al. 2012a).

In a typical Bellman network, each node represents the age and the usage (i.e., mileage/hours) of the asset at that point in time, which is also the state space of the model. Each arc represents the decision to either keep (K) or replace (R) the asset. Keeping the asset connects nodes n (i.e., n -year-old) and $n+1$ (i.e., $n+1$ -year-old) while replacing the asset is shown by an arc connecting n and 0. An optimal policy with this model, in the form (K, K, R, K, K, ...), gives the optimal decision at the beginning of each year. If an asset can be retained for a maximum of N periods, then the maximum number of states in a period is N . For an N -period problem, since there are a maximum of two decisions for any state, the problem can be solved using the following calculation: $O(\text{State of year 1} + \text{State of year 2} + \dots + \text{State of year } N) = O(1 + 2 + 3 + \dots + N - 1 + 1) = O(\frac{N(N+1)}{2} + 1)$. Therefore, the computer complexity of Bellman's algorithm is $O(N^2)$.

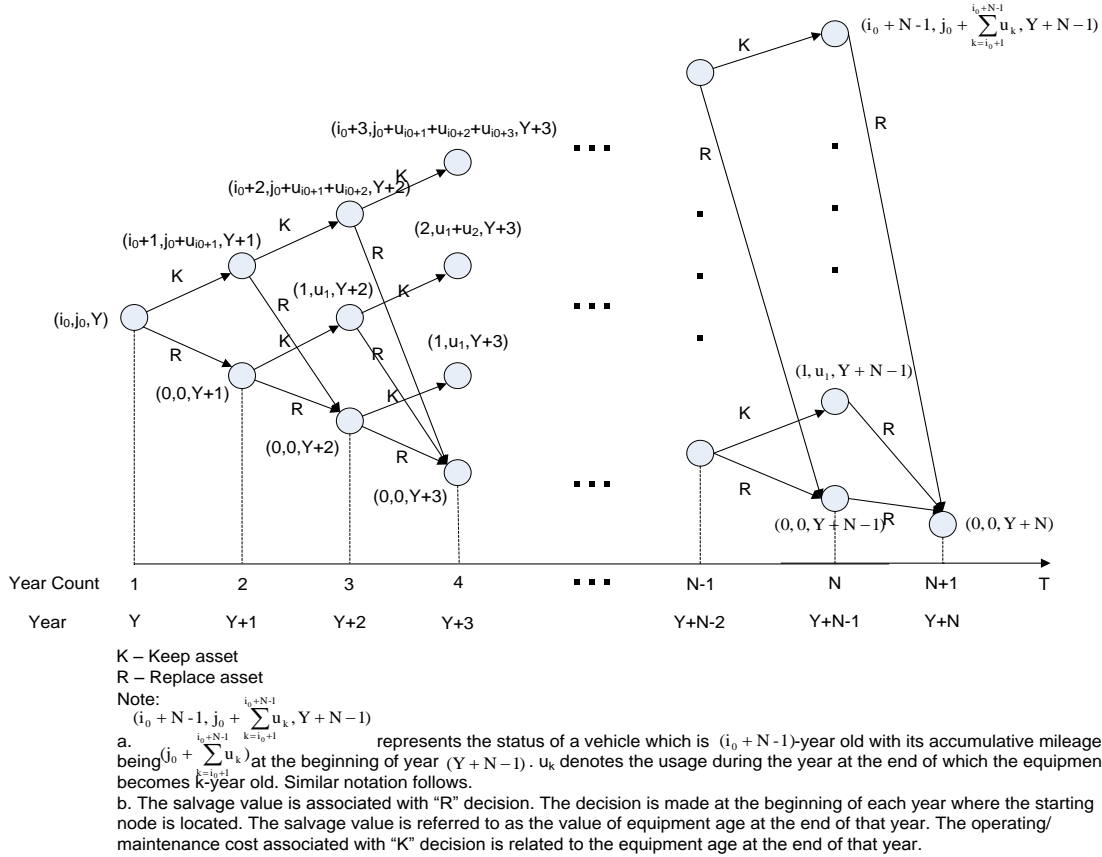
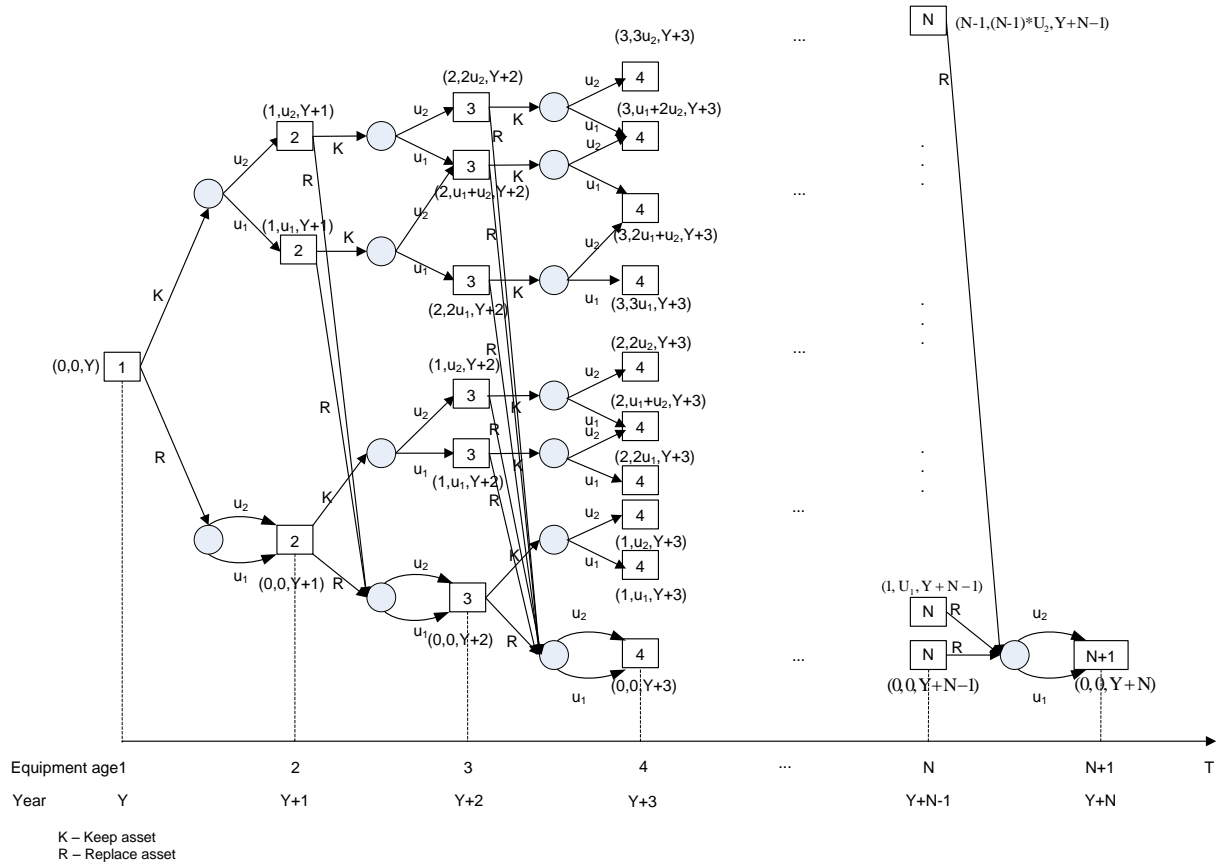


