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## Integrated Assessment of Climate Catastrophes with Endogenous Uncertainty: Does the Risk of Ice Sheet Collapse Justify Precautionary Mitigation?

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## Summary

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**Keywords:** Climate Change Policy, Sea Level Rise, Ice Sheet Collapse, Endogenous Uncertainty, Stochastic Optimization, Greenhouse Gas Mitigation, Risk Management

**JEL Classification:** C61, D61, D81, H23, Q54, Q58

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# Integrated Assessment of Climate Catastrophes with Endogenous Uncertainty: Does the Risk of Ice Sheet Collapse Justify Precautionary Mitigation?

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## Abstract

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# 1 Introduction

Greenhouse gas policies confront the trade-off between the costs of reducing emissions and the benefits of avoided climate change. However, there are unique challenges related to the long time horizon of the problem (with temporal dynamics such as thermal inertia and other lags), the heterogeneous nature of climate impacts, the sizable economic toll of both action and inaction (which occur on different timescales, and are thus contingent on the discount rate), and the pervasive uncertainty around the physical science and the resulting damages.

To help address these multiple dimensions and complexities, researchers in the field of integrated assessment have developed models for interdisciplinary climate and energy policy analysis. Integrated assessment models (IAMs) are a key framework for exploring the implications and consequences of alternative global climate change scenarios (Weyant et al., 1996; Parson and Fisher-Vanden, 1997). These models are widely used and relied upon by policymakers to evaluate climate proposals and inform policy design. In fact, the US government recently used three IAMs to compute the social cost of an incremental carbon emission for use in cost-benefit analysis of federal regulations (IAWG, 2010, 2013).

However, these particular models have faced renewed criticism of their representation of climate damages (Revesz et al., 2014; Pindyck, 2013; Stern, 2013).[1] These damages to society are typically represented in IAMs with a damage function, which in its most basic form equates a future amount of temperature change to an absolute or fractional loss of economic output.[2] Despite the fact that there is a limited empirical basis for the damage functions, they are often extrapolated to different regions and applied beyond current observations of warming. This deficiency is compounded by the fact that most IAMs are deterministic, meaning that all parameter values are fully known, and many optimize with perfect foresight. These characteristics are incongruent with the cascades of uncertainties that permeate the causal chain of climate change (Schneider, Kuntz-Duriseti, and Azar, 2000).

## 1.1 Approaches to Modeling Uncertainty

Uncertainty can be characterized as parametric (epistemic), due to imperfect knowledge, or stochastic (aleatory), due to inherent randomness and natural variability (Kann and Weyant, 2000;

[1]Early IAM studies led to unresolvable debates over distributional concerns related to the aggregation of impacts by region and time (i.e., the discount rate) as well as the sheer uncertainty about impacts. In response, the cost-benefit approach was largely abandoned in favor of the cost-effectiveness approach which sets an environmental goal and reformulates the problem to minimize costs while meeting the target. These controversial topics are now back in the spotlight due to their implications for the social cost of carbon being applied to US regulations (IAWG, 2010, 2013).

[2]Impact sectors such as agriculture and energy have existing markets that inform damage estimates, whereas the valuation of impacts to intangible goods such as ecosystems and human health are even more difficult.

Dessai and Hulme, 2003). While parametric uncertainty can be reduced as knowledge about physical dynamics and social systems improves over time, the variability that characterizes stochastic uncertainty will always exist to some extent. Rather than incorporate these types of uncertainty directly into the model structure, most IAMs have been developed with a deterministic structure, and uncertain parameters are investigated with ex-post analysis. These approaches are classified as uncertainty propagation methods (Kann and Weyant, 2000), and include scenario and sensitivity analysis of key uncertain parameters (e.g., climate sensitivity, damage coefficients, or the discount rate) that are varied in single- or multi-parameter mode, and the Monte Carlo approach, which samples from a range of distributions.[3] However, these uncertainty propagation methods return a collection of possible outcomes, each corresponding to different conditions, without identifying a single strategy that performs well in the face of uncertainty.[4]

In 'Buying Greenhouse Insurance,' Manne and Richels (1991) describe uncertainty propagation models as the 'learn, then act' framework, in contrast to the 'act, then learn' framework, which requires decisions to be made before uncertainty is resolved. The latter is often a more useful and realistic model of decision-making in the real world. 'Act, then learn' frameworks incorporate decision-making under uncertainty directly into the IAM structure, such that a single robust strategy is followed until uncertainty is resolved. This is known as a hedging strategy, because it is designed to perform reasonably well given the range of possible uncertain outcomes.

Often a decision-maker's prior beliefs about uncertainty may change over time. This could take the form of learning about the true value of an unknown parameter in the case of parametric uncertainty, or Bayesian updating of a probability distribution to reflect new evidence in the case of stochastic uncertainty. A framework that recognizes learning over time by a decision-maker is known as sequential decision-making. There are several examples of sequential decision-making applied to certain dimensions of climate change problem, such as technological change, socioeconomic growth, and climate sensitivity (Manne and Richels, 1995; Kelly and Kolstad, 1999; Webster, Jakobovits, and Norton, 2008; Webster, Santen, and Parpas, 2012; Bistline and Weyant, 2013).

Two state-of-the-art stochastic optimization techniques used to model sequential decision-making under uncertainty are stochastic programming and stochastic dynamic programming. Stochastic programs are structured such that parametric uncertainty is resolved after one or more stages. If the model allows recourse, the decision-maker can respond to the information learned and compensate accordingly (Birge and Louveaux, 1997). Dynamic programming is a solution algorithm that decomposes a complex multi-stage optimization problem into a series of simpler subproblems, which are then solved with backward recursion (Bertsekas, 1976). Stochastic dy-

[3] Uncertainty propagation via Monte Carlo simulation is the approach used by two well-known IAMs, PAGE and FUND (Hope, 2011b; Anthoff and Tol, 2013).

[4] There are decision-making frameworks that address the issue of a suite optimal results: a preferred strategy can be selected on the basis of alternative evaluation criteria such as minimax regret or maximin welfare (Bergh, 2004) or robust decision-making (Lempert and Schlesinger, 2000; Lempert, Popper, and Bankes, 2003).

namic programming represents uncertainty with various states that describe the evolution of the system over time. The appropriate method for a particular research question will depend on the type of uncertainty considered (e.g., parametric or stochastic), as well as the dimensionality of the problem.[5]

## 1.2 Stochastic Optimization with Climate Thresholds

A special type of stochastic uncertainty in the greenhouse gas problem is that of a climate catastrophe, a low-probability, high-consequence event that would lead the earth system to shift abruptly in response to climate change (Weitzman, 2009, 2012). Examples of potential catastrophes are ocean thermohaline circulation (THC) disruption, sudden methane release from the oceans or permafrost, disintegration of the polar ice sheets, and widespread forest die-back (National Research Council, 2002; Alley et al., 2003; Overpeck and Cole, 2006; Keller, Yohe, and Schlesinger, 2007; Lenton et al., 2008). The existence of these threats is supported by the geologic record, but the governing dynamics and trigger thresholds are still not fully understood or quantified due to insufficient data and process models limitations (Kriegler and Hall, 2009).

Nevertheless, the often-cited rationale for stringent mitigation targets is the need for an insurance policy against such climatic catastrophes. Yet there is a disconnect between this policy rhetoric and the supporting analysis: the few IAMs that represent catastrophes tend to use the expected value approach, which eliminates the element of risk, and thus they do not call for aggressive mitigation (Nordhaus and Boyer, 2000; Hope, 2006, 2011a). If society is to purchase such an insurance policy by investing in stringent mitigation, that cost will need to be justified by the magnitude of the catastrophic threat. Thus, there is a real need for IAMs to explore the implications of potential catastrophes with more fidelity in a stochastic setting with sequential decision-making, particularly as the frontier of scientific understanding of catastrophe advances over time.

It is not a trivial task to apply stochastic optimization methods to IAMs, which to begin with have non-linear, dynamic structures with long time horizons and many variables in a deterministic mode. An early, pioneering effort to model uncertain climate thresholds in an IAM with global stochastic optimization was Keller, Bolker, and Bradford (2004). Their study investigated the possible collapse of THC, triggered by passing an uncertain threshold level of carbon dioxide concentrations as described in Stocker and Schmittner (1997). The consequence of the catastrophe was a persistent economic loss ranging from 0 to 3% based on Tol (1998). The paper developed several probabilistic approaches to model this uncertain catastrophe, evolving from sensitivity analysis covering the entire parameter space of both climate sensitivity and economic damage ('learn, then act') to a stochastic program with recourse ('act, then learn'). The problem was formulated with three states

[5] Both methods suffer from the curse of dimensionality, however stochastic programs can accommodate high dimension in the decision variables but only a small number of stages, whereas dynamic programs are better suited for many stages but are limited to a smaller state space of the system variables.

of the world with an exogenously specified probability distribution about climate sensitivity and thus the critical $CO_2$ threshold, such that the ultimate likelihood of collapse depended on mitigation. They found that parameter uncertainty about the THC collapse threshold led to decreased near term mitigation (e.g., the optimal hedging path is to wait until uncertainty is resolved before investing in mitigation).

