



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

May 2001
NOT FOR QUOTATION

Griliches' k -shift and Competitiveness: Commodity Progress in U.S. Agriculture¹

Lilyan E. Fulginiti²

¹Prepared for presentation at the AAEA Meetings, Chicago, August 2001.

²Professor, Department of Agricultural Economics, University of Nebraska, Lincoln.
Copyright 2001 by Lilyan E. Fulginiti. All rights reserved.

Griliches' *k*-shift and Competitiveness: Commodity Progress in U.S. Agriculture

Economists have focused considerable attention on the issue of competitiveness in recent years with the objective of understanding the determinants of a nation's ability to compete in world markets. Competitiveness does not have a definition in economic theory and the use of the word to describe export performance of nations was the subject of a heated debate in the 1990's.¹

Most studies describe firms that cannot survive by selling at the going price as not competitive. If they survive and increase market share, they have become more competitive. Trying to define competitiveness for a nation is much more problematic than defining that of a firm. First, countries do not 'go out of business' or disappear as a result of bad economic performance. Second, countries do not compete with each other the way firms do given that they are each others main export markets and suppliers of imports. If anything, economic growth of a trade partner likely means a larger market for a county's exports and cheaper higher quality imports. International trade is not a 'zero-sum game'; when two countries 'compete' through trade they both win. In other words, gains from trade are a well known phenomenon.

Authors have tried to define a county's competitiveness as the combination of favorable trade and domestic performance. D'Andrea Tyson, Chairman of the Council of Economic Advisors in the early 1990's, defines competitiveness as "our ability to produce goods and services that meet the test of international competition while our citizens enjoy a standard of living that is both rising and sustainable." In an economy with little international trade², the growth in living standards, and thus competitiveness would be determined by domestic factors, mainly the rate of productivity growth. The definition of

competitiveness given in the World Competitiveness Report produced by The World Economic Forum states that competitiveness is "the ability of a country to achieve sustained high rates of growth of GDP per capita." In Krugman's words "competitiveness would turn out to be a funny way of saying "productivity" and would have nothing to do with international competition."

This paper uses the word competitiveness to refer to a country's future prosperity depending on its growth in productivity.³ Productivity growth is defined as an increase in output per unit of inputs. It can be represented by a shift up of the production function, or a shift down of the marginal cost of production. When a productivity change is measured in input-output space by a production function shift, it is usually described by technology parameters indicating the rate and input biases of that change. When a productivity change is measured in output-price space as a shift in the marginal cost of the commodity, it is usually described by what has been referred in the literature as Griliches' k -shift.⁴ Even though changes in competitiveness can equivalently be assessed by estimating the relevant technology parameters or by investigating the nature of the changes in the commodity's industry-level supply function, it has been customary in the literature to study one or the other without explicit recognition of their exact relationship.

In this paper a country becomes more competitive in the production of a particular commodity when the product supply curve for that commodity shifts down. Product competitiveness is directly related to the nature and magnitude of the k -shift in the market for that commodity which this paper shows is equivalent to the sum of the rate and bias of technical change. The k -shift so defined is renamed "the rate of commodity progress" to indicate its relationship to the country's competitiveness. This rate is shown to be econometrically estimable.

In this paper, section two summarizes the literature. Section three, introduces the output distance function, characterizes productivity change in terms of the distance function and shows the relationship between the k -shift and the rate of commodity progress. Section four discusses the specification of flexible functional form models for distance functions, and for the rate and bias in technological change. Finally, an example is presented in which the rate of commodity progress for wheat, corn, soybeans, and beef in U.S. agriculture is estimated for the 1960-1995 period. This methodology provides the necessary information to calculate the rate of commodity progress, to simulate the induced supply shifts due to innovation, and to indicate the relative change in competitiveness. Section five is a summary and conclusions.

Other Productivity Studies

A review of the literature shows two types of approaches used to measure the economic consequences of productivity change. The first one includes economy-wide and sectoral studies that use indexes, production, cost and profit functions to estimate the rate and bias of technical change. Studies that measured technical change as a shifter of the production function or as an output over input index start with Tinberger's 1942 effort and include the early works by Schmookler, Fabricant, Kendrick, Abramovitz, Solow, Griliches (1960, 1963), Jorgenson and Griliches and many others that followed.⁵ More recently, introduction of duality theory has provided studies where productivity change is captured as a shifter of the cost, revenue, or profit functions. Along these lines we found the studies by Christensen, Jorgenson, and Lau, Berndt and Khaled, and others.⁶ These studies are done

at a high level of aggregation, providing estimates of the rate of technical change and the input biases at the industry or country level. Information on the implied supply shift, or k -shift, in particular commodity markets is not provided by these studies.

The alternative to the approach above goes beyond the production technology to look at the productivity impact on the firm and industry supply functions. In view of the fact that the ‘output’ of innovative activity does not present itself in countable units, a quantifiable dimension for innovations can be defined directly in value terms, that is, in terms of their impact on social welfare. In other words we seek an answer to the question of how much additional consumer and producer surplus was generated by technical change in a particular commodity market during a period of time. The so called economic surplus approach has been used extensively to evaluate the benefits from a productivity induced supply shift starting with Griliches (1958) study on the social returns to hybrid corn research. Early work includes the evaluation of agricultural research by Peterson and Schmitz and Seckler, of industrial innovations by Mansfield et al, of mainframe computers by Bresnahan and more recently the study by Trajtenberg of computed tomography scanners.⁷ While the last two studies presented hedonic analyses of the impact of changes in *product* qualities on consumers' welfare, all of the former analyze the welfare impact of a *process* innovation as a supply shift and compute the benefits from the implied price reductions. These studies assume an exogenously determined shift of the marginal cost due to innovative activities and calculate the returns to these investments as changes in economic surplus, no reference is made to the technology parameters describing the changes in input-output space.⁸ Critical assumptions in these models include the supply and demand elasticities and the nature of the productivity-induced supply shift. With respect to the k -shift, Alston et al, who present a very comprehensive summary of

the research in this area, state that "This choice in the analysis is crucially important, and by comparison, the choices about functional forms pale into insignificance." In fact, there has been a good deal of discussion in the literature about the effects of different hypothesized supply shifts, parallel or pivotal, on the size and distribution of the benefits from technical change.⁹ And even though researchers agree on the importance of such parameter, all studies have used ad-hoc approximations. It is the general consensus, as expressed in Alston et al that "...with current techniques and typically available data, it is not possible to settle these questions econometrically."(page 64) and "...thus assumptions about the nature of the research-induced supply shift are unavoidable."

It is at this point that this paper makes a contribution. As competitiveness is captured by productivity change that induces reductions in the marginal cost of production of a commodity, it should also be possible to describe such change by the rate and bias in technical change which are specific technology parameters that can be econometrically estimated. We introduce a commodity level indicator of competitiveness, which we name "the rate of commodity progress," and is nothing more than Griliches' k -shift expressed in terms of these two specific and measurable parameters. In empirical analyses, there is no need to make assumptions about the nature of this shift as it can be econometrically estimated.

The Distance Function Approach to Productivity Measurement

In this paper, a change in competitiveness is defined as a **technology-driven** divergence in the sizes of the output bundles obtained from given inputs. This measure is closely related to the downward

shift in the supply function for a commodity, but it can be measured in input-output space, i.e. from quantity data alone. To measure it we need a representation of the technological possibilities of the firm. The technology of a firm that produces a single output may be described by a production function indicating the maximum attainable output as a function of an input vector. If there are many outputs, however, an alternative representation in quantity space is provided by the distance function, sometimes referred to as the transformation function, the gauge function, or the direct cost or revenue function. This turns out to have some advantages. First, only under certain circumstances can a multiple output technology be described by a set of production functions, one for each output. Second, the distance function is a dual to the cost and revenue functions and inherits some of their properties. Third, and most important for this study, changes in the output distance function due to technological innovation exactly captures changes in competitiveness as **technology-driven** changes in the size of the output bundles for given inputs. In general, the distance function allows a very flexible description of the technology.

The first references to this function are Wold who uses it to define a utility function, Debreu who uses it to define a 'coefficient of resource utilization', Malmquist who develops a series of index numbers based on it and Shephard who extensively discusses it in the context of production theory. More recent publications in the production area¹⁰ that use and describe the properties of this function can be found in Fuss and McFadden, Blackorby, Lovell, and Thursby, and Färe and Primont. Much of what follows is derived from one or more of the contributions listed above. However, none of these provides a systematic and complete treatment of the distance function in the context of productivity measurement. The aim of the present paper is to do so.

The output distance function is particularly fit to capture the concept of competitiveness as developed in this analysis given that it provides a clear and exact definition of productivity change. By definition, it allows representation of the maximum amount by which outputs could be expanded given available inputs and the state of the technology. If the technology changes as a result of innovations enlarging the feasible technology set, the distance function will also change capturing in a natural way this change in productivity.

