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# Copula-Based Modeling of Dependence Structure among International Food Grain Markets

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# Copula-Based Modeling of Dependence Structure among International Food Grain Markets

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## Abstracts

This paper examined the dependence structure among global food grain markets that determines the speed of shock/volatility transmission from one market to another. We have applied copula-based models that consider the joint distribution of food grain prices from different markets and three most traded food grains (rice, wheat and corn) are considered for the analysis. Gaussian copulas have shown statistically significant dependence for most price pairs between markets, but with small Kendall's tau values, which imply low dependence among markets. Then we have applied copulas that capture distributions other than the Gaussian and also capture tail dependence and found a significant improvement in Kendall's tau values, implying a strong dependence for price pairs among global food grain markets.

Key Words: Dependence Structure, Commodity Prices, Copulas, Price Boom

## I. Introduction

Food grain prices in global markets surged sharply in late 2000s and the welfare implications of such food price upsurge were huge, especially for net food importing developing countries. The sharp food price hike caused serious food insecurity and even civil unrests in many countries across Africa and Asia (Lagi *et. al.*, 2011). Sudden price surge in international food grain markets also hurts fiscal stability of net food importing countries, especially in countries where food subsidies consist a bulk share of national budget (FAO, 2011). Voluminous studies and reports have been published on what explain the commodity price boom in late 2000s. Carter *et.al.* (2011) presents

a survey of the relevant literature and finds three prominent explanations for the global commodity price boom in late 2000s. They are: i) speculation, ii) export restrictions by exporting countries; and iii) demand shock to corn market due to changes in biofuel policy of united states of America (USA) in 2007.

Speculation, arbitrage activities in commodity markets have been increased in recent years due to future markets trading in commodity exchange markets. Futures attract investors who are not interested in the commodity as such, but in making a speculative profit on future movements in the price of the commodity. The feature of negative correlation between returns to commodity futures and returns to equities and bonds attracts “non-commercial” investors increasingly in commodity futures markets (FAO, 2010). Commodity futures, thus, constitute an attractive vehicle for portfolio diversification. There has been a significant inflow of funds from traditional institutions such as hedge funds and pension funds into commodity futures markets (FAO, 2012). Thus, speculation might play a role in commodity price hike in late 2000s. However, the role of speculation in the price boom has not been confirmed yet in relevant literature due to lack of proper data.

Deliberate restriction on agricultural exports such as export bans, raising or introducing export tax etc. was another key factor behind the commodity price boom in late 2000s. At least 30 countries around the world imposed some forms of export restriction in 2007 and 2008 (IMF, 2008). For example, Argentina raised export tax on soybeans from 35% to 45 %, India banned exports of wheat and non-basmati rice, Vietnam restricted rice exports and Kazakhstan banned wheat exports. Martin and Anderson (2011) estimates that approximately 30% of rice price increase and 25% of wheat price increase in the period of 2005-08 were due to the export control measures of exporting countries.

The most prominent explanations for the global food price hike in late 2000s that emerges in the literature is that ethanol production growth in the USA played crucial role in rising global food prices through excess demand in corn markets. Roberts & Schlenker (2010) claim that 30 percent of the rise of average price of staple food commodities was caused by excess biofuel demand in 2007-2008. Studies dealing with rice price boom in 2008 argue that the role of the change in the biofuel policy in the USA had, even, a broader role in explaining rice price boom (FAO, 2008). The economics of substitution in supply and demand is a major factor that strongly tied linkages among food grain markets due to demand shock in corn markets as a response to a shift in ethanol production policy. Supply substitutability leads to higher corn production and lower wheat and soybeans production in the United States, while demand substitutability transmits corn market shocks into other food grain markets. More than 30 % of corn produced in 2008 was used for ethanol production, while the analogous figure was only 14% in 2005. This diversion causes to raise food prices significantly first through the direct effect on corn prices and second through the economics of substitution with other commodities as the US produces around 40% of global corn production and accounts more than 60% of global corn exports (FAO, 2008). FAO (2008) reports that the global demand for corn had been increased by 40% in 2007; and 75% of the increase were due to ethanol production. The demand shock in corn market due to ethanol policy lowered acreage of wheat and soybeans; and reduced corn inventory level significantly in the U.S.