While the work of Keller, Bolker, and Bradford (2004) was an important initial step toward incorporating endogenous catastrophes into IAMs, it lacked an endogenous link between the contemporaneous climate and the probability of being in a catastrophic state of the world, instead adopting exogenous parametric uncertainty. In this case the hedging path reflects downside risk aversion. However, a characteristic of climate catastrophes is that their likelihood is thought to increase with warming (Kriegler and Hall, 2009; Urban and Keller, 2010). Stochastic optimization with endogenous uncertainty can overcome this limitation because it allows the probability of catastrophe to be controlled by the climate variables (e.g., $CO_2$ concentration or temperature). This endogenous approach introduces a precautionary component to the optimal path, not solely hedging against the catastrophic outcome but also altering the probabilities of that state.

Cai, Judd, and Lontzek (2013) and Lemoine and Traeger (2014) are two recent efforts to apply stochastic dynamic programming to model uncertain climate thresholds (also referred to as tipping points) in an IAM.[6] Both studies use a hazard rate that is endogenously linked to atmospheric temperature to represent the probability of encountering the threshold, which leads to an irreversible regime shift if crossed. Cai, Judd, and Lontzek (2013) (hereafter CJL) consider the possible collapse of THC. If this event occurs there is a transition from the pre-threshold regime, characterized by the baseline damage function, to the post-threshold regime, characterized by a permanent shock to economic output via an upward-shifted damage function. The hazard rate function is calibrated to an expert elicitation of the likelihood of THC collapse temperature change (Zickfeld et al. 2007).

Lemoine and Traeger (2014) (hereafter LT) compare the effect of two potential tipping points, activating just one at a time: the first increases the climate sensitivity and the second prolongs the lifetime of carbon dioxide in the atmosphere. Unlike the economic consequences incurred in the post-tip regime of CJL, crossing the threshold in LT alters the physical response of the climate system. The authors assume a uniform prior distribution for the threshold temperature (e.g., the threshold lies somewhere between current temperature and an arbitrary upper bound with equal likelihood). Over time, as warmer temperatures are observed and tolerated without causing the system to tip, the Bayesian hazard rate is updated by truncating the portion of the probability distribution that has been safely observed.

A major difference between the two designs is how the hazard rate changes over time with

[6]Both of these stochastic dynamic programs are solved with backward recursion by decomposing the problem into the pre-threshold and post-threshold regime and approximating the value functions. Details on the respective algorithms can be found in Cai, Judd, and Lontzek (2012); Traeger (2013).

warming. CJL formulate the hazard rate as a Markovian jump process that is a linear function of current surface temperature. In this way the probability of encountering the tipping point jumps discretely to a new value, to face a higher probability of catastrophe if the climate continues to warm or a lower probability if temperatures have begun to decline. This allows for the fact that a tipping point can occur even after the climate begins to cool (e.g., at a temperature that has already been experienced) because there is still a positive hazard rate. In contrast, LT assume that there is a certain tipping point at some unknown but unique temperature. Accordingly, the Bayesian hazard rate is continually updated,eliminating 'safe' temperatures from the probability distribution and concentrating the weight on the remaining 'risky' temperatures. This formulation guarantees that a tipping point will not occur after the climate has been stabilized or begins to cool. Of these two approaches, the Markovian appears to offer more skill for representing physical system tipping points where risk persists for any forcing over pre-industrial.

A third optimization technique for sequential decision-making under uncertainty is stochastic programming with endogenous uncertainty (Goel and Grossmann, 2006; Dupacova, 2006). This differs from the classic stochastic programming approach, in which the probability distributions of all uncertain parameters are exogenously fixed a priori, because the probability distributions are instead an endogenous function of the decision variables. There is a limited and scattered literature on stochastic programming applications with endogenous uncertainty. It has been applied to the offshore gas field development planning problem (Goel and Grossmann, 2004) and the research and development portfolio management problem (Solak et al., 2010).

The potential for using stochastic programming with endogenous uncertainty to incorporate a generic catastrophe into an IAM has been recently demonstrated by Rutherford (2013), using an alternative name, stochastic control. This particular approach will be discussed in more detail in Section 3. A clear advantage of endogenous uncertainty in stochastic programming is that it overcomes the limitation of traditional stochastic programming to credibly represent the responsive nature of a climate change catastrophe described above. Furthermore, in contrast to dynamic programming with endogenous uncertainty, a stochastic program of this type can often be solved as a single optimization problem. This simpler approach avoids the computational burden of backward recursion and value function approximation required in dynamic programming, and can reduce solution times by more than an order of magnitude.

An important next phase in modeling catastrophes with endogenous hazard in IAMs is to progress beyond the generic, stylized catastrophe. While CJL specify the THC collapse as their particular catastrophe, the resulting consequence is an arbitrary economic penalty, not based on an empirical or economic assessment. LT model abrupt changes that manifest consequences in accordance with the physical system; however, the hazard is characterized with the stylized assumption of a uniform prior, which implies the threshold for an abrupt climatic regime shift is equally likely to happen anywhere between today's temperature and an arbitrary upper bound.

While the contributions of these papers are notable and valuable as methodological demonstrations, the characterization of catastrophes can and should be improved if such analysis is to be useful for policy-makers. For each specific catastrophe that threatens society, both the hazard and the consequence should be characterized with fidelity, based on physical relationships, empirical results, or expert elicitation, and reflect unique features such as hysteresis or irreversibility. To the extent that these key characteristics can be represented, the better will be the policy guidance.

### 1.3 Present Work

This paper demonstrates a stochastic programming framework for modeling catastrophes with endogenous uncertainty in an IAM following the approach of Rutherford (2013). I apply his framework to the uncertain collapse of the West Antarctic Ice Sheet (WAIS). In order to study this particular catastrophe, I modify the stochastic structure and augment the IAM with additional climatic variables so that the ice sheet collapse is characterized in keeping with the uncertain, irreversible nature of the event, as well as the physical system dynamics, accounting for thermal inertia of the oceans, lags in the mitigation signal, and likely hysteresis. This work finds that accounting for catastrophic consequences in a stochastic programming setting with endogenous uncertainty leads to more stringent climate policy recommendations, reflecting the need to hedge against uncertainties with downside risk as well as pursue precautionary mitigation.

The remainder of this paper will be organized as follows. Section 2 provides the relevant scientific background on sea level rise and the role of the cryosphere in particular. Section 3 introduces the stochastic programming framework for a climate catastrophe with endogenous uncertainty, as well as the unique characterization of the risk of WAIS collapse. Section 4 reformulates a benchmark integrated assessment model with the stochastic programming framework and describes modifications to incorporate WAIS climate variables and sea level rise impacts. Section 5 presents results, showing the effect of catastrophe risk on the optimal mitigation policy as well as quantifying the role of precautionary mitigation and the expected value of perfect information. Section 6 discusses policy implications, limitations, and directions for future work.

## 2 Ice Sheets and Global Sea Level Rise

Climate change will directly affect the two main components of global sea level: thermal expansion and loss of land ice (Milne et al., 2009). Thermal expansion occurs when the oceans absorb additional heat from the atmosphere: water expands as it warms which increases the volume of seawater. Loss of land ice, is generally subdivided into two groups: mountain glaciers and small ice caps, and the massive polar ice sheets of Greenland and Antarctica. Mountain glaciers and small ice caps contain enough water to increase sea levels by 0.32 m. Of greater concern is the possibility of rapid melt from the disintegration of the Greenland and Antarctic ice sheets, which store approximately 65 m of sea level rise (SLR) combined (Vaughan, Comiso, and Allison, 2013). Greenland holds over 7 m of SLR. A mountain range in Antarctica separates the smaller West Antarctic Ice Sheet (WAIS), with nearly 5 m of SLR, from the larger and more stable the East Antarctic Ice Sheet (EAIS), which stores over 53 m.

Ice sheets contribute to SLR when annual snowfall additions are less than mass loss to either incremental surface melt, also called ablation or runoff, or dynamic ice discharge (Bamber, Layberry, and Gogineni, 2001; van den Broeke et al., 2009). The former is driven by warming air temperature. In the case of the Greenland there is a known positive feedback cycle in which ablation lowers the surface elevation, thereby exposing the ice sheet to warmer air temperature at the lower altitude (Vaughan, Comiso, and Allison, 2013). The latter is driven by complex ice dynamics, including changes in ocean temperature and circulation (Bamber and Aspinall, 2013). Ice sheets are most vulnerable to discharge at their outer perimeter, where ice shelves serve as a buttress, or support structure, to slow the flow of ice streams into the ocean (Rignot et al., 2004).

WAIS is particularly at risk of dynamic discharge because it is a marine ice sheet. Much of its base is grounded below sea level with floating ice shelves that are exposed to warming subsurface ocean temperatures that cause basal melt (Joughin and Alley, 2011; Church and Clark, 2013). Moreover, it is potentially subject to the 'marine ice sheet instability' (Weertman, 1974), because it has a reverse bedslope that deepens inland, increasing the force from the ice sheet's own weight pushing outward toward the ocean (Joughin and Alley, 2011). WAIS has experienced major ice shelf collapses in the last decade (Shepherd, Wingham, and Rignot, 2004; Bamber et al., 2009), and two recent studies of WAIS by independent scientists argue that the current extent of loss has destabilized the ice sheet irreversibly (Joughin, Smith, and Medley, 2014; Rignot et al., 2014).

