Structural Models of Area Yield Crop Insurance

April 2001

Bharat Ramaswami[#]
Indian Statistical Institute,
7, S.J.S. Sansanwal Marg,
New Delhi 110 016, India,
bharat@isid.ac.in

Terry L. Roe
Department of Applied Economics,
University of Minnesota,
1994 Buford Avenue,
St. Paul, MN, 55108,
U.S.A.
troe@apec.umn.edu

Abstract

Earlier analyses of area yield crop insurance schemes used a reduced form linear relationship between individual and area yield. However, without knowledge of the structural framework, the analysis and design of alternative schemes is problematic. This paper resolves this problem. The paper characterizes the entire class of structural models equivalent to the reduced form. As a result, the beta, the slope coefficient of the reduced form is expressed as a function of structural parameters. Second, the structural model is used to analyze the relation between the aggregation (that determines area yields) and the risk reduction due to area yield insurance. Third, we consider optimal area yield insurance for an important class of structural models not consistent with the reduced form.

Keywords: Area Yield, Beta, Crop Insurance, Systemic Risks.

JEL Classification: G22, Q14

#Corresponding author, Phone: (91)-11-6514594, Fax: (91)-11-6856779, Email: bharat@isid.ac.in

Selected Paper, American Agricultural Economics Association Meetings

Structural Models of Area Yield Crop Insurance

Introduction

A classic issue in agricultural economics is the design of schemes that would offer insurance against production risks in agriculture. The experience with conventional crop insurance has been disappointing as insurers have struggled to obtain reliable actuarial data on individual yields (Skees, Black and Barnett). The primary attraction of area yield insurance schemes is that insurers do not have to contend with the informational problems of moral hazard and adverse selection (Halcrow). These problems can be dismissed because indemnities and premiums are based not on a producer's individual yield but rather on the aggregate yield of a sorrounding geographical area. However, how good are they in reducing the risks faced by producers? What are the structural features of yield risk that determine this effectiveness and are these features important in design?

The answers provided by the literature build on a reduced form linear relationship between individual yield and area yield. The key parameter of the reduced form model is the beta which is the slope coefficient of the linear model. For an exogenously specified insurance contract, Miranda showed the extent of variance reduction for a producer to be proportional to that producer's beta. Consequently, for producers with large enough betas, the risk-reduction from area yield insurance plans may well outweigh the risk reduction from individual yield plans with large deductibles. Mahul showed that if insurance is actuarially fair, then a producer's optimal indemnity schedule contains no deductible and has slope equal to beta. However, the literature does not discuss, except as informal remarks, how the betas are determined. Why do some producers have higher betas than others? How would it depend on individual production functions? And would the level of aggregation matter?

To answer such questions, this paper provides the structure for the reduced form linear model. The structural model is a set of assumptions about individual yield functions. Area yields are obtained as an aggregate of individual yields. Under some conditions, individual yields can be expressed as a linear function of area yield. The paper characterizes the entire class of structural models that are consistent with the reduced form model. The characterization is valuable for several reasons. First, for any member of the general class of structural models consistent with the reduced form, the betas can be readily computed as a function of structural parameters, which can be producer and region specific. This provides insights into the relation between stochastic technologies and producer betas. Second, and as shall be shown, the structural model is useful for analyzing the relation between the level of aggregation (that determines area yields) and the risk reduction due to area yield crop insurance. Third, the characterization points to structural models that are *not* consistent with reduced form models. Previous results do not apply to these models. In this paper, we consider optimal area yield insurance for an important class of structural models that do not imply, and nor are implied by, the reduced form model.

Literature

Both Miranda and Mahul begin by assuming that the expectation of individual yield conditional on area yield is linear in area yield. Thus, we have

(1) $$y_i = m_i + b_i (y - m) + e_i$$

where y_i is producer i's yield, m_i is the unconditional mean of y_i, i.e., $E(y_i)$, y is area yield, $\boldsymbol{b}_i$ is the slope parameter satisfying $\boldsymbol{b}_i = Cov(y_i, y) / \boldsymbol{s}_y^2$, m is the unconditional mean of y and $\boldsymbol{e}_i$ is a mean zero random variable uncorrelated with area yield. Equation (1) decomposes individual yield variation into a systemic component $\boldsymbol{b}_i(y - m)$ perfectly correlated with area yield (since $Cov(\boldsymbol{b}_i(y - m), y)^2 / \boldsymbol{b}_i^2 Var(y)^2 = 1$) and a non-systemic component $\boldsymbol{e}_i$ uncorrelated with area yield. For reasons that will become clear later, we shall refer to (1) as a reduced form model.

Suppose the indemnity schedule is $I(y) = \max(y_c - y, 0)$ where y_c is a yield trigger fixed exogenously. Then Miranda showed

(a) The extent of variance reduction is proportional to $\boldsymbol{b}_i$ and other exogenous parameters that are the same across all farmers.

(b) It thus follows that more highly correlated a producer's yield is to the area yield, greater is the risk reduction.

Mahul considered the choice of an optimal contract $I(y)$. If insurance is actuarially fair, then the optimal contract is characterized by $I(y) = \boldsymbol{b}_i(y_m - y)$ where y_m, the yield trigger, is the maximum possible value of y.[1] Hence the slope of the optimal indemnity schedule is $-\boldsymbol{b}_i$. An aspect of this result, not noted by Mahul but relevant for us, is that the optimal indemnity schedule is independent of the non-systemic risk and its moments (such as $Var(\boldsymbol{e}_i)$).

