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# Dynamic Water Regulation Under Endogenous Irrigation Investment and Production Uncertainty

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# 1 Introduction

Irrigation water that is not absorbed by crops contributes to waterlogging problems. Salt-laden drainage water accumulates under the ground and reduces crop yields. High concentrations in drainage water of naturally occurring trace elements, such as selenium, may also pose a hazard to fish and wildlife when agricultural drainage waters are discharged to the surface (Loehman and Dinar [16]). To sustain agricultural productivity and water quality, some form of water usage regulation is essential. Otherwise, the market failure caused by common access to the drainwater aquifer may cause farmers to over-irrigate, releasing excessive amounts of deep percolating water (Shah et al. [18]). By imposing quotas or taxing water use, the regulator can directly affect the amount of water usage. These policies can also have indirect effects through their influence on the farmers' choice of irrigation technology, e.g. furrow vs. sprinkler or drip (Caswell, et al. [1]).

Most of the existing literature on agricultural drainage management (see Dinar and Letey [4], Dinar and Zilberman [7]) focus on the effects of exogenously specified policies on water conservation and drainage, ignoring the optimal design of regulating policies. The optimal policy choice is complicated by the fact that farmers are likely to have better information about their production profitability than do regulators. Although there is related work on the stock externality regulation under asymmetric information (Hoel and Karp [8][9], Karp and Zhang [10][11]), the role of uncertainty has rarely been addressed in the water management context.

Farmers have private information about the marginal productivity of water (an agricultural input). The regulator treats this private information as a serially correlated random variable, and learns about it by observing farmers' response to previous regulations, thereby reducing the asymmetry of information. The optimal feedback policy design results in a control rule that determines policies as a function of all the available information along time. The policy level is adjusted in light of new information. We compare the welfare effects of using either the optimal tax or the optimal quota on water when farmers do not internalize the environmental damage caused by waterlogging.

The optimal policy choice is further complicated by farmers' irrigation investment behavior response to regulation. However, the models in the literature have not been very successful in capturing the intertemporal technology adoption. A static analysis of optimal investment in irrigation systems is given by Caswell, et al. [1]. Assuming that each irrigation system lasts for only one period, Shah et al. [18] present a dynamic analysis of (myopic) investment decision. However, irrigation systems and drainage facilities which reduce drainage water involve long-term investment. Farmers' decision on the current level of investment is contingent on their expectations about future regulatory policies, and an intertemporal variability in key economic parameters such as output and water prices.

Both individual farmers and society at large benefit from the efficient use of water and irrigation investment. Here we develop, analyze, and apply a dynamic optimization model for irrigation management and investment over a sequence of years. The model considers intertemporal adoption of modern irrigation technology and water application as responses to water regulations, and incorporates the regulator's learning about the farmer-specific environmental conditions such as weather and the salinity of irrigation water. We investigate the sensitivity of the optimal regulatory policy choice, and of farmers' production and their technology investment, to changes in key economic parameters, such as the time preference, agricultural output prices, water prices, marginal water and technology productivity, marginal contribution of a unit increase in water application and technology choice to drainage stock, and marginal environmental damages.

## 2 The Model

For simplicity, we consider a single crop production with a fixed amount of land of uniform quality, and assume that other production inputs have been applied at a level so that water is the limiting factor in crop production. We denote the total output at time  $t$  by  $y_t$ , and the applied water at time  $t$  by  $a_t$ . Determined exogenously, output price at time  $t$  is  $p_t$ , and the water price at time  $t$  is  $w_t$ .