Specialized ice sheet models are used to simulate behavior through either surface mass balance or ice flow dynamics. Early models focused on surface mass balance and often projected that warming would increase net mass due to increased snow fall. Now they focus on representing ice flow at the grounding line, the critical boundary between floating marine ice and grounded ice (Rignot et al., 2014). These efforts are currently thought to be incomplete, facing obstacles such as computational limitations to resolve the migrating calving front or grounding line in three-dimensional space. The bottom line is that despite ongoing research and progress, SLR projections are uncertain and fail to account for abrupt changes from a possible ice sheet collapse, so there may be surprises ahead that are worth preparing for.[7]

Many negative impacts are expected to accompany these increases in sea levels. The scientific

[7] In fact, this has happened in the past: a study of paleo-climatic data found that during the last warm interglacial period (over 100,000 years ago) global average sea level rose at an average rate of 1.5 m per century to a level 5 m above present, when mean temperatures were 2 °C above pre-industrial and the polar ice sheets had melted (Rohling et al., 2008; Church and Clark, 2013).

literature and popular media warn that rising sea levels will affect tens of millions of people who live in low-lying coastal areas, as well as infrastructure and capital assets, vulnerable ecosystems, cultural heritage, and many of the most densely populated and economically productive cities in the world. The potential damages from SLR and storm surge to coasts span a range of impacts including accelerated erosion of beaches and cliffs, permanent inundation of low-lying zones, salinization of water resources, degradation of coastal wetlands and increased flooding events (Dasgupta et al., 2007; Nicholls et al., 2007; Nicholls, 2011; Hinkel et al., 2013). Climate impact assessments consistently find that the coastal sector comprises a significant component of the estimated costs of climate change (Nordhaus and Boyer, 2000; Tol, 2002a,b).

Effective policies for baseline SLR will require risk-management strategies that consider the long-term nature of the threat which will unfold gradually over several centuries due to thermal inertia in oceanic processes such as mixing and circulation. However, the very same inertia that moderates the rate of SLR also ensures that SLR will not be easily reversed even with aggressive greenhouse gas mitigation, a feature known as *commitment* to SLR.[8] This condition is compounded by the abrupt SLR that would be triggered by an ice sheet collapse.

# 3 Stochastic Programming with Endogenous Uncertainty

In the next two sections I present a stochastic programming IAM with endogenous uncertainty that investigates the possible collapse of WAIS and the optimal policy response. In this section I explain the multistage stochastic programming framework, based on the approach of Rutherford (2013), and in Section 4 I use the framework to reformulate a simple IAM and describe modifications to incorporate WAIS climate variables and SLR impacts.

## 3.1 Rutherford's Stochastic Catastrophe Framework

Rutherford (2013) presents a sequential, binomial scenario tree to represent the occurrence of a climate catastrophe in a stochastic programming framework, as depicted in Figure 1 below. This parsimonious description of the catastrophic 'states of the world' allows the optimization problem to be efficiently solved via a deterministic equivalent formulation. The multistage stochastic program imitates the more complex structure of the stochastic dynamic programs employed in Cai, Judd, and Lontzek (2013) and Lemoine and Traeger (2014), but avoids the computational burden of backward recursion and value function approximation, reducing solve times by more than an order of magnitude.

[8] If greenhouse gas concentrations were stabilized by 2100, SLR would continue into the future due to thermal expansion of the oceans; the rate of thermal expansion would decrease only gradually over many centuries (Meehl et al., 2007).

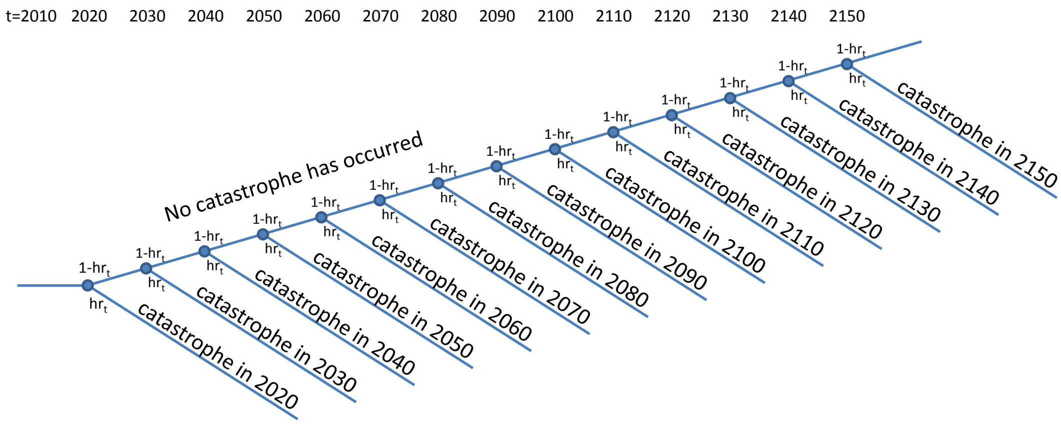


Figure 1: Scenario tree for stochastic programming framework for modeling a climate catastrophe. The hazard rate $hr_t$ gives the probability of catastrophe in period $t$ conditional on there having been no catastrophe up to period $t$. Modified from Rutherford (2013)

A catastrophe can be triggered in any time period $t$, where the hazard rate $hr_t$ represents the conditional probability of a catastrophe occurring based on warming between period $t$ and the next period, assuming one has not yet occurred. This example uses 10 year time periods but the framework can be flexibly defined.

This approach is based on the traditional definition of risk as a function of the hazard (i.e., probability) and the consequence: $Risk = hazard \times consequence$. At the outset, the catastrophic threshold is uncertain, but the decision-maker knows that it is linked to warming and has a prior belief about the probability. The decision-maker can influence current and future probabilities of catastrophe (i.e., hazard rates) by mitigating $CO_2$ to reduce climate warming.

Rutherford classifies his catastrophe framework as stochastic control, to distinguish the fact that the hazard rate probabilities are endogenous to the model, compared to a traditional stochastic program with exogenous probabilities for each period (i.e., fixed from the outset). As described in Section 1.2, this is equivalent to stochastic programming with endogenous uncertainty. I will return to the distinction between exogenous and endogenous uncertainty in stochastic programming in Section 5 as it relates to the precautionary motive for mitigation.

Rutherford adopts the hazard rate formulation of Lemoine and Traeger (2014) (see Section 1.2) and assumes a uniform prior distribution such that the probability of the catastrophe occurring at a unique temperature threshold $T$ is uniformly distributed between the current temperature $T_0$ and some upper bound $\overline{T}$, at which point the catastrophe will certainly have occurred. The Bayesian hazard rate implied by this prior is updated in each time period $t$ if no catastrophe has occurred following $hr_t = \frac{T_{t+1}-T_t}{\overline{T}-T_t}$. Rutherford assumes that the exogenous consequence of the catastrophe is a steeper damage function such that climate change becomes much more costly to society. Specifically, the catastrophe is characterized with a cubic damage function of temperature (in addition to the baseline quadratic damage function).

Rutherford's work demonstrates an elegant and a computationally efficient approach to modeling catastrophes using stochastic programming with endogenous uncertainty. While his framework is a major contribution, the stylized, generic nature of the catastrophe limits the usefulness of the findings for policy insight. The hazard rate assumption of a uniformly distributed threshold oversimplifies the physical dynamics of climatic catastrophes. Similarly, the consequence is characterized with an arbitrary penalty, rather than being calibrated to scientific understanding of a specific catastrophic response.

As raised in Section 1.2, this type of risk management study will provide most insight to policy-makers when it depicts a specific and well-characterized catastrophe. Because Rutherford's flexible and tractable framework allows wide latitude for the catastrophe characterization, it is an excellent point of departure for a more precise investigation of a specific climate threat, which will be presented in the next subsection.

## 3.2 Adaptation of Stochastic Programming Framework to WAIS

I modify Rutherford's framework to model the specific catastrophe of WAIS collapse, capturing the key characteristics described in Section 2. The term 'collapse' suggests a sudden event when in reality the process will unfold over a period of several hundreds of years, slowly raising sea levels. I use collapse to mean the onset of melt from WAIS, and the point of no return.[9] The remainder of this section will describe the characterization of both the hazard and the consequence of such an event.

### 3.2.1 Hazard of WAIS Collapse

In this subsection I explain how the probabilities of WAIS collapse are endogenously determined in a manner consistent with the science of WAIS stability and dynamics as described in Section 2. The collapse is triggered by local ocean warming. Shaffer (2014) finds that melting at the base of the Antarctic marine ice shelf depends on the temperature difference between adjacent subsurface ocean temperature and the ice shelf base, the latter fixed to the freezing temperature of sea water. Thus, I assume that the probability of triggering a collapse is driven by the Antarctic ocean subsurface temperature, given by a quadratic hazard rate

$$hr_t = \min[\beta\,(\Delta T_{O_t})^2\,, 1] \tag{1}$$

[9] These events may not coincide in time. For instance, the point of no return may be crossed before rapid melt begins; similarly, rapid melt may begin, but there is still a window of opportunity to prevent the total disintegration of the sheet. Moreover, it is not guaranteed that the onset of rapid melt will be immediately detected in the observational record.

where $hr_t$ is the hazard rate for period $t$, $\beta$ is the collapse coefficient, and $\Delta T_{Ot}$ is Antarctic ocean subsurface temperature above the freezing temperature of sea water. (Further explanation of local climate variables such as $\Delta T_{Ot}$ will be presented in Section 4.2.)

