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# **Methodology for Integrated Environmental-Economic Analysis of GDP and Productivity**

By

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# Integrated Environmental-Economic Analysis of GDP and Productivity

Shunrong Qi, Jay S. Coggins and Lan Xu

Besides the inputs of capital and labor, the stock of the environment<sup>1</sup> is employed in the production of commodities.<sup>2</sup> The utilization of the environmental stock in production may cause depletion of natural resources and degradation of the environment. Thus, as a country develops there appears to be a trade-off between GDP growth and the quality of the environment. In this sense, GDP is a poor indicator of social welfare at the national level. The traditional productivity analyses, which overlook the environment's contribution to GDP, have a similar problem in measuring 'true' technological change. Therefore, the conventional GDP and productivity indexes are of limited usefulness in making the policies of social welfare and sustainable development.<sup>3</sup>

## 1. Current State of Knowledge and Its Gaps

1.1. GDP Accounts and Green GDP Accounts. When the National Accounts were systematized in the 1940s, environmental issues had a low perceived importance, and the accounting structure adopted simply ignored depletion/degradation of the environment. Since the 1970s, when the gap between economic growth and quality of life began to widen<sup>4</sup>, the conventional System of National Accounts (SNA) has been criticized for distortions regarding the measurement of economic performance, growth and development (e.g., Hueting, 1989; Repetto, Magarh, Wells, Beer and Rossini, 1989; Congressional Budget Office, 1994; Dieren, 1995; and Milton, 1995). One of the key drawbacks of SNA is that GDP, the most widely used measure of aggregate economic activity, fails to account for the impact of economic activity on the environment. Economists have suggested that GDP accounts should be adjusted for the value of environmental damages to constitute integrated environmental-economic (or 'green') GDP accounts (e.g., Harrison, 1989; Hartwick, 1990; and Mäler, 1991). That is,

$$\text{green GDP} = \text{GDP} - \sum_D p_D D; \quad (1)$$

where  $D$  is a vector of indicators of environmental depletion or degradation and  $\sum_D p_D D$  is the vector of shadow prices of such depletion or degradation. The Statistical Division of the United Nations also pursues this line of thought and outlines a System for Integrated Environmental and Economic Accounting (SEEA) (United Nations, 1993). However, the green GDP accounts depend critically upon  $\sum_D p_D D$ , the monetary valuation of the depletion/degradation of the environment. This presents a problem in that the shadow prices  $p_D$  are not easily observable, because the markets of many environmental goods are missing or not competitive.

<sup>1</sup>The environment is broadly defined, including environmental and natural resources.

<sup>2</sup>This point comes originally from an influential paper by Weitzman (1976). He emphasizes that all sources of economic growth must be included in the notion of "capital": physical capital, human capital (labor) and natural capital (the environment).

<sup>3</sup>To quote Aaheim and Nyborg (1995), "£££ much of the demand for a 'green GDP' is caused by the fear that authorities will take no notice of environmental degradation as long as GDP increases, and that a common feature of the proposals of 'greening' the national product is that they are meant to provide a better informational background for evaluating and eventually changing policy".

<sup>4</sup>For example, while per capita income in Oman was more than 17 times higher than in neighboring Sri Lanka in 1985, life expectancy in Sri Lanka was 16 years longer than in Oman (Sen, 1991).

In its handbook the U.N. (1993) proposes three different methods for measuring shadow prices  $p_D$ :

a. Market valuation. This approach assumes that observed market prices do not deviate significantly from the 'true' shadow prices  $p_D$ , and use observed prices for adjustments in the green GDP. This compromise approach is not entirely satisfying because market prices do not necessarily reflect the environmental impacts of economic activities.

b. Contingent valuation. Willingness-to-pay (WTP) information is used to obtain shadow prices for environmental deterioration. Contingent valuation (CV) in this setting would be based on a hypothetical scenario and presents some practical difficulties in its procedure. Other major problems are that WTP is closely related to ability to pay of respondents and that the valuation is probably influenced by distorted market prices.

c. Maintenance valuation. Maintenance cost is defined as the least cost of maintaining the environmental standard unchanged, whether actually incurred or not, during the accounting period. There are similar problems of this hypothetical valuation as in the CV approach.