[1] For the reduced form relation (1), Vercammen considers the optimal design of an area yield crop insurance contract when the yield trigger is constrained, for institutional reasons, to be below the maximum possible value of area yield.

Another implication is that optimal area yield insurance completely eliminates the systemic risk. To see this, note that a producer's revenue with insurance (denoted $\boldsymbol{p}$) is

$$\boldsymbol{p} = y_i + I(y) - P \tag{2}$$

where P is the premium. When a producer chooses the optimal area yield insurance, (2) becomes

$$\boldsymbol{p} = \boldsymbol{m}_i + \boldsymbol{b}_i(y - \boldsymbol{m}) + \boldsymbol{e}_i + \boldsymbol{b}_i(y_m - y) - P \tag{3}$$

where we have used (1). But when insurance is actuarially fair, $P = \boldsymbol{b}_i(y_m - \boldsymbol{m})$. Substituting in (3), we see that the producer bears only the non-systemic risk, i.e.,

$$\boldsymbol{p} = \boldsymbol{m}_i + \boldsymbol{e}_i$$

Thus optimal area yield insurance fully insures against the systemic risk. Since the optimal insurance is independent of the riskiness of the non-systemic risk $\boldsymbol{e}_i$, we have the result that the optimal area yield insurance delivers full insurance against the insured (systemic) risk whatever be the riskiness of the uninsured (non-systemic) risk.

We note a final result regarding the dispersion of betas. Miranda showed that the acreage weighted average of the betas within any area is always one. Hence

$$\sum_i w_i \boldsymbol{b}_i = 1$$

where w_i denotes the ratio of producer i's acreage to total acreage in the area.

A Structural Model of Systemic and Non-Systemic Risks

Consider a region R where there are n producers. Producer i's yield y_i, is given by

$$y_i = \boldsymbol{m}_i \boldsymbol{h}_i \tag{4}$$

where μ_i is producer i's mean yield and η_i is a unit mean random variable capturing the risks of farming. (4) is a standard specification of stochastic technologies where risks are multiplicative to mean yields. η_i is a linear combination of two independent shocks and is given by

(5) $$\eta_i = \alpha e_i + \gamma\theta$$

where e_i is a shock specific to i and θ is a shock common to all producers in region R. We therefore refer to e_i as the non-systemic or individual risk and θ as the systemic or aggregate risk. The individual and aggregate risks satisfy the following properties:

$E(\theta) = 1$, $Var(\theta) = \sigma_\theta^2$, $E(e_i) = 1$, $Var(e_i) = \sigma_e^2$, $Cov(e_i\theta) = 0$ for all i, and $Cov(e_i e_j) = 0$ for all $i \neq j$. To ensure the composite risk η_i has unit mean, we impose the restriction $(\alpha + \gamma) = 1$. Individual yields are, therefore,

(6) $$y_i = \mu_i(\gamma\theta + \alpha e_i)$$

We also assume that individual risks are independent of mean yields, i.e., $E(e_i \mid \mu_i) = E(e_i)$. This completes the description of the structural model.

The area yield for the region R is

$$y = \sum_i w_i y_i = \gamma\theta[(\sum_i w_i \mu_i) + \alpha \sum_i (w_i \mu_i e_i)]$$

where w_i denotes the area share of the ith producer. Let μ denote the mean area yield (i.e., average of the mean yields of producers). Then, $\mu = \sum_i w_i \mu_i$ and

(7) $$y = \gamma\theta\mu + \alpha \sum_i (\mu_i w_i e_i)$$

Now decompose $\sum_i (\mu_i w_i e_i)$ as

(8) $$\sum_i (m_i w_i e_i) = \sum_i w_i (m_i - m)(e_i - \bar{e}) + m\bar{e}$$

where $\bar{e} = \sum_i w_i e_i$ is the area average of individual risks. Note that the first term on the right-hand side of (8) is the sample covariance (weighted) between mean yields and individual risk. If the region contains a large number of producers, and if the law of large numbers applies, the sample covariance will approach (in probability) the population covariance (assumed to be zero). Similarly, $\bar{e}$ in large samples will be close to $E(e_i)$.

When $w_i = (1/n)$, it is straightforward to use the law of large numbers to obtain large sample results. In the case of weighted averages, however, a restriction on the weights is necessary. Essentially, we need to assume that the average yield is not dominated by the yield of any single producer. This requirement is automatically satisfied by the unweighted sum but needs to be explicitly assumed in the case of weighted sums.[2] Assuming this condition to be satisfied, we use large sample approximations to get

(9) $$\sum_i (m_i w_i e_i) = Cov(m_i, e_i) + mE(e_i) = m$$

Substituting in (7), area yield is

(10) $$y = [gq + a]m$$

---

[2] Consider $\sum_i a_i x_i$ where x_i is i.i.d with mean m and $\sum_i a_i = 1$. Then $E(\sum_i a_i x_i) = m$. By Chebychev's inequality, given any $d > 0$, Prob$[|\sum_i a_i x_i - m| > d] \le (Var(x_i)/d^2)\sum_i a_i^2$, the limit of which tends to zero as long as for every n, there exists a bound c such that $a_i \le c$ and $c(n) \to 0$ for large n.