Given the considerable number of alternatives in irrigation technologies (CH2MHILL

[2], UC Committee [17]), we can represent irrigation technology by a continuous index variable with higher levels denoting improved and more costly technologies. (Dinar et al. [3] [5][6], Letey and Dinar [15], Loehman and Dinar [16]) Denoting the existing irrigation technology at the beginning of time  $t$  by  $K_{t-1}$ , and the irrigation technology investment at time  $t$  by  $I_t$ , the irrigation technology at the beginning of the next period is

$$K_t = \delta K_{t-1} + I_t \quad (1)$$

with  $0 \leq \delta \leq 1$  being the fraction of irrigation technology that lasts into the next period. In each period, the irrigation technology cost includes both the operating cost associated with the use of the existing technology  $K_{t-1}$ , and the investment cost associated with investing  $I_t$  units at the existing technology level. Suppose we can decompose these two kinds of costs:

$$C_t = c(K_{t-1}, I_t) = c_1(K_{t-1}) + c_2(I_t).$$

We expect that  $c_K > 0$ ,  $c_I > 0$ ,  $c_{KK} \leq 0$ , and  $c_{II} \geq 0$ .

In each period, water application  $a_t$ , irrigation technology  $K_{t-1}$ , and other relevant factors such as water salinity, temperature, and rainfall determine crop output. The general specification of the per acre production is

$$y_t = f(a_t, K_{t-1}, \theta_t)$$

with  $\theta_t$  being a random production shock to be discussed in more detail later. We suppose that  $f_a > 0$ ,  $f_{aa} < 0$ ,  $f_K > 0$ ,  $f_{KK} < 0$ ,  $f_{aK} \leq 0$ , and  $f_\theta > 0$ ; in other words, marginal productivity of  $a$  and  $K$  are positive but diminishing, and a unit increase in  $\theta$  increases productivity.

The random production shock  $\theta_t$  represents other yield related factors, such as changing weather and soil conditions, salt concentration of applied water, average temperature and precipitation, and pan evaporation. We model the production shock as the farmer's private information. The farmer knows  $\theta_t$  before making its current water usage and irrigation decisions. The regulator does not know the current  $\theta_t$  when deciding the water management policies, but he as well as farmers

knows that the production shock follows an  $AR(1)$  process:

$$\theta_t = \rho\theta_{t-1} + \mu_t, \quad \mu_t \sim iid(0, \sigma_\mu^2), \quad \forall t \geq 1.$$

The autocorrelation parameter satisfies  $-1 < \rho < 1$ .  $\{\mu_t\}$  ( $t \geq 1$ ) is an i.i.d. random process with zero mean and common variance  $\sigma_\mu^2$ . In period  $t = 0$  the regulator has a subjective prior of the initial production shock  $\theta_0$ , with mean  $\bar{\theta}_0$  and variance  $\sigma_0^2$ .  $\theta_0$  is independent of the random process  $\mu_t$  ( $\forall t \geq 1$ ). At  $t \geq 1$ , the regulator is able to learn the value of the previous shock with one period lag by observing farmer response to regulating policies. Consequently the variance for the regulator's belief on the current shock is  $\sigma_\mu^2$ . This endogenous change in the variance under learning causes the regulator's control problem to be nonstationary.

Let  $q_t$  equal deep percolation from irrigation at time  $t$ ;  $q_t$  is a function of water application  $a_t$ , and irrigation technology  $K_{t-1}$ :

$$q_t = h(a_t, K_{t-1}).$$

We suppose that  $h_a > 0$ , and  $h_K < 0$ . With higher irrigation technology, water is applied more uniformly and is used more effectively, and there is less drainage per unit water applied.

Let  $Q_{t-1}$  equal the accumulated stock of drainage water at the beginning of time  $t$ . The increase in the drainage water stock at time  $t+1$  equals the deep percolation from irrigation at time  $t$ :

$$Q_{t+1} = \Delta Q_{t-1} + q_t$$

with  $0 \leq \Delta \leq 1$  being the fraction of deep percolation that lasts into the next period. If the underlain clay layer has a low permeability, there is no further percolation of drainwater and  $\Delta = 0$ .

Shah et al. [18] assume that production is not affected by drainage stock until it reaches a critical level. However, the drainage stock from irrigated production continuously causes environmental damages in the form of high water tables encroaching the root zone and degradation of receiving surface water and ground

water quality (Loehman and Dinar [16]). We assume that environmental damage from water-logging is represented by the increasing convex function:

$$D_t = d(Q_{t-1}), \quad \text{with} \quad d' > 0, \quad d'' \geq 0.$$

### 3 The Theory

Evaluation of specific water management policies requires analysis of farmer response to those policies. We assume there are a large number of competitive farmers who have common access to the subsurface drainwater aquifer. Since an individual farmer cannot ensure a reduction in deep percolation stock through his actions alone, he will put a shadow value of 0 on  $Q_t$ . As a result, the farmers as a group will do the same, even though this is not their collective interest. In this situation, the farmers' profit maximization objective is achieved by maximizing net income, or sales minus private costs, over intertemporal irrigation technology investment and water application.

