



The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search

<http://ageconsearch.umn.edu>

aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

Tony Diana, Ph.D.
Division Manager, NextGen Performance
Office of NextGen Performance and Outreach, ANG-F1
Federal Aviation Administration
Phone: 202-385-7311 E-Mail: Tony.Diana@faa.gov

Predicting Block Time: An Application of Quantile Regression

Abstract: Airlines face three types of delay that make it difficult to build robust schedules and to support block time predictability. These delays can be induced (i.e., ground delays), propagated, or stochastic. With capacity constrained at major airports and regulators facing greater public pressure to alleviate congestion and tarmac delays, aviation practitioners have renewed their interest in the predictability of block time, that is, the time elapsed from gate departure to gate arrival. This study presents a methodology based on the case study of the Seattle/Tacoma International and Oakland International airport city pair to determine a block time. This methodology based on quantile regression models is appropriate for skewed distribution where analysts are interested in the impact of selected operational covariates on the conditional mean of block times at given percentiles.

Words: 3,435

1. Introduction

This article proposes a methodology to determine the predictability of block time based on the case study of the Seattle-Oakland city pair. It relies on quantile regression to determine how some selected operational variables are likely to affect actual block times at different percentiles. This is of importance to aviation practitioners and, especially, airline schedulers who have often resorted to schedule padding in order to make up for ground and en route delays.

Predictability is all the more difficult to achieve as airlines often face three types of delay. First, delays can be induced: The Federal Aviation Administration (FAA) can initiate a ground delay program in case of adverse weather conditions or heavy traffic volume on the ground or en-route. Second, delays can be propagated: In a sequence of legs operated by the same tail-numbered aircraft, a flight may accumulate delays that cannot be recovered by the end of the itinerary. Finally, delays can be stochastic because they are the results of random events such as equipment breakdown or an extreme weather event.

Predictability represents an important key performance indicator in the aviation industry for several reasons.

- For the International Civil Aviation Organization (ICAO), predictability refers to the “ability of the airspace users and ATM service providers to provide consistent and dependable levels of performance.”ⁱ
- One of the goals of the U.S. Next Generation of Air Transportation System (NextGen) is to foster the transition from an air traffic control to more of an air traffic managed system where pilots have more flexibility to select their routes, utilize performance-based navigation and make decisions based on automated information sharing. Presently, it is very difficult to assess the impact of NextGen-related technologies on flight performance because surveillance data do not account for the difference between the use of Required Navigational Requirement (RNP) and Instrument Landing Systems (ILS) when flight tracks for both types of procedure overlay, for instance.
- According to Rapajic (2009:51), “cutting five minutes of average of 50 per cent of schedules thanks to higher predictability would be worth some €1,000 million per annum, through savings or better use of airlines and airport resources.” Unpredictability imposes considerable costs on airlines in the forms of lost revenues, customer dissatisfaction and potential loss of market share.

Recently, much discussion has revolved around the validity of using airlines’ schedules as a measure of on-time performance and the variance of block delay as an indicator of predictability. Both airlines’ limited control over the three types of delay and airport congestion make it difficult to build robust schedules. In this discussion, the predictable block time is located at the percentile where the sign and magnitude of the pseudo coefficient of determination is the highest with all the covariates significant at a given confidence level. Ordinary Least Square (OLS) regression models enable analysts to evaluate the percentage of variation in actual block time explained by

changes in selected operational variables. However, quantile regression is more robust to outliers than the traditional OLS regression because the latter does not focus on the conditional mean. The attributes of quantile regression will be addressed later in the discussion.

This article presents a different perspective on the study of predictability with the intent of helping aviation practitioners achieve the following objectives:

- To assess the impact of selected independent variables at different locations of the distribution of block delays in order to anticipate block time based on selected operational variables.
- To derive more predictable block times based on the impact of operational covariates at various percentiles.
- To test a model without any assumption about the distribution of errors and homoscedasticity.

After a brief background, the discussion will proceed with the methodology, an explanation of the outcomes and eventually some final comments.