Figure 1. Bellman's Formulation

Bellman's Formulation for the ERO SDP Problem

When Bellman's approach is applied to the SDP method to solve the ERO problem, a phenomenon, commonly termed “curse of dimensionality,” appears. For example, the ERO SDP solution procedure, without scenario reduction treatment, has a general state-space issue that can result in exponential growth in the computer memory and software computational time with increases in the time horizon. Careful consideration and special treatments have been used to resolve these issues (Fan et al. 2012b).

Figure 2 shows a complete “Keep-Replace” Bellman formulation example starting with a brand-new equipment unit for the ERO SDP problem, with uncertainty in vehicle utilization for the SDP-2Level case, after conducting the scenario reduction treatment. In Figure 2, the square nodes represent the decision to either keep or replace the equipment unit. The circular nodes represent chance nodes, as the equipment utilization level is uncertain and the path taken from these nodes defines the cumulative equipment utilization in the next stage. The path taken from the circular nodes are defined as u_1 and u_2 which represent two feasible (i.e., the high and low) equipment utilization levels. Additionally, all nodes at time N are connected to a dummy node at time $N+1$, which represents the salvage of the equipment unit after the final stage of the finite horizon problem (Fan et al. 2012b). It should also be noted that the total cost would include the purchase cost, the expected annual O&M cost, and salvage value, as previously mentioned.