In each period the regulator chooses tax  $\tau_t$  or quota  $a_t$  to maximize the expectation of the present value of the difference between crop return net of investment costs and pollution damages, and the (small) representative farmer chooses water usage  $a_t^*$  (if under taxes) and investment  $I_t$  to maximize the expectation of the present value of the stream of crop return from minus investment cost minus pollution taxes (if under taxes). The regulator knows the farmer's reaction functions for water usage and investment response. Asymmetric information on the farmer's production shocks persists. The regulator's current decision affects future irrigation technology and drainage water, thereby affecting future regulatory decisions. However, the regulator is not able to make credible commitments about future policies. That is, the regulator is restricted to Markov perfect policies.

#### 3.1 The Social Optimality of Farmer's Investment

We want to know whether the farmer's investment decision is socially optimal. Otherwise, the regulator needs to set an investment subsidy to induce the farmer

to invest the desired level.

### 3.2 Farmer's Behavior

The (small) representative farmer behaves nonstrategically. That is, the farmer believes that its behavior has no effect on the regulator's future decisions, and takes the sequence of future policies (  $\{\tau_{t+j}\}_{j=1}^{\infty}$  or  $\{a_{t+j}\}_{j=1}^{\infty}$  ) as exogenous.

**Under taxes.** Let  $B(K_{t-1}, \theta_t, a_t) = p_t \cdot y(K_{t-1}, \theta_t, a_t)$ , the crop return from water usage. The dynamic programming equation for the farmer's optimization problem under taxes is

$$V^T(K_{t-1}, \theta_t, \tau_t, t) = \max_{a_t, I_t} \{ B(K_{t-1}, \theta_t, a_t) - \tau_t a_t - C(I_t, K_{t-1}) + \beta E_{F_t} [V^T(K_t, \theta_{t+1}, \tau_{t+1}, t+1)] \}, \quad (2)$$

subject to the equation of motion for the capital stock (equation (1)) and the data generating process for production shocks.  $\beta$  is the constant discount factor. The superscripts  $T$  and  $Q$  distinguish functions and variables under taxes and quotas.

The farmer's return following optimal water use and investments at time  $t$  is  $V^T(K_{t-1}, \theta_t, \tau_t, t)$ , depending on the current irrigation technology, the realized production shock, the current tax, and the farmer's expectation at time  $t$  of future taxes and production shocks which is represented by the last argument ( $t$ ) in the value function. The farmer anticipates future variables based on all the available information, represented by the notation  $E_{F_t}(\cdot)$ . The current revenue—related information set is  $F_t \equiv [K_{t-1}, \theta_t, policy_t]$ , the current irrigation technology, current production shock, and current policy ( $policy_t = \tau_t$  under taxes and  $policy_t = a_t$  under quotas).

The first-order conditions with respect to water usage ( $a_t$ ) and the irrigation technology ( $K_t$ ) are respectively

$$B_x(K_{t-1}, \theta_t, a_t) - \tau_t = 0, \quad (3)$$

$$-C_I(I_t, K_{t-1}) + \beta E_{F_t} [V_K^T(K_t, \theta_{t+1}, \tau_{t+1}, t+1)] = 0. \quad (4)$$

The optimal water usage is a static response while investment is a dynamic response. In each period, the marginal cost saving from optimal water usage equals



the current tax (equation (3)). The water usage response

$$a_t^* = \chi(K_{t-1}, \theta_t, \tau_t), \quad (5)$$

is a function of the current irrigation technology, current production shock, and current tax.