The representation of the output distance function in Figure 1 highlights its interpretation in product space. Let y^0 be an arbitrary reference bundle of commodities. Now consider an arbitrary production frontier, $F(x^0)$, associated with output set $P(x^0)$ representing all output bundles feasible to produce with a particular technology and input bundle x^0 . The bundle y^0 is not achievable with these inputs and technology. However, if the quantities of *all* outputs are decreased by the same proportion (along a ray through the origin) so as to take the firm to $P(x^0)$, then the output bundle h^0 is just attainable. y^0 and h^0 are scalar multiples of one another, so we can define scalar 2^0 such that $y^0 = 2^0 h^0$. If we choose $2 < 2^0$ then $y^0/2$ produces another infeasible bundle. If $2 > 2^0$ the resulting bundle will be smaller than what is feasible with $P(x^0)$. 2^0 can therefore be defined as the minimum value of 2 such that $F(x^0) \ni (y^0/2)$. Its value will depend on the selected output reference bundle y^0 and production set $P(x^0)$. Formally, for given vectors y and x the output distance function is defined as

$$q^* \equiv D_o(x, y) \equiv \min_q \{ q / (1/q) y, x \text{ feasible} \}. \quad (1)$$

Had y^0 and x^0 been chosen so that $y^0 = F(x^0)$ then $D(y^0, x^0) = 1$ because h^0 and y^0 would have

coincided.¹¹ The output distance function is nonincreasing in each input level, nondecreasing in each output level, homogeneous of degree one, convex and superadditive in outputs.¹²

The definition of the distance function makes no mention of, and does not depend on, a price system. It is useful though to provide an alternative characterization in which competitive prices play a role. This displays the duality between the output distance function $D_o(\mathbf{x}, \mathbf{y})$ and the revenue function $R(\mathbf{p}, \mathbf{x})$ and provides a point of departure for obtaining the derivative property of the output distance function, used later in our analysis. Consider a firm using a $(n \times 1)$ vector of inputs \mathbf{x} with prices \mathbf{W} to produce a $(m \times 1)$ vector of outputs \mathbf{y} with prices \mathbf{P} subject to the technology set \mathbf{S} . The revenue function solves the following problem:

$$R(\mathbf{P}, \mathbf{x}) \equiv \max_{\mathbf{y}} \{ \mathbf{P} \cdot \mathbf{y} / (\mathbf{x}, \mathbf{y}) \in S \} \quad (2)$$

which provides the revenue-maximizing bundle \mathbf{y}^* when output prices are \mathbf{P} and inputs are \mathbf{x} . We normalize output prices ($p_i = P_i/R$) so that the maximum revenue obtained when producing the target vector of outputs is unity, that is $R(\mathbf{p}, \mathbf{x}) = 1$.¹³ We note that the revenue function has properties similar to the output distance function. Those properties of $D_o(\mathbf{x}, \mathbf{y})$ that essentially characterize it as a distance function are its homogeneity, superadditivity, and convexity in outputs \mathbf{y} and the same properties are possessed by the revenue function $R(\mathbf{p}, \mathbf{x})$ in \mathbf{p} . In fact, Shephard shows that the revenue function is a distance function in price space.¹⁴

The revenue function is a distance function for the revenue structure $V(\mathbf{x})$ and it is a dual of the production structure $P(\mathbf{x})$ distance function $D_o(\mathbf{x}, \mathbf{y})$.¹⁵ If we consider the maximization with respect to

the output price vector \mathbf{p} for any input vector \mathbf{x} (the dual operation to the one that defined $R(\mathbf{p}, \mathbf{x})$ in (2))

$$D_o(\mathbf{x}, \mathbf{y}) = \max_{\mathbf{p}} \{ \mathbf{p} \cdot \mathbf{y} / \mathbf{p} \in V(\mathbf{p}), \mathbf{p} \geq 0 \}, \mathbf{x} \geq 0 \quad (3)$$

this price maximal revenue function is identical to the distance function defined in (1) and has the same properties. Then we can alternative define the output distance function as

$$D_o(\mathbf{x}, \mathbf{y}) \equiv \max_{\mathbf{p}} \{ \mathbf{p} \cdot \mathbf{y} / R(\mathbf{p}, \mathbf{x}) = 1 \}. \quad (4)$$

For an intuitive interpretation we use Figure 1. Assuming convexity of the technology set, to every hypothetical bundle \mathbf{h} obtained by using the given technology there corresponds a price vector \mathbf{p} such that the implied revenue line is tangent to the production possibilities frontier at \mathbf{h} , so that $R(\mathbf{p}, \mathbf{x}) = \mathbf{p} \cdot \mathbf{h} = 1$. In Figure 2, the intersection between such a revenue line and the ray OM is denoted by N. Let d^* be the ratio OM/ON. Then it can be expressed as

$$d^* = OM / ON = \mathbf{p} \cdot \mathbf{y}^0 / \mathbf{p} \cdot \mathbf{h} = \mathbf{p} \cdot \mathbf{y}^0. \quad (5)$$

The value d^* is maximized when the normalized price vector touches the production possibilities frontier at the point T, at which $\mathbf{h} = \mathbf{h}^0$. In short, from among all the normalized price vectors that just make that output bundle attainable from the given input bundle, (5) picks out the one that maximizes the value of the reference bundle \mathbf{y} .

Now recall the definition of $R(\mathbf{p}, \mathbf{x})$. We know that if the vectors of inputs and outputs are on

the boundary of the production set, then $D_O(\mathbf{x}, \mathbf{y}) = 1$. Hence the revenue function originally defined in (2) can be expressed alternatively as

$$R(\mathbf{p}, \mathbf{x}) \equiv \max_{\mathbf{y}} \{\mathbf{p} \cdot \mathbf{y} | D_O(\mathbf{x}, \mathbf{y}) = 1\}. \quad (6)$$

Equations (4) and (6) express Shephard duality. Equation (4) tells us that given the information summarized by the revenue function in price space, a simple optimizing problem will generate a representation of the technology in terms of quantities, using the output distance function. Equation (6) expresses the opposite. The revenue function and the output distance function are dual functions that contain equivalent information when the technology is convex.

An important property of the output distance function for our purpose is its derivative property. Whenever they are defined, the partial differentials of $D_O(\mathbf{x}, \mathbf{y})$ with respect to outputs, which we write $R_m(\mathbf{x}, \mathbf{y})$, are the prices normalized with reference to total revenues. Hence, writing p_m for P_m/R ,

$$\nabla D(\mathbf{x}, \mathbf{y}) / \nabla y_m \equiv y_m(\mathbf{x}, \mathbf{y}) = p_m \equiv \frac{P_m}{R} \text{ for } m = 1, \dots, M. \quad (7)$$

Differentiation of the output distance function with respect to outputs gives input compensated inverse supply functions with revenue normalized price (marginal cost) as a function of outputs and inputs. The function $R_m(\mathbf{x}, \mathbf{y})$ is a conditional valuation, or an inverse supply function when shadow and market prices coincide, and it expresses the cost to the firm of an extra unit of the commodity m as a function of the input and output bundle. It is the marginal cost or virtual price of output m .

Price differentiation of the revenue function $R(\mathbf{p}, \mathbf{x})$ gives the input compensated supply functions $y_m(\mathbf{p}, \mathbf{x})$, with quantity supplied as a function of prices and inputs. Note also from (7) that

$$\frac{\mathbb{I} \ln D_o(\mathbf{x}, \mathbf{y})}{\mathbb{I} \ln y_m} = s_m(\mathbf{x}, \mathbf{y}) = s_m(\mathbf{x}, \mathbf{p}) = \frac{\mathbb{I} \ln r(\mathbf{x}, \mathbf{p})}{\mathbb{I} \ln p_m} \quad (8)$$

where $s_m(\mathbf{x}, \mathbf{y}) = R_m y_m / D_o(\mathbf{x}, \mathbf{y})$ is, at the optimum i.e. $D_o(.) = 1$ $R_m = p_m / R$, the revenue share of output m , $s_m(\mathbf{x}, \mathbf{p}) = p_m y_m / R$.

The partial differentials of $D_o(\mathbf{x}, \mathbf{y})$ with respect to inputs, which we write $N_n(\mathbf{x}, \mathbf{y})$, are the marginal revenue product of inputs which at the optimum equal the input prices W , normalized with reference to total revenues. Hence, writing w_n for W_n / R

$$\mathbb{I} D(\mathbf{x}, \mathbf{y}) / \mathbb{I} x_n \equiv f_n(\mathbf{x}, \mathbf{y}) = w_n \equiv \frac{W_n}{R} \text{ for } n = 1, \dots, N. \quad (9)$$

The Radial Rate of Technical Change.

The set of relationships of interest in this paper have to do with the use of the output distance function to measure Griliches' k -shift described as a combination of the rate and biases of technological change. We first show how the distance function is used to capture these specific technology parameters. We start with the rate of technological change.

Under technical change, each observation is possibly associated with a different technology. We consider the case of time series observations where technical progress can shift the production frontier across observations. Technical change is characterized by moving the technological frontier through innovations of different forms, which may be disembodied or embodied in inputs used for production. It will be represented by the technology index A_t that can be characterized by increasing

the effectiveness of inputs in the production of outputs. Modifying (1) to include a technology index, the output distance function can be written

$$D_O^t(\mathbf{x}_t, \mathbf{y}_t, A_t) = \min_q \{q > 0: (\mathbf{y}_t / q) \in P(\mathbf{x}, A_t)\}. \quad (10)$$

where the output set $P(\mathbf{x}, A_t)$ is defined as the set of all output bundles that can be produced from input bundle \mathbf{x} and technology A_t . Technological change is progressive if for $A_{t+1} > A_t$, it expands the output set and allows output bundles formerly infeasible with inputs \mathbf{x} to be in the new feasible set, or $P(\mathbf{x}, A_t) \subset P(\mathbf{x}, A_{t+1})$. It is regressive if for $A_{t+1} > A_t$, $P(\mathbf{x}, A_t) \not\subset P(\mathbf{x}, A_{t+1})$, it shrinks the output set by eliminating feasible output bundles. The behavior of $D_O(\mathbf{x}, \mathbf{y}, A_t)$ in A_t is easy to categorize. If technical change is progressive the output distance function is nonincreasing in A_t , if it is regressive it is nondecreasing in A_t . This is because technical change expands the production set so the minimum achieved on the expanded set cannot be larger than the minimum achieved on the original output set since the original output bundle remains feasible. The same argument establishes the relationship between regressive technical change and the output distance function.

