

A Dynamic Model of Microlending in the Developing Countries

Ani L. Katchova, Claudio Gonzalez-Vega, and Mario J. Miranda

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Ani L. Katchova is assistant professor in the Department of Agricultural and Consumer Economics at the University of Illinois at Urbana-Champaign, Claudio Gonzalez-Vega and Mario J. Miranda are professors in the Department of Agricultural, Environmental, and Development Economics at The Ohio State University.

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Abstract

In this paper, we examine the contract design problem of banks that extend loans to poor borrowers and seek to maximize outreach while remaining financially sustainable. A dynamic model is developed that shows how interest rates can be determined based on information about productivity and diligence characteristics of borrowers, investment opportunities, correlation of business activities, peer monitoring costs, and social sanctions. The results indicate that relative to the traditional static models, the dynamic model explains better the current experience in individual and group lending in developing countries.

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Poor households in developing countries have enjoyed extremely limited access to formal financial services. Greater access, however, would appear to be welfare-improving (Morduch). Potential suppliers of financial services, nevertheless, have historically encountered substantial difficulties in overcoming obstacles in this market segment and thereby profitably responding to latent demands (Chaves and Gonzalez-Vega).

Microfinance appears to be a solution to this challenge, where several types of microfinance organizations (MFOs), including banks, extend financial services to poor households while remaining financially sustainable. The success of microfinance programs has been attributed to several innovations in lending technologies (Navajas and Gonzalez-Vega). Most of these innovations have resulted from trial and error processes. The expectation is that the most successful innovations will pass the test of time.

The most commonly cited innovation is *group lending*, where members of a borrowing group accept joint liability for repayment of a loan. After group lending was shown to work reasonably well in a few places, standardization and replication became a priority for many MFOs. This lending technology became widely adopted by MFOs, including well-known organizations such as Grameen Bank in Bangladesh and BancoSol in Bolivia (Gonzalez-Vega *et al.*). Other MFOs, however, such as the BRI Unit Desa in Indonesia, Caja Los Andes in Bolivia, and Financiera Calpia in El Salvador, continued to develop individual lending technologies and successfully survived the systemic shocks of the turn of the century.

There is still much debate, in any case, about the relative merits of the two lending technologies. In developing countries, dominance of one lending technology over the other may depend on particular circumstances, unique to each country and region. Moreover, these innovations may not be sufficient to overcome the existing obstacles and, as a result, most banks and other formal financial intermediaries may neither reach these clienteles with financial services nor become financially sustainable when they attempt it.

Addressing these financial market problems would require further development of new lending technologies, based on economic models better tailored to the economic environment where MFOs operate. This paper develops models to show how a bank can determine optimal interest rates based on information about the productivity and diligence characteristics of borrowers, their investment opportunities, degree of correlation of their business activities, and levels of peer monitoring. The bank's objective is to extend loans to a large number of poor households (*breadth* of outreach), especially to the very poor (*depth* of outreach), while remaining financially sustainable (Yaron).¹

An important determinant of bank sustainability is the ability of the bank to correctly model borrower characteristics and behavior, in order to secure high repayment rates. While group lending is often associated with high repayment rates, forming groups will not always guarantee repayment. Two other mechanisms ensuring high repayment rates are dynamic incentives and regular repayment schedules (Gonzalez-Vega *et al.*; Morduch). A strong dynamic incentive is the credible threat to cut the borrowers off

¹ For an excellent discussion about other dimensions of outreach, see Navajas *et al.*

from any future lending when loans are not repaid. When borrowers lack access to credit from other sources, this mechanism may overcome information and incentive problems in both individual and group lending. Regular repayment schedules reduce monitoring costs for the lender (Navajas). Several papers have attempted to model borrowers' behavior under individual and group lending in an uncertain economic environment. These static models have taken into consideration moral hazard (Conning; Ghatak; Ghatak and Guinnane; Navajas), adverse selection (Ghatak and Guinnane; Navajas, Conning, and Gonzalez-Vega; Van Tassel), correlated returns of the borrowers' projects (Armendariz de Aghion), and strategic default, where borrowers are able but unwilling to repay their loans (Armendariz de Aghion; Ghatak and Guinnane).

The basic static model used in this paper was developed by Navajas for the case of simultaneous moral hazard and adverse selection in individual lending in a static framework. Following Armendariz de Aghion, we extend the basic static model to incorporate strategic default, correlated risks, and different group sizes in group lending.

A problem with static models is that they attempt to model dynamic interactions, such as dynamic incentives and regular repayment schedules, in a static framework. In this paper, we propose a new dynamic model of individual and group lending that incorporates moral hazard, adverse selection, strategic default, and correlated risks. The difference between the static and dynamic models here is that, in the dynamic model, borrowers take into account the outcomes of their projects and of their peers' projects before they make their decisions about strategic default. Static models provide useful statistics concerning the outcomes of borrower-bank interactions. Dynamic models are needed, however, if one wants to capture the more complex dynamic interactions that

underpin repayment behavior. We believe that static models are only a first step in the economist's attempt to understand strategic lender-borrower interactions. Despite this belief, a substantial fraction of this paper is devoted to static models. The reason for this detailed treatment is that many issues are more clearly and simply observable in the static model, while the complicated and computationally intensive treatment of the dynamic model is avoided. Using the static model, furthermore, extensive sensitivity analyses are performed for group size, the determinants of strategic default, and the degree of correlation of project returns. Finally, the results from the dynamic and the static models are compared and conclusions are drawn.

The Static Model

Individual liability contracts with moral hazard and adverse selection

This section describes the basic model developed by Navajas. Consider an individual liability loan contract between a borrower and a bank. The loan is of size I and the principal plus interest due next period is RI . It is assumed that repayment is an all or nothing decision, *i.e.*, the borrower either repays RI or nothing. The borrower uses the loan to invest in a one-period project that yields a random return $F(z,I)$ or, for simplicity, $zf(I)$, depending on the borrower's productivity type z and loan size I . It is assumed that if the borrower's project is successful, it will yield $zf(I)$, and that if the project fails, it will yield zero returns. Productivity type is assumed to be distributed as a uniform variable for the whole population. Adverse selection may occur if productivity type is not observable by the bank. The production function parameter f is assumed to be constant, as different loan sizes are not considered in this paper.