The marginal cost of the optimal investment equals the discounted shadow value of irrigation technology (equation (4)). Differentiating both sides of (??) with respect to  $K_{t-1}$  and applying the Envelope Theorem, we have

$$V_K^T(K_{t-1}, \theta_t, \tau_t, t) = B_K(K_{t-1}, \theta_t, a_t) + \delta C_I(I_t, K_{t-1}). \quad (6)$$

Using the first-order condition (4) and the envelope condition (6) and applying the iterated expectations, we have the stochastic Euler equation for the farmer's optimal investment

$$-C_I(I_t, K_{t-1}) + \beta E_{F_t} \{B_K[K_t, \theta_{t+1}, \chi(K_t, \theta_{t+1}, \tau_{t+1})] + \delta C_I(I_{t+1}, K_t)\} = 0, \quad (7)$$

which is a second-order difference equation in terms of irrigation technology. To solve this second-order difference equation, we need two boundary conditions. One boundary condition is supplied by the current irrigation technology  $K_{t-1}$ , and the other by the transversality condition

$$\lim_{T \rightarrow \infty} \beta^{T-t} E_{F_t} \{C_I(I_T, K_{T-1}) K_T\} = 0. \quad (8)$$

The transversality condition is a necessary condition for optimizing (2). We obtain this condition by considering a finite  $T$  horizon version of problem (2), multiplying the first-order necessary condition for investment in the last period ( $I_T$ ) by the terminal stock of irrigation technology  $K_T$  and the discount factor  $\beta^{T-t}$ , and taking the limit of the resulting equation as  $T \rightarrow \infty$ . Solving the Euler equation (7) we know the investment response is a function of the current irrigation technology, the realized production shock, and expected future taxes and production shocks:<sup>1</sup>

$$K_t^* = \kappa^T \left( K_{t-1}, \theta_t, E(\tau_{t+j})_{j=1}^{\infty}, E(\theta_{t+j})_{j=1}^{\infty} \right). \quad (9)$$

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<sup>1</sup>With general function forms for the abatement cost and environmental damages, we can also assume that cost shocks follow a more general data generating process than AR(1).

The current tax has no effect on current investment.

**Under quotas.** We assume that farmers are homogeneous and quotas are not bankable. Thus, under a quota policy, there is no incentive for tradable permits and the quota level is binding. The dynamic programming equation for the farmer's optimization problem under quotas is

$$V^Q(K_{t-1}, \theta_t, a_t, t) = \max_{I_t} \{B(K_{t-1}, \theta_t, a_t) - C(I_t, K_{t-1}) + \beta E_{F_t} V^Q(K_t, \theta_{t+1}, a_t, t+1)\}.$$

The farmer's optimal return at time  $t$  under a quota policy is  $V^Q(K_{t-1}, \theta_t, a_t, t)$ , a function of current irrigation technology, the realized production shock, the current quota, and the farmer's expectation at time  $t$  of future quotas and production shocks (captured by the last argument  $t$  in the value function). The optimal investment under quotas equates the marginal investment cost and the discounted shadow value of irrigation technology, and satisfies the stochastic Euler equation

$$-C_I(I_t, K_{t-1}) + \beta E_{F_t} \{B_K[K_t, \theta_{t+1}, a_t] + \delta C_I(I_{t+1}, K_t)\} = 0, \quad (10)$$

and the transversality condition (8). The resulted farmer's investment response under quotas is a function of the current irrigation technology, the realized production shock, and expected future quotas and production shocks:

$$K_t^* = \kappa^Q \left( K_{t-1}, \theta_t, E(a_t)_{j=1}^\infty, E(\theta_{t+j})_{j=1}^\infty \right). \quad (11)$$

The current quota has no effect on current investment.

The proposition below summarizes the farmer's optimal water usage and investment responses.

**Proposition 1** *The regulator's current policy (tax or quota) influences the farmer's current water usage, and expected future policies influence the farmer's current investment. Optimal investment equates the marginal cost of investment to the discounted shadow value of capital, which depends on expected future policies and production shocks.*

### 3.3 Regulator's Behavior with Direct Control on the Investment

To check the social optimality of farmer's investment, we look at the regulator's investment decision supposing that he can choose investment directly.