**1.2. Productivity Measurement.** Though GDP represents the level of economic activity, productivity is often of greater interest to economists and policy-makers because productivity growth is the source and the determinant of economic growth and welfare improvement. Two main methods of productivity measurement are the growth accounting approaches<sup>5</sup>, of which the Solow residual is the basic approach, and productivity index approaches, including the Malmquist, Fisher, and Törnqvist productivity indexes.

**Solow residual.** Following the pioneering work of Solow (1957), observed economic growth is broken down into contributions from associated changes in factor inputs and a Solow residual that reflects technological progress. The analysis starts with the neoclassical production function,

$$GDP = Y = A \cdot f(x); \quad (2)$$

where  $A$  is an index of the level of technology and  $x$  is the vector of quantities of the input factors.

In the conventional growth accounting, GDP is regarded as a function of two input factors, capital  $K$  and labor  $L$ , as well as technology level  $A$ , i.e.,  $GDP = Y = A \cdot f(K; L)$ : Here  $A$  is called total factor productivity (TFP). TFP growth or the Solow residual can be calculated by

$$\frac{\dot{TFP}}{TFP} = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} - \frac{\frac{\partial Y}{\partial K} K}{Y} \frac{\dot{K}}{K} - \frac{\frac{\partial Y}{\partial L} L}{Y} \frac{\dot{L}}{L}; \quad (3)$$

If the depletion/degradation of the environment  $D = (D_1; D_2; \dots; D_N)$  is included in the vector of input factors, i.e.,  $x = (K; L; D)$ , GDP growth can be disaggregated into the contributions from changes in capital  $K$ , labor  $L$  and environmental depletion/degradation  $D$  and the growth of  $A$ . We define this  $A$  in the GDP function  $GDP = Y = A \cdot f(K; L; D)$  as green TFP.

$$\frac{\text{green } \dot{TFP}}{\text{green } TFP} = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} - \frac{\frac{\partial Y}{\partial K} K}{Y} \frac{\dot{K}}{K} - \frac{\frac{\partial Y}{\partial L} L}{Y} \frac{\dot{L}}{L} - \sum_{n=1}^N \frac{\frac{\partial Y}{\partial D_n} D_n}{Y} \frac{\dot{D}_n}{D_n}; \quad (4)$$

<sup>5</sup>The basics of growth accounting are presented in Solow (1957), Kendrick (1961), Denison (1962), and Jorgenson and Griliches (1967). Griliches (1997) provides an overview of the intellectual history of growth accounting, with particular stress on the development of the Solow residual. Barro (1998) provides another excellent reference on growth accounting. See also Qi (1999a).

Note from equation (4) that not only the quantities of  $D$ , but also the shadow prices  $r_D Y$ , are required for calculating green TFP growth. As stated above, these shadow prices are typically unobservable, which restricts the feasibility of using the growth accounting approach in calculating green TFP growth.

**Malmquist productivity index.** The Malmquist productivity index is introduced by Caves, Christensen and Diewert (1982a, b).<sup>6</sup> Let  $x \in \mathbb{R}_+^N$  denote a vector of inputs and  $y \in \mathbb{R}_+^M$  denote an output vector. The production technology  $S^t$  is defined by the production possibility set

$$S^t = \{ (x; y) : x \text{ can produce } y \text{ at period } t \} \quad (5)$$

The output distance function, due to Shephard (1970), is defined by

$$D^t(x; y) = \inf_{\mu} \left\{ \mu : (x; \frac{y}{\mu}) \in S^t \right\} \quad (6)$$

This function is the reciprocal of the maximal radial expansion of the output vector  $y$  consistent with technological feasibility, given the inputs  $x$ .