Thus, area yield is random only because of aggregate systemic shocks as individual risks cancel out in the aggregate. Since area yield is a monotonic function of $\boldsymbol{q}$, the inverse function exists and is given by

$$\boldsymbol{q} = [y - m\boldsymbol{a}]/m\boldsymbol{g}$$

Substituting for $\boldsymbol{q}$ in (6), we obtain producer yield as a function of area yield, i.e.,

$$y_i = (m_i / m)(y - m\boldsymbol{a}) + m_i \boldsymbol{a} e_i \text{ or}$$

$$y_i = (m_i / m)(y - m) + m_i - m_i \boldsymbol{a} + m_i \boldsymbol{a} e_i \text{ or}$$

(11) $$y_i = m_i + (m_i / m)(y - m) + m_i \boldsymbol{a}(e_i - 1)$$

which is identical to the reduced form model (1) if we denote $(m_i / m) = \boldsymbol{b}_i$ and $m_i \boldsymbol{a}(e_i - 1) = \boldsymbol{e}_i$. Hence we have the following result.

Proposition 1: If the structural model is described by equations (4) to (6), then it has a reduced form representation (1) with the following relationships between the structural and reduced form parameters:

(a) $\boldsymbol{b}_i = (m_i / m)$

(b) $\boldsymbol{e}_i = m_i \boldsymbol{a}(e_i - 1)$

From part (a), we see that for any individual producer the $\boldsymbol{b}$ parameter is the ratio of that individual's mean yield to the mean of area yield. It follows immediately that $\sum_i w_i \boldsymbol{b}_i = 1$. This result was noted earlier by Miranda. From part (b), we see that the error term in the linear projection of individual yield on area yield is heteroscedastic. In particular, $Var(e_i) = m_i^2 a^2 s_e^2$ which varies across producers even though the non-systemic risk in the structural model is assumed to be homoscedastic.

In an empirical analysis of 102 cotton farms in Kentucky, Miranda observed that the distribution of the empirical betas possesses a regular, bell shape centred on 1. Proposition 1 says that this property is inherited from the distribution of average yields. Since the distribution of average yields depends on the dispersion of soil and climatic conditions in the region, Proposition 1 provides the formal basis for Miranda's conjecture that "..the more homogenous are the soil and climatic conditions faced by producers in a given area, the more closely the b_i 's will cluster around one." (pp 236). To this, we can add that the dispersion of betas will also depend on the heterogeneity in the other factors that determine yield such as management practices, farming skills and capital assets. In the extreme when all farmers have the same mean yield, they will also have betas identically equal to one. We now turn to the implications of our results for area yield insurance.

Proposition 2: Suppose the area-yield indemnity schedule is $I(y) = \max(y_c - y, 0)$ where y_c is a yield trigger fixed exogenously. Then for a given region, the extent of variance-reduction due to area yield insurance is directly proportional to mean yields.

Miranda showed the extent of variance reduction to be proportional to b_i. Since m is fixed for a given region, the result follows from Proposition 1. The implication is that if producers are restricted to insurance contracts as specified above, a producer would like to be grouped with other producers who have lower mean yields. Conversely, producers with low mean yields relative to the average will have little interest in area yield insurance.

However, if producers are allowed to choose optimal insurance plans then we know from Mahul's analysis that the yield trigger will be chosen to be the maximum area yield and the slope of the indemnity schedule (i.e., coverage) would be $-b_i$. The following result is therefore immediate.

Proposition 3: Producers with higher mean yields will choose higher coverage in an optimal area yield insurance plan.

A General Structural Model

The earlier section presented a structural model that led to the reduced-form equation (1) used in evaluations of area-yield insurance. However, more than one structural model might be consistent with the reduced form model in (1). In this section, we characterize the entire class of structural models that imply the reduced form model. We do this in two steps. Proposition 4 below identifies the class of structural models implied by the reduced form relation (1). Then in Proposition 5, we show that every member of this class implies (1). It thus follows that no structural model outside the class identified in Proposition 4 can imply reduced form model (1).

Suppose a general structural model of the form

$$y_i = f(\mathbf{z}_i, e_i, q)$$

where, as before, e_i and q are the random realizations of individual risk and aggregate shock and f is a function that maps the individual risk, the aggregate shock and a vector of parameters $\mathbf{z}$ into realized yields. In the previous section, $\mathbf{z}_i$ consisted of a single parameter m_i, the i'th producer's mean yield. Suppressing $\mathbf{z}_i$, we can write the model as

$$y_i = f_i(e_i, \boldsymbol{q}) \tag{12}$$

where the function f_i is now specific to producer i. If the relationship between individual yield and area yield is linear as in (1), then what restrictions must the function f_i satisfy?

Proposition 4: If the relationship between individual and area yields is described by (1), the structural model (12) necessarily satisfies the following:

(a) For all i, $y_i = f_i(e_i, \boldsymbol{q}) = h_i(e_i) + g_i(\boldsymbol{q})$ where h_i and g_i are functions that map non-systemic shocks and systemic shocks respectively into individual yields.

(b) For all i, there exists a function $k(.)$ and a parameter $\boldsymbol{l}_i$ such that,

$g_i(\boldsymbol{q}) = \boldsymbol{l}_i k(\boldsymbol{q}) + c_i$ where c_i is a constant of integration that possibly varies with i.