**Under taxes.** Knowing the farmer's water usage response (5), the regulator in each period solves the following dynamic programming equation:

$$\begin{aligned}
J^T(K_{t-1}, S_{t-1}, \theta_{t-1}) &= \max_{I_t, \tau_t} E_{\theta_t|\theta_{t-1}} \{B(K_{t-1}, \theta_t, a_t^*) - D(S_{t-1}) - C(I_t, K_{t-1}) \\
&\quad + \beta J^T(K_t, S_t, \theta_t)\} \\
s.t. \quad S_t &= \Delta S_{t-1} + a_t^*, \\
a_t^* &= \chi(K_{t-1}, \theta_t, \tau_t).
\end{aligned} \tag{12}$$

The first-order conditions with respect to  $K_t$  and  $\tau_t$  are respectively

$$-C'(I_t, K_{t-1}) + \beta E_{\theta_t|\theta_{t-1}} J_K^T(K_t, S_t, \theta_t) = 0 \tag{13}$$

$$E_{\theta_t|\theta_{t-1}} \left\{ [B_x(K_{t-1}, \theta_t, a_t^*) + \beta J_S^T(K_t, S_t, \theta_t)] \cdot \frac{\partial \chi(K_{t-1}, \theta_t, \tau_t)}{\partial \tau_t} \right\} = 0 \tag{14}$$

Differentiating both sides of (??) with respect to the state variable  $K_{t-1}$  and using the Envelope Theorem, we have

$$\begin{aligned}
&J_K^T(K_{t-1}, S_{t-1}, \theta_{t-1}) \\
&= E_{\theta_t|\theta_{t-1}} B_K(K_{t-1}, \theta_t, a_t) + \delta C'(I_t, K_{t-1}) \\
&\quad + E_{\theta_t|\theta_{t-1}} \left\{ [B_x(K_{t-1}, \theta_t, a_t) + \beta J_S^T(K_t, S_t, \theta_t)] \cdot \frac{\partial \chi(K_{t-1}, \theta_t, \tau_t)}{\partial K_{t-1}} \right\}.
\end{aligned} \tag{15}$$

The first-order condition (13) and the envelope condition (15) imply the Euler equation for the regulator's investment decision:

$$\begin{aligned}
&-C'(I_t, K_{t-1}) \\
&+ \beta E_{\theta_t|\theta_{t-1}} E_{\theta_{t+1}|\theta_t} \left\{ B_K(K_t, \theta_{t+1}, \chi(K_t, \theta_{t+1}, \tau_{t+1})) + \beta \delta C'(I_{t+1}, K_t) \right. \\
&\quad \left. + [B_x(K_t, \theta_{t+1}, a_t) + \beta J_S^T(K_{t+1}, S_{t+1}, \theta_{t+1})] \cdot \frac{\partial \chi(K_t, \theta_{t+1}, \tau_{t+1})}{\partial K_t} \right\} = 0.
\end{aligned} \tag{16}$$

We obtain the transversality condition

$$\lim_{T \rightarrow \infty} E_{\theta_T | \theta_{t-1}} \left\{ \beta^{T-t} C' (I_T, K_{T-1}) K_T \right\} = 0. \quad (17)$$

as discussed in the previous section for farmer's investment.

**Under quotas.** The regulator in each period solves the following dynamic programming equation:

$$\begin{aligned} J^Q (K_{t-1}, S_{t-1}, \theta_{t-1}) &= \max_{K_t, a_t} E_{\theta_t | \theta_{t-1}} \{ B (K_{t-1}, \theta_t, a_t) - D (S_{t-1}) - C (I_t, K_{t-1}) \\ &\quad + \beta J^Q (K_t, S_t, \theta_t) \}, \\ \text{s.t.} \quad S_t &= \Delta S_{t-1} + a_t. \end{aligned}$$

The first-order conditions with respect to  $K_t$  and  $a_t$  are respectively

$$\begin{aligned} -C' (I_t, K_{t-1}) + \beta E_{\theta_t | \theta_{t-1}} J_K^Q (K_t, S_t, \theta_t) &= 0, \\ E_{\theta_t | \theta_{t-1}} \left\{ B_x (K_{t-1}, \theta_t, a_t) + \beta J_S^Q (K_t, S_t, \theta_t) \right\} &= 0. \end{aligned}$$