Figure 2 illustrates the case where the output set is enlarged from $P(\mathbf{x}^0, A_t)$ to $P(\mathbf{x}^0, A_{t+1})$ as a result of technical change. Let the bundle \mathbf{y}^0 be attainable with period t technology and inputs but not in the production frontier $F(\mathbf{x}^0, A_t)$. Let \mathbf{h}^0 be the output bundle just attainable in this period, then $\mathbf{h}^0 = \mathbf{y}^0 / \mathcal{Z}^0$, where \mathcal{Z}^0 is the smaller scalar by which all outputs are expanded to reach the frontier. After technical change, the output set is enlarged and the new frontier is $F(\mathbf{x}^0, A_{t+1})$, with the output bundle \mathbf{h}^1 just attainable with the new technology. In this case $\mathbf{h}^1 = \mathbf{y}^0 / \mathcal{Z}^1$, the smaller scalar \mathcal{Z}^1 by which all outputs should be expanded in order to reach the new frontier is smaller than the one before the

expansion ($2^1 < 2^0$.) So progressive technical change represented by $P(\mathbf{x}, A_t) \neq P(\mathbf{x}, A_{t+1})$ as $A_{t+1} > A_t$, implies $2^1 < 2^0$.

For observations on the frontier:

$$D_O^t(\mathbf{x}_t, \mathbf{y}_t, A_t) = 1 \quad (11)$$

where we can interpret $\ln D_O^t / \ln A_t$ as the rate of technological change in a multiple-output, multiple-input environment which can be expressed as

$$-\frac{\ln D_O^t}{\ln A_t} = \sum_{n=1}^N \frac{\ln D_O^t}{\ln x_{nt}} \frac{d \ln x_{nt}}{d A_t} + \sum_{m=1}^M \frac{\ln D_O^t}{\ln y_{mt}} \frac{d \ln y_{mt}}{d A_t}. \quad (12)$$

Consistent with the definition of the output distance function, \mathbf{x} is held constant then $d\mathbf{x} = 0$. Given the radial nature of the distance function in output space $dy_1/dA_t = dy_2/dA_t = \dots = dy_M/dA_t$ and a common scalar is computed so that

$$\frac{-\ln D_O^t / \ln A_t}{\sum_{m=1}^M (\ln D_O^t / \ln y_{mt}) (1/y_{mt})} = \frac{dy_{mt}}{dA_t} \quad (13)$$

Using the property of linear homogeneity of the distance function we can rewrite the left-hand side of (13) and obtain

$$\frac{-\ln D_O^t(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\ln A_t} = \frac{d \ln y_{mt}}{d A_t} = d(\mathbf{x}_t, \mathbf{y}_t, A_t). \quad (14)$$

Equation (14) indicates that $\hat{\lambda}$, the rate of technical change obtained from an output distance function, equals the common rate of expansion of outputs along a ray through the origin due to an increase in the technology index A when inputs are not allowed to change. Once a particular parametric specification for the distance function is chosen, $\hat{\lambda}$ can be econometrically estimated.

It is also of interest to note how productivity changes are captured in price space using the revenue frontier by augmenting it to include a technology index A_t . In particular, due to the duality between the output distance function and the revenue function it is possible to establish an equivalence between the primal and dual measures of technical change. Figure 3 illustrates in price space the relationship of the revenue function to this index. For progressive technical change, represented by $A_{t+1} > A_t$, the price frontier moves inward and the radial revenue function is nondecreasing in A as seen by the segment OA/OB being smaller than OA/OC . For observations on the price frontier

$$R^t(\mathbf{p}_t, \mathbf{x}_t, A_t) = 1 \quad (15)$$

where we can interpret $\ln R / \ln A_t$ as the dual rate of technological change, and obtain

$$-\frac{\ln R^t}{\ln A_t} = \sum_{n=1}^N \frac{\ln R^t}{\ln x_{nt}} \frac{d \ln x_{nt}}{d A_t} + \sum_{m=1}^M \frac{\ln R^t}{\ln p_{mt}} \frac{d \ln p_{mt}}{d A_t}. \quad (16)$$

Using the definition of the radial revenue function, we hold \mathbf{x} constant ($d\mathbf{x} = \mathbf{0}$) and compute a common scalar for \mathbf{p} so that $dp_1/dA_t = dp_2/dA_t = \dots = dp_M/dA_t$ (radial change in \mathbf{p} space)

$$\frac{-\ln R^t / \ln A_t}{\sum_{m=1}^M (\ln R^t / \ln p_{mt})(1/p_{mt})} = \frac{dp_{mt}}{dA_t} \quad (17)$$

and by linear homogeneity of this function in prices we can write the dual rate of technical change as

$$\frac{\nabla \ln R^t(\mathbf{x}_t, \mathbf{p}_t, A_t)}{\nabla A_t} = - \frac{d \ln p_{mt}}{dA_t} = m(\mathbf{x}_t, \mathbf{p}_t, A_t). \quad (18)$$

This equation indicates that the dual rate of technical change \hat{A}_t that is obtained from a normalized revenue function, equals the common rate of change of output prices along a ray through the origin in price space, when inputs are not allowed to change. Once the revenue function is specified, \hat{A}_t can be estimated.

At this point it is useful to note the relationship between the primal and dual rates of technical change. From equation (6), using the envelope theorem we see that

$$\frac{\nabla R^t(\mathbf{p}_t, \mathbf{x}_t, A_t)}{\nabla A_t} = - \frac{\nabla D_o^t(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\nabla A_t} \quad (19)$$

then for $D_o(\cdot) = 1$ ¹⁶, and using the definition of the revenue function, when inputs are constant so $d\mathbf{x} = 0$ and for a radial change in price and quantity space,

$$m = \frac{\nabla \ln R^t(\mathbf{p}_t, \mathbf{x}_t, A_t)}{\nabla A_t} = - \frac{\nabla \ln D_o^t(\mathbf{y}_t, \mathbf{x}_t, A_t)}{\nabla A_t} = d \quad (20)$$

which establishes the equivalence between the *radial* primal and dual rate of technical change.

So we define the rate of technical change as the rate of contraction of the output distance function or equivalently as the rate of expansion of the revenue function. It is clear from Figures 2 and 3 that the

radial rates only allow measurement of neutral technical change.

Hicksian and Overall Biases of Technical Change.

In a multiple-output production process, technological change may privilege some outputs resulting on some outputs growing faster than others. This is of particular importance in this paper given that relative changes in outputs will indicate if technological change has modified their opportunity cost differentially and therefore their relative competitiveness.

Hicks introduced the definition of neutral and bias technological change for input pairs. He suggested that inventions could be classified in terms of their effects on the marginal product of one factor relative to another, or on the marginal rate of substitution between two factors. We follow Blackorby, Lovell, and Thursby's interpretation of Hicks neutrality as the invariance of the marginal rate of technical substitution at different points on the firm's expansion path. Technical change may shift isoquants but in doing so, the marginal rate of technical substitution is not affected. In other words, the technology is separable with respect to the technical change index and then it is said to be Hicks neutral. We have shown above that the radial primal rate of technical change θ^* is particularly suitable to capture Hick's neutral technical change. When technical change is not Hick's neutral we need to resort to additional developments and we show here the usefulness of the output distance function in doing so.

Using the output distance function and extending Fulginiti, consider the following Hicksian pairwise measure of output bias of technological change:

$$B_{ij}(\mathbf{y}, \mathbf{x}, t) \equiv \frac{\mathbb{J} \ln(\text{MRT}_{ij})}{\mathbb{J} A_t} = \frac{\mathbb{J} \ln D_{oi}(\mathbf{x}, \mathbf{y}, t)}{\mathbb{J} A_t} - \frac{\mathbb{J} \ln D_{oj}(\mathbf{x}, \mathbf{y}, t)}{\mathbb{J} A_t} = \frac{\mathbb{J} \ln(\mathbf{y}_i / \mathbf{y}_j)}{\mathbb{J} A_t} \quad (21)$$

$i, j = 1, \dots, M, i \neq j$

where the subscripts on the output distance function indicate first derivatives. The distance function gives us a direct way of measuring the marginal rate of transformation (MRT) as a ratio of its first derivatives, the virtual prices. This bias concept measures the rotation of the production possibilities frontier at a point in output space in response to technical change. As illustrated in Figure 2, the firm is producing at \mathbf{h}^0 on the initial expansion path. After technological change has occurred, the firm produces at \mathbf{h}^2 , on a new expansion path. This movement can be decomposed into a Hicks neutral change from \mathbf{h}^0 to \mathbf{h}^1 and a substitution change from \mathbf{h}^1 to \mathbf{h}^2 . B_{ij} measures the change in slope of the production frontiers through \mathbf{h}^1 on the initial expansion path. Hicks neutrality is captured by $B_{ij} = 0$, when technical change does not change the expansion path. If $B_{ij} > 0$ the opportunity cost of output j in terms of output i for given inputs has decreased and the technological change is biased toward the production of output j relative to output i . $B_{ij} < 0$ when as a result of technical change, production of one more unit of output j with the same inputs, requires the firm to give up more units of output i than before the technological change, so that it is j th output reducing relative to the i th output.