Proof: The structural model (12) satisfies

$$(\partial y_i / \partial e_i) = (\partial y_i / \partial \boldsymbol{e}_i)(\partial \boldsymbol{e}_i / \partial e_i)$$

But from the reduced form model (1), $\partial y_i / \partial \boldsymbol{e}_i = 1$. Hence

$$(\partial y_i / \partial e_i) = (\partial \boldsymbol{e}_i / \partial e_i)$$

Notice, that the reduced form model splits the variation in individual yields into variation in area yield y and an individual-specific risk $\boldsymbol{e}_i$. By assumption, y and $\boldsymbol{e}_i$ are orthogonal. It follows that area yield y is a function of $\boldsymbol{q}$ alone while $\boldsymbol{e}_i$ is a function of e_i alone. Hence

$$(\partial^2 y_i / \partial e_i \partial \boldsymbol{q}) = (\partial^2 \boldsymbol{e}_i / \partial e_i \partial \boldsymbol{q}) = 0$$

i.e., the cross-partial derivatives of (12) are zero. Since this can be true only if (12) is additive in the two risks, we have the result in part (a).

We now turn to the proof of part (b) of Proposition 4. Define the parameter d_i = $\partial y_i / \partial q$. d_i measures the sensitivity of producer *i*'s yield to aggregate shocks. Also define d as the sensitivity of area yield to aggregate shocks, i.e., $d = \partial y / \partial q$. Since

$\partial y / \partial q = \sum w_i (\partial y_i / \partial q)$, we have $d = \sum_{i=1}^{n} w_i d_i$. Now

$$d_i = \partial y_i / \partial q = (\partial y_i / \partial y)(\partial y / \partial q) = d(\partial y_i / \partial y). \tag{13}$$

Hence, for all *i*,

$$\partial y_i / \partial y = \frac{d_i}{d} \tag{14}$$

Fix a producer *j* and define, for all *i*, $l_i = (\partial y_i / \partial y)/(\partial y_j / \partial y)$. Clearly l_j is 1. Using (14) we obtain, $d_i = l_i d_j$. Using part(a) of Proposition 4, this can be written as

$$\partial g_i / \partial q = l_i (\partial g_j / \partial q). \tag{15}$$

l_i does not vary with the aggregate shock q. This can be seen from the reduced form model (1), where for all $i, \partial y_i / \partial y$ is a parameter that is independent of the realization of q. Integrating both sides of (14) with respect to q, we therefore find that, for all *i*, the structural model satisfies $g_i(q) = l_i g_j(q) + c_i$ where c_i is a constant of integration that varies with *i*. Since *j* is arbitrarily chosen, we define $k(q)$ to be $g_j(q)$. This proves part (b).

Proposition 4 specifies the class of structural models implied by equation (1). The next result shows that the relationship runs the other way too, i.e., every member of the class identified in Proposition 4 implies (1).

Proposition 5: The structural model in (12) has a reduced form representation as in (1) provided the structural model satisfies

(16) $$y_i = f_i(q, e_i) = a_i + b_i k(q) + h_i(e_i)$$

where $k_i(.)$ and $h_i(.)$ are monotone functions, a_i and b_i are parameters that possibly vary with *i*.

Proof: From (16), mean producer yield is

(17) $$m_i = a_i + b_i E[k(q)] + E[h_i(e_i)]$$

and area yield is

$$y = \sum w_i a_i + k(q) \sum w_i b_i + \sum w_i h_i(e_i)$$

Denote $a = \sum w_i a_i$ and $b = \sum w_i b_i$. Using the weak law of large numbers, $\sum_i w_i h_i(e_i)$ can be approximated in large samples by $E[h_i(e_i)]$. Hence

(18) $$y(q) = a + bk(q) + Eh_i(e_i)$$

Mean area yield is therefore

(19) $$m = a + bEk(q) + Eh_i(e_i)$$

Adding and subtracting m_i to the right-hand side of (16), we get

$$y_i = m_i + b_i[k(q) - Ek(q)] + [h_i(e_i) - Eh_i(e_i)]$$

where we have used (17). But from (18) and (19), $y - \mu = b[k(\theta) - Ek(\theta)]$. Denoting $(b_i / b) = \beta_i$ and $(h_i(e_i) - Eh_i(e_i)) = \varepsilon_i$, we get

$$y_i = \mu_i + \beta_i (y - \mu) + \varepsilon_i$$

where ε_i is a mean zero random variable uncorrelated with area yield.

From Propositions 4 and 5, we conclude that the class of structural models that satisfy (16) constitutes the *entire* class of structural models that has a reduced form representation (1). Depending on the choice of functions g and h_i, and the parameters a_i and b_i, there can be many special cases of (16). However, as we have seen, in all models satisfying (16), the beta parameter will be related to the structural parameters in the following manner.

Proposition 6: In the general structural model that is equivalent to the reduced form model in (1), the parameters satisfy

(a) $b_i / b = \beta_i$.