The hazard rate formulated in Equation 1 is conceptually similar to the one employed by Cai, Judd, and Lontzek (2013), in that it assumes a Markovian jump process. Unlike the Bayesian hazard rate in Rutherford (2013) and Lemoine and Traeger (2014), there is no 'safe' level of warming after stabilization; the catastrophe is always a threat. This hazard rate formulation is the more appropriate choice for characterizing ice sheet disintegration, as ice sheets are inherently unstable (Mercer, 1978). This reflects the fact that basal melt and the marine ice sheet instability will continue to threaten WAIS even if $T_O$ is stabilized. Specifically, it ensures that there is a positive chance of triggering a collapse for any amount of $\Delta T_O$ above the freezing temperature of sea water.

Calibrating such a function as Equation 1 requires some subjective belief about the probability of WAIS collapse for a given climate scenario. This likelihood assessment cannot be based on recent results from Atmosphere-Ocean General Circulation Model or Earth System Model projections because of the limitations discussed in Section 2. There are three published expert surveys of the likelihood of WAIS melt. Most recently, Bamber and Aspinall (2013) elicited expert opinion about the contribution of the three polar ice sheet components to global SLR in 2100 and found a mean melt rate for WAIS of 3 mm/yr (with the 95th percentile melt rate of 11.8 mm/yr). Kriegler and Hall (2009) conducted an expert elicitation on the likelihood of five potential climate tipping points occurring by 2200. For WAIS disintegration they found a mid-range probability of 0.6 from core experts assuming a high temperature scenario (3-5.5°C above pre-industrial in 2100). Finally, a risk estimation study by Vaughan and Spouge (2002) assessed an expert panel about the probability of WAIS collapse by 2200. The result of this study was a probability of 0.05 that the melt rate of WAIS would be 10 mm/yr, with a probability of 0.3 that it would be 2 mm/yr. Overall there is a trend of the likelihoods increasing over time. Moreover, two recent observational and modeling studies suggest that a WAIS collapse is nearly certain at some point in the future, assuming a business-as-usual emissions pathway (Joughin, Smith, and Medley, 2014; Rignot et al., 2014).

I perform the calibration based on the Coupled Model Intercomparison Project Phase 5 (CMIP5) representative concentration pathways (RCPs) (Meinshausen et al., 2011). Specifically, I assume that the highest-emitting radiative forcing scenario, RCP8.5, corresponds to a cumulative probability of 0.5 that the onset of WAIS collapse has been triggered between 2010 and 2100. While this probability is arbitrary, it is in the range of the expert elicitations described above for a high warming scenario.

To calibrate the hazard function of ocean temperature change in Equation 1, I solve a system of equations for $\beta$, the desired hazard rate calibration coefficient:

$$P_s = hr_t \Pi_t \quad \forall s, t : s = t \tag{2}$$

$$hr_t = \min[\beta\,(\Delta T_{O_t})^2\,, 1] \tag{3}$$

$$\Pi_t = \prod_{i<t} (1 - hr_i) \tag{4}$$

$$\sum_{s \leq 2100} P_s = 1 \tag{5}$$

where state $s$ and time period $t$ are defined in 10-year steps between 2010 and 2100, $P_s$ is the probability of being in state $s$, $hr_t$ is the collapse hazard rate, $\Pi_t$ is the probability that the collapse has not yet occurred at start of time t, and $\Delta T_O$ is the local ocean temperature above freezing given by RCP8.5.[10] Solving this system of 31 equations and 31 unknowns (i.e., $P_s$, $hr_t$, $\Pi_t$, and $\beta$) gives $\beta = 0.008$, which means if the climate were to follow an RCP8.5 trajectory, the probability of a collapse having been triggered would be 16.8% by 2050 and 27.9% by 2070.[11]

This calibration process yields a logistic cumulative distribution function (CDF) of the collapse probability in terms of $\Delta T_O$ that is depicted in Figure 2.

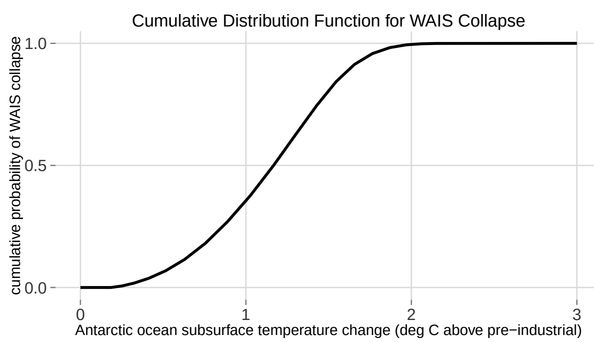


Figure 2: Cumulative distribution function (CDF) of the WAIS collapse probability.

### 3.2.2 Consequence of WAIS Collapse

The consequence of the WAIS collapse is represented in the stochastic framework as a climate system regime shift. Specifically, when the WAIS collapse has been triggered, there is an irreversible change in the SLR dynamics from the baseline process to an altered state in which there is an additional source of melt from WAIS. The timeframe of complete disintegration of WAIS is on the order of 1,000 years, with expert estimates ranging from 400 to 2400 years (National Research Council,

[10] Note that physical constraints ensure $\Delta T_O \geq 0$, as the ocean temperature will always be above freezing.

[11] It is worth noting that the RCP8.5 trajectory assumes higher emissions that many model baseline assumptions. For example, $\beta = 0.008$ in the DICE model business-as-usual scenario with the mitigation level fixed to 0 implies the probability of a collapse having been triggered is 6.8% by 2050, 16.4% by 2070, and 39.2% by 2100.

2013; Oppenheimer, 1998). In the collapse state of the world, the baseline annual rate of WAIS melt is assumed to be

$$\frac{\mathrm{d}SLE_{WAIS_t}}{\mathrm{d}t} = \frac{5}{\tau} \tag{6}$$

where $SLE_{WAIS_t}$ is the SLR-equivalent melt in m, and $\tau$ is the lifetime of WAIS, which I assume to be 500 years. This irreversible design intends to reflect current understanding of ice sheet dynamics (e.g., ice sheet growth occurs over tens of thousands of years through accumulated snowfall, and will not be easily re-formed; dynamic instabilities suggest it is not possible to halt the process of ice flow once underway).

Moreover, the baseline rate is assumed to accelerate over time to represent the likely effect of hysteresis. Shaffer (2014) reports that the WAIS melt rate is found to be 'proportional to the product of ocean flow speed and ocean temperature beneath the ice shelf, both of which increase linearly with ocean warming' as found in Holland et al. (2008). In this way, the baseline rate of melt is assumed to increase linearly with the change in the Antarctic ocean subsurface temperature above today, as depicted in Figure 3.

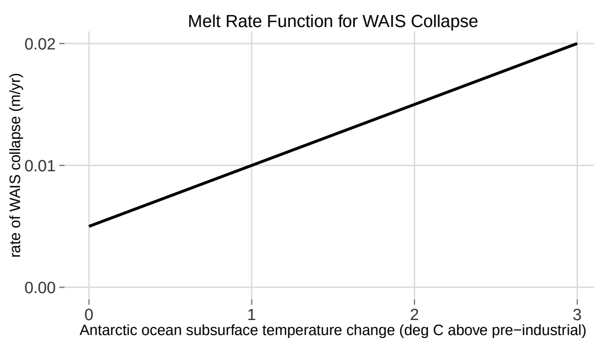


Figure 3: Annual melt rate of WAIS as a function of Antarctic ocean subsurface temperature.

The direct physical consequence of WAIS collapse is the additional rate of SLR described above.[12] In an IAM framework, this physical consequence of SLR has important implications in terms of social welfare. These costs to society are represented through the coastal damage function of SLR, presented in the following section (see Equation 13).

[12] The physical consequence of WAIS collapse is limited to additional SLR. This constrained scope does not fully capture the effect of WAIS collapse on climate system dynamics, which could alter the local weather patterns as well as the albedo (surface reflectivity). In future work it would be interesting to explore this additional dimension of the climate system.

# 4 Integrated Assessment Model with Stochastic Framework

I now use the stochastic programming framework with endogenous uncertainty described in Section 3 to reformulate a benchmark integrated assessment model (IAM) of optimal growth, the Dynamic Integrated Climate-Economy (DICE) model, version 2013R (Nordhaus and Sztorc, 2013), with essential modifications to incorporate ice sheet climate variables and SLR impacts.

## 4.1 Dynamic Integrated Climate-Economy (DICE) model

DICE is a transparent and tractable intertemporal optimization model of economic growth and climate impacts for a single region, the world. DICE solves the optimal Pareto problem, which sets the level of greenhouse gas mitigation such that marginal cost of mitigation is equal to the marginal benefit of avoided climate impacts over the model time path.