Thus, demand shock in the corn market of the United States shoots the overall food commodity prices in international food grain markets in late 2000s. Rice markets around the world, even, responded more swiftly and spurred. While net rice exporting countries could exploit the returns from a high global rice price, instead these countries imposed various restrictions on rice export to insulate domestic market from the global markets to curb domestic food inflation. In fact, some of

net rice exporting countries have been engaged in ‘panic buying’ as they feared that food crisis could lead their countries into political destabilization. The question that remains unexplored in the literature is that, while correlation among prices in food grain markets is historically quite low, why are demand shocks in corn markets transmitted to other food grain markets so swiftly? This paper examines this issue with a novel approach. The dependence structure among global food grain markets determines the speed of shock transmission from one market to another. Insufficient understanding about dependence structures among international food grain markets make it difficult to forecast food grain prices. Predicting future commodity price booms requires a careful examination of dependence structure among food grain markets and of price transmission mechanisms.

Efficient trade and arbitrage activities should ensure that prices of related goods in the major food markets are well integrated through a common long-run equilibrium. However, the dependence structure among spatially separated global food grain markets has not been examined yet with proper analytical tools such as the copula; in fact studies on the dependence structure among major global financial markets are not handful. While major stock markets around the globe are more dependent at the state of ‘crash’ than at the state of ‘boom’, global food grain markets appear to be more dependent on ‘up days’ than on ‘down days’. As price elasticity of demand for a staple food is usually low, overall food prices do not respond much for a small price change of a specific food grain. A large price change of a specific grain, however, could stimulate panic responses from agents involved in the food grain markets due to the economics of substitutions in supply and demand.

The link among spatially separated agricultural commodity markets around the world has received considerable attention in recent years due to the issue of food security. While correlations among

price changes of food grains are historically quite low, many studies on the price boom in late 2000s conclude that growth in the ethanol industry, through the demand shock in corn markets, has strengthened the links among commodity markets around the world. Despite studies on co-movement of prices among global food grain markets somewhat limited, there are numerous studies on the notions of price parity, price transmission and price arbitrage relationships for tradable homogeneous goods. The main idea of these studies is that efficient functioning of market should ensure stable links across spatially separated regional markets and eliminates any potential for persistent spatial arbitrage profits. This fundamental condition is known as the "Law of One Price" (LOP) and the general implication is that prices in spatially separated markets should not vary by no more than the transport and transaction cost. The existing literature on spatial price linkages extends from simple tests of correlation among prices, to recent sophisticated time series regression models that address the issues of nonstationarity, nonlinearities, and threshold behavior in price linkages among spatially separated markets.

Empirical support to LOP is rather mixed. While early studies ( see Isard (1977), Thursby, Johnson, and Grennes (1986), and Benninga and Protopapadakis (1988)) fail to confirm the LOP; Goodwin, Grennes, and Wohlgenant (1990) do, however, find some support in favor of the LOP when price expectations was taken into account instead of observed prices. Cointegration techniques have been adopted extensively evaluating LOP as a long-run concept following the seminal paper Engle and Granger (1987), and more convincing evidence were established in favor of the LOP ( see Buongiorno and Uusivuori (1992), Bessler and Fuller (1993), and Jung and Doroodian (1994)). Smooth or discrete threshold time series models get prominence among the most recent literature on the LOP with the underlying assumption that adjustments to equilibrium may not be linear, and that this nonlinearity may, in turn, be associated with hard-to-observe

transactions costs associated with arbitrage. Studies that use this line of methodology find nonlinearity is an important feature of price relationships in these markets and that the price parity relationships implied by economic theory and efficient arbitrage are generally supported by the threshold models ( Holt, Prestemon, and Goodwin (2011), Goodwin and Piggott (2001), Lo and Zivot (2001), Sephton (2003), Balcombe, Bailey, and Brooks (2007), and Park, Mjelde, and Bessler (2007). Very recently, researchers have started to use the copulas in modeling spatial price linkages. Goodwin et.al (2011) estimates copula-based nonlinear models for pairs of North American orient strand board (OSB) prices and finds even stronger evidence of nonlinearities in spatial market linkages.

Most studies in the relevant literature have generally examined the notion of price parity, price transmission and arbitrage activities for tradable homogeneous good (Goodwin, Grennes, and Wohlgenant (1990), Goodwin and Piggott (2001); Goodwin et.al (2011)). Studies on price transmission and price linkages among the international food grain markets are limited. Despite progress, the understanding about commodity prices and the ability to forecast commodity prices remains seriously scarce and this insufficient understanding make it difficult to construct good policy rules (Deaton, 1999). This paper intends to improve the understanding about the dependence structure among global food grain markets, and hence advances the ability to forecast food grain prices. In this paper, we examine the links among international food grain markets and propose an alternative and novel approach to analyzing dependence structures among international food grain markets. We develop copula-based models that consider the joint distribution of prices from different markets and apply them to weekly prices for food grains at geographically distinct global markets. To improve the capability of precise forecasts of food grain prices in future, a careful examination of the dependence structure among food grain markets is needed. Evaluation



of the dependence structure among international food grain markets could serve dual purposes: as risk management tool for speculators operating in the commodity exchange markets to forecast future price movements and as better forecasting tool for policy makers around the world dealing with policies to ensure food security.