Note:

- (i, U, Y) represents the status of a vehicle which is i -year old with its accumulative mileage used being U , during the i years at the beginning of year Y . Similar notation follows.
- The salvage value is associated with "R" decision. The decision is made at the beginning of each year where the starting node is located. The salvage value is referred to as the value of equipment age at the end of that year. The operating/maintenance cost associated with "K" decision is related to the equipment age at the end of that year.

Figure 2. A Complete "Keep-Replace" SDP Formulation for the ERO Problem with Uncertainty in Asset Utilization: the Two-Level Case after Conducting the Scenario Reduction Treatment

SOFTWARE DEVELOPMENT AND FUNCTIONALITIES

SDP Computer Implementation Techniques

To successfully implement the Bellman formulation to solve the ERO SDP problem, an efficient and effective data structure is designed and then implemented by developed Java computer codes. The model year-based equipment purchase cost, the equipment age-, and model year- based salvage value, and the equipment age- and mileage-based annual operating and maintenance cost data along with corresponding probability distribution for each year that come from SAS (Fan et al. 2011) are read and processed by the Java codes through three steps/layers within the Optimization Engine. The first layer is reading the classcode, the second layer is reading the equipment age, and the third layer is reading the equipment utilization and associated probability (to accommodate the different equipment utilization levels). A series of dynamically allocated arrays are developed to store the data (Fan et al. 2012b). The Bellman approach as presented earlier is then solved backward and the recursive functions are called efficiently.

SDP Software Development and Functionalities

The developed DDP software considers two approaches for the ERO problem: 1). Assume “Current Trend” --- Take all the information from current TERM data that are “error- and outlier- free” and assume that the same trend will continue for all future years. For example, the current TERM data shows that equipment utilization decreases as equipment gets older and therefore we assume this trend will continue (Fan et al. 2011); and 2) Assume “Equal Utilization” --- Take the average mileage across all equipment with same classcode and use this number for the utilization for all equipment during that year. Note that year-to-year utilization for the same classcode can still be different under this assumption. In subsequent sections, numerical results will be presented to show an example of the differences in the equipment keep/replacement decisions between these two approaches.

Many other functions have been incorporated in the DP-based ERO software including the following: 1) The software allows the user to specify budget constraints, as well as the time window that the programming will use during optimization. 2) The software allows users to selectively “Clean the data.” 3) The user can choose to run the software using SAS automatically generated cost data or use the Editable cost data that they have provided manually at the beginning of each year. 4) The user can choose from several different approaches, namely: Cost Current Trend or Cost Equal Mileage; DDP or SDP, and Bellman or Wagner. 5) The user can also choose to delay the replacement of equipment or replace it early by specifying a positive or negative delay time. 6) The software can also run optimization on a single used piece of equipment from a specific classcode, on all equipment units from either one specific classcode or from all classcodes, or on brand new equipment units from either one specific classcode or from all classcodes. 7) The software gives an EXCEL report for the cost savings by comparing the optimal solution with the benchmark rules, and it provides an EXCEL report summarizing the cost savings by comparing the optimal solution with the “delay by N years” option or the “ignore the optimized decision” option. 8) Finally, users can add new annual TERM data at the beginning of each year and make dynamic keep/replacement decisions for any chosen classcode or equipment unit (Fan et al. 2012b).

OPPORTUNITIES AND CHALLENGES

The developed ERO solution software in this paper is very general and can be used to make optimal keep/replacement decisions with or without uncertainty in vehicle utilization for both brand-new and used vehicles, both with or without annual budget considerations. In other words, the developed solution methodology can be used to: 1) Provide a general guide for the equipment keep/replacement decisions (i.e., how many years to keep) for a particular classcode containing brand-new equipment without considering any budget constraints; 2) Select the equipment units for annual replacement from a solution space that is composed of all the candidate equipment units that are eligible for replacement based on the annual budget and other constraints, if any. Also, it should be noted that all numerical results are essentially

dependent upon the specific classcode chosen. However, after comprehensive testing it was found that numerical results of all classcodes seem to follow similar patterns and exhibit some shared general characteristics. In this regard, the following section uses the real TxDOT TERM data (TERM 2004) and describes some interesting and representative numerical results using two classcodes, 420010 and 520020, as an example for the light vehicle and heavy vehicle classes respectively. Related characteristics are discussed as follows.