Hicks defined factor biases in terms of a two-input production function. This definition is not very useful in a multiple-output, multiple-input framework described by the output distance function of this paper because it provides $(m^2 - m)/2$ potential forms of relative bias. For example, technical change could enhance the production of corn relative to that of wheat while diminishing the production of corn

relative to soybeans. This definition does not give a clear interpretation as to whether technical change is expanding or contracting in each output.

An overall measure of bias, in the manner of Antle and Capalbo, defined in product space with the use of the distance function is possible

$$B_i(\mathbf{x}_t, \mathbf{y}_t, A_t) \equiv \sum_{j \neq i=1}^M S_{jt}^v B_{ij}(\mathbf{x}_t, \mathbf{y}_t, A_t) = \frac{\mathbb{J} \ln y_i}{\mathbb{J} A_t} - \sum_{j \neq i=1}^M S_{jt}^v \frac{\mathbb{J} \ln y_j}{\mathbb{J} A_t} \quad (22)$$

where $S_{jt}^v = R_j y_j / D_o$ is the virtual share of output j . It can also be shown that

$$B_{jt}(\mathbf{x}_t, \mathbf{y}_t, A_t) = \frac{\mathbb{J} \ln S_{jt}^v}{\mathbb{J} A_t} \bigg|_y \quad (23)$$

which provides a convenient taxonomy of effects associated with technical change. Equation (22) indicates that if the marginal input requirement of output i is increasing relative to all others, then $B_i > 0$ and the technological change is output- i reducing overall or bias against the production of this output (*anti-output i biased*) as its virtual share increases due to increases in its opportunity cost of production. If $B_i = 0$ then technical change is Hicks neutral. $B_i < 0$ indicates that, an additional unit of output i requires less inputs than other outputs after the technical change has taken place, therefore the technological change has been output- i augmenting and its virtual share decreases as its cost of production has done so. More of the i th output can be produced now with the same inputs and

technological change is *pro-output i biased*. It follows that if technical change is neutral, all B_{ij} 's are 0 and all B_i 's are also 0. Since the S_i^r sum to unity it is also true that $\sum_i G_i S_i^r B_i$ is 0 so if technical change is biased at least one B_i must be positive and one must be negative. Once a parametric specification of the distance function is chosen, pairwise and overall biases can be econometrically estimated.

If the optimal output mix is produced, shadow and actual prices coincide and

$$MRT_{ij} = \frac{y_i(x_t, y_t, A_t)}{y_j(x_t, y_t, A_t)} = \frac{p_{it}}{p_{jt}} \quad (24)$$

then we can rewrite the overall output bias as

$$B_{it} = \frac{\ln p_{it}}{\ln A_t} - \sum_{j=1}^M S_{jt} \frac{\ln p_{jt}}{\ln A_t} = \frac{\ln S_{it}}{\ln A_t} \Big|_y \quad (25)$$

which is readily obtainable from market data.

Commodity Progress: Griliches' k-shift.

We have shown above how the output distance function provides information about the rate and biases in technological change. The task now is to relate these concepts to Griliches' k -shift. In the 1958 paper "Research Costs and Social Returns: Hybrid Corn and Related Innovations," Griliches defines the parameter k used in its welfare analysis as '...the relative shift in the supply curve,...' due to the introduction of the new varieties. As we can see in Figure 4, this is a shift in the marginal cost curve of

commodity i and can be represented by the percentage change in the virtual price of that commodity measured at the original equilibrium value of y .¹⁷ Griliches' k -shift is then

$$k_{it} = \frac{\mathbb{I} \ln y_{it}}{\mathbb{I} A_t} \Big|_y \quad (26)$$

We know from above that the overall bias is

$$B_{jt}(\mathbf{x}_t, \mathbf{y}_t, A_t) = \frac{\mathbb{I} \ln S_{jt}^v}{\mathbb{I} A_t} \Big|_y = \left[\frac{\mathbb{I} \ln y_{it}}{\mathbb{I} A_t} - \frac{\mathbb{I} \ln D(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\mathbb{I} A_t} \right] \quad (27)$$

and from here we obtain that

$$\frac{\mathbb{I} \ln y_{it}}{\mathbb{I} A_t} = \frac{\mathbb{I} \ln S_{it}^v}{\mathbb{I} A_t} \Big|_y + \frac{\mathbb{I} \ln D(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\mathbb{I} A_t} = B_{it}(\mathbf{x}_t, \mathbf{y}_t, A_t) - d(\mathbf{x}_t, \mathbf{y}_t, A_t). \quad (28)$$

In (28) we have expressed the shift of the marginal cost curve of commodity i due to technological innovation as the addition of two terms, the bias and the radial rate of technological change. We refer to this concept as ‘the rate of commodity progress.’ This rate can be estimated for each commodity directly from the technology once a representation of the output distance function is chosen.

Combining (26) and (28) we have shown that Griliches’ k -shift is the rate of commodity progress that can be readily estimated on a commodity basis from a specific output distance function

$$k_{it} = \frac{\mathbb{I} \ln y_{it}}{\mathbb{I} A_t} = B_{it} - d_t = \text{bias} + \text{rate} = \text{commodity progress} \quad (29)$$

Equation (29) is the main result of this paper. Alternatively we can interpret the k -shift or the rate of commodity progress as indicating the effect of technological change on inverse output supplies for a given set of outputs and factor endowments. These elasticities are not independent of one another. It follows from linear homogeneity of the output distance function in \mathbf{y} that

$$\frac{\mathbb{I} D(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\mathbb{I} A_t} = - \sum_i \frac{\mathbb{I}^2 D(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\mathbb{I} A_t \mathbb{I} y_{it}} y_{it} = - \sum_i \frac{\mathbb{I} y_{it}(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\mathbb{I} A_t} y_{it} \quad (30)$$

Dividing through by $D(\mathbf{x}, \mathbf{y}, A)$ we obtain

$$d = - \sum_i S_{it}^v \frac{\mathbb{I} \ln y_{it}}{\mathbb{I} A_t}, \quad (31)$$

where \star is the radial rate of technological change, S_{it}^v is the share of commodity i , and $\mathbb{I} \ln R_{it} / \mathbb{I} A_t$ is the k -shift for the same commodity. This shows that the radial rate of technological change is a weighted average of the (negative) rates of commodity progress or of the individual commodities k -shifts for given outputs and factor endowments.

While the B_i 's from equation (22) can be of either sign we generally expect the $\mathbb{I} \ln R_{it} / \mathbb{I} A_t$ to be negative since their weighted average equals $-\mathbb{I} \ln D(\mathcal{C}) / \mathbb{I} A_t < 0$. However for many commodities some $\mathbb{I} \ln R_{it} / \mathbb{I} A_t$ may well be positive. This would reveal that technological change is strongly biased

against the corresponding output. Thus we may say that technological change is *ultra anti-output i* biased if $\frac{\partial \ln R_{it}}{\partial A_t} > 0$. In particular, if technological change were completely unbiased (i.e. $B_{it} = 0$) so that all output prices change at the same rate, (31) would reduce to

$$d = - \frac{\sum \ln y_{it}}{\sum A_t}. \quad (32)$$

Equation (32) indicates that under Hicks neutrality, the radial rate of technical change equals the decrease in virtual prices common to all commodities. In other words, the rate of technical change is the k -shift and this is the same for all commodities.¹⁸

There is a close link between the effects of technological change on output shadow prices (or marginal input requirements of output i), on one hand, and the impact of changes in output quantities on the radial rate of technological change, on the other. This relationship results from the symmetry of the Hessian of the distance function. Let $\frac{\partial^2 \ln D(\mathbf{C})}{\partial A_t \partial \ln y_i} = -\frac{\partial^2 \ln D(\mathbf{C})}{\partial \ln y_i \partial A_t}$ be the impact described above. One can then easily show that

$$\frac{\sum \frac{\partial^2 \ln D(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\partial A_t \partial \ln y_i}}{\sum A_t} = \frac{\sum S_{it}^v}{\sum A_t} = S_{it}^v \left[\frac{\sum \ln y_{it}}{\sum A_t} - \frac{\sum \ln D(\mathbf{x}_t, \mathbf{y}_t, A_t)}{\sum A_t} \right] = S_{it}^v B_{it} \quad (33)$$

Thus, the share-weighted technological bias indices capture the effects of changes in output quantities on the rate of technological progress. It results immediately from the linear homogeneity of the distance function in outputs and from the definition of B_{it} that

$$-\sum_i \frac{\mathbb{I}d}{\mathbb{I} \ln y_i} = \sum_i S_i^v B_i = \sum_i \frac{\mathbb{I}^2 \ln D}{\mathbb{I}A_t \mathbb{I} \ln y_i} = \sum_i \frac{\mathbb{I}S_i^v}{\mathbb{I}A_t} = 0 \quad (34)$$

that is, the rate of technical change is homogeneous of degree 0 in output quantities.¹⁹ However, changes in *relative* output quantities do normally affect the rate of technological change.