Caves et al. define their Malmquist productivity index as

$$M_L^t = \frac{D^t(x^{t+1}; y^{t+1})}{D^t(x^t; y^t)} \quad (7)$$

In this formulation,  $M_L^t$  is a Laspeyres-type index which uses technology in period  $t$  as the reference technology. Alternatively, one could define a Paasche-type index  $M_P^t$  which uses technology in period  $t + 1$  as the reference technology:

$$M_P^t = \frac{D^{t+1}(x^{t+1}; y^{t+1})}{D^{t+1}(x^t; y^t)} \quad (8)$$

Färe, Grosskopf, Lindergren and Roos (1989) specify the output-oriented Malmquist productivity index as the geometric mean of  $M_L^t$  and  $M_P^t$ :<sup>7</sup>

$$M^t(x; y) = \left( \frac{D^t(x^{t+1}; y^{t+1})}{D^t(x^t; y^t)} \right)^{\frac{1}{2}} \left( \frac{D^{t+1}(x^{t+1}; y^{t+1})}{D^{t+1}(x^t; y^t)} \right)^{\frac{1}{2}} \quad (9)$$

Färe et al. decompose this productivity index into two components, an efficiency change component (EFFCH) and a technical change component (TECH):

$$M^t(x; y) = \text{EFFCH} \times \text{TECH} \quad (10)$$

where efficiency change

$$\text{EFFCH} = \frac{D^{t+1}(x^{t+1}; y^{t+1})}{D^t(x^t; y^t)}$$

and technical change

$$\text{TECH} = \left( \frac{D^t(x^{t+1}; y^{t+1})}{D^{t+1}(x^{t+1}; y^{t+1})} \right)^{\frac{1}{2}} \left( \frac{D^t(x^t; y^t)}{D^{t+1}(x^t; y^t)} \right)^{\frac{1}{2}} :$$

<sup>6</sup> Surveys of productivity indexes which include the Malmquist index are Diewert (1992a, 1993), Roos (1993), Studit (1995), and Färe, Grosskopf and Roos (1998).

<sup>7</sup> Clearly, this is in the spirit of Fisher (1922) who defines his ideal price index as the geometric mean of the Laspeyres and Paasche indexes.

EFFCH measures the change in relative efficiency, i.e., the change in how far observed production is from maximal potential production, between periods  $t$  and  $t + 1$ . TECH captures the shift in technology between the two periods  $t$  and  $t + 1$  evaluated at  $x^t$  and  $x^{t+1}$ . These two components lend themselves in a natural way to the identification of catching up and the identification of innovation, respectively. Catching up and technological innovation are two key factors to productivity growth. They are associated with different sources, and so different policies may be required to address them.<sup>8</sup> Therefore, it is important to decompose productivity growth into these two components.

Besides the advantage of decomposition, the Malmquist productivity index is less demanding in terms of data requirements. Growth accounting approaches and the Törnqvist and Fisher productivity indexes<sup>9</sup> utilize proxies for the shadow prices of all inputs and outputs in order to form a TFP growth or a productivity index, while no prices are required in the Malmquist index. Furthermore, the Malmquist index is more general and includes the Solow residual and the Fisher and Törnqvist indexes as special cases (see Färe, Grosskopf and Roos, 1998).

The Malmquist productivity index has been used in a variety of empirical studies on country comparisons of productivity. For example, Färe, Grosskopf, Norris and Zhang (1994), Perelman (1995), and Gouyette and Perelman (1995) apply this technique to the analysis of productivity growth in OECD countries; Chambers, Färe and Grosskopf (1996) compute the Malmquist productivity indexes in APEC countries; and Taskin and Zaim (1995) provides an international productivity comparison for a sample of both developed and less developed countries. The above empirical studies do not incorporate the environmental impact of economic growth. Therefore the productivity indexes they computed are in the sense of conventional TFP rather than green TFP.

There has been an increasing interest in incorporating environmental damages into productivity measurement. Originating with the work by Pittman (1983), depletion/degradation indicators  $D$  are treated by many researchers as undesirable by-products (bad outputs)  $b$  in conjunction with the desirable outputs (good outputs)  $y$ .<sup>10</sup> However, in the presence of undesirable outputs, the Malmquist productivity index may not be computable because the distance function, e.g.  $D^t(x^{t+1}; y^{t+1}; b^{t+1})$  where  $(x^{t+1}; y^{t+1}; b^{t+1})$  is at point B in Figure 1, may be undefined. Following Chambers, Chung and Färe (1996), Chung, Färe and Grosskopf (1997) suggest using the directional distance function rather than the original distance function (equation 6) to remedy the difficulty of undefined distance at some observations.<sup>11</sup> The directional distance function is defined as

$$D^t(x; y; b; y; b) = \sup_{\alpha} : (x; (y; b) + \alpha (y; b)) \in S^t : \quad (11)$$

But the possibility of an undefined distance function at some observations still remains for the directional distance function. For example,  $D^t(x^{t+1}; y^{t+1}; b^{t+1})$  is still undefined if  $(x^{t+1}; y^{t+1}; b^{t+1})$  is at point B' in Figure 1.