DICE chooses the optimal path of consumption (trading off carbon mitigation and capital investment) that maximizes the social welfare objective function.[13]

Welfare is the discounted sum of utility over time, where the isoelastic (i.e., constant relative risk aversion) utility function expresses preferences over per capita consumption:

$$W = \sum_{t \in T} \frac{1}{(1+\rho)^t} \left[ l_t \frac{(\frac{C_t}{l_t})^{(1-\eta)} - 1}{1-\eta} \right] \tag{7}$$

where $W$ is total social welfare, $C_t$ is the level of consumption, $l_t$ is the population, $\rho$, the pure rate of social time preference, and $\eta$ is the consumption elasticity parameter. Utility, the second term in the formula, is weighted by the social discount factor, the first term. The parameter $\rho$ reflects intertemporal preferences for comparing utility across different generations.

In DICE utility increases in population and per capita consumption, with diminishing marginal utility from the latter. $\eta$, the elasticity of marginal utility of consumption, captures aversion to inequality in per capita consumption levels. These two preference parameters in DICE have been calibrated in accordance with the Ramsey growth equation to observed economic outcomes (e.g., interest rates, rates of return on capital) such that $\rho$ is 1.5 and $\eta$ is 1.45 (Nordhaus and Sztorc, 2013).

A stylized representation of DICE is shown in Figure 4 below.

[13]DICE is a neoclassical model of optimal economic growth premised on the Ramsey rule $r_t = \rho + \eta g_t$, where $r_t$ is the discount rate, $\rho$ is the pure rate of time preference, $\eta$ is the marginal utility of consumption, and $g_t$ is the per capita growth rate of consumption (Ramsey 1928).

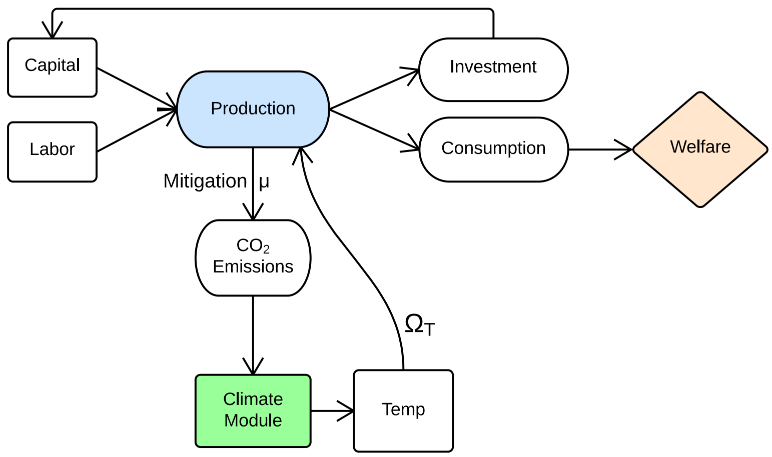


Figure 4: Stylized diagram of the DICE integrated assessment model.

Economic output is determined by a Cobb-Douglas production function of endogenous capital and exogenous labor, with exogenous Hicks-neutral technological change represented by total factor productivity. This output, if unmitigated, has an associated carbon intensity, resulting in greenhouse gas emissions that warm the atmosphere.

The decision variable for carbon mitigation, $\mu$, equals the fraction of emissions from the business-as-usual emissions projection that are avoided through decarbonization. The cost of mitigation (as a proportion of output) is given by a convex power function of $\mu$ in which the marginal cost of mitigation increases more than linearly with $\mu$.

Climate damages $\Omega$ act as a claim on output, reducing the amount that can be spent on either welfare-improving consumption today or investment in the future capital stock. DICE uses an aggregate damage function that gives the fraction of economic output lost to temperature during time period t, formulated as a quadratic function of $\Delta T_t$, the equilibrium change in global mean surface temperature above preindustrial. The damage functions will be discussed in more detail in the following section. The DICE-2013R model documentation and GAMS code are described in Nordhaus and Sztorc (2013).

### 4.2 DICE-WAIS: Augmented IAM with WAIS Collapse

I augment DICE to model SLR, local WAIS climate variables, and coastal damages in order to integrate it with the stochastic programming framework presented in Section 3.2.

The modified model, DICE-WAIS, introduces two additional variables related to the local WAIS climate (which drive WAIS hazard), and four variables related to SLR and associated damages from coastal impacts (which define WAIS consequence). The ocean temperature threshold that triggers the collapse is an uncertain parameter, but the social planner knows the relationship governing the probability of this threat (Equation 1) as well as the consequence of the event (Equation 6).

The DICE-WAIS model is solved along each one of the collapse branches of the stochastic scenario tree presented in Figure 1. The key difference among the different scenario branches is a shift in the SLR regime. The DICE-WAIS diagram in Figure 5 below depicts the blue 'pre-collapse' pathway that corresponds to the upper branch of the scenario tree, while the red 'post-collapse' pathway corresponds to the offshoot collapse scenarios.

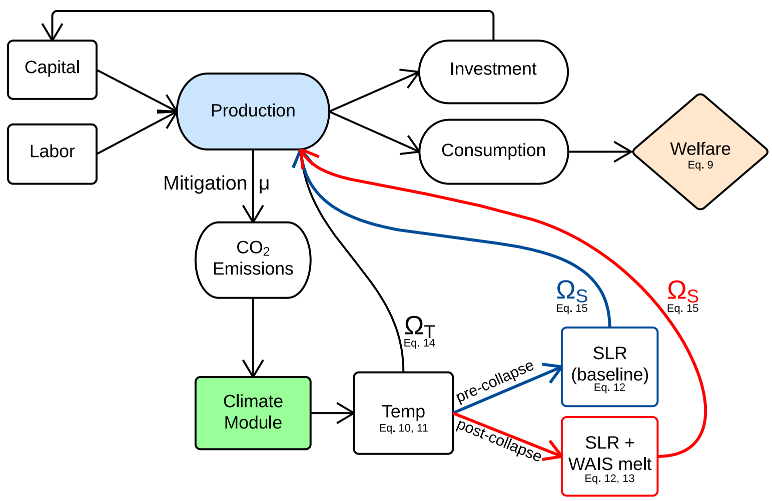


Figure 5: Stylized diagram of the DICE-WAIS model.

The climate module in DICE-2013R computes $CO_2$ concentrations, radiative forcing, and atmosphere and ocean warming. I introduce two local climate variables for WAIS, the Antarctic air temperature, $T_A$, and the ocean subsurface temperature adjacent to the Antartica, $T_O$, which drives the hazard of WAIS collapse.

Antarctic air temperature change at sea level can be derived from global mean temperature change using the southern hemisphere polar amplification factor, $\alpha_s$. The polar amplification effect describes the fact that warming is more pronounced at high latitudes than at the equator. The magnitude of the effect is commonly represented by applying a polar amplification factor to global mean temperature. I define $\alpha_s$ to be 1.2 in accordance with Holland and Bitz (2003):

$$\Delta T_A = \alpha_s \Delta T \tag{8}$$

The local ocean subsurface temperature, $T_O$, has been empirically estimated from Antarctic air temperature (reduced to sea level), $T_A$, in Shaffer (2014). Shaffer uses temperature reconstructions over the past 240,000 years to relate Antarctic surface air temperatures to the average high-latitude (52-70°S) ocean subsurface (200-800m) temperatures. He derives the equation

$$T_O = 0.00690{T_A}^2 + 0.439T_A + 6.39 \tag{9}$$

I also introduce SLR dynamics that is based on RICE-2010, the first and only vintage of the DICE model to explicitly include an SLR module. RICE-2010 decomposes SLR into contributions from four major processes: thermal expansion, melt from glaciers and small ice caps, Greenland Ice Sheet melt, and WAIS melt (Nordhaus, 2010b). I use the first three components to describe a 'baseline SLR' function of temperature:

$$SLR_{t+1} = (r_{TE} + r_{GSIC} + r_{GIS})\,\Delta T_t + SLR_t \tag{10}$$

The fourth component, SLR from WAIS melt, is only introduced in the collapse states of the world. It is represented with WAIS rapid melt regime described in the stochastic programming framework Section 3.2.2:

$$\frac{\mathrm{d}SLE_{WAIS_t}}{\mathrm{d}t} = \frac{5}{\tau}\left(T_{Ot} - T_{Ot_0}\right) \tag{11}$$

There are two types of damages in DICE-WAIS, temperature damages and SLR damages. Both damage functions are based on the DICE-2010 model, which distinguishes two categories of impact sectors, coastal impacts and all other noncoastal impacts.(Nordhaus, 2010a).