We have seen a tremendous increase in the application of copulas in the financial literature recently and they have proven to be a very useful tool in modeling the dependence structures among financial markets. Like financial markets, international food grain prices tend to exhibit asymmetric dependence. This asymmetry implies that in times of upward trend, prices tend to be more dependent than they are in times of downward trend. This phenomenon has important implications for the risk of speculators and arbitragers who operate in commodity futures market and for the risk of policy-makers who formulates policy to ensure food security for a country.

This paper uses the copula models to study the comovement and the tail dependence of food grain prices using three most traded food grains in the World: Rice, Wheat and Corn. The approach used in this paper is a natural extension of the existing time-series evaluations of spatial price linkages. Our contribution is two-fold. First, we use copulas which allow modeling the dependence in a much more flexible and realistic way than models based on the Gaussian distribution which have been previously implemented. The use of copulas makes it possible to separate the dependence model from the marginal distributions. Second, copulas allow us to have asymmetric tail dependence, which means that, unlike with the Gaussian distribution, the dependence does not vanish as we consider increasingly higher or lower price changes.

The remainder of the paper is organized as follows. Following introductory discussions in Section I, we discuss methodology and models in section II. We describe the two-step estimation

procedures of the model in this section. First, we discuss various forms of copulas and compare them in terms of capturing the dependence among international food grain prices. Then we present the GARCH model to be used for the marginal. In Section III, we discuss data sources and summary statistics of the price series as it is useful to describe the data and make initial decisions about the choice of GARCH process and copula selection. In section IV, we present the results and discussions of GARCH models for the marginal. This section also presents the results of the copula models. Section V concludes the paper.

## II. Specifications and Methodology

In the last decade, copula modeling has become a frequently used tool in financial economics<sup>2</sup>. The empirical approach adopted in this paper involves considering the joint distribution function of  $\Delta(p_t^i - p_{t-1}^i)$  and  $\Delta(p_t^j - p_{t-1}^j)$ . To implement an empirical model we use the copula approach. The fundamental of copulas dates to work by Sklar (1959). Sklar (1959) theorem implies that any joint probability function can be represented in terms of the marginal densities and a function known as ‘Copula’.

### 2.1. Copulas

Copula models have been used extensively in recent empirical literature on risk management and financial economics (see, among others, Hu (2006), Patton (2006), and Jondeau and Rockinger (2006)). Copula models are used in empirical models of joint probability distributions and the model use a “copula” function to link together two marginal probability functions which may (or may not) be linked with each other.

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<sup>2</sup> A nice accounts of copula theory are available in Joe (1997) and Nelsen (2006)

A  $p$ -dimensional copula,  $C(u_1, u_2, \dots, u_p)$ , is a multivariate distribution function in the unit hypercube  $[0, 1]^p$  with uniform  $U(0, 1)$  marginal distributions. For the continuous marginal distributions, Sklar (1959) has demonstrated that a unique copula is linked with the joint distribution,  $F$ , that can be found as:

$$C(u_1, u_2, \dots, u_p) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \dots, F_p^{-1}(u_p)) \quad (1)$$

Similarly, given a  $p$ -dimensional copula,  $C(u_1, u_2, \dots, u_p)$ , and  $p$  univariate distributions,  $F_1(x_1), F_2(x_2), \dots, F_p(x_p)$ , equation (1) is a  $p$ -variate distribution function with marginals  $F_1, F_2, \dots, F_p$  whose corresponding density function can be expressed as:

$$f(x_1, x_2, \dots, x_p) = c(F_1(x_1), F_2(x_2), \dots, F_p(x_p)) \prod_{i=1}^p f_i(x_i) \quad (2)$$

Then, the density function of the copula ( $c$ ), given that it exists, can be derived using equation (1) and marginal density functions,  $f_i$ :

$$c(u_1, u_2, \dots, u_p) = \frac{f(F_1^{-1}(u_1), F_2^{-1}(u_2), \dots, F_p^{-1}(u_p))}{\prod_{i=1}^p f_i(F_i^{-1}(u_i))} \quad (3)$$

Copulas differ in terms of how the dependencies among variables are represented. For example, a Gaussian copula assumes linear correlation and imposes zero dependence in the tails of the distributions. A  $t$  copula allows for non-zero tail dependence but imposes symmetry in the dependence relationships in alternate tails of the distributions. Archimedean copulas typically allow for dependence in only one tail and represent the dependence relationship by using a single parameter. Thus, the choice of a copula function determines the nature of the relationships among dependent random variables.