<sup>8</sup>Färe, Grosskopf, Norris and Zhang (1994) find that all of US productivity growth is due to technical change while almost half of Japan's productivity growth is due to efficiency change.

<sup>9</sup>Refer to Balk (1993), Diewert (1992b), and Färe and Grosskopf (1992, 1996) for the Fisher index, and refer to Caves, Christensen and Diewert (1982b) for the Törnqvist index.

<sup>10</sup>See, e.g., Färe, Grosskopf, Lovell and Pasurka (1989), Brännlund, Färe and Grosskopf (1995) and Hetemaki (1996). Tyteca (1996) gives an overview with a bibliography.

<sup>11</sup>Chavas and Cox (1999) recently introduce the concept of generalized distance function. The computation of their distance function is based on nonlinear programming rather than linear programming.

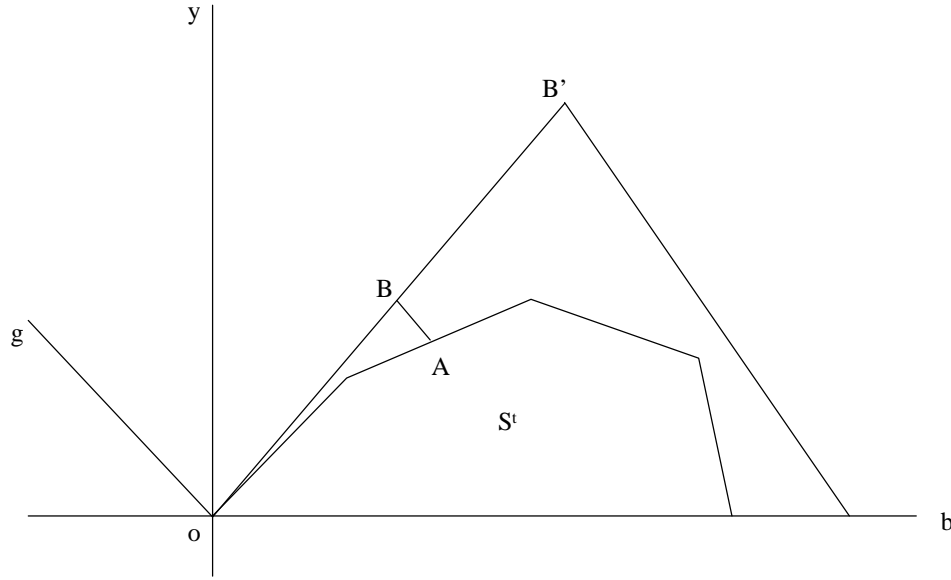


Figure 1:

1.3. **Data Envelopment Analysis.** The Data Envelopment Analysis<sup>12</sup> (DEA) approach is applied to compute the distance functions that make up the Malmquist productivity index. The DEA approach consists of solving a linear programming problem for each producer (each country in this paper) in each period. Suppose that the technologies  $(x^i; y^i)$  ( $i = 1; 2; \dots; I$ ) are technologically feasible in period  $t$ , and that a country uses inputs  $x$  to produce outputs  $y$  in this period. The production possibility set is

$$S^t = \left\{ (x; y) : y \leq \sum_{i=1}^I z^i y^i, \sum_{i=1}^I z^i x^i \leq x, z^i \geq 0 \right\} \quad (12)$$

where  $z^i$  is the intensity variable indicating at what intensity technology  $i$  may be employed in production. This activity-analysis model is originated by von Neumann (see Karlin, 1959). It satisfies constant returns to scale<sup>13</sup> and free disposability of inputs and outputs (Färe, Grosskopf and Roos, 1998). Distance functions are computed relative to the reference technology  $S^t$  by

$$f_{D^t}(x; y)^{\mu, 1} = \max_{z^i \geq 0} \left\{ \mu : \mu y \leq \sum_{i=1}^I z^i y^i, \sum_{i=1}^I z^i x^i \leq \mu x \right\} \quad (13)$$