(b) $h_i(e_i) - Eh_i(e_i) = \varepsilon_i$

We may note couple of implications of Proposition 6. b_i measures the sensitivity of producer *i*'s yield to aggregate shocks while b is the sensitivity of area yield to aggregate shocks. Part (a) of Proposition 6 therefore states that β_i, the sensitivity of producer *i*'s yield to area yield is that producer's sensitivity to aggregate shocks relative to the sensitivity of area yield to aggregate shocks. Also recall that when area yield insurance is optimal, the producer bears only the risk ε_i. From part (b) of Proposition 6, it can be seen therefore that, with optimal area yield insurance, the variability of producer profits is $Var(h_i(e_i))$.

Special Cases

Given Proposition 6, it is easy to compute the betas for special cases. We consider a few specifications that are popular in the literature.

Case (a): $y_i = m_i(gq + ae_i)$

This is the multiplicative specification considered earlier. It is additive in the interaction of systemic and non-systemic shocks. Fix any j and define $k(q) = m_j gq$. Define b_i = (m_i/m_j) and $h_i(e_i) = m_i ae_i$. Then, individual yields can be written as $y_i = b_i k(q) + h_i(e_i)$, which is a special case of the structural model (16). Here, b = m/m_j. Applying Proposition 6, we can compute $\boldsymbol{b}_i$ as m_i / m.

Case (b): $y_i = m_i + e_i + q$

In this specification, risks are additive to mean yield. It clearly satisfies (16). Here $k(q) = q$, $b_i = 1$ and so $b = 1$. Hence $\boldsymbol{b}_i = 1$ for all i. Note this result obtains even though producers are heterogenous in mean yields. What is important for there to be heterogeneity in betas is heterogeneity in the way the aggregate shock affects mean yields.

Case (c): $y_i = m_i + s_i(q + e_i)$

This is the specification of a stochastic production function due to Just and Pope. This is also a special case of (16) where $k(q) = q$, b_i = s_i and therefore b = s where $s = \sum_i w_i s_i$

Therefore $\boldsymbol{b}_i$ = s_i/s.

Systemic Risks, Non-Systemic Risks and Aggregation

A design issue is the selection of the area that should be used as the basis for computing area yields. To maximize correlation of producer yield with area yield, it has been suggested that "the area or zone boundaries for an area yield contract should be selected so as to group together the largest possible number of farms with similar soils and climate" (Skees, Black and Barnett). To evaluate this recommendation, we turn to a structural model with different levels of aggregation.

Suppose producer yields can be averaged at two levels of aggregation.[3] For convenience, call the smaller aggregation as a cluster and the larger aggregation as a county. Yield of producer i in cluster c of county k is given by

(20) $$y_{ick} = \mu_{ick}\eta_{ick} \text{ where}$$

$$\eta_{ick} = \alpha_1 e_{ick} + \alpha_2\theta_{1ck} + \alpha_3\theta_{2k}$$

where e_{ick} is a shock specific to i, θ_{1ck} is a shock specific to all producers in cluster c of county k and θ_{2k} is a shock common to all producers in county k. In other words, e_{ick} is the individual risk, θ_{1ck} is the cluster-specific risk and θ_{2k} is the county-specific risk. The risks have unit means, constant variances and are stochastically independent. Also assume $\sum \alpha_i = 1$. This ensures the mean of y_{ick} is μ_{ick}. The individual risk e_{ick} is distributed independently of the individual mean yield μ_{ick}.

[3] Extension to many levels is straightforward.

Consider first area yield insurance schemes where the indemnity schedule is contingent on cluster yields. The average yield of cluster c in county k can be calculated as

$$\sum_{i\in c} w_{ick}\, y_{ick} = a_1 \sum_{i\in c} w_{ick}\, m_{ick}\, e_{ick} + (a_2 q_{1ck} + a_3 q_{2k}) \sum_{i\in c} w_{ick}\, m_{ick}$$

where w_{ick} is the share of the ith producer in the area of cluster c. Denote cluster c's yield as y_{ck} and its mean as m_{ck}. By arguments similar to that in preceding sections, substitute $\sum_i w_{ick}\, m_{ick}\, e_{ick}$ by its large sample approximation m_{ck}. Hence

(21) $$y_{ck} = [a_1 + a_2 q_{1ck} + a_3 q_{2k}] m_{ck}$$

Thus, cluster yields are random because of cluster-specific risk and county-specific risk. Area yield insurance schemes at the cluster level would therefore offer protection against both these risks. Write $q_k = (a_2 q_{1ck} + a_3 q_{2k})$. q_k denotes the systemic risk at the cluster level. Hence, for the cluster yield insurance scheme, we can write the equations of the structural model as

(22) $$y_{ick} = m_{ick}\, h_{ick} = m_{ick}\,(a_1 e_{ick} + q_k) \text{ and}$$

(23) $$y_{ck} = (a_1 + q_k) m_{ck}$$

Since (22) satisfies the structure of (16), the relationship between individual and cluster yields can be represented by a reduced form model like (1). In particular, we can write

$$y_{ick} = m_{ick} + b_{ick}\,(y_{ck} - m_{ck}) + e_{ick}$$

where, by Proposition 6, the beta of an individual producer can be computed $b_{ick} = m_{ick} / m_{ck}$. By the same proposition, the disturbance term in the linear model is $e_{ick} = a_1 m_{ick}\,(e_{ick} - 1)$. Because optimal area insurance (on the basis of cluster yields) eliminates systemic risk (at the cluster level), the variance of profits for a producer with insurance is $Var(e_i) = (a_1 m_{ick})^2 Var(e_i)$. The reduction in variance due to cluster yield insurance is $m_{ick}^2 Var(h_i) - Var(e_i) = Var(q_k)$.