There are several advantages of using copula models to analyze the dependence structure among pairs of prices in spatially distinct global agricultural commodity markets. First, copulas allow us to model the marginal behavior and the dependence structure of commodity prices separately and, thus, allow for greater flexibility in estimating margins. Second, copula models provide both the degree and the structure of dependence among pairs of prices, while the simple correlation coefficient only look at the degree of dependence among the marginal distributions assuming multivariate normality. Third, Kendall's tau is a copula-based dependence measure which relies on the notion of concordance while it does not depend on the marginal distributions. The Kendall's tau is exclusively a function of the copula

$$\tau = \int_{[0,1]^2} C(v_1, v_2) dC(v_1, v_2) - 1 \quad (4)$$

While the Kendall's tau measures overall dependence, tail dependence, another copula-based measure, captures the dependence between extremes. Intuitively, tail dependence measures the propensity of geographically distinct global agricultural commodity markets to go up or down together. The coefficients of upper tail dependence,  $\lambda_U$ , and lower tail dependence,  $\lambda_L$ , between  $X_1$  and  $X_2$  can be expressed in terms of the copula, given that it exists, between  $X_1$  and  $X_2$  as:

$$\lambda_U = \lim_{u \rightarrow 1^-} P(X_2 > F_{X_2}^{-1}(u) \mid X_1 > F_{X_1}^{-1}(u)) = \lim_{u \rightarrow 1^-} \frac{1-2u+C(u,u)}{1-u} \quad (5)$$

$$\lambda_L = \lim_{u \rightarrow 0^+} P(X_2 \leq F_{X_2}^{-1}(u) \mid X_1 \leq F_{X_1}^{-1}(u)) = \lim_{u \rightarrow 0^+} \frac{C(u,u)}{1-u} \quad (6)$$

Where  $F^{-1}$  is the marginal quantile function and  $\lambda_U, \lambda_L \in [0,1]$ . There is no upper (lower) tail dependence if  $\lambda_U=0$  ( $\lambda_L=0$ ). Different copulas allow for different degrees of tail dependence<sup>3</sup>. The

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<sup>3</sup> Range of parameters, Tail dependence, and Kendall's tau for some common copulas are provided in a table in the annex.

Gaussian copula captures no tail dependence, which would imply that extreme price changes in spatially distinct commodity markets are independent.

In this paper, to empirically estimate copula models of the joint distribution of log price changes between different markets, we consider a large number of bivariate copula specifications that permit considerable flexibility in explaining the relationships between price changes in spatially distinct markets around the world. We first estimate models with one symmetric Gaussian copula. Then we choose a copula family from a wide range of copula models, using AIC model-fitting criterion that capture asymmetries for a bivariate analysis of 15 different price pairs<sup>4</sup>. We estimate Kendall's tau value for both Gaussian copula and the selected copula family to compare the dependence structure based on traditional Gaussian assumptions and based on flexible distributional assumption that capture asymmetries and tail dependence.

## **2.2. The Marginal Model: GARCH Process**

We need to find the appropriate marginal distributions for the copula model. Usually a specific form of heteroscedasticity is observed in price series. Today's price volatility will lead to a higher volatility tomorrow and, thus, variances over time somewhat are related. This type of heteroscedasticity implies autocorrelation in squared price changes. GARCH is a model of stochastic process that allows such type of heteroscedasticity (Engle, 1982; Bollerslev, 1986). A GARCH is a stationary stochastic time series process and as prices are non-stationary (ADF and PP tests suggests so), we use changes in log prices and these price changes were found stationary.

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<sup>4</sup> As asymmetric tail dependence is one of our goals, we considered those copula families that can capture asymmetric tail dependence well and exclude Gaussian, Student t and Frank copulas from consideration in this stage.

We use GARCH model instead of ARMA-GARCH as price change series are demeaned which remove the autocorrelation components from price changes.

Specifically, the GARCH process is expressed as follows:

$$y_t = \sqrt{h_t} \cdot \varepsilon_t$$

Where  $\varepsilon_t$  is a white noise with  $\sigma_\varepsilon^2 = var(\varepsilon_t) = 1$ , and

$$h_t = \alpha_0 + \sum_{i=1}^p \beta_i h_{t-i} + \sum_{i=1}^q \alpha_i y_{t-i}^2$$

With parameters  $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_q, \beta_1, \beta_2, \dots, \beta_p \geq 0$  and  $\sum_{i=1}^q \alpha_i + \sum_{i=1}^p \beta_i < 1$

$\varepsilon_t$  accounts for ‘idiosyncratic’ shocks, news etc.