Suppose that country  $k$  ( $k = 1; \dots; K$ ) uses inputs  $x^{k;t} \in \mathbb{R}_+^N$  to produce outputs  $y^{k;t} \in \mathbb{R}_+^M$  in period  $t$ . The cross-country (world) technology set (Färe, Grosskopf and

<sup>12</sup>The expression Data Envelopment Analysis is coined by Charnes, Cooper and Rhodes (1978).

<sup>13</sup>Constant returns to scale (CRS) is defined as  $\mu S^t = S^t$  for any  $\mu > 0$ : CRS is a necessary condition for the resulting productivity indexes to be true total factor productivity indexes (Färe and Grosskopf, 1996; Chung, Färe and Grosskopf, 1997).

Lovell, 1985; Färe and Grosskopf, 1996) in period  $t$  is

$$S^t = \left( (x; y) : y \leq \sum_{k=1}^K z^{k;t} y^{k;t}, \sum_{k=1}^K z^{k;t} x^{k;t} \leq x, z^{k;t} \geq 0 \right) \quad (14)$$

The advantage of the cross-country technology set is that the set constructs the world production frontier based on the data of all countries in the sample. The world frontier, as an explicit benchmark, is used in the calculation of the Malmquist productivity index. Each country is compared to that frontier. A country's movement toward the world frontiers over time is called "catching up"; a shift of the world frontiers over time is called "technical change" or "innovation". The product of these two components yields the productivity change of the country.

1.4. Shadow Prices of Depletion/degradation of the Environment. Treating the depletion/degradation  $D$  as undesirable outputs, Färe, Grosskopf, Lovell and Yaisawarng (1993) and also Färe and Grosskopf (1998) provide a practical method for computing shadow prices of nonmarketed undesirable goods.<sup>14</sup> For two different outputs,  $m$  and  $m^0$ , where one of the outputs can be an undesirable output, their relative price equals the corresponding ratio of distance function derivatives:

$$\frac{p_{m^0}}{p_m} = \frac{\partial D(x; y) / \partial y_{m^0}}{\partial D(x; y) / \partial y_m} \text{ for all } m; m^0 (m; m^0 = 1; 2; \dots; M) \quad (15)$$

If  $m$  is a desirable output with an observable market price  $p_m$  and  $m^0$  is a nonmarketed (undesirable) output such as pollution, the price  $p_{m^0}$  can be determined from equation (15).

Of course, this approach yields a unique price  $p_{m^0}$  only if the distance function  $D(x; y)$  is differentiable. In fact,  $D(x; y)$  is not differentiated everywhere if we use a nonparametric estimation of  $D(x; y)$ , e.g. the equation (13) of distance function in DEA. Typically this approach has been in conjunction with parametric distance functions to estimate shadow prices of undesirable goods. A specific functional form for the underlying production function  $D(x; y)$ , a translog function, is proposed in Färe, Grosskopf, Lovell and Yaisawarng (1993). This assumption of the parametric distance function is not consistent with the nonparametric version of distance function  $D(x; y)$  in DEA.

## 2. Methodology and Its Rationale

It seems evident that social welfare policies and market mechanisms for environmental management should be improved by basing them on measurements of green GDP and green TFP growth. But, as noted above, there are some gaps in the literature on the green GDP accounting and productivity measurement. The attempt to fill these gaps will be accomplished in the following ways.

### 2.1. Integrated Environmental-Economic Measurement of Productivity.

<sup>14</sup>See also Coggins and Swinton (1996), who use this approach to estimate the shadow price of  $SO_2$  abatement for Wisconsin electric utilities.



**Measurement of green MPI.** The Malmquist productivity index approach has an obvious advantage over growth accounting and other productivity index approaches on measurement because the Malmquist approach does not require estimates of shadow prices. If the depletion/degradation  $D$  is treated as undesirable outputs in the Malmquist approach, however, problems can arise. The distance function may be undefined that causes the MPI incomputable (see Figure 1). In our methodology, the elements of  $D$  are treated as inputs rather than as undesirable outputs.<sup>15</sup> For our purposes, regarding the depletion/degradation as inputs is preferred for several reasons.