The borrower chooses a level of effort to exert once he takes the loan. If the borrower exerts high effort H , his project will be successful with probability P^H , and if he exerts low effort L , his project will be successful with probability P^L . It is assumed that $P^H > P^L$; *i.e.*, if the borrower exerts high effort, his chances of having a successful project are higher than if he exerts low effort.

When the borrower exerts low effort, he enjoys additional leisure benefits BI . The moral hazard problem arises as the borrower enjoys leisure benefits from exerting low effort and passes the costs of doing so to the lender as a lower probability of a successful outcome (Conning). The borrower may also choose not to apply for a loan (this choice is indicated as U), if his reservation utility Y is higher than the project returns less the loan repayment. The borrower maximizes his project returns less his repayment of the loan under both the high and low effort levels and compares them with his reservation utility:

$$(1) \quad \max_{H,L,U} \{P^H zf - P^H RI, P^L zf - P^L RI + BI, Y\} .$$

Given the loan size I and the interest rate $r=R-1$ offered by the bank, the borrower chooses whether to not take the loan U and, if he takes the loan, whether to exert high (H) or low (L) effort.

Joint liability contracts

In the following sections, the basic model for individual lending is extended to examine group lending. A joint liability loan is a contract between a bank and a group of borrowers which includes a joint responsibility default clause. If two borrowers are in a group, face independent project returns, and both of them exert high effort, then, with

probability $P^H(1-P^H)$, one of the borrowers will earn positive returns and will be able to repay the loan share of the other borrower when the latter earns zero returns. Each borrower maximizes his project returns less the repayment of his share of the loan and a fraction c of the shares of those other borrowers with zero returns. Given group size of n , the maximization problem is as follows (Ghatak):

$$(2) \quad \max_{H,L,U} \left\{ P^H z f - P^H R I - P^H (1 - P^H) c R I (n - 1), \right. \\ \left. P^L z f - P^L R I + B I - P^L (1 - P^L) c R I (n - 1), Y \right\}$$

The individual liability loan problem is a special case of the joint liability loan problem, when $c=0$.

Correlated returns

Consider a joint liability contract when the investment returns are correlated among the borrowers. Following Armendariz de Aghion, let $P_{s|s}^{HH}$ be the conditional probability that the first borrower, who exerts high effort, is successful and has positive returns, given that the second borrower, who also exerts high effort, is successful. Similarly, let $P_{s|f}^{HH}$ be the conditional probability that the first borrower, who exerts high effort, is successful and has positive returns, given that the second borrower, who also exerts high effort, has zero returns. The probabilities $P_{s|s}^{HH}$ and $P_{s|f}^{HH}$ are linked by the following equation:

$$(3) \quad P^H P_{s|s}^{HH} + (1 - P^H) P_{s|f}^{HH} = P^H .$$

Then, $P_{s|s}^{HH} = P_{s|f}^{HH} = P^H$ corresponds to the case where the borrowers' returns are not correlated and $P_{s|s}^{HH} > P^H > P_{s|f}^{HH}$ corresponds to the case where the borrowers' returns are positively correlated.

The borrower's problem is modified to reflect the possibility of correlated returns among the borrowers in the group:

$$(4) \quad \max_{H,L,U} \left\{ P^H z f - P^H P^{HH} RI - P^H (1 - P^{HH}) c RI (n-1), \right. \\ \left. P^L z f - P^L P^{LL} RI - P^L (1 - P^{LL}) c RI (n-1) + BI, Y \right\}.$$

For convenience, the subscript s/s is dropped from $P_{s/s}^{HH}$ for the rest of the paper.

Strategic default

Previous sections of this paper discussed the case in which a borrower is *unable* to repay his loan due to project failure and zero returns. Let us now suppose that the borrower is successful and can repay his loan but is *unwilling* to do so. A borrower has two choices: (a) he may repay his share of the loan and the share of the other borrowers who are unable or unwilling to repay their share, or (b) he may default, keep the returns from his project, and forfeit future access to credit. In order to gain future access to credit, the borrower needs to repay not only his share but also the shares of all other borrowers who do not repay.

The value that the borrower derives from being able to re-access future financing is denoted by V . This value is lost if the borrower does not repay his loan and the loans of the other defaulting members in the group. In the static model, the value V is assumed to be exogenous.. It is also assumed that the borrowers decide whether to repay or to strategically default *before* they realize whether their projects are successful.

Joint liability contracts are used to transfer screening and monitoring costs from the bank to the borrowers. Peer monitoring helps to deter strategic default if a borrower who strategically defaults is subject to social sanctions. Following Armendariz de

Aghion, let W denote the (private) cost that social sanctions impose upon a borrower who strategically defaults and let γ denote the probability that a borrower is monitored by another member in the group. The borrower solves the following optimization model:

$$(5) \quad \max_{N,D} \left\{ \begin{aligned} &P^H z_f - P^H P^{HH} RI - P^H (1 - P^{HH}) cRI(n-1) + P^H V, \\ &P^H z_f - P^H [1 - (1 - \gamma)^{n-1}] W + P^H P^{HH} V \end{aligned} \right\} .$$

The borrower can choose to not default (N), to repay his share and the share of the other defaulting members and gain the value of re-accessing future credit, or to strategically default (D) and keep his project returns, but incur social sanctions, and gain the value of re-accessing future credit only if another group member repays his share of the loan.

Note that the probability of being monitored $\Gamma(n) = 1 - (1 - \gamma)^{n-1}$ and, therefore, the decision of strategic default depend on group size n .

A comprehensive static model

The model developed in this section is a comprehensive model that includes all of the dimensions discussed in previous sections. The borrower has five choices: (1) take the loan, exert high effort, and, if he is successful, repay his share and the share of other defaulting members, (2) take the loan, exert high effort, and strategically default, if he is successful, (3) take the loan, exert low effort, and if he is successful, repay his share and the share of other defaulting members, (4) take the loan, exert low effort, and strategically default, if he is successful, and (5) not apply for the loan.