Taking into consideration the characteristics of price changes in Table 1, which are generally non-normal and skewed, we employ the GARCH (1,1) model with the skewed student-t to capture the time-varying volatility and leverage effect, and to fit the marginal distributions for the copula model.<sup>5</sup> Skewed t distribution of Hansen (1994) has two shape parameters: a skewness parameter which controls the degree of asymmetry, and a degrees of freedom parameter which controls the thickness of the tails. Following the GARCH fit, the cumulative distributions of standardized residuals are formed to plug into the copula model.

### III. Data Source and Descriptive Statistics:

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<sup>5</sup> I also estimated GARCH(1,1) model with normal distribution for the comparison purpose and we found that GARCH (1,1) with skewed student t performs better as AIC value is lowest for all marginal.

To model dependence structures among global food grain markets, we consider the three most traded food grains: rice, wheat and corn. To capture the dynamics within each food grain, we use two price series from each cereal grain. There are quality differences within each food grain markets and consumers' preferences vary over the quality of grains. Substitution between food grains depends, somewhat, on quality of the grains. For example, price change of A1 super rice which is traded in Thailand is more correlated with either hard wheat or soft wheat, two variation of wheat traded in the US, than the correlation of 100% broken rice traded in Thailand with either types of wheat. Thus, using a single series for a food grain would lead to misleading conclusion about dependencies among commodity markets. For each food grain, we select two key price series. The rice market is represented by the type of 100% Broken in Thailand (R1) and by the type of A1 Super Quality (R2). For wheat, the study uses prices of Winter Hard Wheat (W1) and Winter Soft Wheat (W2) traded in US export markets. For corn, we use US corn price (C1) and Argentine corn price (C2). Selection of commodities and price series have been somewhat restricted by the data availability. We use weekly price data from Food and Agriculture Organization (FAO) of United Nations (UN) for the period of January, 2000 to February, 2014, which yields 735 weekly observations. We replace missing values using commonly used cubic spline interpolation<sup>6</sup>. The percentage of missing observations varies from 0.7 % to 6.6% of total sample.

Table 1 presents the time series properties of the price series. Based on augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979), I find that two price series of corn and the hard wheat price are non-stationary in levels. Other three price series, two price series of rice and soft wheat price, are non-stationary at conventional 5 percent level; but these prices are stationary at 10 percent

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<sup>6</sup> For details of Cubic Spline intrapolation, see 'A Practical Guide to Spline' by C.De Boor, New York, Springer, 1978.

level. Figure 1 depicts presence of structural shift for most prices, and we consider Phillips-Perron (PP) test (Phillips & Perron, 1988) to check stationarity of the price series. PP test suggests all prices are non-stationary even at 10 percent level, except winter soft wheat which is stationary at 5 percent level. Then we examine stationarity of price series in first difference and find all series are stationary at conventional 5 percent level in both ADF and PP tests. As price series in levels are mostly non-stationary and price series in first difference are stationary, we use changes in weekly prices. The price change ( $pc_t$ ) is calculated as  $pc_t = \log(p_t / p_{t-1})$ , where  $p_t$  and  $p_{t-1}$  are current and one period lagged weekly spot prices respectively. All price series are expressed as current US Dollar per metric ton.

Table 2 provides descriptive statistics of the price series in return form. We find that all prices have some positive returns and positive skewness (with the exception of corn prices in Argentina) which indicates that the right tail of the density function of price changes is fatter or longer than the left side. The right-skewed nature of price changes implies that dependence of commodity prices may be present and prices are more dependent on each other when move upward than when they move downward.

Asymmetric dependence nature of price changes has also been observed as Kurtosis values for all price changes except corn price in Argentina are either well above of 3 or well below of 3. This implies that empirical distribution of price changes may not be well described by the widely used normal distributions. From figure 2, it is also obvious that volatility in price series are somewhat clustered at least for rice and corn markets which imply that large changes in prices followed by large change as well. To have the appropriate marginal distributions for the copula models, we use GARCH model as our price change series are demeaned series which remove the autocorrelation components from price changes. Taking into consideration the characteristics of log price changes,



which are generally non-normal and skewed, we employ the GARCH (1,1) model with the skewed student-t to capture the time-varying volatility and leverage effect, and to fit the marginal distributions for the copula model<sup>7</sup>. Following the GARCH fit, the cumulative distributions of standardized residuals are formed to plug into copula model.