Opportunities

The computational time of the ERO software for all classcodes and each solution approach was examined. It was found that the computational time is very uniform for the DDP and SDP 2-Level approaches and it takes an average of 10 seconds for the software to provide the best optimized decision for each classcode. It takes a total of about 32 minutes to loop through all (i.e., 194) classcodes and output all optimized solutions in an EXCEL file for either “Current Trend” or “Equal Utilization” approach. However, the SDP 3-Level approach appears to be less uniform and most classcodes take more time to run; the average for this approach was nearly 30 seconds for the ERO software to provide the best optimized decision for each classcode with probabilistic vehicle utilization. Therefore, it takes a total of about 97 minutes to loop through all (i.e., 194) classcodes and output all optimized solutions in an EXCEL file for the “current trend” approach in which the probability distribution of the vehicle utilization is forecasted based on the historical data.

A comparison of the solution quality for the DDP solution, the SDP 2-Level and 3-Level optimization solutions, and the current benchmark solutions for classcodes 420010 and 520020 is given in Table 1. As can be seen, the objective function values (represented in \$ value) for each DP approach are smaller (more desirable) than for the corresponding benchmark solutions for both classcodes. This is expected because each DP approach ensures that all solution paths (which certainly include the current purely experience-based replacement benchmark solution) are explored by solving backward. This guarantees that the best solution is also found by selecting the solution path with minimum total cost over the definite horizon (determined by the benchmark year).

In addition, one may notice that the total cost of the benchmark solutions for the DDP, SDP 2-Level and SDP 3-Level approaches are all different. This is expected because the DDP approach uses the classcode-level cost/mileage forecast for all future years to calculate the benchmark decision year. On the other hand, both SDP approaches generate and use cost/mileage forecasts for each individual and all the vehicle utilization levels (low-high for 2-Level, or low-medium-high for 3-Level) and their associated probability distributions for all future years to determine the benchmark decision year. This can cause the expected cost/mileage data to be slightly different between the different solution approaches.

Table 1. Solution Quality Comparisons between the SDP and DDP Optimized Solutions and the Current Benchmark Solutions for Classcodes 420010 and 520020