The borrower's problem becomes:

$$\begin{aligned}
(6) \quad & \max_{HN,HD,LN,ND,U} \left\{ P^H z f - P^H P^{HH} RI - P^H (1 - P^{HH}) c RI (n-1) + P^H V, \right. \\
& P^H z f - P^H \Gamma W + P^H P^{HH} V, \\
& P^L z f - P^L P^{LL} RI + BI - P^L (1 - P^{LL}) c RI (n-1) + P^L V, \\
& P^L z f - P^L \Gamma W + P^L P^{LL} V, \\
& \left. Y \right\}
\end{aligned}$$

This model incorporates the adverse selection problem arising from the fact that productivity type z is not observable by the bank. It also incorporates the moral hazard problem through the borrower's choice of either a high and low effort level and the resulting probabilities of success (P^H and P^L). The model applies to individual liability contracts, when $c=0$, and to joint liability contracts, when $c>0$. The question of how group size n affects decisions can also be addressed. The model can be applied to the agricultural sector, where usually returns are positively correlated ($P_{s|s}^{HH} > P^H > P_{s|f}^{HH}$) and to other sectors where returns are independent among borrowers ($P_{s|s}^{HH} = P_{s|f}^{HH} = P^H$). Finally, the model incorporates the decision of strategic default when social sanctions W are imposed on a borrower who, with probability Γ , is monitored by at least one of his peers.

Individual and joint liability contracts offered simultaneously

Suppose that the bank chooses to offer both individual and joint liability contracts with interest rates $r^i = R^i - I$ for the individual liability contract and $r^j = R^j - I$ for the joint liability contract. The borrower's choices increase to nine: high or low effort level, default or not default for both individual and joint liability contracts (a combination of eight choices), and not to take the loan. The borrower solves the following problem:

$$\begin{aligned}
(7) \quad & \max_{\substack{HN_i, HD_i, LN_i, ND_i, \\ HN_j, HD_j, LN_j, ND_j, U}} \left\{ \begin{aligned}
& P^H z_f - P^H R^i I + P^H V, \\
& P^H z_f - P^H \Gamma W, \\
& P^L z_f - P^L R^i I + BI + P^L V, \\
& P^L z_f - P^L \Gamma W, \\
& P^H z_f - P^H P^{HH} R^j I - P^H (1 - P^{HH}) c R^j I (n-1) + P^H V, \\
& P^H z_f - P^H \Gamma W + P^H P^{HH} V, \\
& P^L z_f - P^L P^{LL} R^j I + BI - P^L (1 - P^{LL}) c R^j I (n-1) + P^L V, \\
& P^L z_f - P^L \Gamma W + P^L P^{LL} V, \\
& Y \}
\end{aligned} \right.
\end{aligned}$$

Bank profits

The bank and the borrowers engage in the following sequential game (Ghatak). First, the bank offers an individual or a joint liability contract, specifying loan size I , interest rate $r=R-1$, and level of joint liability c . For simplification, $c=0$ or $c=1$, although the model can be easily extended for other degrees of joint liability. The bank can offer an individual liability contract, a joint liability contract, or both. Second, the borrowers decide whether to take the loan and, if they do, they make their choices regarding their level of effort and possible strategic default. Finally, the projects are carried out and outcome-contingent transfers as specified in the contracts are met.

This paper assumes that the bank's objective is to choose an interest rate that maximizes the borrowers' residual returns (project returns minus loan repayments), subject to the constraint that the bank's expected profit per loan is zero (Navajas; Conning; Ghatak; Ghatak and Guinnane). The maximization problem considered here extends Navajas' model of adverse selection and moral hazard in individual lending to incorporate correlated returns and strategic default in group lending.

Borrowers of different productivity type z choose whether to participate in the loan market and, if the loan is taken, they select different levels of effort and decide whether to strategically default. Let P^k be the probabilities of loans being repaid corresponding to the choices that borrowers of type z make, where

(8)

$$P^k = \begin{cases} P^H & \text{if the borrower exerts high effort and does not default} \\ P^L & \text{if the borrower exerts low effort and does not default} \\ P^H P^{HH} & \text{if the borrower exerts high effort and defaults but his partner repays for him} \\ P^L P^{LL} & \text{if the borrower exerts low effort and defaults but his partner repays for him} \\ 0 & \text{if the borrower defaults on a single liability loan or does not take a loan} \end{cases}$$

Suppose that productivity type z in the population is uniformly distributed on the interval zero to one, with a distribution function $G(z)=\text{uniform}[0,1]$.² The optimization problem determines the choices that borrowers of different type z make regarding taking loans, exerting high or low effort, and defaulting strategically. Let z^k be the breakpoints at which borrowers of different type change their choices. If a borrower of type z chooses not to take a loan, then $G(z)=0$. The number of borrowers β as a proportion of the population is given by:

$$(9) \quad \text{borrowers} = \sum_{k=1}^K [G(z^k) - G(z^{k-1})] \equiv \beta$$

where $0 \leq \beta \leq 1$.

² Navajas; Navajas, Conning, and Gonzalez-Vega claim that productivity type is distributed as a standard normal variable (this is equivalent to assuming that project returns are zero on average). Since we assume that project returns are non-negative and, on average positive, a uniform distribution of this variable is more appropriate.

For each choice of participation, level of effort, and strategic default, there is a corresponding probability of repaying the loan P^k and a proportion of borrowers β^k having made the same choice ($\sum_{k=1}^K \beta^k \equiv \beta$). Assuming that there are no partial loan collections and that group members receive loans of equal size, the default rate for the bank is calculated as the probability that a borrower will be unwilling or unable to repay his loan ($1-P^k$), weighted by the proportion of borrowers β^k having made the same choice.

$$(10) \quad \text{default rate} = \frac{\sum_{k=1}^K (1-P^k) \beta^k}{\beta}$$

The bank obtains outside funding with interest rate ρ and has total handling costs m , which are diluted among all borrowers. This is the number of borrowers as a proportion of the population β times the number of agents in the population b . The bank sets the lowest interest rate such that its operations are financially sustainable, *i.e.*, the bank's expected profit per loan is zero:

$$(11) \quad \frac{\sum_{k=1}^K P^k \beta^k RI}{\beta} - (1 + \rho)I - \frac{m}{b\beta} = 0.$$

This equation represents bank profits as repayments on the loans minus the interest rate paid on bank funds and the average (per borrower) handling costs that the bank incurs. The first term is the repayments (RI) received from the borrowers, weighted by their probability of success for the corresponding level of effort and by their default decision, added over the number of borrowers having made the same choice, as a proportion of the total number of borrowers. The second term is the bank's cost of obtaining outside

financing, at interest rate ρ . The third term is the total handling costs for the bank divided by the number of borrowers. The bank chooses an interest rate $r = R - 1$ such that its profits are zero. The zero profit condition can be generalized to the case in which the bank offers both individual and joint liability contracts with interest rates $r^i = R^i - 1$ and $r^j = R^j - 1$, respectively.