**Table 1: Time Series Properties of the Prices**

Variables	Augmented Dickey-Fuller (ADF) Test		Phillips-Perron (PP) Test	
	Test Statistics	P-Value	Test Statistics	P-Value
<b>A. Levels</b>				
Rice Price (100%B, Thailand)	-3.2417	0.08	-11.3707	0.47
Rice Price (A1 Super, Thailand)	-3.1747	0.09	-12.8922	0.39
Wheat Price (Hard Wheat, USA)	-2.8656	0.21	-17.5175	0.13
Wheat Price (Soft Wheat, USA)	-3.2712	0.07	-20.4879	0.06
Corn Price (USA)	-2.5681	0.34	-12.3446	0.42
Corn Price (Argentina)	-2.4344	0.39	-15.8153	.23
<b>B. Returns</b>				
Rice Price (100%B, Thailand)	-6.7567	0.01	-545.9469	0.01
Rice Price (A1 Super, Thailand)	-6.7996	0.01	-606.4759	0.01
Wheat Price (Hard Wheat, USA)	-8.4482	0.01	-747.9682	0.01
Wheat Price (Soft Wheat, USA)	-8.2113	0.01	-724.5378	0.01
Corn Price (USA)	-7.8713	0.01	-777.9462	0.01
Corn Price (Argentina)	-8.7185	0.01	-746.1828	0.01

Note: ADF test applies 9<sup>th</sup> lag order and truncation lag parameter for PP test was 6 for each series.

**Table 2: Summary Statistics of the Prices Returns.**

Variables	Mean	Standard Dev.	Skewness	Kurtosis	Observation
<b>Sample Period: January, 2000 to February, 2014</b>					
<b>Return</b>					
Rice Price (100%B, Thailand)	0.09	2.23	2.19	21.56	735
Rice Price (A1 Super, Thailand)	0.10	2.38	0.45	4.74	735
Wheat Price (Hard Wheat, USA)	0.14	3.58	0.02	1.28	735
Wheat Price (Soft Wheat, USA)	0.14	4.43	0.03	1.46	735
Corn Price (USA)	0.11	3.89	0.02	1.97	735
Corn Price (Argentina)	0.13	3.91	-0.20	3.02	735

**Table 3: Spearman and Pearson's Correlation Coefficients for the Prices**

<sup>7</sup> I have estimated GARCH (1,1) with normal distribution as well and I find loglikelihoods are higher for models with skewed student t distribution than for models with normal distributions for each price series. AIC values were also higher for the GARCH models with student t-distribution. (Results are presented in the Appendix)

Variables	Rice (100%B, Thai)	Rice (A1 Super, Thai)	Wheat (Hard, USA)	Wheat (Soft, USA)	Corn (USA)	Corn (Argentina)
<b>Spearman's Correlation Coefficients</b>						
Rice (100%B, Thai)	1.00					
Rice (A1Sup, Thai)	0.56	1.00				
Wheat (Hard, USA)	0.04	0.08	1.00			
Wheat (Soft, USA)	0.02	0.05	0.80	1.00		
Corn (USA)	0.08	0.10	0.47	0.50	1.00	
Corn (Argentina)	0.04	0.07	0.44	0.47	0.75	1.00
<b>Pearson's Correlation Coefficients</b>						
Rice (100%B, Thai)	1.00					
Rice (A1Sup, Thai)	0.61	1.00				
Wheat (Hard, USA)	-0.05	0.04	1.00			
Wheat (Soft, USA)	-0.03	0.01	0.80	1.00		
Corn (USA)	0.04	0.08	0.49	0.50	1.00	
Corn (Argentina)	0.01	0.04	0.45	0.45	0.76	1.00