A Quadratic Specification

A flexible representation of the technology that embodies the regularity conditions required by theory is desirable for implementation of this model. The translog functional form has been used in a number of studies (Lovell et al, Grosskopf et al, Coelli and Perelman) in the distance function context. Here, a generalized quadratic form is used. This specification has the flexibility needed to capture the different functional relationships in output and input space without restrictive assumptions about the technology. We assume that the output distance function can be approximated by a normalized quadratic functional form.

In general,

$$D_o^* = \mathbf{a}_0 + \mathbf{a}' \mathbf{d}^* + \frac{1}{2} \mathbf{d}^{*'} \mathbf{\Gamma} \mathbf{d}^*,$$

where

$$D_o^* = D_o / y_1$$

and

$$\mathbf{d}^* = \begin{bmatrix} \mathbf{y}^* / y_1 \\ \mathbf{x} \\ \mathbf{z} \end{bmatrix} \quad (35)$$

where \mathbf{y} is an M output vector, \mathbf{y}^* is a vector of $M-1$ outputs (output one used for normalization), \mathbf{x} is an N vector of inputs and \mathbf{z} is a K vector of exogenous variables such as A_t (technical change), and α_0 , α , and β are parameters to be estimated (a scalar, a vector and a matrix, respectively). A convenient partition consists of $\alpha' = (\alpha_{y^*} \ \alpha_x \ \alpha_z)'$, and

$$\Gamma = \begin{bmatrix} \Gamma_{y^*y^*} & \Gamma_{y^*x} & \Gamma_{y^*z} \\ \Gamma_{xy^*} & \Gamma_{xx} & \Gamma_{xz} \\ \Gamma_{zy^*} & \Gamma_{zx} & \Gamma_{zz} \end{bmatrix}. \quad (36)$$

Theoretically required regularity conditions for this function include homogeneity of degree one in outputs and symmetry. This functional form maintains linear homogeneity of the output distance function. Symmetry requires the constraints:

$$g_{ij} = g_{ji} \quad \forall i \neq j \quad (37)$$

for all outputs, inputs, and other exogenous variables and their cross products.

First order differentiation of the normalized output distance function with respect to outputs yields a system of marginal input requirement equations that are linear in normalized output quantities, in input quantities and in other exogenous variables

$$\mathbf{y} = \mathbf{a}_y + \Gamma_{y^*y^*} \mathbf{y}^* + \Gamma_{y^*x} \mathbf{x} + \mathbf{g}_{y^*z} \mathbf{z}, \quad (38)$$

where \mathbf{R} is a column vector consisting of marginal valuations ($MD_{\mathcal{O}} \mathcal{M}_{y_m^*} = \mathbf{R}_m(\cdot)$).

Using equation (9), the marginal revenue product of inputs is

$$f = -(\mathbf{a}_x + \Gamma_{y^*x} \mathbf{y}^* + \Gamma_{xx} \mathbf{x} + \mathbf{g}_{xz} \mathbf{z}), \quad (39)$$

where \mathbf{N} is a $n \times 1$ vector of marginal input revenues. Note that Γ_{zz} , which is needed to evaluate technical change, cannot be estimated from these sets of equations. The output distance function must be estimated either alone or jointly with these marginal valuation equations.

If, in addition, monotonicity and convexity of the technology is assumed, the output distance function is dual to the revenue function. By the envelope theorem, \mathbf{R} in equation (28) is a vector of inverse conditional supply functions. Convexity in output quantities implies a positive semidefinite matrix of second order derivatives of the output distance function with respect to outputs (Γ_{yy} , the Antonelli matrix) while for concavity in inputs the Hessian implied by the estimated parameters in inputs, Γ_{xx} , must be negative semidefinite. These properties are maintained in estimation of this system. Monotonicity is satisfied if the predicted prices are positive. This property is not maintained but evaluated after estimation.

Equation (14) indicates how the output distance function provides a measure of the common rate of technical change in all outputs. If $z = A_t$, this rate is obtained as

$$d_t = -(\mathbf{a}_z + \Gamma_{yz} \mathbf{y}_t + \Gamma_{xz} \mathbf{x}_t + \Gamma_{zz} z_t) \cdot \frac{z_t}{D_O(\mathbf{x}_t, \mathbf{y}_t, z_t)}, \quad (40)$$

where Δ is the radial rate of technical change and all outputs, including the numeraire, are contained in the vector \mathbf{y} and the parameter matrix Γ_{yz} . Equation (41) can be evaluated for given values of outputs, inputs and other exogenous variables by using the estimated coefficients. The coefficients for the

numeraire output are retrieved from the homogeneity condition.

Hicksian pairwise measures are obtained for the quadratic distance function using equation (21)

which gives

$$B_{ijt} = \left[\frac{\mathbf{g}_{iz}}{\mathbf{y}_{it}} - \frac{\mathbf{g}_{jz}}{\mathbf{y}_{jt}} \right] z_t \quad (41)$$

for all outputs $i, j = 1, \dots, M$, $t = 1, \dots, T$, and when $z = A_t$. If $B_{ij} = 0$, then technical change does not bias the optimal mix between outputs, while $B_{ij} > 0$ implies a bias toward the production of the j th output, and $B_{ij} < 0$ implies a bias toward the production of the i th output. Overall biases are obtained using the pairwise biases and equation (22). Alternatively, in terms of the parameters of the normalized quadratic output distance function

$$B_{ti} = \sum_j S_{jt} B_{ijt} = \left[\frac{\mathbf{g}_{iz}}{\mathbf{y}_i} z_t + \mathbf{d}_t \right] \quad (42)$$

for all $i, j = 1, \dots, M$, $t = 1, \dots, T$, and $z = A_t$. Once the radial rate of technical change and the overall biases per commodity are obtained using equations (40) and (42) they are combined to obtain the rate of commodity progress giving, according to equation (29) the respective k -shifts

$$k_{it} = B_{it} - \mathbf{d}_t = \frac{\mathbf{g}_{iz}}{\mathbf{y}_i} z_t \quad (43)$$

for all $i = 1, \dots, M$, $t = 1, \dots, T$, and $z_t = A_t$. For the quadratic distance function, the k -shifts are simply a

normalization of the coefficient of each output and technical change while the rates of technological bias are obtained by adding to this the aggregate radial rate of technical progress. Moreover, the pairwise biases of equation (41) can be alternatively defined in terms of the k -shifts as $B_{ijt} = (k_{it} - k_{jt})$. Another parameter of interest is the percentage change in the rate of technical change due to changes in output quantities

$$-\frac{\eta d_t}{\eta \ln y_{it}} = S_{it} B_{it} = \frac{y_{it} y_{it}}{D_{ot}} \left[\frac{g_{iz}}{y_{it}} + d_t \right]. \quad (44)$$

For a quadratic distance function the same type of information would be obtained in a simpler form from the elasticity that indicates the percentage change in the marginal valuation of technical change (MV_z) as a consequence of percentage changes in quantities of outputs

$$x_{zy} = \frac{\eta \ln MV_z}{\eta \ln y_i} = \frac{g_{iz}}{MV_z} y_i \quad (45)$$

where $MV_z = MD_0 / M_z$ where $z = A_i$, for all $i = 1, \dots, M$; $t = 1, \dots, T$. We can also found in a similar way, using the appropriate parameters, the percentage change in the marginal valuation of technical change as the technology index and inputs change.

An Application: Estimating 'Commodity Progress' in U.S. Agriculture

As stated in section I, changes in competitiveness of a country in a specific product market are

traced to changes in productivity. Innovations change the characteristics of the technology in use and the efficiency with which inputs are used in production. This results in potential changes in productivity with possible impacts on exports. The rate of technological change along with the output biases implied by this change are indicators of changes in competitiveness in the markets of interest. In this section we utilize the theory developed to analyze technical change and its characteristics in production of wheat, corn, soybeans and livestock in U.S. agriculture.

There have been numerous studies of productivity growth²⁰ at the aggregate, sectoral, and industry level for the U.S. These studies have used a number of different approaches to productivity measurement, including parametric and non-parametric, stochastic and deterministic. Productivity studies that focus on the agricultural sector include Gollop and Jorgenson, Capalbo and Vo, Ball et al., Pardey et al., Huffman and Evenson, Chavas and Cox, and Lim and Shumway, among others. These studies estimate productivity growth using index numbers, estimates of production, cost and profit functions, and non-parametric approaches. Most of these studies obtain estimates of the rate of agricultural productivity growth for the sector as a whole. They are consistent at estimating positive rates of productivity growth in U.S. agriculture in the last forty years. Few are able to differentiate productivity growth in the crops and livestock subsectors. None of these studies have obtained estimates of productivity growth at the commodity level. Similarly, none of these studies have econometrically estimated Griliches' k -shifts nor the rate of commodity progress.

A. The Data

The five commodities chosen in this study constitute 100 % of the value of all U.S. agricultural

production. U.S. agricultural exports have dropped from approximately 30% of total value of exports in 1950 to 10% in 1995. Wheat exports contributes approximately 14% to the total value of agricultural exports while corn and soybeans contribute 15% and 13 % respectively. The data used in the analysis consists of annual observations on quantity and price indexes from 1950 to 1995 obtained from a number of sources. I estimate a structure with five outputs (corn, wheat, soybeans, beef cattle, and all other commodities), one input (all production inputs), and a time trend as a proxy for technological change. These variables are described in Table 1.