2. Background

Robust airline scheduling is the outcome of four sequential tasks as schedule generation, fleet assignment, aircraft routing and crew pairing/rostering (Wu 2010; Abdelghany and Abdelghany 2009). Fleet assignment models (FAM) are often used to determine how demand for air travel is met by available fleet (see Abara 1989 and Hane et al. 1995). Moreover, the fleet assignment models present two challenges: complexity and size of the problem that the FAM can handle.

Rapajic (2009) identified network structure and fleet composition as sources of flight irregularities. Wu (2010) provided an excellent exposition of issues related to delay management, operating process optimization, and schedule disruption management. Wu explained that "airline schedule planning is deeply rooted in economic principles and market forces, some of which are imposed and constrained by the operating environment of the [airline] industry" (2010:11). He presented a schedule optimization model to improve the robustness of airline scheduling. However, such a model does not consider how selective operational variables are likely to impact scheduling.

Morrisset and Odoni (2011) compared runway system capacity, air traffic delay, scheduling practices, and flight schedule reliability at thirty-four major airports in Europe and the United States from 2007 to 2008. The authors explained that European airports limit air traffic delay through slot controls. The other difference is that declared capacity (therefore, the number of available slots) is based mainly on operations under instrument meteorological conditions. By not placing restrictions on the number of operations, schedule reliability in the United States depends more on weather conditions as at European airports.

3. Methodology

3.1 The Sample and the Assumptions

The sample includes the month of June to August in 2000, 2004, 2010 and 2011 (by day) for the Seattle/Tacoma International (SEA)-Oakland International (OAK) city pair. The summer season is usually characterized by low ceiling and visibility (instrument meteorological conditions) and weather events such as thunderstorms—all likely to skew the distribution of block times. Secondly, it is also part of the high travel season when demand is at its peak, which is likely to increase airport congestion and subsequently impact travel time. Finally, the years were selected to account for (1) pre- and post-September 11, 2001 traffic, (2) lower traffic demand resulting from the 2008-2009 recession, and (3) the introduction of NextGen-related initiatives after 2010, such as Green Skies over Seattleⁱⁱ.

The sample does not include a variable that measures performance based navigation. The available surveillance data such as Traffic Flow Management System (TFMS) do not capture whether a pilot had requested a required navigation performance procedure, whether air traffic control had granted the request, and whether the procedure had actually been implemented. Moreover, it is often difficult to differentiate flown RNP from instrument landing system (ILS) approaches in the case of flight track overlay.

Secondly, the availability of Q-routes makes it possible for the two airlines' RNAV/RNP capable aircraft to reduce mileage, to minimize conflicts between routes and to maximize high-altitude airspace. Q-routes are available for use by RNAV/RNP capable aircraft between 18,000 feet MSL and FL 450 inclusive. They help minimize mileage and reduce conflicts between routes.

Thirdly, block time as a measure of gate-to-gate performance is sensitive to delays on the ground and en route as the model outputs will later show. Therefore, airborne delay represents a surrogate for enroute congestion, while increases in taxi times imply surface movement congestion.

3.2 Sources and Definition of the Variables

The sources for the variables are ARINCⁱⁱⁱ's Out-Off-On-In times and the Federal Aviation Administration's Traffic Flow Management System (TFMS). The directional city pair data originated from the 'Enroute' and 'Individual Flights' data marts of the ASPM data warehouse^{iv}.

The choice of variables reflects operational and statistical considerations. On the one hand, some model variables represent core factors in airport congestion (taxi times) and enroute performance (airborne delays). On the other hand, the model with the highest values for the Akaike Information Criterion (AIC)^v and Bayesian Information Criterion (BIC)^{vi} was selected in order to prevent overfitting and to reduce the number of covariates.