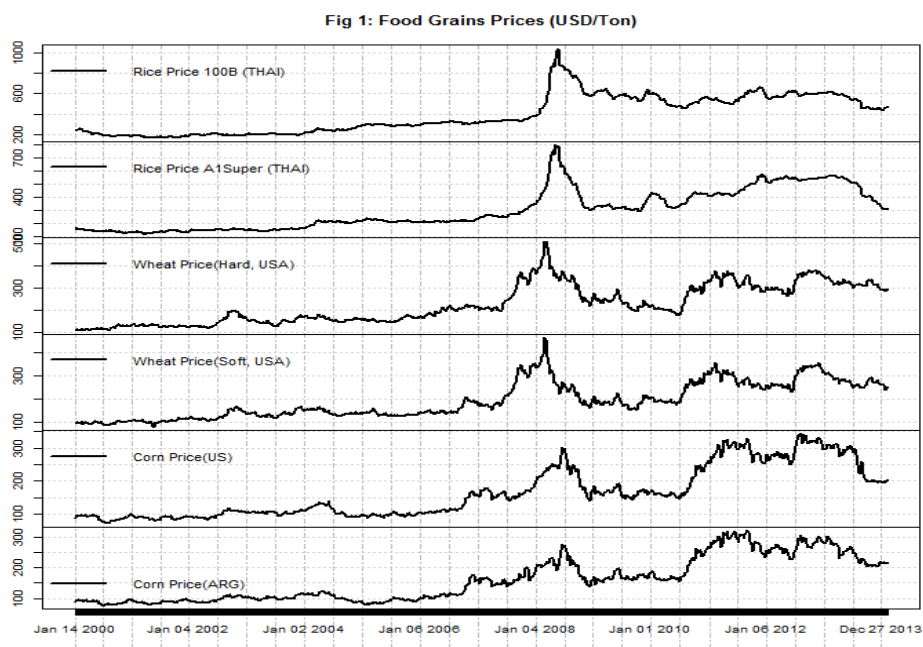


Fig 2: Change in Log of Food Grains Prices

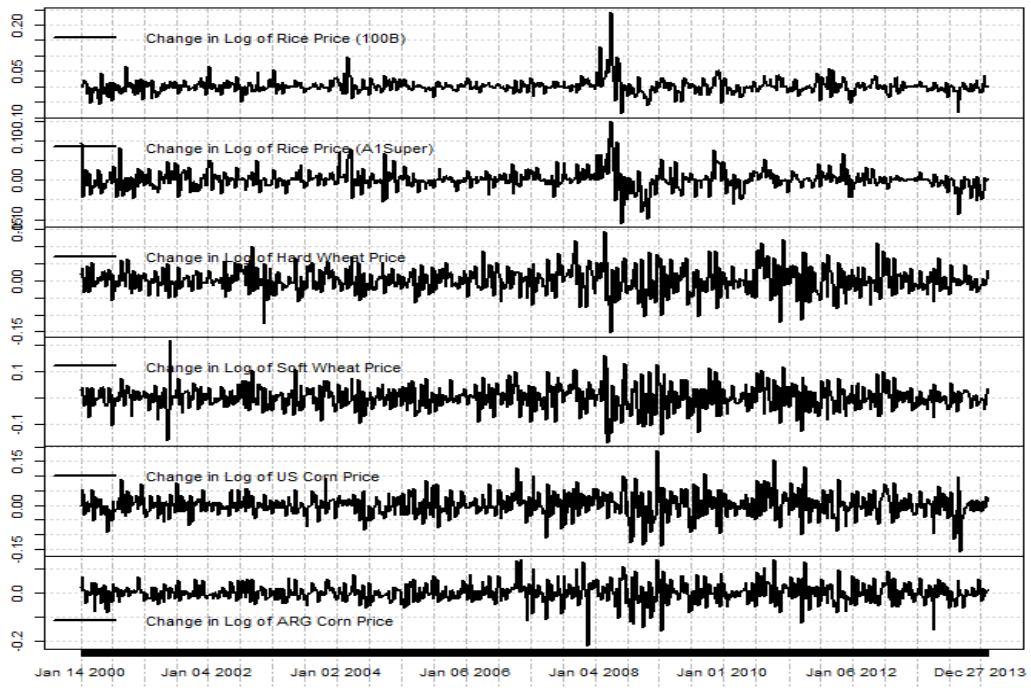
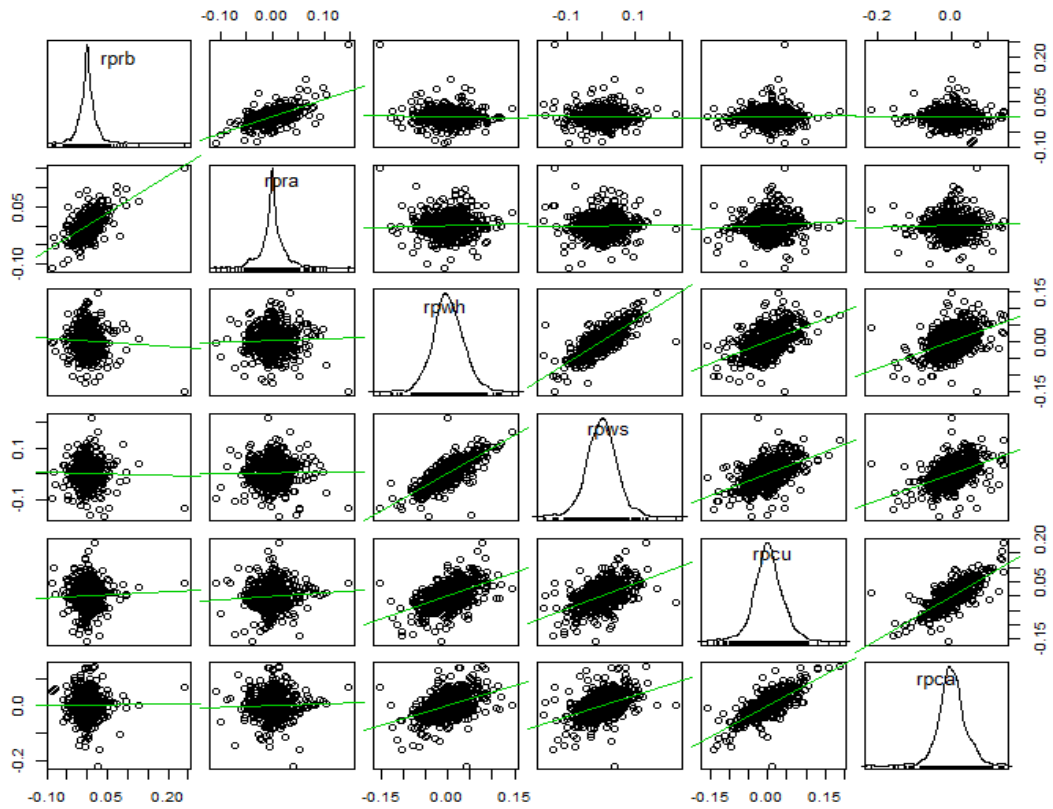