Firstly, production of commodities not only consumes capital and labor but also causes depletion/degradation of the environment. If the quality of the environment is regarded as a stock, the depletion/degradation  $D$  is the utilization of the environmental stock in process of production, and then it is natural to treat the elements of  $D$  as inputs. Secondly, environmental damages, including the emissions of pollutants, can be modeled as normal inputs because any increase of damages (e.g., emissions) will free up capital and labor, which would otherwise be devoted to damage control (e.g., pollution abatement), for production of market goods. In other word, environmental damages act in production process like normal inputs with positive marginal products. Thirdly, depletion/degradation of the environment  $D$  should meet some constraints in regulated economy like other input constraints. For example, the constraint on air pollutants reflects total emission allowance. In this sense,  $D$  is properly considered as inputs rather than outputs. Finally and most importantly, the distance functions are always well defined in the case of a single output  $y$  that is GDP. Thus, we are able reliably to compute the productivity index.

For a single output  $y$  and multiple inputs  $x$ , the output distance function may be written as

$$D^t(x; y) = \frac{y}{F^t(x)}; \quad (16)$$

where  $F^t(x) = \max \{y : (x; y) \in S^t\}$  is the production function. The Malmquist productivity index based on (16) is

$$M^t(x; y) = \frac{y^{t+1}}{y^t} \frac{F^t(x^t)}{F^t(x^{t+1})} \frac{F^{t+1}(x^t)^{-\frac{1}{2}}}{F^{t+1}(x^{t+1})^{-\frac{1}{2}}}; \quad (17)$$

In the traditional way of productivity measurement, depletion/degradation of the environment  $D$  is excluded. We denote  $M^t(x; y)$  in which  $x = (K; L; D)$ , so that the environmental effects are included into measurement, as "green MPI".

As in equation (10), this  $M^t(x; y)$  (equation 17) can be decomposed into two components: efficiency change (EFFCH) and technical change (TECH). These are defined as

$$EFFCH = \frac{y^{t+1}}{y^t} \frac{F^t(x^t)}{F^{t+1}(x^{t+1})}$$

and

$$TECH = \frac{F^{t+1}(x^{t+1})}{F^t(x^{t+1})} \frac{F^{t+1}(x^t)^{-\frac{1}{2}}}{F^t(x^t)^{-\frac{1}{2}}};$$

<sup>15</sup>Though treating pollution as an output is common in the productivity literature based on the Malmquist index, pollution is often treated as an input as well. For example, in their survey of environmental economics, Cropper and Oates (1992) state that a standard treatment is to regard pollution as an input to the firm. See also Reinhard, Lovell and Thijssen (1999). There appears to be no settled view on whether pollution should be treated as an output or as an input.

For the cross-country (world) technology set  $S^t$  in period  $t$ , the computation of the production function  $F^t(x)$  can be carried out using linear programming by solving, for each country,

$$F^t(x) = \max_{z^{k;t} \geq 0} \left( \sum_{k=1}^K z^{k;t} y^{k;t} : \sum_{k=1}^K z^{k;t} x^{k;t} \leq x \right) \quad (18)$$

Relationship between the productivity index and TFP growth. Suppose that the technology can be represented by a production function,  $y = F^t(x) = A(t) f(x)$ . Following equation (17),

$$M^t(x; y) = \frac{y^{t+1}}{y^t} \frac{F^t(x^t)}{F^t(x^{t+1})} \frac{F^{t+1}(x^t)}{F^{t+1}(x^{t+1})}^{-\frac{1}{\alpha}} = \frac{y^{t+1}}{y^t} \frac{f(x^t)}{f(x^{t+1})} = \frac{A(t+1)}{A(t)}.$$