The dependent (response variable) and independent variables (covariates) are defined as follows:

- **Actual Block Time** (ACTBLKTM) is the dependent variable. It refers to the time from *actual* gate departure to *actual* gate arrival.
- **Block Buffer** (BLKBUFFER) represents the difference between planned and optimal block time. The latter is the sum of unimpeded taxi-out times and filed estimated time enroute. Block buffer is the additional minutes included in planned block time in order to take into account potential induced, propagated and stochastic delays. It has also been defined as “the additional time built into the schedule specifically to absorb delay whilst the aircraft is on the ground and to allow recovery between the rotations of aircraft” (Cook, 2007:105). Donohue et al. (2001:113) explained that “to obtain their desired on-time performance, airlines will add padding into a schedule to reflect an amount above average block times to allow for delay and seasonally experienced variations in block times.”
- **Departure Delay** (DEPDEL) corresponds to difference between the actual and planned gate departure time at the departure airport in a city pair.
- **Arrival Delay** (ARRDEL) represents to the difference between the actual and planned gate arrival time at the arrival airport in a city pair.
- **Airborne Delay** (AIRBNDEL) accounts for the total minutes of airborne delay. It is the difference between the actual airborne times (landing minus takeoff times) minus the filed estimated time enroute.
- **Taxi-Out Time** (TXOUTTM) refers to the duration in minutes from gate departure to wheels-off times.

3.2 Quantile Regression

Quantile regression features several advantages compared with the traditional ordinary-least-square (OLS) regression in assessing the influence of selected operational factors on the variations of block time at various locations of its distribution:

- Quantile regression specifies the conditional quantile function and, therefore, a way to assess the probability of achieving a certain level of performance. It permits the analysis of the full conditional distributional properties of block delays as opposed to ordinary-least-square (OLS) regression models that focus on the mean.
- It defines functional relations between variables for all portions of a probability distribution. Quantile regression can improve the predictive relationship between block times and selected variables by focusing on quantiles instead of the mean. As Hao and Naiman (2007:4) pointed out, “While the linear regression model specifies the changes in the conditional mean of the dependent variable

associated with a change in the covariates, the quantile regression model specifies changes in the conditional quantile.”

- It determines the effect of explanatory variables on the central or non-central location, scale, and shape of the distribution of block times.
- It is distribution-free, which allows the study of extreme quantiles. Outliers influence the length of the right tail and make average block time irrelevant as a standard for identifying the best-possible block time. A single rate of change characterized by the slope of the OLS regression line cannot be representative of the relationship between an independent variable and the entire distribution of block time. In the quantile regression, the estimates represent the rates of change conditional on adjusting for the effects of the other model variables at a specified percentile. Therefore, the skewed distribution of block times calls for a more robust regression method that takes into account outliers or the lack of sufficient data at a particular percentile (especially at the extremes of the distribution) and generates different slopes for different quantiles.

4. Outcomes and Implications

The estimates as well as the key regression outputs at the 5th, 25th, median, 75th and 95th percentile are summarized in appendix 1. The 50th quantile estimates can be used to track location changes. According to Hao and Naiman (2007: 55), the 5th and 95th percentiles “can be used to assess how a covariate predicts the conditional off-central locations as well as shape shifts of the response.” Based on the graphs in appendix 2, the coefficient estimates show a positive relationship between the quantile value and the estimated coefficients at higher percentiles for scheduled block times, taxi out times and airborne delay.

If we take the example of the 50th percentile in summer 2011, the quantile regression model for at $\tau = 0.50$ (50th percentile) is as follows:

$$\begin{aligned} \text{Block Time}_{\tau = 0.50} = & -0.9105 * X_{\text{BLKBUFFER}} + 0.8888 * X_{\text{SCHEDBLKTM}} - 0.3090 * X_{\text{DEPDEL}} \\ & + 0.2702 * X_{\text{ARRDEL}} + 1.1015 * X_{\text{AIRBNDEL}} + 0.1372 * X_{\text{TXOUTTM}} + \varepsilon \end{aligned}$$

In the above equation, 1.1372 represents the change in the median of block time between SEA and OAK corresponding to a one minute change in taxi-out time at SEA. Since the p value is zero, we reject the null hypothesis, at a 95 percent confidence level, that taxi-out times at SEA has no effect on the median block time between SEA and OAK in summer 2010. The pseudo coefficient of determination is a goodness-of-fit measure^{vii}.