Consider next area yield insurance schemes where the indemnity is contingent on county yield rather than cluster yield. The average yield of county k can be calculated by using (21) to average across clusters within the county. Hence

$$\sum_c w_{ck} y_{ck} = a_1 \sum_c w_{ck} m_{ck} + a_2 \sum_c w_{ck} q_{1ck} m_{ck} + a_3 q_{2k} \sum_c w_{ck} m_{ck}$$

where w_{ck} is the share of cluster c in area of county k. Denote y_k to be county yield and m_k to be its mean. Because q_{1ck} is a cluster specific risk, averaging across clusters should lead this risk be approximately equal to its expected value. Using this approximation and arguments similar to that in equations (7) to (9), $\sum_c w_{ck} q_{1ck} m_{ck} = m_k$. Substituting,

$$y_k = (a_1 + a_2 + a_3 q_{2k}) m_k$$

Denoting $a_1 + a_2$ as a, and $a_1 e_i + a_2 q_{1ck}$ as v_{ick}, the structural equations for the county yield insurance scheme are

(24) $$y_{ick} = m_{ick}(v_{ick} + a_3 q_{2k}) \text{ and}$$

(25) $$y_k = (a + a_3 q_{2k}) m_k$$

Compare (24) and (22). At the county level, the systemic risk is q_{2k} while it is q_k at the cluster level. The non-systemic individual specific risk changes too. At the county level, what is measured as the non-systemic risk is $a_1 e_{ick} + a_2 q_{1ck}$ while it is $a_1 e_{ick}$ at the cluster level. Higher aggregation reduces systemic risk and increases non-systemic individual specific risk. In the extreme, averages at the level of nation or group of nations may be so stable that the systemic risk component of a producer's yield might be close to zero. In such a case, all producer risk would be non-systemic individual specific risk.

(24) is additive in systemic and non-systemic risks and satisfies (16). Thus (24) can also be represented by a reduced form linear relationship (1) such that

$$y_{ick} = m_{ick} + \boldsymbol{b}_{ick}(y_k - m_k) + \boldsymbol{e}_{ick}$$

where, by Proposition 6, the beta is now $\boldsymbol{b}_{ick} = m_{ick} / m_k$ and the disturbance term is $\boldsymbol{e}_{ick} = m_{ick}(v_{ick} - \boldsymbol{a})$. Since optimal area insurance (on the basis of county yields) eliminates systemic risk (at the county level), the variance of profits for a producer with optimal county yield insurance is $Var(\boldsymbol{e}_{ick}) = m_{ick}{}^2 Var(v_{ick}) =$ $(\boldsymbol{a}_1 m_{ick})^2 Var(e_{ick}) + (\boldsymbol{a}_2 m_{ick})^2 Var(\boldsymbol{q}_{1ck})$. Consequently, the reduction in variance due to county yield insurance schemes is $Var(\boldsymbol{h}_i) - Var(\boldsymbol{e}_i) = m_i^2 \boldsymbol{a}_3^2 Var(\boldsymbol{q}_{2k})$.

Compared with the reduction achieved by cluster yield insurance, we see that the cluster yield insurance achieves an additional variance reduction of $Var(m_{ick}\boldsymbol{a}_2\boldsymbol{q}_{1ck})$. This happens because, while $\boldsymbol{q}_{1ck}$ is a systemic risk at the cluster level, it becomes a non-systemic risk at the county level and is therefore not insured by the county yield insurance scheme.[4] The division of producer risk into systemic and non-systemic risks is therefore dependent on the level of aggregation. Higher is the level of aggregation, greater are individual risks, smaller are systemic risks and hence smaller are the risk reduction impacts of area-yield insurance.

[4] It is easy to show that cluster yields are more correlated with producer yields than county yields.

Skees, Black and Barnett are right in emphasizing that farms with similar soils and climate should be grouped together. In terms of the structural model, such a grouping would face risks that do not cancel out in the aggregate and hence would qualify as systemic risks. However, what our analysis has pointed out is that more risks are likely to survive aggregation (and hence be regarded as systemic) when the farmer groups are as small as possible. Hence, for area yield insurance to have the maximum impact on risk reduction, the area boundaries for an area yield contract should be selected so as to group together the *smallest* (and not the largest) number of farms with similar soils and climate.[5]

The Multiplicative Case

Consider a structural model where, for a given level of aggregation, individual yields are described by

(26) $$y_i = \boldsymbol{m}_i \boldsymbol{h}_i \text{ and } \boldsymbol{h}_i = e_i \boldsymbol{q}$$

where the variables continue to have the same meaning and properties as before. The difference from (4) lies in the multiplicative interaction of risks. Such a specification is natural whenever the yield impacts of one risk depend on the realization of the other risk as well. For instance, even with a positive systemic shock due to say excellent rainfall, the impact on an individual producer's yield might be negligible because of a local risk such a pest or fungal infestation. Conversely, very adverse aggregate shocks could

[5] Informational problems aside, the smallest possible group consists of only a single producer. With the informational problems of moral hazard and adverse selection, the smallest group would be the minimum group size in which group outcomes are immune to the actions of any one individual. The smallest group size is therefore greater than one.