Thus, green TFP growth is given by

$$\frac{\text{green TFP}^2}{\text{green TFP}} = \text{green MPI} \quad i = 1:$$

**2.2. Shadow Prices of Depletion/degradation and Green GDP Accounts.** Much of the current debate in the literature is on the question of the suitability of green GDP as an indicator of social welfare or as an indicator of sustainability.<sup>16</sup> Probably influenced by the earlier Hicksian concept of income (Hicks, 1947)<sup>17</sup>, some economists argue that green GDP is an indicator of sustainability, since it is a number representing the amount of welfare which can be enjoyed over a period of time and leave the economy with the capacity to enjoy that same amount of welfare for the next period of time. Thus sustainability is defined as constant instantaneous welfare over time, which might not be something the economy is aiming at. The economy's objective might be maximizing the total discounted utility flow over time. Weitzman (1976) defines welfare as the present value of future consumption and demonstrates that green GDP can be interpreted as a measure of welfare if the economy is on the optimal growth path.

There is an extensive theoretical literature aimed at modeling the relationship between economic growth and environmental quality.<sup>18</sup> A number of studies focus on the optimal growth path on which a country maximizes its discounted social welfare over time subject to the accumulation of stocks of capital, human capital and natural capital (the environmental stock). Social welfare includes utility from commodity goods and disutility from environmental damages. On the optimal growth path, the country achieves optimal trade-off between current welfare and the stocks of all capital left to next period. The stocks are not necessary to keep an unchanged level in the Hicksian sense. Green GDP is viewed in our methodology as an indicator of social welfare.

<sup>16</sup>See, e.g., Aaheim and Nyborg (1995), Asheim (1994), Brekke (1994), Hartwick (1990, 1994), Lintott (1996), Mäler (1991), Pemberton and Ulph (1997), Solow (1986), and Vellinga and Withagen (1996).

<sup>17</sup>Hicks defines that an individual's income is "the maximum value which he can consume during a week and still expect to be as well off at the end of the week as he was in the beginning".

<sup>18</sup>Early contributions to this literature include articles by Forster (1972, 1973), Gruver (1976), Keeler, Spence and Zeckhauser (1971), Smith (1977), and Stephens (1976). Recent contributions include the work of Beltratti (1996), Bovenberg and Smulders (1995, 1996), Elbasha and Roe (1996), Hofkes (1996), Michel and Rotillon (1995), Mohtadi (1996), Qi and Coggins (1999), Selden and Song (1995), Smulders and Gradus (1996), Stokey (1998), Tahvonen and Kuuluvainen (1993), and Withagen (1995).

The shadow prices are the equilibrium prices that will ensure that decentralized, general equilibrium outcomes are socially optimal. Thus, the shadow prices are the values of marginal products where the technology is efficient, or say, at the production frontier.

For the general technology set  $S^t$  in period  $t$  (equation 12), the production function is

$$F^t(x) = \max_{z^i \geq 0} \left( \sum_{i=1}^I z^i y^i : \sum_{i=1}^I z^i x^i \leq x : \right) \quad (19)$$

The Lagrangian for this problem  $L = \sum_{i=1}^I z^i y^i + \sum_{j=1}^J \mu_j \left( x_j - \sum_{i=1}^I z^i x_j^i \right)$ . By the Envelope theorem,  $\partial F^t(x) / \partial x_j = \mu_j$ . The dual values for the environmental constraints,  $\mu_j$ , are the shadow prices of depletion/degradation  $D$  in period  $t$  with respect to the constraints of  $D$ . The dual values  $\mu_j$  can be computed directly using standard linear programming software. The estimates of the shadow prices  $\mu_j$  can be thought of as the socially optimal tax rates or the prices that should prevail in emissions permit markets. They can also be thought of as the unit costs of abatement in competitive economy.

After measuring the shadow prices  $\mu_j$ , green GDP can be easily calculated as

$$\text{Green GDP} = \text{GDP} - \sum_j \mu_j D_j$$

### 3. Concluding Remarks

This paper illustrates a methodology which is feasibly implemented for integrated environmental-economic GDP accounting and productivity measurement. The development of the statistics of green GDP and green TFP growth provides a useful input to the formation of policies regarding socially optimal growth. Deriving the shadow prices for environmental inputs will enable us to calculate green GDP and to formulate the market mechanisms for environmental management, like a pollution tax scheme or a tradable permit system for social optimum.

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