Overall, summer 2011 is the only period when all the covariates have a significant effect on block times at all the considered percentiles. Remarkably, block buffer, scheduled block time, departure, arrival and airborne delays, as well as taxi-out times are significant at the 95th percentile at a 95% confidence level in summer 2011, 2010, 2004, and 2000. This suggests that the difference between actual and planned departure and arrival times are more likely to have an incidence on the conditional mean of block times at the highest percentile as a result of surface area movement

congestion. Moreover, the magnitude of block buffer and departure delays have a negative impact on the conditional mean of block time for all samples at all selected percentiles. This calls for airline schedulers to understand the reasons for the gap between planned and actual block time and for airport analysts to evaluate the times and conditions when departure operations are delayed.

The appendix 1 table shows that 95 percent of the distribution of block times between SEA and OAK was below 129.14 minutes in the June-to-August time period based across the samples. While the standard distribution is appropriate to measure the spread of a symmetric distribution, interquartile ranges are more indicative of spread changes in skewed distributions. One benefit of quantile regression is that it facilitates the evaluation of scale and magnitude changes across samples and percentiles.

In a comparison of summer 2000 with summer 2011, there has been an increase of 2.21 minutes in block times at the 95th percentile. The SEA-OAK city pair has been mainly operated by Southwest Airlines (SWA) and Alaska Airlines (ASA) with a fleet of Boeing 737s. The total number of ASA operations declined to 356 in summer 2011 from 693 in summer 2000, while 91 flights were operated by Horizon's Bombardier Q400 on behalf of ASA. Nevertheless, ASA operated larger capacity models such as the dash 400, 800 and 900 series, while SWA utilized a combination of dash 300, 500 and 700 models. The reason for the increase in block time may be attributed to airlines' corporate policy to slow aircraft speed in order to save on fuel costs. Weather conditions characterized by the percentage of operations in instrument meteorological conditions (IMC) did not vary substantially at OAK: It was respectively 29.78, 29.86, 29.71 and 29.49% in summer 2011, 2010, 2004 and 2000. At SEA, the percentages were respectively 22.83, 30.29, 8.33 and 10.58% during the same time periods.

5. Final Comments

Predictability is a key performance area identified by the International Civil Aviation Organization. Moreover, it is a corner stone of the Next Generation of Air Transport System (NextGen) initiatives in the U.S. to ensure the transition from an air traffic controlled to a more air traffic managed environment. As air transportation regulators are under public pressure to crack down on tarmac and other types of delays, it has become imperative for airline schedulers to evaluate models that reflect the influence of key operational variables on actual performance. The complexity of the air traffic system, the inability for airline schedulers to fully anticipate both airport and en route congestion, the imbalance between travel demand and capacity that result in delay all make it more significant for aviation practitioners to assess the impact of operational variables at different locations of the distribution of block times.

Based on the analysis of the SEA-OAK city pair case study, this article showed how quantile regression can help aviation practitioners develop more robust schedules. First, it enables aviation analysts to consider the impact of covariates on different locations of the distribution of block times. Secondly, the significance of the selected variables and the strength of the impact of selected covariates on block times make it possible to assess the probability that gate-to-gate operations is likely to reach a specific duration. This is made possible by looking at the conditional mean in the case

of quantile regression as opposed to the mean of the distribution of block times in the case of OLS models. Thirdly, quantile regression makes it easier to evaluate the scale and magnitude of changes across specific percentiles over a sample. Finally, quantile regression can help analysts study the impact of covariates from different perspectives. In the present case, the quantile regression models focused on constraining factors such as airborne, departure and arrival delays on the conditional means of block times, which explains the identification of the 95th percentile optimal values as the expected block time given the impact of the covariates and the strength of the pseudo R-square.

Although the quantile regression models could provide some indication as to the scale and magnitude of change, they would have benefited by measuring the impact of technical changes introduced by NextGen between summer 2000 and summer 2011. However, the assessment of such changes requires that available surveillance data keep track of the use of procedures such as RNAV/RNP, optimal profile descents, among other key technological improvements related to NextGen.