		Year	DDP Approach				SDP 2-Level Approach				SDP 3-Level Approach			
			DDP Solution		Benchmark Solution		SDP Solution		Benchmark Solution		SDP Solution		Benchmark Solution	
			Decision	Cost	Decision	Cost	Decision	Cost	Decision	Cost	Decision	Cost	Decision	Cost
Classcode	420010	1	K	\$2,881.39	K	\$2,881.39	R	\$5,269.29	K	\$2,469.76	K	\$2,469.76	K	\$2,469.76
		2	R	\$9,050.29	K	\$3,320.66	R	\$6,101.20	K	\$3,448.38	R	\$8,794.86	K	\$3,065.23
		3	K	\$2,881.39	K	\$3,782.13	K	\$2,469.76	K	\$3,696.17	K	\$2,469.76	K	\$3,724.82
		4	K	\$3,320.66	K	\$4,256.11	K	\$3,448.38	K	\$4,038.96	K	\$3,065.23	K	\$4,198.20
		5	K	\$3,782.13	K	\$4,732.92	K	\$3,696.17	K	\$4,503.90	K	\$3,724.82	K	\$4,783.81
		6	K	\$4,256.11	K	\$5,202.88	K	\$4,038.96	K	\$5,070.60	R	\$15,601.30	K	\$4,967.72
		7	R	\$17,989.34	K	\$5,656.32	R	\$17,760.33	K	\$5,556.50	K	\$2,469.76	K	\$5,478.87
		8	K	\$2,881.39	K	\$6,083.55	K	\$2,469.76	K	\$6,007.50	K	\$3,065.23	K	\$5,779.37
		9	K	\$3,320.66	K	\$6,474.89	K	\$3,448.38	K	\$6,474.89	K	\$3,724.82	K	\$6,151.15
		10	K	\$3,782.13	R	\$25,673.63	K	\$3,696.17	R	\$25,478.75	K	\$4,198.20	R	\$25,413.79
		11	K	\$4,256.11	K	\$2,881.39	K	\$4,038.96	K	\$2,469.76	K	\$4,783.81	K	\$2,469.76
		12	K	\$4,732.92	K	\$3,320.66	K	\$4,503.90	K	\$3,448.38	R	\$21,279.03	K	\$3,065.23
		13	R	\$21,887.57	K	\$3,782.13	R	\$21,755.29	K	\$3,696.17	K	\$2,469.76	K	\$3,724.82
		14	K	\$2,881.39	K	\$4,256.11	K	\$2,469.76	K	\$4,038.96	K	\$3,065.23	K	\$4,198.20
		15	K	\$3,320.66	K	\$4,732.92	K	\$3,448.38	K	\$4,503.90	K	\$3,724.82	K	\$4,783.81
		16	K	\$3,782.13	K	\$5,202.88	K	\$3,696.17	K	\$5,070.60	K	\$4,198.20	K	\$4,967.72
		17	K	\$4,256.11	K	\$5,656.32	K	\$4,038.96	K	\$5,556.50	K	\$4,783.81	K	\$5,478.87
		18	K	\$4,732.92	K	\$6,083.55	K	\$4,503.90	K	\$6,007.50	K	\$4,967.72	K	\$5,779.37
		19	K	\$5,202.88	K	\$6,474.89	K	\$5,070.60	K	\$6,474.89	K	\$5,478.87	K	\$6,151.15
		20	R	\$26,202.97	R	\$29,674.69	R	\$26,103.16	R	\$29,479.81	R	\$27,230.39	R	\$29,414.86
			Total	\$135,401.15	Total	\$140,130.02	Total	\$132,027.48	Total	\$137,491.88	Total	\$131,565.38	Total	\$136,066.51
			Cost Savings	\$4,728.87			Cost Savings	\$5,464.40			Cost Savings	\$4,501.13		
	520020	1	K	\$1,865.53	K	\$1,865.53	K	\$1,865.53	K	\$1,865.53	K	\$1,865.53	K	\$1,865.53
		2	K	\$2,915.71	K	\$2,915.71	K	\$2,915.71	K	\$2,915.71	K	\$2,915.71	K	\$2,915.71
		3	K	\$3,916.86	K	\$3,916.86	K	\$3,916.86	K	\$3,916.86	K	\$3,916.86	K	\$3,916.86
		4	K	\$4,864.60	K	\$4,864.60	K	\$4,864.60	K	\$4,864.60	K	\$4,864.60	K	\$4,864.60
		5	K	\$5,754.55	K	\$5,754.55	K	\$5,754.55	K	\$5,754.55	K	\$5,754.55	K	\$5,754.55
		6	K	\$6,582.32	K	\$6,582.32	R	\$39,399.23	K	\$6,582.33	K	\$6,582.32	K	\$6,582.32
		7	K	\$7,343.55	K	\$7,343.55	K	\$1,865.53	K	\$7,343.55	K	\$8,567.48	K	\$8,567.48
		8	K	\$8,033.85	K	\$8,033.85	K	\$2,915.71	K	\$10,042.31	K	\$8,033.85	K	\$8,033.85
		9	R	\$47,607.00	K	\$8,648.84	K	\$3,916.86	K	\$10,090.31	R	\$47,607.00	K	\$8,648.83
		10	K	\$1,865.53	K	\$9,184.14	K	\$4,864.60	K	\$11,152.17	K	\$1,865.53	K	\$8,309.46
		11	K	\$2,915.71	R	\$52,129.15	K	\$5,754.55	R	\$53,735.05	K	\$2,915.71	R	\$49,987.96
		12	K	\$3,916.86	K	\$1,865.53	K	\$6,582.33	K	\$1,865.53	K	\$3,916.86	K	\$1,865.53
		13	K	\$4,864.60	K	\$2,915.71	R	\$47,495.25	K	\$2,915.71	K	\$4,864.60	K	\$2,915.71
		14	K	\$5,754.55	K	\$3,916.86	K	\$1,865.53	K	\$3,916.86	K	\$5,754.55	K	\$3,916.86
		15	K	\$6,582.32	K	\$4,864.60	K	\$2,915.71	K	\$4,864.60	K	\$6,582.32	K	\$4,864.60
		16	K	\$7,343.55	K	\$5,754.55	K	\$3,916.86	K	\$5,754.55	K	\$8,567.48	K	\$5,754.55
		17	K	\$8,033.85	K	\$6,582.32	K	\$4,864.60	K	\$6,582.33	K	\$8,033.85	K	\$6,582.32
		18	K	\$8,648.84	K	\$7,343.55	K	\$5,754.55	K	\$7,343.55	K	\$8,648.83	K	\$8,567.48
		19	K	\$9,184.14	K	\$8,033.85	K	\$6,582.33	K	\$10,042.31	K	\$8,309.46	K	\$8,033.85
		20	R	\$60,198.47	R	\$57,327.35	R	\$53,674.70	R	\$58,768.83	R	\$58,057.28	R	\$57,327.35
			Total	\$208,192.39	Total	\$209,843.42	Total	\$211,685.59	Total	\$220,317.24	Total	\$207,624.37	Total	\$209,275.40
			Cost Savings	\$1,651.03			Cost Savings	\$8,631.65			Cost Savings	\$1,651.03		