The Envelope condition on  $K_{t-1}$  is

$$J_K^Q (K_{t-1}, S_{t-1}, \theta_{t-1}) = E_{\theta_t | \theta_{t-1}} B_K (K_{t-1}, \theta_t, a_t) + \delta C' (I_t, K_{t-1}),$$

implying the Euler equations for the regulator's investment under quotas

$$-C' (I_t, K_{t-1}) + \beta E_{\theta_t | \theta_{t-1}} E_{\theta_{t+1} | \theta_t} \left\{ B_K (K_t, \theta_{t+1}, a_t) + \delta C' (I_{t+1}, K_t) \right\} = 0. \quad (18)$$

The transversality condition is (17).

### 3.4 A Theorem

**Theorem 1** *The farmer's investment will necessarily be socially optimal under quotas, but not so under taxes. A sufficient condition for zero subsidy under taxes is that the marginal water usage responses with respect to both the tax and the irrigation technology are independent of the production shock.*

We first consider quota. The farmer's investment in equilibrium, characterized by the Euler equation (10) and the transversality condition (8), satisfies the necessary conditions for the regulator's direct investment optimization, characterized by the Euler equation (18) and the transversality condition (17). If the farmer has rational expectations about future quotas, it sets investment at the socially optimal level. (More on the information set)

Under a tax policy the farmer's investment may not be socially optimal. The Euler equation (7) for farmer's investment may fail to satisfy the Euler equation (16) for the regulator's direct investment. The optimal investment subsidy  $\tau$  equals the last term in (16):

$$\tau = \beta E_{\theta_t|\theta_{t-1}} E_{\theta_{t+1}|\theta_t} \left\{ \left[ B_x(K_t, \theta_{t+1}, a_t) + \beta J_S^T(K_{t+1}, S_{t+1}, \theta_{t+1}) \right] \cdot \frac{\partial \chi(K_t, \theta_{t+1}, \tau_{t+1})}{\partial K_t} \right\} \geq 0.$$

$\tau$  could be positive, zero, or negative. Sufficient conditions to ensure a zero subsidy are that both  $\frac{\partial \chi(K_{t-1}, \theta_t, \tau_t)}{\partial \tau_t}$  and  $\frac{\partial \chi(K_{t-1}, \theta_t, \tau_t)}{\partial K_{t-1}}$  are independent of  $\theta_t$ .

To confirm this claim, note first that the independence of  $\frac{\partial \chi(K_{t-1}, \theta_t, \tau_t)}{\partial \tau_t}$  on  $\theta_t$  implies

$$E_{\theta_t|\theta_{t-1}} [B_x(K_{t-1}, \theta_t, a_t) + \beta J_S^T(K_t, S_t, \theta_t)] = 0$$

by the first-order condition (14). This equation with the independence of  $\frac{\partial \chi(K_{t-1}, \theta_t, \tau_t)}{\partial K_{t-1}}$  on  $\theta_t$  implies  $\Gamma = 0$  so that the farmer's investment is social optimal.

From the first-order condition (3) for the optimal water usage, we can get

$$B_{xx}(K_{t-1}, \theta_t, a_t) dx_t + B_{xK}(K_{t-1}, \theta_t, x_t) dK_{t-1} - dp_t = 0.$$

It implies

$$\begin{aligned} \frac{\partial x_t^*}{\partial p_t} &= \frac{1}{B_{xx}(K_{t-1}, \theta_t, x_t^*)} \\ \frac{\partial x_t^*}{\partial K_{t-1}} &= -\frac{B_{xK}(K_{t-1}, \theta_t, x_t^*)}{B_{xx}(K_{t-1}, \theta_t, x_t^*)} \end{aligned}$$