6. Appendix 1

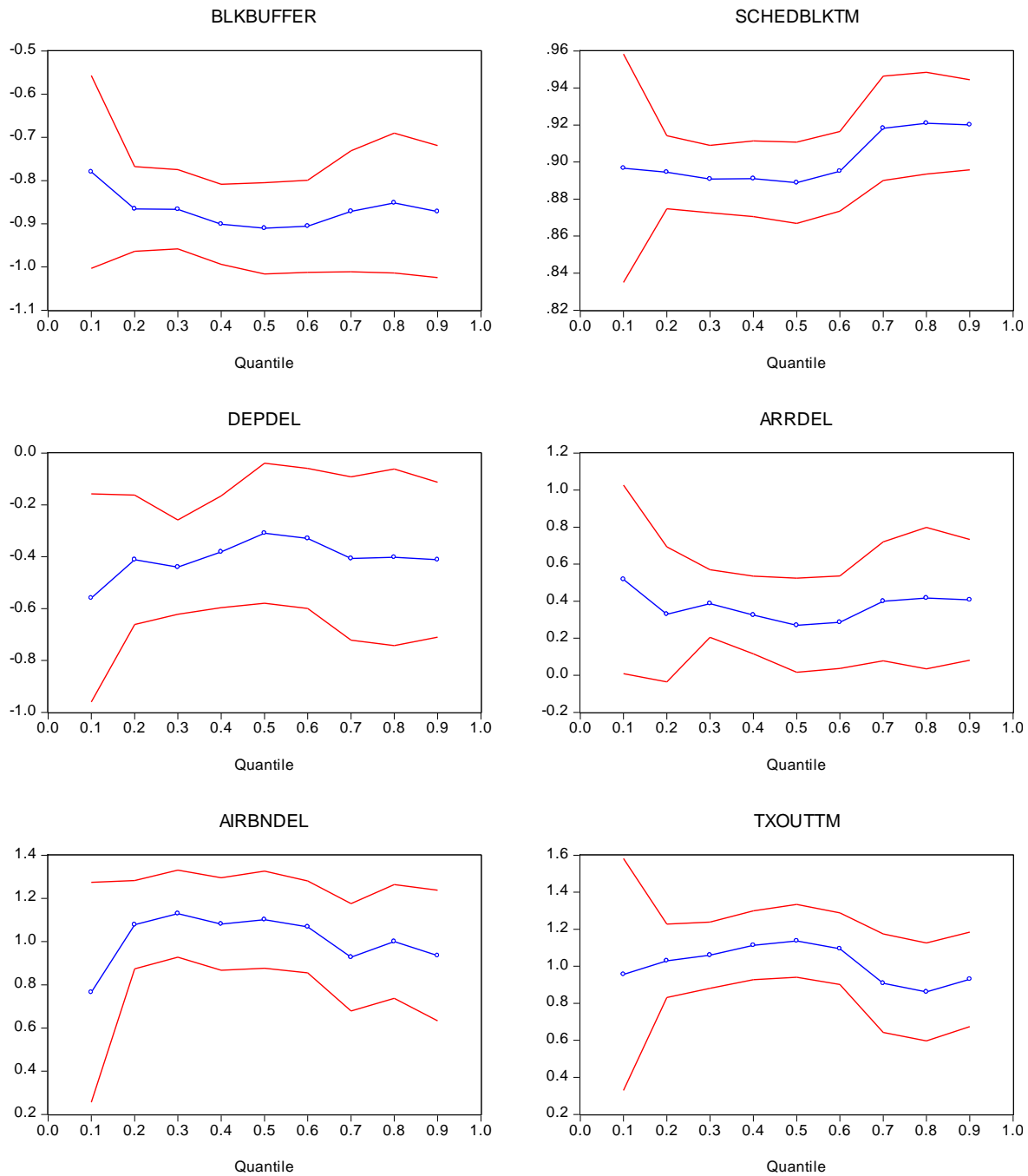
THE QUANTILE REGRESSION OUTPUTS

Alpha = .95	2011		2010		2004		2000	
	Coefficient	Probability	Coefficient	Probability	Coefficient	Probability	Coefficient	Probability
5th Percentile								
BLKBUFFER	-0.7594	0.0000	-1.0083	0.0000	-0.9693	0.0000	-0.9904	0.0000
SCHEDBLKTM	0.9263	0.0000	0.9006	0.0000	0.9316	0.0000	0.9051	0.0000
DEPDEL	-0.7318	0.0000	-0.0404	0.3929	-0.0448	0.5115	-0.0616	0.5885
ARRDEL	0.7623	0.0001	0.0788	0.1598	0.0621	0.4139	-0.0131	0.9229
AIRBNDEL	0.7751	0.0000	1.0951	0.0000	0.8766	0.0000	1.2568	0.0000
TXOUTTM	0.5935	0.0127	1.0119	0.0000	0.9374	0.0000	1.0042	0.0000
Pseudo R-squared	0.7014		0.8893		0.8878		0.8758	
Adjusted R-squared	0.6841		0.8829		0.8813		0.8686	
S.E. of regression	2.4713		1.0580		1.0013		1.3161	
Quantile dependent var	112.8600		107.5600		113.0000		110.9400	
Sparsity	12.6438		3.4399		2.8121		4.0616	
25th Percentile								
BLKBUFFER	-0.8525	0.0000	-1.0084	0.0000	-0.9134	0.0000	-0.8522	0.0000
SCHEDBLKTM	0.8916	0.0000	0.9066	0.0000	0.9253	0.0000	0.9230	0.0000
DEPDEL	-0.4393	0.0000	-0.0439	0.4714	-0.1207	0.1369	-0.1860	0.0400
ARRDEL	0.3976	0.0000	0.0411	0.6096	0.1134	0.1995	0.1727	0.0665
AIRBNDEL	1.1222	0.0000	1.0805	0.0000	0.9252	0.0000	0.9989	0.0000
TXOUTTM	1.0369	0.0000	1.0048	0.0000	0.9607	0.0000	0.8326	0.0000
Pseudo R-squared	0.7694		0.8952		0.8753		0.8843	
Adjusted R-squared	0.7560		0.8891		0.8680		0.8776	
S.E. of regression	1.2083		0.7264		0.7354		0.7577	
Quantile dependent var	117.7100		115.0000		116.5300		117.2100	
Sparsity	2.4094		1.8315		1.5958		1.8171	
50th Percentile								
BLKBUFFER	-0.9105	0.0000	-0.9907	0.0000	-0.9152	0.0000	-0.7903	0.0000
SCHEDBLKTM	0.8888	0.0000	0.9074	0.0000	0.9383	0.0000	0.9285	0.0000
DEPDEL	-0.3090	0.0275	-0.0187	0.8068	-0.1183	0.1299	-0.2783	0.0260