Dynamic Model

In the static model, the value that a borrower derives from being able to re-access future credit was assumed to be exogenous. It was also assumed that the borrower decides whether to repay or strategically default *before* he observes whether he has been successful. In this section, we develop a dynamic model, where the value of re-accessing future credit is assumed to be endogenous and where the borrower decides whether to repay or to strategically default *after* he observes whether his project has been successful.

Consider the following dynamic two-stage, two-person game.³

Stage 1

In the first stage, the borrowers are either *in* (never defaulted on a loan before) or *out* (defaulted on a loan before). They choose whether to not take the loan U , take the loan and exert low effort LL , or take the loan and exert high effort HH . The first index refers to the first borrower and the second index refers to the second borrower. Only the symmetric cases of HH (high effort and high effort) or LL (low effort and low effort) are

³ Rodriguez-Meza also discusses a case of strategic default in a dynamic setting. His model and implications, however, are different from the model and implications in this paper.

considered; otherwise, each borrower has an incentive to choose a low level of effort for himself and a high level of effort for the other borrower. States, actions, and transition probabilities are defined as follows:

$$\begin{array}{l}
 \text{state} \left\{ \begin{array}{l}
 \textit{in} - \text{never defaulted on a loan before} \\
 \textit{out} - \text{defaulted on a loan before}
 \end{array} \right. \\
 \\
 \text{action} \left\{ \begin{array}{l}
 \textit{U} - \text{do not get a loan, get a reservation utility} \\
 \textit{LL} - \text{get a loan and exert low effort} \\
 \textit{HH} - \text{get a loan and exert high effort}
 \end{array} \right. \\
 \\
 \text{transition probabilities} \left\{ \begin{array}{l}
 P^{HH}(ss) \text{ probability of } ss \text{ if } HH \\
 P^{HH}(sf) \text{ probability of } sf \text{ if } HH \\
 P^{HH}(fs) \text{ probability of } fs \text{ if } HH \\
 P^{HH}(ff) \text{ probability of } ff \text{ if } HH \\
 P^{LL}(ss) \text{ probability of } ss \text{ if } LL \\
 P^{LL}(fs) \text{ probability of } sf \text{ if } LL \\
 P^{LL}(sf) \text{ probability of } fs \text{ if } LL \\
 P^{LL}(ff) \text{ probability of } ff \text{ if } LL \\
 1 \quad \text{probability of } out \text{ if } out \\
 0 \quad \text{probability of } in \text{ if } out
 \end{array} \right.
 \end{array}$$

The probability $P^{HH}(ss)$ is the joint probability of the state ss (both the first and the second borrowers are successful) in the second stage, given that HH (both the first and the second borrowers exert high effort) were chosen as actions in the first stage. The same logic applies to the other probabilities. Given that the borrower is out this period, the probability of being out next period is 1, and the probability of being in next period is 0.

Stage 2

In the second stage, the borrowers observe the outcomes of their projects (success or failure) and then decide whether to default. If at least one of the borrowers decides not to default and repay the loan, then the probability of them being *in* next period is 1. If both borrowers decide to default, then the probability of them being in state *out* next period is

$$\text{state} \begin{cases} ss - \text{success and success} \\ sf - \text{success and failure} \\ fs - \text{failure and success} \\ ff - \text{failure and failure} \end{cases}$$

$$\text{action} \begin{cases} NN, ND, DN, DD \text{ if } ss \\ ND, DD \text{ if } sf \\ DN, DD \text{ if } fs \\ DD \text{ if } ff \end{cases}$$

$$\text{transition probabilities} \begin{cases} 1 & \text{probability of } in \text{ if } NN, ND, DN \\ 1 & \text{probability of } out \text{ if } DD \\ 0 & \text{otherwise} \end{cases}$$

Here N is not default, and D is default. Again, the first index refers to the state or action of the first borrower and the second index refers to the state or action of the second borrower. Note that some actions are not applicable in some states. If the state is f (failure), the only possible action is D (default). Therefore, there are 16 possible combinations of actions, contingent on the states occurred.

Bellman's equation for stage 1

The value of being in state *in* is the maximum value of either taking the reservation utility and staying out of the loan market this period, or taking a loan, choosing a high or low effort level, and receiving the value of being in different states in stage 2 (ss, sf, fs, ff).

$$(12) \quad V(in) = \max_{U, LL, HH} \left\{ P^{HH}(ss)V(ss) + P^{HH}(sf)V(sf) + P^{HH}(fs)V(fs) + P^{HH}(ff)V(ff), \right. \\ \left. P^{LL}(ss)V(ss) + P^{LL}(sf)V(sf) + P^{LL}(fs)V(fs) + P^{LL}(ff)V(ff), \right. \\ \left. Y + \delta V(in) \right\}$$

The value of being in state *out* is the reservation utility and remaining in state *out* next period.

$$(13) \quad V(out) = Y + \delta V(out)$$

Bellman's equation for stage 2

A Bellman's equation captures the borrower's problem of balancing the immediate reward of defaulting against future rewards of being able to re-access the loan market.

$V(ss)$ is the maximum attainable sum of current and expected future rewards, given that the borrowers are currently in the state success and success. If the borrowers are in state *ss* (success and success), they have four choices: both of them repay their shares, one of them repays both loans when the other strategically defaults, or both of them default and are removed from the loan market. The same logic applies for the other states. The rest of the notation is the same as in the static model, where

$$(14) \quad V(ss) = \max_{NN, ND, DN, DD} \left\{ zf - RI + \delta V(in), zf - R(1+c)I + \delta V(in), \right. \\ \left. zf - \gamma W + \delta V(in), zf - \gamma W + \delta V(out) \right\}$$

$$(15) \quad V(sf) = \max_{ND, DD} \left\{ zf - R(1+c)I + \delta V(in), zf - \gamma W + \delta V(out) \right\},$$

$$(16) \quad V(fs) = \max_{DN, DD} \left\{ \delta V(in), \delta V(out) \right\}, \text{ and}$$

$$(17) \quad V(ff) = \delta V(out).$$

Note that in the dynamic model, the choice of whether to default depends on the success or failure of the borrowers, whereas, in the static model, the choice to default or not is made prior to the realization of the success or failure state.