Temperature damages are a function of $\Delta T$ as given in Equation 12. These damages include all market and nonmarket impacts excluding coastal impacts. For these damages, $\Omega_{TEMPt}$ gives the fraction of economic output lost to temperature during time period t, formulated as a quadratic function of $\Delta T_t$, the equilibrium change in global mean surface temperature above preindustrial. The severity of the damage function is specified by the linear coefficient $a_0$ and quadratic coefficient $a_1$:

$$\Omega_{TEMPt} = \alpha_1 \Delta T_t + \alpha_2 {\Delta T_t}^2 = 0.00008\Delta T_t + 0.002{\Delta T_t}^2 \tag{12}$$

The other type of damages is SLR damages. $\Omega_{SLRt}$ gives the fraction of economic output lost to coastal damages during time period t, a function of $SLR_t$.

$$\Omega_{SLR_t} = \beta_1 SLR_t + \beta_2 {SLR_t}^2 = 0.00518SLR_t + 0.00306{SLR_t}^2 \tag{13}$$

One limitation of the DICE model is the simple representation of mitigation, which allows the fractional level of emissions controlled to fluctuate freely, with no expansion constraint from period to period (Keller, McInerney, and Bradford, 2008). This simplification fails to capture real-world inertia that limits the rate of decarbonization due to delayed availability of low-emitting technologies, construction lead times, stranded assets, or other capital turnover factors (Ha-Duong, Grubb, and Hourcade, 1997; Richels and Blanford, 2008). Technology inertia is represented in more sophisticated energy system IAMs, but those are not well-suited for the stochastic catastrophe framework presented here because they are too complex and often ignore climate impacts entirely. The final modification to the DICE model is to impose more realistic limits to the mitigation level $\mu_t$ with an expansion constraint, similar to that proposed by Keller, McInerney, and Bradford (2008):

$$\mu_{t+1} \leq \mu_t + c \tag{14}$$

where the $c$ is the maximum amount of additional mitigation during a 10-year period. $c$ is calibrated to be 0.2 based on analysis of decarbonization rates found in an inter-comparison of 18 energy-economy and integrated assessment models in Stanford Energy Modeling Forum Study 27 (Kriegler et al., 2014).

# 5 Results

## 5.1 Main Results

Figure 6 illustrates two optimal levels of mitigation, one corresponding to a world that is ignorant of the possibility of WAIS collapse and another that understands the nature of that threat, as described in Sections 3 and 4.

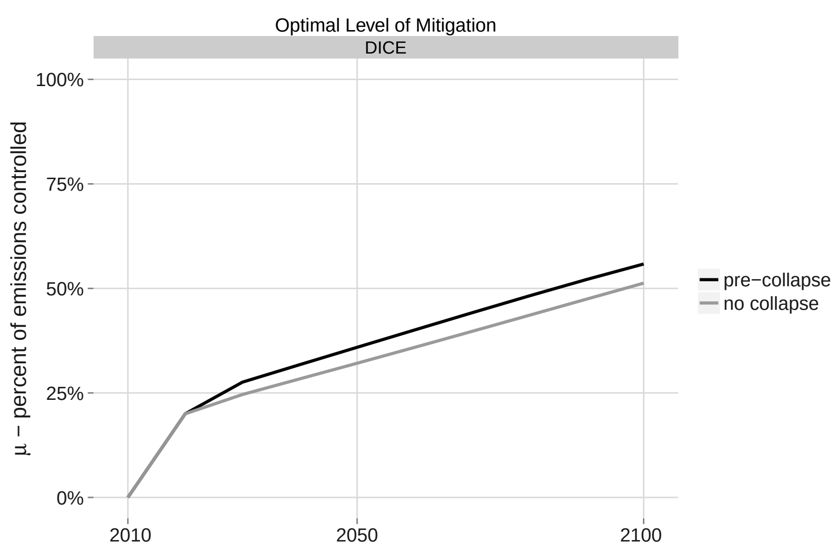


Figure 6: Optimal mitigation pathway.

The risk of ice sheet collapse justifies a more stringent climate policy. Specifically, it is optimal to mitigate an additional 4% of global emissions. This is shown by the increase from the optimal level of mitigation when there is no threat of collapse (gray line) up to the optimal pre-collapse path (black line), in which the collapse has not yet occurred but is still a threat.

This pre-collapse mitigation path result is an important finding for policy-makers because it gives the optimal pre-collapse mitigation, where the policy should be set in order to avoid, or delay, the collapse. The difference in the optimal mitigation policies with and without the WAIS risk can be understood by examining the marginal cost of carbon dioxide emissions, also known as the social cost of carbon (SCC), illustrated in Figure 7.

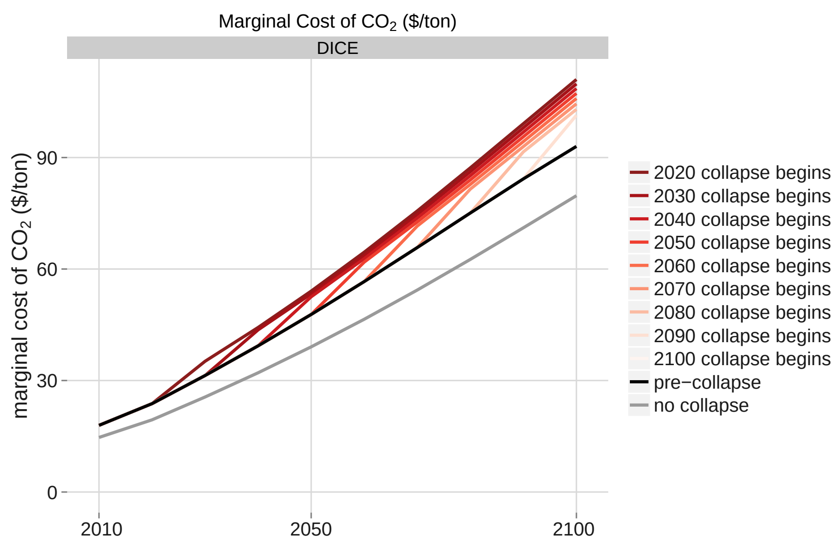


Figure 7: Marginal cost of carbon dioxide for each collapse scenario.

As in Figure 6, the gray line shows the case with no collapse threat, and the black line shows the case in which the collapse is an active threat but has not been triggered. The red lines correspond to the states of the world in which WAIS collapse has been triggered. In each of these collapse scenarios, the marginal cost of carbon departs from the hedging path, rising in response to the acceleration in SLR, which corresponds to a steeper slope on the quadratic damage function given in Equation 13. Because the marginal cost of $CO_2$ (equivalently the marginal benefit of avoided $CO_2$) will equal the marginal cost of mitigation along the optimal policy path, each of these red lines show that the value of mitigation becomes even greater after a collapse. The post-collapse marginal cost illustrates the motivation for hedging against the collapse.

Figure 8 shows the temperature and sea level rise trajectories corresponding to each collapse scenario.

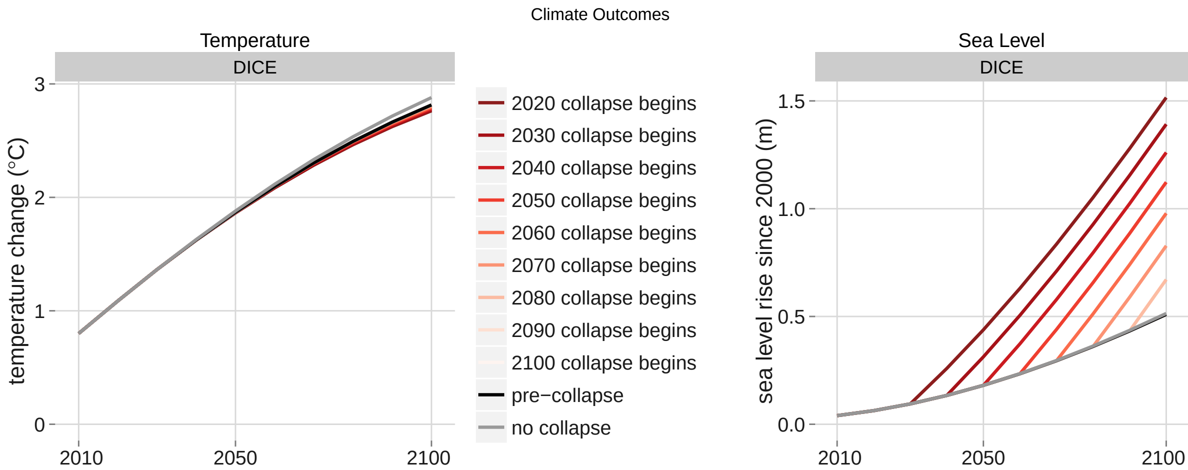


Figure 8: Climate outcomes corresponding to each collapse scenario.

Despite the slightly lower temperatures in 2100 in the collapse cases (left), the corresponding SLR outcome for those states is much greater than the baseline pathway (right). The right panel illustrates that the earlier WAIS collapses, the more SLR that society will face, leading to increased damages in the coastal sector.

I also model an alternative scenario tree, in which all uncertainty about the ice sheet collapse is eliminated. There is only one state of the world and it suffers an ice sheet collapse in 2080 with certainty.