The independence of  $\frac{\partial x_t^*}{\partial p_t}$  on  $\theta_t$  means the independence of  $B_{xx}(K_{t-1}, \theta_t, x_t^*)$  on  $\theta_t$ , which in turn requires the independence of  $B_{xK}(K_{t-1}, \theta_t, x_t^*)$  on  $\theta_t$  to satisfy

the independence of  $\frac{\partial x_t^*}{\partial K_{t-1}}$  on  $\theta_t$ . Thus the sufficient condition for zero-subsidy under taxes is equivalent to that both  $B_{xx}(K_{t-1}, \theta_t, x_t^*)$  and  $B_{xK}(K_{t-1}, \theta_t, x_t)$  are independent of  $\theta_t$ . Note that for the zero-subsidy under taxes, there is no requirement on the environmental damage functions. For a special case which satisfies the sufficient conditions, we have

**Proposition 2** *With the linear quadratic specification for both abatement cost function and environmental damage function, the firm's investment is socially optimal under both taxes and quotas. No investment subsidy is needed.*

## 4 Empirical Model

We illustrate the theoretical results of the preceding sections with a specific example based on data from cotton production in the San Joaquin Valley in California. We report data and results on a per acre basis. We discuss functional forms and data sources below, present results for the baseline model, and conduct sensitivity analysis.

Different irrigation technologies typically have different water application/infiltration uniformities. Following the literature (Dinar et al. [3][5], Letey and Dinar [15]), we use water application uniformity as a surrogate for irrigation technology and irrigation management activities, with a more advanced technology being associated with a higher uniformity. We measure the irrigation uniformity by the Christiansen Uniformity Coefficient (CUC). The set of CUC corresponds to a continuum of possible technologies.

CUC ranges from 0 to 100, with 100 representing perfect uniformity. Table 1 lists CUC values for five types of irrigation technologies, along with capital and operating and maintenance costs, and maximum physical system life. We calibrate parameters in the technology cost function using CUC and cost values. The best fit operating cost function and investment cost function are both linear:

$$\begin{aligned} c_1(K_t) &= c_{1,0} + c_{1,1}K_t, \\ c_2(I_t) &= c_{2,0} + c_{2,1}I_t. \end{aligned}$$

Table 1: Irrigation Technology Data

Irrigation Technologies	CUC <sup>a</sup>	Capital Cost, \$/ac	OM Cost, <sup>b</sup> \$/(ac yr)	Maximum Life, years
Furrow, 1/2-mile	70	219	7	12
Furrow, 1/4-mile	75	266	8	12
Hand move sprinkler	80	306	16	12
Linear move sprinkler	90	586	30	12
LEPA <sup>c</sup>	85	585	30	12
Subsurface drip	90	915	47	8

Source: University of California Committee of Consultants on Drainage Water

Reduction [17]

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<sup>a</sup>Christiansen uniformity coefficient.

<sup>b</sup>Operating and maintenance (OM) costs, including taxes, maintenance, and insurance.

<sup>c</sup>Low-energy precise application (LEPA) system.

We set the technology persistence  $\delta = 0.16$  so that the life expectancy is about 10 years. ( $0.16^{10} = 1.0995e - 008$ )

Assuming irrigation with nonsaline water, we use the production function parameters from Letey and Dinar (1986) relating crop yield and drainage quality, to the amount of applied water, its quality (salinity), and the water application uniformity, to develop the cotton production function and deep percolation function:

$$\begin{aligned} y_t &= f_0 + (f_1 + \theta_t) a_t - \frac{f_2}{2} a_t^2 + (f_3 + \kappa \theta_t) K_t - \frac{f_4}{2} K_t^2, \\ q_t &= h_0 + h_1 a_t + \frac{h_2}{2} a_t^2 + h_3 K_t + \frac{h_4}{2} K_t^2. \end{aligned}$$

We adopt a linear environmental damage function associated with drainage stock (Knapp [12][13][14]):

$$D_t = gQ_t$$

with  $g = \$15/AF$  from Loehman and Dinar [16].

For the baseline model, using values from Loehman and Dinar [16], we let output price  $p_t = \$0.75/lb$ , water price  $w_t = \$60/AF$ , and the annual discount factor  $\beta = 0.9512$ .

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