Equations (14), (15), (16), and (17) are substituted into equation (12), so that $V(in)$ is a function of only $V(in)$ and $V(out)$. Equations (12) and (13) are functional equations for $V(in)$ and $V(out)$ and can be solved using a dynamic programming approach. Given a loan of size I , an interest rate $r=R-1$ offered by the bank, and a level of joint liability c , the borrowers choose whether to take the loan, exert high or low effort, and whether to default based on the current state of being *in* or *out* in stage 1 and success or failure in stage 2.

Bank profits

The analysis of bank profits in the dynamic model is the same as in the static model, except for the probabilities associated with different choices. Let P^k be the probability corresponding to the choices made by borrowers of type z . Then, $P^k = P^k(ss) + P^k(sf) + P^k(fs) + P^k(ff)$. That is,

$$(18) P^k(xx) = \begin{cases} P^{HH}(xx) & \text{if the borrowers exert high effort and at least one does not default} \\ P^{LL}(xx) & \text{if the borrowers exert low effort and at least one does not default} \\ 0 & \text{if both borrowers default or do not take a loan} \end{cases}$$

where xx is ss , sf , fs , or ff . The rest of the analysis is the same as in the static model.

Solving the Models

The static and dynamic models described above are solved using a three-step procedure (Ghatak). First, the bank offers a loan contract of size I with interest rate $r=R-1$ and a level of joint liability $c=0$ or $c=1$. The bank can offer an individual liability contract, a joint liability contract, or both. Second, the borrowers select different contracts and decide what level of effort to exert and whether to default or not. Third, the bank takes into consideration the choices of the borrowers and calculates profits. The bank sets the interest rate, the joint liability level, and whether it offers one or both contracts, such that its operations are financially sustainable.

Table 1 shows the parameters used in the static and the dynamic models. The parameters corresponding to the static model with moral hazard and adverse selection for individual lending are from Navajas, with some modifications to fit our model. The rest of the parameters are set using economic intuition. Sensitivity analyses and robustness checks are performed for different values of the parameters. The parameters are the same for both the static and the dynamic models, with the exception of group size and the value of re-accessing future credit. The value of re-accessing future credit is assumed to be exogenous in the static model and endogenous in the dynamic model. In the dynamic model, group size is either 1 (individual liability) or 2 (joint liability). Building a dynamic model with more players significantly complicates the game, as each action is state contingent.

Static Model Results

The static optimization model is solved using the Matlab software. The next section presents the basic results and the following sections perform sensitivity analyses on the parameters used in the model.

Individual and joint liability contracts

Suppose that the bank offers either an individual liability contract or a joint liability contract but not both contracts. Table 2 shows the interest rate required for bank sustainability, the number of borrowers reached, the default rates, the borrowers' decisions of participating in the credit market, and their choices of effort levels.

Given these parameters, if bank operations are to be sustainable, the bank must charge interest rates of 61 percent on the individual liability contracts, 37 percent on the joint liability contracts with independent returns, and 54 percent on the joint liability contracts with correlated returns. Thus, the interest rate required for bank sustainability is lower for group lending than for individual lending. The bank can offer lower interest rates on joint liability contracts because those borrowers with failed projects may use other members of the group to repay their loan shares, an option not available under individual liability contracts. That is, there is risk sharing between the bank and repaying group members.

The crucial assumptions for this result are that group lending does not destroy the borrowers' incentives, by leading to low effort levels (moral hazard) and/or to a desire to free ride on other group members either in monitoring their peers or for loan repayment (strategic default). Ghatak formally shows that the interest rate under group lending will

always be lower than the interest rate under individual lending in the absence of moral hazard and strategic default. This may no longer be the case if these assumptions are abandoned.

The results for group lending with correlated returns will always be between the results for the two extreme cases: group lending with independent returns and individual lending, which can be thought of as group lending with perfectly correlated returns. The interest rate that the bank must charge in group lending where returns among borrowers are independent is lower than in group lending where returns are correlated. The bank can offer lower interest rates when returns are independent because, when a borrower's project fails, the chance that another borrower's project will fail is lower for independent than for positively correlated project returns. The finding that the interest rate in group lending may be lower than in individual lending does not necessarily mean that all borrowers will be better off under group lending (Ghatak). When making decisions about taking loans, borrowers not only consider differences in repayment costs associated with interest rates. In contrast to individual credit, in group lending borrowers must also balance the benefit of having someone else repay their loan share (the benefit of being partially insured against default) and the cost that they may incur to repay the loan shares of other borrowers in case of their inability or unwillingness to repay loans (the cost of insuring other members against default). They will also incur peer monitoring costs inexistent in individual loans.

In fact, under the parameters of the model, for low productivity borrowers with low project returns, the cost of insuring others is higher than the benefit of being insured plus the benefit of paying a lower interest rate. These agents will choose not to take joint

liability loans, even if they would have taken individual liability loans at higher interest rates.

Both the breadth and depth of outreach are lower in group lending, since the lowest productivity borrowers willing to participate in individual lending will not to take group loans. The number of borrowers reached by the bank is highest in the case of individual lending (90 percent of the population), lower for group lending with correlated returns (87 percent), and lowest for group lending with independent returns (80 percent). Equivalently, 10 percent (3 percent) of the population, precisely the lowest productivity borrowers who would take individual loans will not take group loans when their returns are independent (correlated). This result seems to contradict the usual observation that poorer borrowers take group rather than individual loans. The reason may be that individual lenders require some form of non-traditional collateral beyond the expectation of a future loan. The cost of insuring others against default versus the benefits of being insured against default also influences the default rates for the bank. The default rates for the bank are 28 percent for the individual liability contracts, 25 percent for the joint liability contracts with correlated returns, and 15 percent for the joint liability contracts with independent returns. Group lending has a positive effect on repayment rates, resulting from the possibility that a successful borrower may repay the loan of a defaulting partner. There is also a potential negative effect, which arises if the entire group defaults, when at least some members would have repaid if they were not burdened with the liability of other members (Besley and Coate). For group lending with two borrowers, as long as moral hazard is not increased by group lending, the positive effect dominates the negative effect, and repayment rates are higher for group compared to

individual lending. This may not be the case when joint liability influences the possibility of strategic default.