As one can see from Table 1, using classcode 420010 with the “current trend” approach as an example, the SDP 2-Level approach results in the most savings and suggests 5 replacements over the 20 year window, while the benchmark solution suggests replacement at years 10 and 20 only. While the SDP 3-Level solution and the DDP solution offer similar replacement strategies, the difference in savings comes from the difference in the expected costs associated with each approach; these results indicate that using the developed SDP-based ERO software can significantly improve the replacement procedures and can result in substantial cost savings every year. Specifically, for classcode 420010, the estimated savings is about $\$5,464.40/20 = \273.22 per year for a single piece of equipment. For classcode 520020 the SDP 2-Level solution estimates the cost savings with replacement on years 6, 13, and 20 of $\$8,631.65/20 = \431.58 per year, which is much greater than either of the DDP or the

SDP 3-Level solutions. The average of the cost savings for both classcodes is estimated at $(\$273.22 + \$431.58)/2 = \$352.40$ per year. Considering there are 194 classcodes used by TxDOT and on average each classcode includes 84 pieces of equipment, a cost savings of $\$352.40 \times 194 \times 84 = \$5,742,730.77$ might be expected. As can also be seen from Table 1, an also-significant cost savings of \$2,506,389.98 for the SDP 3-Level approach can be estimated using the same calculation method. Therefore, one might expect a cost savings of several million dollars annually for the agency using the SDP approaches.

The results provided here were run without explicitly considering the annual budget constraints, which may exist in the real world for government agencies and private fleet sectors. However the developed solution methodology in this paper can also be used to select the equipment units for annual replacement based on the annual budget constraints and possibly some other constraints specified by the fleet manager. To solve the ERO problem under such constraints, the following steps are required.

First, the cost of NOT replacing an equipment unit when it should be replaced is estimated by comparing the total cost of the optimal solution to the minimum total cost incurred when delaying the replacement of equipment by a certain number of years. The increases in cost are quantified for each feasible replacement year and are used as inputs to the second round of optimization. Next, the second round of optimization is used to select the equipment units for annual replacement from all equipment units that are eligible for replacement. The main objective of this step is to maximize the benefits produced, in order to embody a mixture of both TxDOT's short-term and long-term interests. Preliminary results indicate that a significant amount of cost savings can be estimated by using our developed solution methodology when using an annual budget of 15 million dollars for TxDOT's current TERM data.

Challenges

After conducting comprehensive testing, all three approaches have produced promising results and can yield significant cost savings compared to the current TxDOT benchmark decision process. However, because the probabilistic nature of vehicle utilization is explicitly considered, the formulated SDP approach appears to be more practically feasible than the DDP approach. However, the lack of large enough and dependable data sets for some classcode/equipment units may prevent the SDP software from generating as reliable of solutions as possible. In this regard, the SDP approach is still in somewhat of an early development stage and will be more promising for a future application as this line of research matures and the data collection effort progresses. The impact of the uncertain future purchase cost, the down time cost, and O&M cost on the ERO keep/replacement decision and its total cost also needs further investigation.

SUMMARY AND FUTURE RESEARCH

In this paper, a comprehensive review of the state-of-the art and state-of-the practice literature for the equipment replacement optimization (ERO) problem is first conducted. A dynamic programming (DP) based optimization solution methodology is then presented to solve the ERO problem. The Bellman's formulation for the ERO deterministic (DDP) and stochastic dynamic programming (SDP) problems are discussed in detail. Finally, comprehensive ERO numerical results and implications are given along with the opportunities and challenges associated with the equipment replacement optimization problem. The software computational time and solution quality have been demonstrated to be very promising and encouraging, and substantial cost-savings are estimated using this ERO software. The computational experience with the ERO problem also indicates some challenges with data collection efforts need to be met in the future. Other issues with forecasting future purchase cost, the down time cost, and O&M cost must also be addressed. As this line of research matures and data accumulates, the software can be of immediate use to provide even more reliable and better results.

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