ARRDEL	0.2702	0.0398	-0.0170	0.8388	0.1205	0.1602	0.2709	0.0433
AIRBNDEL	1.1015	0.0000	0.9973	0.0000	0.7753	0.0000	0.9507	0.0000
TXOUTTM	1.1372	0.0000	1.0547	0.0000	0.9383	0.0000	0.7760	0.0000
Pseudo R-squared	0.7994		0.8922		0.8631		0.8689	
Adjusted R-squared	0.7877		0.8859		0.8551		0.8613	
S.E. of regression	1.1793		0.6220		0.6015		0.6263	
Quantile dependent var	120.4300		119.0000		118.9300		119.8600	
Sparsity	2.2739		1.4886		1.5010		1.7504	
75th Percentile								
BLKBUFFER	-0.8864	0.0000	-0.9900	0.0000	-0.9185	0.0000	-0.7780	0.0000
SCHEDBLKTM	0.9255	0.0000	0.9140	0.0000	0.9344	0.0000	0.9385	0.0000
DEPDEL	-0.3590	0.0340	-0.0885	0.3238	-0.0900	0.2008	-0.2509	0.0089
ARRDEL	0.3720	0.0402	0.0561	0.5520	0.1094	0.1546	0.2650	0.0144
AIRBNDEL	0.9501	0.0000	0.9861	0.0000	0.8565	0.0000	0.8778	0.0000
TXOUTTM	0.8492	0.0000	1.0249	0.0000	0.9496	0.0000	0.7251	0.0000
Pseudo R-squared	0.8138		0.8954		0.8548		0.8695	
Adjusted R-squared	0.8030		0.8893		0.8464		0.8619	
S.E. of regression	1.4336		0.7203		0.6748		0.7868	
Quantile dependent var	123.8000		121.9200		121.6000		122.1500	
Sparsity	2.6969		1.6380		1.9727		2.0322	
95th Percentile								
BLKBUFFER	-0.8970	0.0000	-0.9377	0.0000	-0.7299	0.0000	-0.6846	0.0000
SCHEDBLKTM	0.9308	0.0000	0.9097	0.0000	0.9750	0.0000	0.9456	0.0000
DEPDEL	-0.3469	0.0108	-0.2347	0.0023	-0.3725	0.0002	-0.3753	0.0000
ARRDEL	0.4219	0.0047	0.2056	0.0127	0.3844	0.0001	0.4111	0.0000
AIRBNDEL	0.6948	0.0000	1.0246	0.0000	0.6003	0.0000	0.7797	0.0000
TXOUTTM	0.9168	0.0000	1.0483	0.0000	0.5678	0.0000	0.6431	0.0000
Pseudo R-squared	0.8430		0.8986		0.8761		0.9103	
Adjusted R-squared	0.8339		0.8927		0.8689		0.9051	
S.E. of regression	1.9295		1.0992		1.2589		1.1371	
Quantile dependent var	129.1400		127.4300		126.0600		126.9300	
Sparsity	4.8819		3.2418		3.6042		3.4603	