The borrowers self-select into the following groups: non-applicants, for agents with low levels of productivity; borrowers who exert low effort and do not default, for agents with medium levels of productivity; and borrowers who exert high effort and do not default, for agents with high levels of productivity. This is a standard incentive compatibility result, due to the assumption of different productivity types. Low productivity agents prefer to enjoy their reservation utility, medium productivity agents prefer to borrow but exert low effort and enjoy leisure benefits, and high productivity agents prefer to take a loan and exert high effort, because their returns are most sensitive to an increase in effort. Generally, the results found in Navajas and here, regarding the borrowers' choices of participation and exerted effort, for the case of individual lending also extend to group lending.

Group lending may not only attract more productive borrowers than individual lending but, in small groups, it may also reduce moral hazard, due to increased monitoring. The proportion of the population that takes loans and exerts low effort is 55 percent in individual lending, 43 percent in group lending with correlated returns, and 14 percent in group lending with independent returns. As shown below, this result does not hold in large groups.

Sensitivity analysis on strategic default

The main reason for the success of group lending is often attributed to peer monitoring. Compared to the bank, borrowers have a comparative advantage in monitoring their peers

and enjoy a superior enforcement technology, as they can impose social sanctions to peers who strategically default. If a borrower is caught in strategically defaulting, he will incur social sanctions W with a probability Γ that he is being monitored.

If the social sanctions for default and/or the monitoring probability are low, then the borrower will strategically default. For example, if the social sanctions decrease from \$200 (as was assumed in the previous section) to \$100 or, equivalently, if the monitoring probability decreases in half, all group members will default and the bank's operations will not be sustainable at any interest rate. Therefore, bank group lending operations can only be sustainable in communities with high social sanctions and high probabilities of peer monitoring (low transaction costs of monitoring). Moreover, if the social sanctions are sufficiently severe, group lending will yield higher repayment rates than individual lending (Beasley and Coate).

Sensitivity analysis on correlated returns and moral hazard

When strategic default is prevented by high social sanctions, individual lending can be considered as group lending with perfectly correlated returns. Therefore, as the correlation of borrower returns increases from zero to one, the results trace all values between the two extreme cases of group lending with independent returns and individual lending.

Since it was found here that lower interest rates are required to ensure sustainability in group lending than in individual lending, an increase in the degree of correlation in the borrowers' project returns, as is typically the case in agriculture, will lead to higher interest rates being charged. The ambiguity found in the literature on this

issue is not found here. In general, the degree of positive correlation has an ambiguous effect on the interest rates charged by the bank (Armendariz de Aghion). On the one hand, an increase in correlation increases the borrower's incentive to invest in monitoring efforts, since a borrower will often try to avoid having to repay the debt of his peer. On the other hand, a higher correlation will increase the borrower's incentive to strategically default, for a given monitoring probability, and instead rely on his peers to repay his debt.

An increase in the probability of success of the project, when the borrower exerts high effort, tends to encourage diligence and this, in turn, leads to lower interest rates and lower default rates and to increased bank outreach. For example, if the probability of success P^H increases from 0.9 to 1, the interest rate declines to 29 percent (16 percent), the default rate declines to 11 percent (zero percent), and the number of borrowers increases to 100 percent (95 percent) for individual lending (group lending).

If the probability of success of the project, when a borrower exerts low effort, increases, it becomes more beneficial for borrowers to exert low effort, because of the leisure benefits that they obtain. In general, the effect on interest rates and on default rates of an increase in P^L is ambiguous. The higher probability of success reduces default rates for borrowers exerting low effort, but more borrowers will shift from exerting high effort to exerting low effort.

Sensitivity analysis on group size

Armendariz de Aghion shows that, when group size increases, there are four effects that influence the decision of strategic default for the members of the group. A free-riding effect discourages individual monitoring efforts in a larger group, due to the increased

probability that at least one borrower will be successful and able to repay for defaulting members. A joint-responsibility effect encourages more intense monitoring, since monitoring will not only reduce the possibility of having to repay for a peer but will also force that peer to repay for other defaulting borrowers. A cost-sharing effect increases monitoring, since the cost of monitoring another borrower declines in larger groups, given that monitoring is shared among an increasing number of peers. Finally, a commitment effect encourages peer monitoring, because each borrower becomes more fearful about strategic default by peers, since he has to repay for a larger number of defaulting borrowers.

As table 3 shows, a larger group size initially increases monitoring and risk sharing within the group. As this prevents defaults and reduces moral hazard (the choice of low level of effort), the bank offers a lower interest rate. The bank is able to charge the lowest interest rate, of 30 percent, for group size of three borrowers with independent returns and for group size of seven borrowers with correlated returns.

As group size increases even further, however, the free-riding effect starts to dominate and the bank has to charge increasingly higher interest rates. For a group of six borrowers with independent returns and a group of 11 borrowers with correlated returns, the free-riding effect becomes so adverse that all borrowers default and the bank's operations are not sustainable. Therefore, the relative benefits from monitoring and risk sharing are maximized when group size is neither too small, due to a joint-responsibility, cost-sharing, and commitment effects, nor too large, due to a free-riding effect (Armendariz de Aghion).

Individual and joint liability contracts offered simultaneously

The results regarding the sustainability and outreach of the bank are valid only if the bank offers either individual or joint liability contracts but not both of them simultaneously. If the bank (or two banks competing for the same clients) offers both individual and joint liability contracts at the same time, the borrowers may choose the type of contract.

Suppose that the bank offers both individual and joint liability contracts, at the interest rates required for sustainability for each type of contract, if they were offered separately. The borrowers decide whether to take the loan and, if they take the loan, they self-select into one of the two contract types and make their choices regarding the effort to be exerted and the default decision. Given the decisions that the borrowers make, the bank recalculates its profits.