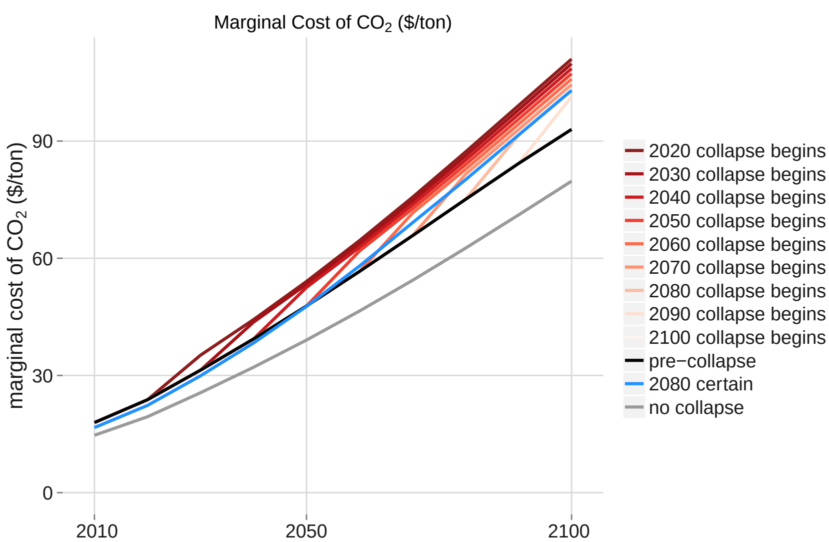


Figure 9: Marginal cost of carbon dioxide for certain collapse in 2080.

This single state is given by the blue line. The initial SCC is lower than along the hedging path. This implies there is some additional early mitigation over the no threat case, and mitigation becomes increasingly more beneficial in the periods approaching the collapse.

#### 5.1.1 Robustness Test

The model presented in Sections 3 and 4 is designed to be a convex problem. However, stochastic optimization with endogenous probabilities is computationally challenging and can result in nonconvexities. In order to test that the combination of an endogenous hazard with endogenous consequence has not inadvertently created this situation, I perform a robustness test to show that the solution is indeed the global optimum. Specifically, I test a wide range of initial conditions for the decision variable for carbon mitigation, $\mu$, exploring the entire solution space. Each of these initial conditions returns the same optimal mitigation path, as seen in Figure 10, confirming the robustness of the result.

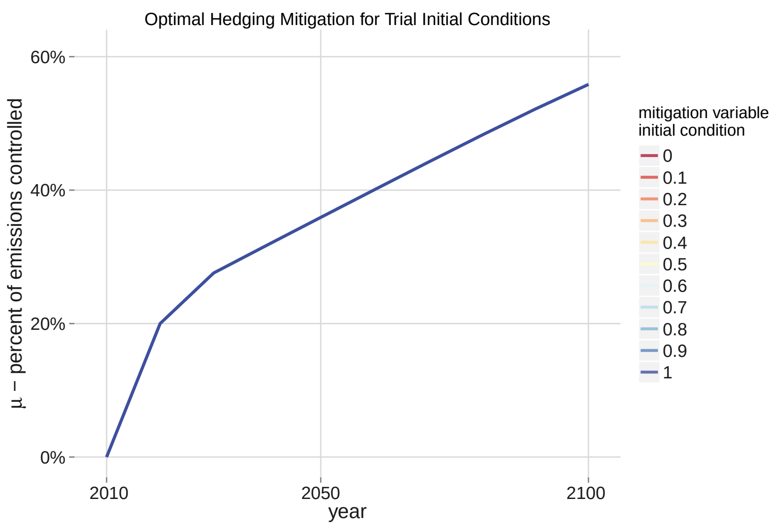


Figure 10: Robustness test of decision variable initial conditions.

### 5.2 Precautionary Motivation for Mitigation

The increased mitigation effort from the case of ignorance about the collapse to the pre-collapse level (Figure 6) is due to two factors. The first is the hedging motive due to the fact that the WAIS collapse introduces downside risk from additional SLR damages, so additional mitigation reduces the consequence of entering the collapse state. The second is the precautionary motive due to the fact that the optimizing agent is able to reduce the probability of collapse through additional mitigation that slows the subsurface warming of the Antarctic ocean.

These two factors have been shown analytically by Lemoine and Traeger (2014) to be additively separable. This section investigates the relative contribution of the two motives through a numerical experiment. When the model is formulated with endogenous uncertainty, both motives are present. In contrast, in a stochastic program with exogenous uncertainty there is no opportunity for precaution. These two modes can be compared to separate the influence of the two factors.

At the outset of the stochastic program with endogenous uncertainty, the hazard rate at each timestep is unknown, as it depends on the warming that is allowed to occur by the social planner. After solving the model for the optimal pathway, it is possible to observe what the hazard rate was for each time step. I then reformulate the model as a classic multistage stochastic program with probabilities specified exogenously, corresponding to the hazard rates from the original model. This exogenous setting eliminates the precautionary motive.

Figure 11 compares the two formulations of endogenous and exogenous hazard relative to the ignorant case, in terms of the level of mitigation on the left and the marginal cost of carbon dioxide on the right.

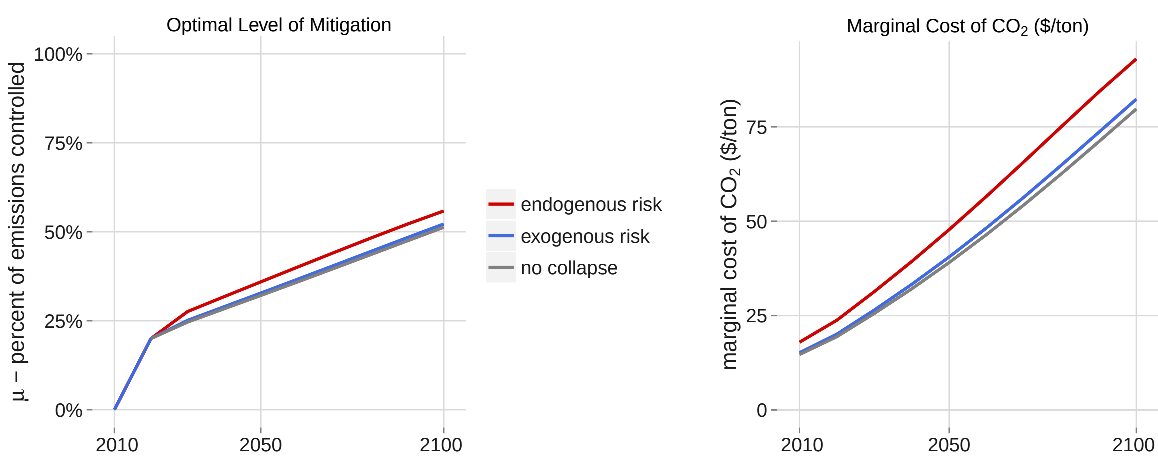


Figure 11: Comparison of endogenous and exogenous hazard formulation with optimal mitigation pathway (left) and marginal cost of carbon dioxide (right).

The optimal path can be decomposed into the hedging motive and the precautionary motive based on the relative location of the exogenous hazard (blue line). The collapse risk motive is given by the increase from the result with no threat of collapse (gray line) up to the exogenous hazard hedging path (blue line). The precautionary motive is then given by the increase from exogenous hazard hedging path (blue line) up to the endogenous hazard path (red line). Both panels illustrate that what matters most for mitigation policy is not the existence of the hazard itself, but the fact that it is endogenous and therefore can be reduced.

### 5.3 Expected Value of Perfect Information

In this section I present the expected value of perfect information (EVPI) about the likelihood of WAIS collapse. The EVPI is defined as the difference between the expected value of the objective function obtained from the 'learn, then act' solution for all states of the world, compared to the expected value of the 'act, then learn' stochastic problem. In a convex maximization problem the former is higher than the latter, since the decision-maker has perfect information for a given state and can behave optimally for that particular scenario in isolation (e.g., without having to satisfy nonanticipativity constraints for coincident states).

In order to quantify this metric in a meaningful unit, the objective function must be converted from units of utility to consumption in a way that preserves the planner's preferences over risk, consumption elasticity of substitution, and time. This requires computing the certain-equivalent consumption in each period $t$, according to the DICE utility function defined earlier in Equation 7:

$$U_t = \frac{(\frac{C_t}{l_t})^{(1-\eta)} - 1}{1-\eta} \tag{15}$$

Accordingly, $\tilde{C}_t$, the certain-equivalent consumption, is given by

$$\tilde{C}_t = (\mathbb{E}[U_t^{\omega}](1-\eta) + 1)^{\frac{1}{(1-\eta)}}\, l_t \tag{16}$$

Thus, the expected net present value of certain-equivalent consumption, discounted at the consumption discount rate $r$, is given by

$$z^* = \sum_{t \in T} \frac{1}{(1+r)^t}\, \tilde{C}_t \tag{17}$$

The expected value of discounted consumption for the DICE-WAIS stochastic model with endogenous uncertainty is $z^* =$ \$2,405 Trillion in discounted certain-equivalent consumption.

This $z^*$ can then be compared to the expected value of discounted consumption given perfect information, $z_{ws} = \mathbb{E}[z^{\omega}] = \sum_{\omega \in \Omega} z^{\omega} p(\omega)$, where $z^{\omega}$ corresponds to the resulting net present value of consumption for each $\omega \in \Omega$ and $p(\omega)$ is the corresponding state probability realized in the stochastic formulation. Practically, this is computed by solving the 'learn, then act' version of the model for each collapse branch of the scenario tree independently. The results are summarized in Table 1 below.