B. Econometric Estimation

Equations (35), (38) and (39) are estimated with slight modifications for estimation purposes. In time series estimation it is maintained that all observations are efficient, so for each year the production bundle is not only feasible but it is on the frontier. This amounts to assuming that $D_O(\mathbf{x}, \mathbf{y}) = 1$ in every period. Equation (35) regresses $(y_I)^{-1}$ on all other output and input quantities and other exogenous variables. Random disturbances are also added to the normalized distance and normalized price equations. These disturbances represent the effect of random weather conditions and approximate error; they are assumed homoscedastic and uncorrelated within equation. Contemporaneous cross-equation correlation of the disturbance terms is permitted.

If besides satisfying the above assumptions, the vector of disturbances is multnormally distributed, maximum likelihood estimation can be performed. Under the stated stochastic assumptions, the maximum likelihood estimators are consistent, asymptotically normal, and asymptotically efficient. The IML procedure in SAS was used for estimation.

Using the data described in the previous section, equations (35), (38) and (39) are estimated by the method of maximum likelihood. Cross-equation symmetry and identity restrictions are imposed on the parameters at estimation. Linear homogeneity in outputs is imposed by normalizing all outputs by the index of all other outputs. Convexity in outputs is not satisfied by this system. The output distance function will be convex in outputs if \mathbf{H}_{yy} is a positive semidefinite matrix implying that the diagonal elements of this matrix are nonnegative. Convexity is imposed by estimating the system subject to nonnegativity constraints on these parameters. This is done using the NLPQM (Dual Quasi Newton Method) optimization subroutine in the IML procedure in SAS, version 6.10. This approach allows estimation of the parameters in the system by maximizing the likelihood function subject to equality and inequality, linear and nonlinear constraints on the parameters. Once these parameters are estimated, their standard errors are obtained from running one iteration of the SUR option of the MODEL procedure in SAS with all parameter values restricted to the values estimated by the previous approach.

The system has six equations, the dependent variables being the inverse of the numeraire output and the normalized prices of corn, wheat, soybeans, beef cattle, and the negative of the marginal revenue of the input aggregate. The stacked model has 276 observations and 28 estimated parameters.

Collinearity diagnostics developed by Belsley, Kuh, and Welsch indicate an absence of strong multicollinearity. Because time-series data are used, the presence of autocorrelation in the residuals is possible. Simple Durbin-Watson statistics for each of the equations in the system fall in the inconclusive range. Guilkey's likelihood ratio test statistic for a system of simultaneous equations that do not contain lagged endogenous variables as regressors is calculated as 34.8. For thirty six degrees of freedom, the

Chi-square critical value at the 5% level is 50.71. Therefore this statistic does not lead to rejection of the hypothesis that the matrix of first order vector autoregressive coefficients is zero. Estimation proceeds under the assumption of serially independent errors. R^2 obtained from OLS residuals are 0.77 for the distance function equation, 0.71 for the price of wheat equation, 0.68 for the price of corn equation, 0.75 for the price of soybeans equation, 0.59 for the price of beef cattle equation, and 0.82 for the input equation. Table 2 presents the parameter estimates of the restricted model. The table contains a total of twenty eight estimated parameters, eight of which are significant at the 1% level, five at the 5% level, and four at the 10% level. The signs of the estimated parameters are in general consistent with the theoretical model. The own-quantity responses of the inverse output supply equations are positive, while the own-quantity response of the input marginal revenue is negative. Monotonicity is satisfied at the point of expansion and at the mean of the data but violated in 36 of the 276 data points.

Among the most significant estimated parameters are those of the time variable, indicating a strong autonomous component in the trend of the inverse supply and inverse demand equations. In all cases this trend is associated with a decrease in normalized prices of outputs, suggesting the presence of technical change.

C. Estimates of Technical Change, Bias and Griliches' k shifts.

Equation (41) with predicted output distance function evaluated at the mean values of variables, to calculate the estimated radial rate of technical change for given levels of outputs and inputs. The results indicate that U.S. agriculture has grown at an average radial rate of 2.8% per year. This rate is a

larger than expected, and it is larger than recent estimates of 1.90% to 2.50% by Ball, et al., Pardey et al., and Huffman and Evenson, who used an index number approach covering a slightly different time period.

The elasticities of the marginal valuation of technical change given by equation (45) are estimated for changes in outputs, inputs, and technical change index. Evaluated at the mean they indicate that the marginal value of technical change decreases with corn, and wheat; while it increases with soybeans and beef livestock. Our calculations also indicate that the implicit value of technical change goes down through time (Table 3.)

Pairwise biases are obtained from equation (41) where the predicted inverse supply is evaluated at the mean of the exogenous variables. The results, shown in Table 4, indicate that technical change has not been Hicks neutral, in fact it has been biased in favor of soybeans relative to corn, wheat, and beef, in favor of corn relative to wheat and beef, and in favor of wheat relative to beef. The overall bias measure, B_{it} , calculated according to equation (42) and evaluated at the mean of the variables, indicate that technological change has been biased in favor of corn, soybeans and wheat and against beef livestock.

Finally, the rate of commodity progress is estimated using equation (43) as the sum of the radial rate of technical change and the overall bias per commodity. These results are presented in Table 5. On average, the marginal cost of corn, soybeans, and wheat, conditional on output and input levels, has decreased during the 1950-1995 period while that for beef livestock has slightly increased. Griliches k -shift indicate that the percentage reduction in the marginal cost of corn has been more than that of soybeans which has also been bigger than the percentage cost reduction in production of wheat. These

figures might be taken to indicate a increase in relative competitiveness in corn relative to soybeans, wheat and beef ; an increase in relative competitiveness of soybeans realtive to wheat and beef and of wheat relative to beef livestock in U.S. agriculture during the last fifty years.

Summary and Conclusions

I have discussed the relationship between the firm's technology and Griliches' k -shift and have shown how the output distance function and the inverse supplies or marginal valuations obtained from it may be used to specify this concept in terms of rates of commodity progress. Among the implications of the results are the fact that these rates are equal to the addition of two technological parameters, the rate and bias of technological change. This information is important because it allows productivity measurement by commodity and enables estimation of the downward shift of the marginal cost per commodity. Griliches' k -shifts, a crucial parameter in the welfare evaluation of technological change usually obtained in an ad-hoc manner can now be econometrically estimated based on the theory of the firm.

I have also discussed the dual relationship between the output distance function and the normalized revenue function in this context, establishing the similarities and differences between the radial dual and primal rates of technical change and the respective biases. It is possible to define a dual rate of commodity progress which describes the horizontal shift of the marginal for each commodity due to technological change. This is Griliches' K -shift, concept used in the welfare evaluation literature and usually picked in ad-hoc manner.

We use the approach in estimation of the rate of commodity progress for wheat, corn,

soybeans, and beef livestock in U.S. agriculture for the 1950-1995 period. The radial rate of technical change is estimated at about 2.8% per year, slightly higher than that estimated by others using very different approaches. The k -shift for corn is about 8.1%, with a 6.2% for soybeans, a 3.9% for wheat and a -0.6% for beef. This shows that U.S. agriculture has become more competitive in the production of corn relative to all other commodities, in the production of soybeans relative to wheat and beef, and in the production of wheat relative to beef.

References

- Abramovitz, M.. "Resource and Output Trends in the United States since 1870," *American Economic Review*, May 1956, 46(2), pp. 5-23.
- Alston, J., G. Norton, P. Pardey. *Science Under Scarcity: Principles and Practice for Agricultural Research Evaluation and Priority Setting*, Ithaca: Cornell University Press, 1995.
- Antle, J. and S. Capalbo. "An Introduction to Recent Developments in Production Theory and Productivity Measurement," in S. Capalbo and J. Antle, eds., *Agricultural Productivity: Measurement and Explanation*, Washington: Resources for the Future, 1988.
- Ball, E., J. Bureau, R. Nehring, and A. Somwaru. "Agricultural Productivity Revisited," *American Journal of Agricultural Economics*, November 1997, 79(4), pp. 1045-1063.
- Barro, R. and X. Sala-i-Martin. *Economic Growth*, New York: McGraw-Hill, Inc., 1995.
- Belsley, D., D. Kuh, and R. Welsch. *Regression Diagnostics*, New York: John Wiley & Sons, 1980.
- Berndt, E. *Practice of Econometrics: Classic and Contemporary*, New York: Addison Wesley, 1991.
- Berndt, E. and M. Khaled. "Parametric Productivity Measurement and Choice among Flexible Functional Forms," *Journal of Political Economy*, Dec. 1979, 87(6), pp. 1220-1245.
- Blackorby, C., C.A.K. Lovell, and M. Thursby. "Extended Hicks Neutral Technical Change," *Economic Journal*, 1976, 86, pp. 845-852.
- Bresnahan, T. "Measuring the Spillovers from Technical Advance: Mainframe Computers in Financial Services," *American Economic Review*, September 1986, 76, pp. 742-755.
- Bresnahan, T. and R. Gordon. *The Economics of New Goods*, Chicago: University of Chicago Press, 1997.
- Capalbo, S. and T. Vo. "A Review of the Evidence on Agricultural Productivity and Aggregate Technology," in S. Capalbo and J. Antle, eds., *Agricultural Productivity: Measurement and Explanation*, Washington: Resources for the Future, 1988.
- Christensen, L., D. Jorgenson, and L. Lau. "Transcendental Logarithmic Production Frontiers." *Review of Economics and Statistics*, February 1973, 55(1), pp. 28-45.