If the borrowers are given a choice between individual liability contracts and joint liability contracts, with group size smaller than four (nine) borrowers for the case of independent (correlated) returns, the bank operations are no longer sustainable. The reason is that, on the one hand, for high productivity borrowers, the benefit of being insured against default is greater than the cost of insuring peers against default, causing them to choose the joint liability contracts. On the other hand, for the low productivity borrowers, however, the cost of insuring others against default is greater than the benefit of being insured against default, and therefore they choose the individual liability contracts. The bank operations cannot be sustainable for reasons similar to the lemon's market argument – the high productivity potential borrowers no longer choose the individual liability contracts, so that the interest rate no longer reflects the underlying risk profiles of the borrowers.

In contrast, if the borrowers are given a choice between individual liability contracts and joint liability contracts with group size larger than four (nine) borrowers for the case of independent (correlated) returns, the bank operations are sustainable but only one type of contract will emerge. Borrowers of all productivity types will self-select into the individual liability contract only. As discussed in the previous section, borrower incentives to default and to free ride on their peers increase in larger groups. In larger groups, high productivity borrowers choose individual liability contracts, because of the increased costs of insuring peers. The bank operations are sustainable, since the interest rate was set to make the individual liability contracts sustainable, if offered separately.

Suppose that there are two banks. One of the banks offers an individual liability contract and the other bank offers a joint liability contract. Each bank sets the interest rate so that its bank operations are sustainable, ignoring the presence of the other bank. A problem appears when the two banks compete for clients. Borrowers of different productivity types self-select into one of the two contracts and thus alter the pool of borrowers that the bank would face if the other bank did not exist (Navajas, Conning, and Gonzalez-Vega). Therefore, banks that are in competition with each other for clients need to set their interest rates taking into consideration how the borrowers will be distributed between the two banks.

Dynamic Model Results

The dynamic model developed in this paper makes a realistic assumption that the value of future access to credit is endogenous (Rodriguez-Meza). Borrowers have the option of defaulting and keeping their project returns but losing the value of being able to re-access

the market for future credit. This value of future credit is found from the optimization problem to be an increasing function of the borrower's productivity type. In the static model, the value of future credit was assumed to be constant. Even though the parameter values in the static and the dynamic models are the same, the results are different. This is due to the assumption concerning the value of future credit and the time when the decisions to default are made (before or after the realization of the success or failure states).

Individual liability contracts

In a dynamic setting, the interest rate required for bank sustainability in individual lending is 67 percent and the number of borrowers is 98 percent of the population (table 4). The results in the dynamic model are somewhat similar to the results in the static model; however, a direct comparison is not possible, since this would require testing for the underlying assumptions of the models rather than comparing the results.

Agents with low productivity do not take loans and enjoy their reservation utility. Agents with medium productivity take loans and exert low effort. Agents with high productivity take loans and exert high effort. In the dynamic model, the borrowers' decisions are state contingent. If a borrower's project is observed to be successful, the borrower does not strategically default and repays his loan. If a borrower's project fails, the borrower defaults because of inability but not of unwillingness to repay his loan. These results represent a dynamic version of the static model results where borrowers do not strategically default.

Joint liability contracts

In a dynamic setting, the interest rate required for bank sustainability in group lending is 37 percent for borrowers with independent returns and 55 percent for borrowers with correlated returns (table 4). The interest rates in group lending are lower than the interest rates in individual lending because the bank can diversify risk across borrowers.

Given the values of the parameters, the number of borrowers is 100 percent of the population for group lending (table 4). This result simply implies that if a bank opens operations in a new town or a village, everyone will form groups and take loans. This result, however, does not imply that 100 percent of the borrowers will repay and remain borrowers in the future.

In a static setting, a low productivity borrower chooses not to take a loan and enjoys his reservation utility. In the dynamic model, however, a low productivity borrower will take the loan and invest it into a project while exerting low effort and enjoying leisure benefits. After the realization of his project, he will have the following state contingent strategy: repay, if both his and his partner's projects are successful, and strategically default, if he is successful but his partner is not. From the optimization problem, it is determined that when a borrower is of low productivity type, he does not value his future access to credit as much as a high productivity borrower. The benefit of being insured against default in order to secure future access to credit less the cost of insuring his partner becomes less than his reservation utility, when he needs to repay two shares, and he then defaults.

The dynamic model, unlike the static model, can explain more reasonably the borrowers' behavior regarding strategic default. In the static model, strategic default is a

predetermined decision to always or never default under all circumstances. The dynamic model has a potential to explain that, when a borrower is burdened with default by several of his partners, he might choose to default even though he would have repaid if fewer partners defaulted. Therefore, the dynamic model can explain a situation where strategic default occurs in groups when adverse shocks affect several members. The bank will be able to continue operations on a sustainable basis even in this case, since it has already taken this effect into account when setting interest rates.

The results for the medium and high productivity borrowers are the same as in individual lending. Medium and high productivity borrowers take loans and exert low and high effort, respectively. They do not strategically default, because their value of future access to credit is higher than their reservation utility, even if they have to repay for defaulting partners. Overall, the results in the static and the dynamic models are similar, except for the case of low productivity borrowers in group lending.

Conclusions

The analysis developed in this paper is an example of how a bank may determine interest rates based on information about peer monitoring costs and social sanctions, correlation of business activities, investment opportunities, productivity and diligence characteristics of borrowers. Following Armendariz de Aghion, we extend Navajas' static model for individual lending with adverse selection and moral hazard to account for correlated returns and strategic default in group lending. We develop a dynamic model that allows for default decisions to be made conditional on borrowers' project outcomes.

We find that, in a static framework, interest rates, default rates, and breadth and depth of outreach are lower in group lending than in individual lending. These results hold as long as joint liability does not destroy borrowers' incentives and does not lead to moral hazard and strategic default. Enforcement of repayment in group lending crucially depends on peer monitoring and social sanctions; otherwise, borrowers strategically default and bank operations are not sustainable at any interest rate. The lower interest rates charged in group lending do not necessarily make the joint liability contracts more attractive, since borrowers also incur the cost of monitoring and insuring their peers against default, while gaining the benefit of being insured against default. In fact, some low productivity borrowers may become worse off under group lending and choose to drop out of the loan market altogether. If low productivity is a signal of poverty, breadth as well as depth of outreach decreases. When project returns are independent (correlated), group lending with group size of up to three people (eight people) is preferred by high productivity borrowers, as the benefit of being insured outweighs the cost of insuring peers; otherwise, if group size is larger, high productivity borrowers shift to individual liability contracts.