Table 1: Discounted global consumption 2010-2250 (Trillion \$).

| $\omega$, state of the world | $p(\omega)$ | $z^{\omega}$ |
|---|---|---|
| collapse in 2020 | 0.6% | \$2984 |
| collapse in 2030 | 1.2% | \$2982 |
| collapse in 2040 | 1.9% | \$2980 |
| collapse in 2050 | 2.7% | \$2978 |
| collapse in 2060 | 3.6% | \$2976 |
| collapse in 2070 | 4.4% | \$2975 |
| collapse in 2080 | 5.2% | \$2973 |
| collapse in 2090 | 5.8% | \$2972 |
| collapse in 2100 | 6.2% | \$2971 |
| collapse in 2110 | 6.4% | \$2970 |
| collapse in 2120 | 6.4% | \$2969 |
| collapse in 2130 | 6.3% | \$2968 |
| collapse in 2140 | 6% | \$2968 |
| collapse in 2150 | 5.6% | \$2967 |
| no collapse | 37.6% | \$2966 |

The expected value of discounted consumption with perfect information about the WAIS collapse is $z_{ws} = \$2,969$ Trillion.

Finally, the difference between these two quantities gives the EVPI $= z_{ws} - z^* = \$564$ Trillion. The EVPI can be interpreted as the price that one would be willing to pay in order to gain access to perfect information. This is nearly twenty percent of the total value. This additional value reflects the benefit of delaying mitigation efforts slightly when the precautionary motive is removed and when the collapse period is known with certainty.

# 6 Discussion

## 6.1 Summary and Policy Implications

Integrated assessment models have long played an important role in policy analysis for climate change. Recent model inter-comparison exercises showcase development in many dimensions (e.g., improved representation of energy technologies, agriculture, and land use) (Edenhofer et al., 2014; Kriegler et al., 2014), yet there has been much less progress in the area of decision-making under uncertainty. At the same time, there is a growing demand for climate policy analysis that addresses the need for iterative risk management. As noted earlier, IAMs have attracted recent criticism for focusing on deterministic assessments and omitting catastrophic impacts (Revesz et al., 2014; Pindyck, 2013; Stern, 2013); these critiques threaten the relevance of IAMs in the policy arena.

This paper seeks to address these limitations by applying a stochastic programming framework with endogenous uncertainty to a benchmark IAM in order investigate the specific catastrophe of WAIS collapse. This work finds that accounting for the potential ice sheet collapse leads to more stringent climate policy recommendations, increasing the mitigation effort by an additional 4% of global emissions and raising the social cost of carbon by $10. This analysis is also able to decompose the relative contribution of the hedging and precautionary motive for mitigation and quantify the value of information about this climate uncertainty.

Moreover, this study moves beyond several recent catastrophe or tipping point studies with arbitrary risk, instead investigating the specific threat of the WAIS collapse, characterizing both the hazard and the consequence in accordance with recent expert elicitations, empirical results, and physical relationships. This additional specificity is necessary to move beyond a conceptual demonstration to numerical results that are relevant for policy insight.

This work highlights the fact that the optimal response is strongly influenced by the nature of the physical and social systems that will be affected (details that are unavoidably absent in an arbitrary catastrophe). To illustrate this point, Figure 12 is a diagnostic plot of the response functions of four climate outcomes, air temperature, sea level (non-WAIS components), deep ocean temperature, and subsurface Antarctic ocean temperature, assuming some fixed level of mitigation over the entire century, ranging from 0 to 100%.

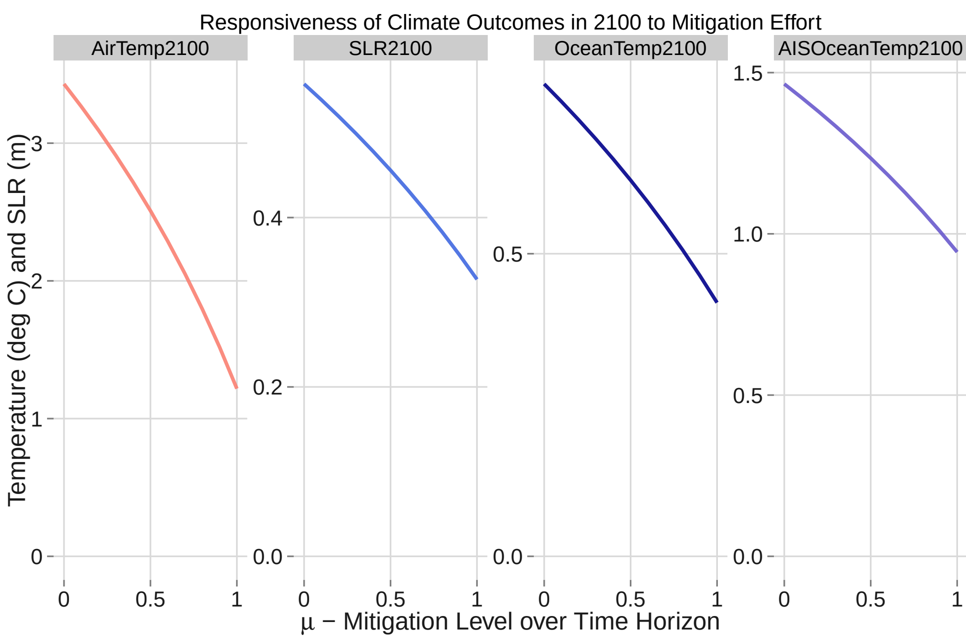


Figure 12: Responsiveness to mitigation effort of physical outcomes in 2100.

The slope of each line measures the responsiveness of each climate outcome. The greatest amount of responsiveness is in air temperature. However, for concern about damages driven by SLR, in the case of coastal impacts, or ocean temperature, in the case of ice sheet collapse, there is much less responsiveness to mitigation. Physical inertia and lags in the natural system play a key role in climate policy design.

### 6.2 Limitations and Future Work

One notable limitation underlying this work is the simplified representation of learning about the climate catastrophe. This study makes the optimistic assumption that the planner is able to detect whether a catastrophe has been triggered in the current period. In reality a catastrophic event may have three distinct moments: when the catastrophe is triggered (which may or may not be irreversible), when the consequence begins, and when the catastrophe can be clearly detected. These events will likely not coincide in time. The collapse (as modeled) triggers the onset of an additional 1 cm/yr of sea level rise, which would amount to an additional 10 cm in the first period. By comparison, there is no guarantee that existing observational methods, which include remotely-sensed satellite measurements of surface ice-sheet extent and gravitational changes as well as RSLR measurements from the global tide gauge network, will be able to detect this event. More sophisticated models of learning about climate parameters have been demonstrated in Kelly and Kolstad (1999) and Urban et al. (2014). Integrating such frameworks into the catastrophe model would improve the relevance and realism of the results.

Another limitation of these results is that they are contingent on the standard DICE expected utility model. However, this simplified model conflates preferences for risk and intertemporal consumption in a single parameter. A current approach for overcoming this limitation is to apply Epstein-Zin utility, which separates the treatment of risk aversion and consumption smoothing (Epstein and Zin, 1989). While other studies referenced in the literature review do address Epstein-Zin preferences (Jensen and Traeger, 2012; Rutherford, 2013; Cai, Judd, and Lontzek, 2013; Lemoine and Traeger, 2014), the primary focus of this paper was to improve the characterization of the catastrophe in terms of the physical system and scientific understanding about risk. Future extensions to this study will address these utility function concerns.

An ongoing direction for similar studies of this kind is to continue to improve the characterization of catastrophes, both in terms of the hazard and the consequence, so that findings are relevant to policy-makers. This study has attempted to do so for WAIS, but necessary simplifications limit the usefulness of these results. For example, the chosen probability distribution governing the likelihood of collapse (i.e., the hazard rate) is a key assumption that will benefit from scientific advances in ice sheet modeling and studies of past behavior. Similarly, the representation of the WAIS climate dynamics can be improved to better capture the lagged response of subsurface ocean temperature to local WAIS atmospheric warming. This would affect the ability for mitigation to affect the collapse. In addition to these local WAIS dynamics, the global climate module in DICE is a vast simplification of the actual climate system, and introduces its own limitations to this work (van Vuuren et al., 2009).

Finally, the paper demonstrates an approach that could be replicated for other catastrophes. Topics for future studies include ocean circulation disruption, Greenland ice sheet collapse, and carbon-cycle tipping points such as sudden methane release from the oceans or permafrost or widespread forest die-back. For each specific catastrophe that threatens society, both the hazard and the consequence should be accurately characterized based on physical relationships, empirical results, and expert elicitation. To the extent that these characteristics can be represented in the modeling framework, the better the policy guidance will be. Furthermore, modeling singular tipping points overlooks the fact that triggering one could increase the likelihood of another. For example, melting permafrost could lead to a runaway albedo change feedback cycle. It is doubtful that the climate system will only ever experience a single climate tipping point, and to oversimplify the model is unrealistic and also sometimes introduces a perverse incentive for the system to increase emissions after a tipping point has been triggered. Although this adds considerable computational burden, the IAM community can pursue model developments to accommodate this need.

The results of this stochastic programming IAM with endogenous uncertainty demonstrate the importance of incorporating the risk of catastrophe directly into the planner's objective function in a realistic characterization. This type of study is useful for policymakers, so they can begin to understand potential hedging strategies that balance the costs of mitigation investments today with the downside risk of future climate change, as well as the precautionary motive.

# 7 Acknowledgments

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