nullify a good outcome in terms of local risks. In an additive structure, on the other hand, the impact of rainfall is invariant to local risks and vice-versa.[6]

(26) does not satisfy (16) and thus does not possess a reduced form representation (1). Earlier results of Miranda and Mahul are therefore not applicable to (26). To see how the multiplicative structure makes a difference, refer to (4) as the additive model and (26) as the multiplicative model. As noted earlier, in the additive model, the slope of the optimal indemnity schedule is $-\boldsymbol{b}_i = (m_i / m)$ and is invariant to the non-systemic risk and its moments. Since at this level of insurance, all systemic risk is eliminated, it is optimal to fully insure against systemic risks in the additive model. To see whether these results extend to the multiplicative model, it is necessary to directly analyze the structural form (26) as the reduced form (1) is unavailable.

The area yield associated with (26) is $y = \boldsymbol{q} \sum_i m_i e_i$. By using large sample approximations, we can express area yield as

(27) $$y = m\boldsymbol{q}$$

Substituting in (26),

(28) $$y_i = (m_i / m) y e_i = \boldsymbol{b}_i y e_i$$

where we have denoted (m_i / m) by $\boldsymbol{b}_i$. Notice that, when the non-systemic risk is absent and is equal to its expected value 1, (28) is identical to the reduced form of the additive model (11). From the results that apply to the additive model, we therefore have that the insurance schedule satisfies $I'(y) = -\boldsymbol{b}_i$ whenever there is no non-systemic risk. Now

[6] For an analysis of multiplicative structures arising from the interaction of price and quantity risks, see Mahul (2000) and Ramaswami and Roe (1992).

suppose e_i is a random variable that takes values other than one with nonzero probability. Using (2), we can write producer i's revenue with insurance as

$$\pi_i = y_i + I(y) - P = \beta_i y e_i + I(y) - P$$

An actuarially fair optimal insurance contract maximizes expected utility of producer i subject to the break-even constraint of the insurers. Hence it solves

(29) $\underset{I(y)}{Max} \int_{e_i}\int_{y} U(\pi_i)dG(y)dF(e_i)$ subject to $P = \int_{y} I(y)dG(y)$

where U is an increasing, concave and thrice differentiable utility function, F is the cumulative density of the non-systemic shock, and G is the cumulative density of area yield derived from the probability distribution of the systemic shock θ (from (27)). Note that since area yield is a function of θ alone, it is distributed independently of the non-systemic risk.

Let λ be the Lagrange multiplier associated with the break-even constraint. Then the optimal function $I(.)$ satisfies for every y

(30) $\int_{e_i} U'(\pi_i) f(y)dG(e_i) = \lambda f(y)$

where $f(y) = dF(y)/dy$. Clearly (30) can also be written as

$$E[U'(\pi_i) \mid y] = \lambda$$

i.e., the optimal insurance equalizes the expected marginal utility in every state of area yield, y. Differentiating the first order condition with respect to y,

$$E[U'(\pi_i)(\beta e_i + I'(y))] = 0$$

from which we can solve for the slope of the indemnity schedule as

(31) $I'(y) = -\beta_i[1 + \dfrac{Cov(U''(\pi), e_i)}{EU''(\pi)}]$

$EU'' < 0$ and so the sign of $\dfrac{Cov(U''(\boldsymbol{p}), e_i)}{EU''(\boldsymbol{p})}$ is opposite to the sign of the covariance term. Since $\partial(U''(\boldsymbol{p}_i)/\partial e_i) = U'''(\boldsymbol{p}_i)\boldsymbol{b}_i y$, the covariance term is positive, equal to zero or negative as U''' is positive, zero or negative. A risk-averse agent with a positive third derivative of utility function has been referred to as prudent (Kimball,). It is easy to show that an agent with non-increasing risk-aversion must be prudent. U''' is zero for an agent with a quadratic utility function. Since constant or decreasing risk-aversion is a reasonable restriction on risk-averse behaviour, we concentrate below on the case when $U''' > 0$.

Proposition 8: If systemic and non-systemic risks interact multiplicatively, the optimal insurance for a prudent producer *I* satisfies $-I'(y) < \boldsymbol{b}_i$.

The proof is immediate from (31). Recall, that when non-systemic risk is absent, $-I'(y) = \boldsymbol{b}_i$. This can also be seen directly from (31). Thus, in the presence of an uninsured non-systemic risk, it is optimal for a producer to choose a lower level of coverage as compared to the case where non-systemic risk is absent. This is unlike the additive case where the demand for insurance against the systemic risk is unaffected by non-systemic risk.

To analyse local changes in risk, consider a one term expansion of U'' as

$$U''(\boldsymbol{p}) = U''(E(\boldsymbol{p})) + (\boldsymbol{p} - E(\boldsymbol{p}))U'''(E(\boldsymbol{p})) \text{ or}$$

$$U''(\boldsymbol{p}) = U''(E(\boldsymbol{p})) + \boldsymbol{b}_i y(e_i - 1)U'''(E(\boldsymbol{p}))$$

Substituting in (31),

$$I'(y) = -\beta_i [1 + \beta_i y Var(e_i) \frac{U'''(E(\pi))}{U''(E(\pi))}] \tag{32}$$

Greater is the riskiness of the non-systemic risk, smaller is the optimal coverage for a prudent producer. The demand for area yield insurance depends therefore on the uninsured non-systemic risks faced by an individual producer. As seen earlier, the classification of risks as either systemic or non-systemic changes with the area size used for computing area yields. In a multiplicative model, therefore, the demand for area yield insurance will depend on the level of aggregation at which area yields are determined. Since higher aggregations increase non-systemic risk, they would reduce the demand for area yield insurance.