The results in the dynamic model indicate that state-contingent default decisions allow borrowers to choose when to default depending on how many other peers default. The dynamic model can better explain the real world situation where default occurs when adverse conditions affect several group members. The bank can continue its operations even in this case if it has already accounted for that possibility when setting interest rates.

The findings in this paper have important policy implications for banks willing to extend financial services to poor producers without traditional collateral. The

conventional wisdom that peer monitoring in group lending substitutes effectively for the lack of collateral may not always be valid. High repayment rates are ensured only when borrowers can impose severe social sanctions on defaulting peers, when the transaction costs of monitoring are low (the probability of monitoring is high), and when groups are not too large.

As more banks offer different types of contracts in the same town or village, it is important to understand how potential borrowers will self-select into individual or group loans. Failure to account for the presence of other banks may adversely change the riskiness of the pool of borrowers and threaten bank sustainability.

Group lending is usually implemented in an effort to alleviate poverty by serving a large number of poor people (breadth of outreach) and especially the poorest of the poor (depth of outreach). Yet, the version of the widely used static model explored here predicts the opposite effect – that outreach in group lending is lower than outreach in individual lending. The dynamic model developed in this paper predicts, instead, that outreach in group lending is higher than in individual lending. Banks that want to extend their outreach to poor producers in developing countries and still remain financially sustainable will have to take into account the issues explored in this paper.

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Table 1. Microlending Parameters

Variables	Parameter	Values in the Static Model	Values in the Dynamic Model
Loan size	I	100	100
Interest rate	$r=R-I$	endogenous	endogenous
Joint liability component	c	0 or 1	0 or 1
Group size	n	1, 2, 3,...	1 or 2
Value of re-accessing market for future credit	V	300	endogenous
Productivity type	z	[0,1]	[0,1]
Production constant	f	300	300
Reservation utility	Y	-100	-100
Leisure Benefits	B	1	1
Probability of success if high effort	P^H	0.9	0.9
Probability of success if low effort	P^L	0.6	0.6
Probability of success and success if high effort and high effort	P^{HH}	0.97	0.97
Probability of success and success if low effort and low effort	P^{LL}	0.89	0.89
Social sanctions	W	200 or 100	200 or 100
Probability of monitoring	γ	0.9	0.9
Population	b	100	100
Total handling costs for the bank	m	500	500
Bank's outside funding interest rate	ρ	0.1	0.1

Table 2. Static Model with Individual or Joint Liability Contracts

Results	Individual Liability Contracts	Joint Liability Contracts	
		Independent Returns	Correlated Returns
Interest rate	61%	37%	54%
Borrowers	90%	80%	87%
Default rate	28%	15%	25%
Productivity type and decision	0-0.10 <i>U</i>	0-0.20 <i>U</i>	0-0.13 <i>U</i>
	0.11-0.65 <i>LN</i>	0.21-0.34 <i>LN</i>	0.14-0.56 <i>LN</i>
	0.66-1 <i>HN</i>	0.35-1 <i>HN</i>	0.57-1 <i>HN</i>

U – non-applicant, *L* – low effort, *H* – high effort, *N* – not default, *D* – default.

Table 3. Sensitivity Analysis on Group Size

Group Size	Independent Returns			Correlated Returns		
	Interest Rate	Borrowers	Effort and Default	Interest Rate	Borrowers	Effort and Default
2	37%	80%	<i>U LN HN</i>	54%	87%	<i>U LN HN</i>
3	30%	73%	<i>U HN</i>	48%	84%	<i>U LN HN</i>
4	31%	69%	<i>U HN</i>	42%	81%	<i>U LN HN</i>
5	31%	64%	<i>U HN</i>	36%	79%	<i>U LN HN</i>
6	32%	59%	<i>U HN</i>	32%	76%	<i>U LN HN</i>
7	-	-	<i>U HD</i>	30%	74%	<i>U HN</i>
8	-	-	<i>U HD</i>	30%	73%	<i>U HN</i>
9	-	-	<i>U HD</i>	30%	72%	<i>U HN</i>
10	-	-	<i>U HD</i>	31%	70%	<i>U HN</i>
11	-	-	<i>U HD</i>	-	-	<i>U HD</i>

U – non-applicant, *L* – low effort, *H* – high effort, *N* – not default, *D* – default.

Table 4. Dynamic Model with Individual and Joint Liability Contracts

Results	Individual Liability Contracts	Joint Liability Contracts	
		Independent Returns	Correlated Returns
Interest rate	67%	37%	55%
Borrowers	98%	100%	100%
Default rate	31%	16%	26%
Productivity type and decision	0-0.02 <i>U</i>	0-0.2 <i>LL</i> $\begin{cases} NN \text{ if } ss \\ DD \text{ if } sf \\ DN \text{ if } fs \\ DD \text{ if } ff \end{cases}$	0-0.29 <i>LL</i> $\begin{cases} NN \text{ if } ss \\ DD \text{ if } sf \\ DN \text{ if } fs \\ DD \text{ if } ff \end{cases}$
	0.03-0.76 <i>L</i> $\begin{cases} N \text{ if } s \\ D \text{ if } f \end{cases}$	0.21-0.68 <i>LL</i> $\begin{cases} NN \text{ if } ss \\ ND \text{ if } sf \\ DN \text{ if } fs \\ DD \text{ if } ff \end{cases}$	0.3-0.72 <i>LL</i> $\begin{cases} NN \text{ if } ss \\ ND \text{ if } sf \\ DN \text{ if } fs \\ DD \text{ if } ff \end{cases}$
	0.77-1 <i>H</i> $\begin{cases} N \text{ if } s \\ D \text{ if } f \end{cases}$	0.69-1 <i>HH</i> $\begin{cases} NN \text{ if } ss \\ ND \text{ if } sf \\ DN \text{ if } fs \\ DD \text{ if } ff \end{cases}$	0.73-1 <i>HH</i> $\begin{cases} NN \text{ if } ss \\ ND \text{ if } sf \\ DN \text{ if } fs \\ DD \text{ if } ff \end{cases}$

U – non-applicant, *L* – low effort, *H* – high effort, *N* – not default, *D* – default, *s* – success, *f* – failure. The first index represents the choice for the first person, the second index represents the choice for the second person.