- Coelli, T. and S. Perelman. "Efficiency Measurement, multiple-output technologies and distance functions: with applications to European Railways," *CREPP 96/05 Working Paper*, University of Liège, 1996.
- Cox, T. and Chavas, J.P. "A Nonparametric Analysis of Productivity: The Case Of U.S Agriculture." *European Review of Agricultural Economics*, 17 (1990), pp. 449-464.
- D'Andrea Tyson, L. *Who's Bashing Whom: Trade Conflict in High-Technology Industries*, Washington: Institute for International Economics, 1992.
- Deaton, A. "The Distance Function in Consumer Behaviour with Applications to Index Numbers and Optimal Taxation," *Review of Economic Studies*, July 1979, 46(3), pp. 391-405.
- Debreu, G. "The Coefficient of Resource Utilization," *Econometrica*, 1951, 19, pp. 273-292.
- Fabricant, S. *Economic Progress and Economic Change*, 34th Annual Report, New York: NBER, 1954.
- Färe, R. and D. Primont. *Multioutput Production and Duality: Theory and Applications*, Boston: Kluwer Academic Press, 1995.
- Fulginiti, L. "From Red to White Meats: The Role of Technology," presented at the Winter Meetings of the Econometric Society, Boston, December 1993, Dept. of Agricultural Economics, University of Nebraska, Lincoln.
- Fuss, M. and D. McFadden. *Production Economics: A Dual Approach to Theory and Applications*, Amsterdam: North Holland, 1978.
- Gollop, F. and D. Jorgenson. "U.S. Productivity Growth by Industry. 1947-73," in J. Kendrick and B. Vaccara, eds., *New Developments in Productivity Measurement and Productivity Analysis*, pp. 17-124, Chicago: University of Chicago Press, 1980.
- Gorman, W. "Tricks with Utility Functions," in Artis, M. and Nobay, R., eds., *Essays in Economic Analysis*, 1976.
- Griliches, Z. "Research Cost and Social Returns: Hybrid Corn and Related Innovations," *Journal of Political Economy*, October 1958, 66(5), pp. 419-431.
- _____. "Measuring Inputs in Agriculture: A Critical Survey," *Journal of Farm Economics*, December 1960, 42(5), pp. 1411-1427.

- _____. "The Sources of Measured Productivity Growth: United States Agriculture, 1940-1960," *Journal of Political Economy*, August 1963, 71(4), pp. 331-346.
- _____. "The Discovery of the Residual: A Historical Note," *Journal of Economic Literature*, September 1996, 34(3), pp.1324-1330.
- Grosskopf, S., K. Hayes, and J. Hirschberg. "Fiscal Stress and the Production of Public Safety: A Distance Function Approach," *Journal of Public Economics*, 1995, 57, pp. 277-296.
- Guilkey, D. "Alternative Tests for a First-Order Vector Autoregressive Error Specification," *Journal of Econometrics*, 1974, 2, pp. 95-104.
- Hicks, J. *The Theory of Wages*, New York: St. Martins Press, 1963.
- Huffman, W. and R. Evenson. *Science for Agriculture: A Long Term Perspective*, Ames: Iowa University Press, 1993.
- Jorgenson, D. and Z. Griliches. "The Explanation of Productivity Change," *Review of Economic Studies*, June 1967, 34(3), pp. 249-283.
- Kendrick, J. "Productivity," in *Government in Economic Life*, ed. S. Fabricant, 35th Annual Report, New York: NBER, 1955, pp. 44-47.
- Krugman, P. *Pop Internationalism*, Cambridge: MIT Press, 1996.
- Lim, H. and R. Shumway, "Technical Change and Model Specification: U.S. Agricultural Production," *American Journal of Agricultural Economics*, 79 (1997): 543-554.
- Lovell, C.A.K., S. Richardson, P. Travers, and L. Wood. "Resources and Functionings: A New View at Inequality in Australia," in W. Eichhorn, ed., *Models and Measurement of Welfare and Inequality*, Berlin: Springer-Verlag, 1994.
- Malmquist, S. "Index Numbers and Indifference Surfaces," *Trabajos de Estadística*, 1953, 4, pp. 209-241.
- Mansfield, E., J. Rapoport, A. Romeo, S. Wagner, and G. Beardsley. "Social and Private Rates of Return from Industrial Innovations," *Quarterly Journal of Economics*, May 1977, 91, pp. 221-240.
- Pardey, P., J. Alston, and B. Craig. "Postwar Productivity Patterns in U.S. Agriculture: Influences of Aggregation Procedures in State Level Analysis," mimeo, University of California, Davis, 2000.

- Paul, C. *Cost Structure and the Measurement of Economic Performance*, Boston: Kluwer Academic Publishers, 1999.
- Peterson, W. "Returns to Poultry Research," *Journal of Farm Economics*, August 1967, 49(3), pp. 656-670.
- Schmitz, A. and D. Seckler. "Mechanized Agriculture and Social Welfare: The Case of the Tomato Harvester," *American Journal of Agricultural Economics*, November 1970, 52(4), pp. 569-577.
- Schmookler, J. "The Changing Efficiency of the American Economy: 1869-1938," *Rev. of Econ. and Statistics*, 1952, 34(3), pp. 214-321.
- Shephard, R. *Costs and Production Functions*, Princeton: Princeton University Press, 1953, 1970.
- Solow, R. "Technical Change and the Aggregate Production Function," *Review of Economic and Statistics*, 1957, 39(3), pp.312-320.
- Tinberger, J. "Zur Theori der Langfristigen Wirtschaftsentwicklung," *Weltwirst. Archiv*. 1, Amstrdam: North Holland Pub. Co., 1942, pp. 511-549; reprinted in English translation in J. Tinberger, "Selected Papers," North Holland, 1959.
- Trajtenberg, M. "The Welfare Analysis of Product Innovations with an Application to Computed Tomography Scanners," *Journal of Political Economy*, 1989, 97(2), pp. 444-479.
- Wold, H. "A Synthesis of Pure Demand Analysis, Parts I and II," *Skandinavisk Aktuarietidskrift*, 1943, 26, pp. 85-144 and 220-272.

Table 1. Variables Describing the Agricultural Sector

$D_o(x,y,A)$	output distance function: value of one as efficiency is assumed in estimation.
y	vector of outputs: Tornquist-Theil index of production for corn, wheat, soybeans, beef livestock, and aggregate of all other products. Prices are corresponding implicit prices.
x	vector of inputs: Tornquist-Theil index of all inputs used from Ball, et.al. Implicit price index from the same source.
A	time trend used as proxy for technical change, 1950-1995.

Table 2. Parameter Estimates (t-ratios in parentheses), symmetry, homogeneity and convexity imposed, 1950-1995, U.S. Agriculture.

Prices	Second Order Coefficients						
	First Order Coefficients		Quantities				
		Corn	Soybeans	Wheat	Beef	Input	Time
Corn	-3.53 (-3.91)	0.03 (2.95)	-0.0167 (-7.01)	0.027 (5.46)	-0.021 (-1.49)	-0.078 (-4.39)	0.002 (3.96)
Soybeans	5.26 (6.36)		0.03 (5.36)	-0.016 (-4.87)	-0.038 (-4.38)	-0.006 (-0.42)	-0.003 (-6.09)
Wheat	-0.95 (-0.77)			0.03 (4.67)	-0.027 (-1.58)	-0.048 (-1.93)	0.0005 (0.85)
Beef	16.45 (6.78)				0.377 (3.31)	0.419 (10.44)	-0.009 (-6.87)
Inputs	-23.35 (-10.69)					-0.999 (-7.88)	0.013 (11.80)
Time	-2.42 (-14.26)						0.001 (14.32)

Table 3. Elasticities of the Marginal Valuation of Technical Change, U.S. Agriculture, 1950-1995.

Outputs	MV_A
Corn	-0.128
Soybeans	0.388
Wheat	-0.033
Beef	0.434
Others	-0.66
Time	-117.15

Table 4. Pairwise and Overall Output Biases, U.S. Agriculture, 1950-1995

i\j	Soybeans	Wheat	Beef	Others	Overall
Corn	0.087	-0.042	-0.087	0.026	-0.053
Soybeans		-0.045	-0.0005	0.061	-0.034
Wheat			-0.046	0.016	-0.011
Beef				0.061	0.0342
Others					0.027

Table 5. Griliches' k -shift or Rates of Commodity Progress, U.S. 1950-1995.