To see this, denote I_1 and I_2 as the optimal insurance contracts at the cluster and county levels of aggregation. Suppose also that the mean yields of all producers are equal. Then $\beta_i = 1$, irrespective of the level of aggregation. In an additive model, the optimal coverage would satisfy $-I_1'(y_{ck}) = -I_2'(y_k) = 1$ where y_{ck} and y_k are cluster and county yields.

In a multiplicative model, individual yields, cluster yields and county yields are given by $y_{ick} = \mu_{ick} e_{ick} \theta_{1ck} \theta_{2k}$, $y_{ck} = \mu_{ck} \theta_{1ck} \theta_{2k}$ and $y_k = \mu_k \theta_{2k}$. Hence the non-systemic risk for cluster insurance is $e_{ick} \theta_{1ck}$ but is only e_{ick} for a county yield insurance. The variance of non-systemic risk is therefore greater with county yield insurance. From Proposition 8 and (32), it follows that the optimal coverage for a prudent producer satisfies $1 > -I_1'(y_{ck}) > I_2'(y_k)$.

Conclusions

From previous literature, we know that the extent of risk-reduction achievable by an area yield insurance plan is proportional to $\boldsymbol{b}_i$ which is the slope coefficient in a linear regression of individual yields on area yields. In this paper, such a relationship is derived on the basis of a structural model that described the interaction of individual non-systemic risks and aggregate systemic risks in determining individual yields. As a result, this paper was able to throw light on the structural determinants of the betas. The major insight is that the betas are determined by the sensitivity of individual yields to aggregate shocks *relative* to the sensitivity of area yields to aggregate shocks. In the special case when aggregate shocks affect all producers identically even when they are otherwise heterogenous, all producers have betas identically equal to 1. Comparison of betas across regions is therefore not meaningful.

The implications for policy are the following. Firstly, if the coverage in an area yield insurance plan is restricted, then it hurts producers who are the most vulnerable to aggregate shocks since they are the ones likely to have betas greater than the permissible coverage. Secondly, since the betas are not comparable across areas, a coverage restriction that is uniform across areas hurts those high risk producers who are unfortunate to find themselves grouped with other low risk producers. On the other hand, if all producers are prone to high risks, then their betas will be clustered around 1 and restriction of coverage levels to 100% of loss will not affect them.

If insurance coverage can be freely chosen, area insurance will eliminate systemic risks for all producers provided such risks interact additively. In this case, the risk reduction impacts of area insurance depend on the size of systemic risks which, in turn, depends on the level of aggregation. In general, smaller aggregations are preferable to larger areas, as more risks are likely to survive aggregation at lower levels. This consideration gains strength if risks interact multiplicatively. In such a set-up, area insurance does not eliminate all systemic risk. Moreover, the demand for insurance is not independent of the non-systemic risk. Greater is the non-systemic risk, lower is the demand for insurance. The feasibility of small area aggregations also depends on the size of farms. Smaller are farm sizes (as in developing countries), more feasible will be smaller area aggregations and hence greater will be the risk reduction impacts of area yield crop insurance.

Firms whose profits depend on area yield such as insurance companies seeking re-insurance or processors and firms that transact with large number of farms in a given area will benefit the most from area yield insurance. Since their operations encompass large number of farms, they are largely exempt from the non-systemic risks. Irrespective of whether the interaction of risks is additive or multiplicative, such firms will be able to use area yield insurance to fully eliminate systemic risks.

References

Halcrow, H. G. "Actuarial Structures for Crop Insurance", *Journal of Farm Economics*, 21 (May 1949): 418-43.

Just, R. E. and R. D. Pope, "Production Function Estimation and Related Risk Considerations," *American Journal of Agricultural Economics*, 61 (1979): 277-284.

Kimball, M, "Precautionary saving in the small and in the large," *Econometrica*, 58, (1990): 53-73.

Mahul, O., "Optimum Area Yield Crop Insurance", *American Journal of Agricultural Economics*, 81 (February 1999): 75-82.

Mahul, O., "Optimum Crop Insurance under Joint Yield and Price Risk," *Journal of Risk and Insurance*, 67 (March 2000): 109-122.

Miranda, M. J. "Area Yield Crop Insurance Reconsidered", *American Journal of Agricultural Economics*, 73 (May 1991): 233-42.

Ramaswami, B. and T. L. Roe, "Crop Insurance in Incomplete Markets", *Contributions to Insurance Economics,* ed. G. Dionne, Kluwer Academic Publishers, Boston, 1992.

Skees, J. R., J. R. Black and B. J. Barnett, "Designing and Rating an Area Yield Crop Insurance Contract", *American Journal of Agricultural Economics*, 79 (May 1997): 430-438.

Vercammen, James A., "Constrained Efficient Contracts for Area Yield Crop Insurance", *American Journal of Agricultural Economics*, 82 (November 2000): 856-864.