Not significant at $\alpha = .95$

7. Appendix 2

QUANTILE PROCESS ESTIMATE GRAPHS (95% CONFIDENCE LEVEL)



8. Bibliography

- Abara, J., 1989. Applying integer linear programming to the fleet assignment problem. *Interfaces* 19(4), 20-28.
- Abdelghany, F. and Abdelghany, K., 2009. Modeling applications in the airline industry. Ashgate Publishing Company: Burlington, Vermont.
- Cook, A., 2007. European Air Traffic Management: principles, practices, and research. Ashgate Publishing Company: Burlington, Vermont.
- Donohue, G.L., Zellweger, A., Rediess, H., and Pusch, C., 2001. Air transportation system engineering: progress in astronautics and aeronautics. American Institute of Aeronautics and Astronautics: Danvers, Massachusetts.
- Hane, C.A., Barnhart, C., Johnson, E.L., Marsten, R.E., Nemhauser, G.L., and Sigismondi, G., 1995. The fleet assignment problem: solving a large scale integer programming. *Mathematical Programming*, 70(2), 211-232.
- Hao, L. and Naiman, D. Q., 2007. Quantile Regression. Sage Publications: Thousand Oaks, CA.
- Koenker, R. and Machado, J., 1999. Goodness of fit and related inference processes for quantile regression," *Journal of the American Statistical Association*, 94(448), 1296-1310.
- Morrisett, T. and Odoni, A., 2011. Capacity, Delay, and Schedule Reliability at Major Airports in Europe and the United States. *Transportation Research Record: Journal of the Transportation Research Board*, 2214, 85-93.
- Rapajic, J., 2009. Beyond airline disruptions. Ashgate Publishing Company: Burlington, Vermont.
- Wu, C.-L., 2010. Airline Operations and Delay Management: Insights from Airline Economics, Networks and Strategic Schedule Planning. Ashgate Publishing Company: Burlington, Vermont.

ⁱ Henk J. Hof, Development of a Performance Framework in support of the Operational Concept, ICAO Mid Region Global ATM Operational Concept Training Seminar, Cairo, Egypt, November 28–December 1, 2005, p. 36.

ⁱⁱ The Green Skies over Seattle program includes initiatives such as reduced track mileage to minimum possible distance to protect the environment, optimized profile descent, reduction or elimination of low altitude radar vectoring, as well as required navigational performance.

ⁱⁱⁱ AIRINC stands for Aeronautical Radio, Inc. (<http://www.arinc.com>).

^{iv} The TFMS (formerly ETMS) and ARINC data as well as the ASPM delay metrics are available at <http://aspm.faa.gov>.

^v The Akaike Information Criterion is defined as $2k - 2 \ln(L)$ where k is the number of parameters and L the maximized value of the likelihood function for the estimated model.

^{vi} The Bayesian Information Criterion is $-2 \ln(L) + k \ln(n)$ where n is the number of observations.
^{vii} See Koenker and Machado 1999 for further explanations.