Outputs	<i>k</i>-shift (%)
Corn	8.1
Soybeans	6.2
Wheat	3.9
Beef	-0.6
Others	0.1

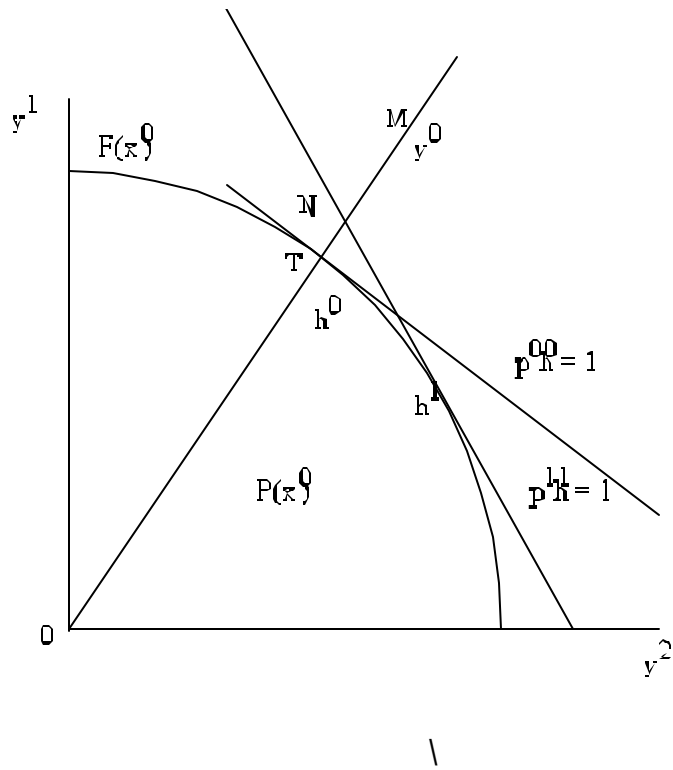


Figure 1. The output distance function

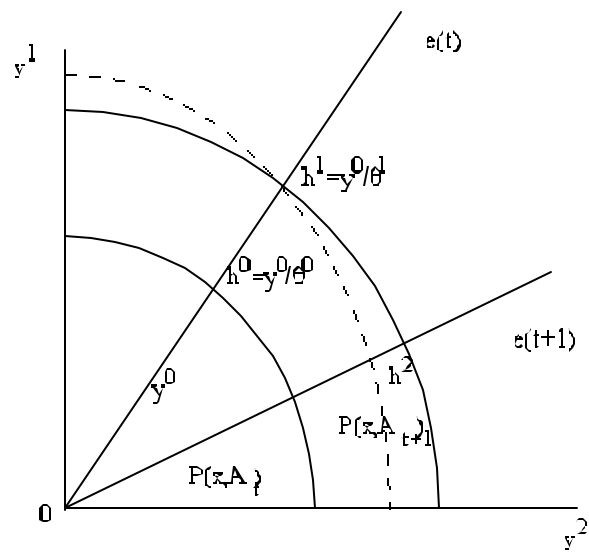


Figure 2. The radial primal rate of technical change and output biases.

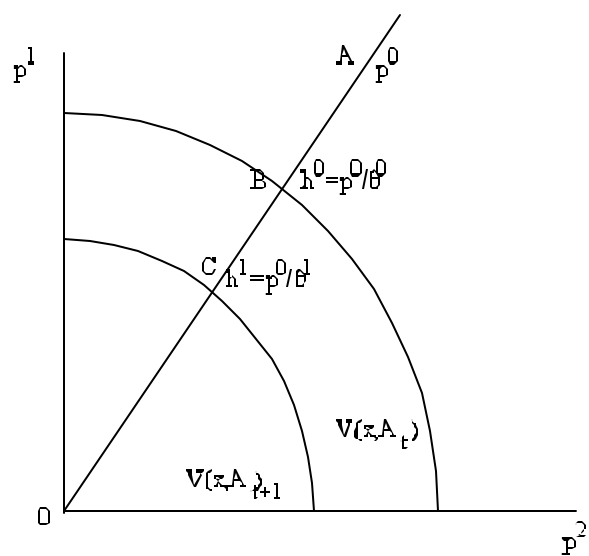


Figure 3. The normalized revenue function and the radial dual rate of technical change.

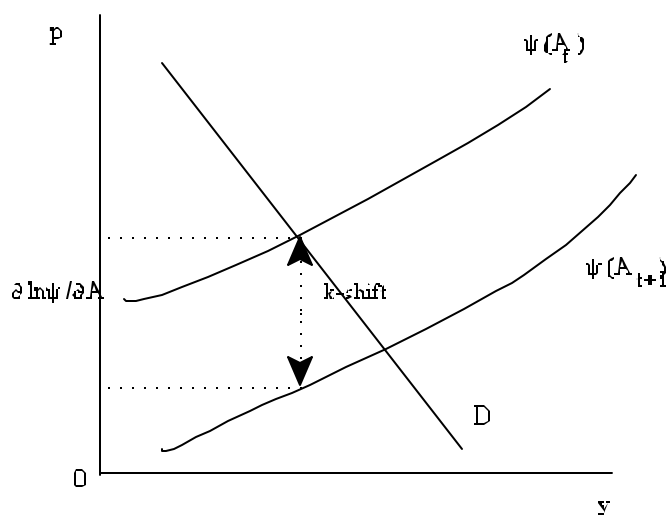


Figure 4. Griliches' k-shift.

Endnotes.

1. See Krugman (1996).
2. U.S. exports are approximately 10% of GNP. When trade becomes more important, changes in the terms of trade could outweigh domestic factors as determinants of the standard of living.
3. Productivity change and technical change are used interchangeably in this paper.
4. Introduced in Griliches seminal 1958 article on social returns to hybrid corn to describe the relative shift in the supply curve (page 423) and later used by many others. For a review of this literature see Alston et al.
5. A summary of the early work can be found in Griliches (1996.) For more recent attempts see Barro and Xala-i-Martin.
6. A summary of the efforts along these lines can be found in Berndt, Morrison-Paul, and Alston et. al.
7. See Bresnahan and Gordon for papers along this line.
8. For a detail review of this literature see Alston et al and Bresnahan and Gordon. Studies of immiserating growth in the trade literature could be consider as early precursors of this approach.
9. See Alston et al for papers using this methodology.
10. Uses of the distance function in demand theory are found in Gorman (1976), and a seminal article by Deaton (1978.)
11. Shephard's (1970) definition of the output distance function makes it obvious that it is a ratio of Euclidean distances from the origin

$$D_o(\mathbf{x}, \mathbf{y}) = \frac{\|\mathbf{y}\|}{\|\mathbf{h}(\mathbf{x}, \mathbf{y})\|}$$

where $\|\cdot\|$ represents the norm of a vector, $\mathbf{h}(\mathbf{x}, \mathbf{y}) = 2(\mathbf{x}, \mathbf{y})^{-1} \mathbf{y}$, and $2(\mathbf{x}, \mathbf{y}) = \min \{2\|(\mathbf{2}^{-1} \mathbf{y}) - \mathbf{P}(\mathbf{x}), \mathbf{2} > 0\}$.

12. For a complete description and proofs see Shephard (1970.)

13. The revenue function $R(p, x)$ is nondecreasing in output prices, nondecreasing in input quantities, homogeneous of degree one in prices, and convex in prices. For more details on these properties and the duality between these functions see Shephard (1970).

14. If the revenue structure is defined as a family of price vectors given by

$$V(x) = \{p / R(p, x) \geq 1; p \geq 0\}$$

the revenue function is

$$R(p, x) = \frac{\|p\|}{\|x(p, x)\|}$$

where $x(p, x) = 2(p, x)^{-1} p$ and $2(p, x) = \min \{2 / (2^{-1} p), V(p)\}$, the vector being the intersection of the ray $\{2^{-1} p / 2 \geq 0\}$ with the boundary of the set $V(x)$ when $x > 0$. The distance ratio is given then by

$$\frac{\|p\|}{\|x(p, x)\|} = q(p, x)$$

But

$$R(p, x) = R(q(p, x)x(p, x), x) = q(p, x)R(x(p, x), x) = q(p, x) = \frac{\|p\|}{\|x(p, x)\|}$$

with $R(p, x) = 1$ by continuity of the revenue function in p . Figure 3 illustrates the set $V(x)$ and the norms of p and $x(p, x)$. These two bundles are scalar multiples of one another, so we can define a scalar 2^0 such that $p^0 = x^0(p, x) = 2^0 h^0$ or $(p^0 / 2^0) = h^0$. If $2^0 > 2$ is chosen then $(p^0 / 2)$ is not in the revenue structure set, if $2^0 < 2$ the resulting price vector is in the set but supports a revenue smaller than the maximum attainable. 2^0 can therefore be defined as the maximum value of 2 such that $2p$ is in $V(x)$. This scalar is a distance function.

15. For a complete proof see Shephard (1970.)

16. From the first order conditions of problem (6) we obtain that $p_m = \theta MD_o(.) / My_m$, so that revenues can be expressed as $R(.) = p y(p, x, A) = \theta \sum y_m MD_o(.) / My_m$ and when $D_o(.) = 1$ then $R(.) = \theta$ due to linear homogeneity of the distance function in outputs.

17. An alternative way of measuring the shift of the marginal cost would be to represent the proportional shift to the right of this curve from the original equilibrium value of p . This is sometimes referred to in the literature as the K -shift and measures the proportional change in quantities for given price. In his original study, Griliches does not use this concept but the one we develop in the text.

18. If one were interested in obtaining the K -shift instead of the k -shift, one can do so using the dual of the output distance function, the normalized revenue function. It is then possible to obtain the horizontal shift of the marginal cost as

19. Follows from $d = -\frac{\mathbb{I} \ln D}{\mathbb{I} A_t} = -\frac{\mathbb{I} D / \mathbb{I} A_t}{D}$ where numerator and denominator are linear

homogeneous in outputs.

20. We will only refer to total factor productivity studies. There have been many more partial factor productivity studies.