Fig 3: Changes in Log Prices



### **III. Results and Discussions**

In this section we present and discuss the results. First, we discuss the results of GARCH (1, 1) for the marginal models; and the discussion on the dependence results follows.

#### **3.1. Marginal: GARCH Process**

The parameter estimates and standard errors of the univariate skewed Student t GARCH models for marginal distribution are presented in Table 4. The skewness coefficients, that capture asymmetry in the distribution, are significant for each series which justify the rationale of using skewed student t GARCH process. Skewness coefficient for each price series is positive which depicts the fact that tail of marginal distribution is longer in the right side. This implies that large positive price changes, as observed during price boom, are more likely than large negative price changes of the same magnitude. This corroborates the descriptive statistics presented in Table 2. The shape parameter estimates portray that wheat prices have most fat tails with coefficients of 10.0 followed by corn prices of USA and Argentina. Table 4 also presents results for a bunch of tests that checks whether models for the marginal are well specified. The p-values of those tests are presented in Table 4. Jerque-Bera (JB) test and Shapir-Wilk (SW) test confirm that marginal models are well specified for each price series. Table 4 also presents the p-values for Ljung-Box test of autocorrelation in the squared residuals of the skewed student t GARCH fits and the p-values for the LM-Arch test. All these results imply that marginal model for each series is well specified. Thus, the residuals from GARCH fits are extracted and are transformed into cumulative distribution function of uniform distribution with range between 0 and 1.

**Table 4: GARCH (1,1) Results**

Variables	$\alpha$	$\beta$	skew	shape	JB Test	SW Test	Ljung-Box Test			LM ARCH
							10	15	20	
Rice (100%B, Thai)	0.26** (0.13)	0.72*** (0.14)	1.06*** (0.005)	3.80*** (0.66)	0.00	0.00	0.17	0.21	0.42	0.16
Rice (A1Super, Thai)	0.27** (0.12)	0.84*** (0.03)	1.06*** (0.004)	2.64*** (0.35)	0.00	0.00	0.12	0.05	0.06	0.09
Wheat (Hard, USA)	0.01* (0.005)	0.88*** (0.007)	1.1*** (0.006)	10.0*** (0.003)	0.00	0.00	0.27	0.41	0.29	0.45
Wheat (Soft, USA)	0.15*** (0.004)	0.79*** (0.006)	1.01*** (0.006)	10.0*** (2.79)	0.00	0.03	0.58	0.71	0.81	0.65
Corn (USA)	0.15*** (0.004)	0.77*** (0.007)	1.05*** (0.005)	7.51*** (2.05)	0.00	0.00	0.95	0.28	0.26	0.23
Corn (Argentina)	0.003*** (0.001)	0.96*** (0.001)	1.09*** (0.006)	5.51*** (1.16)	0.00	0.00	0.06	0.20	0.27	0.11

### 3.2. Copula Results

We estimate copula models to analyzing the dependence structure in spatially distinct global food grain markets. Empirical estimation of copula models proceeds along the maximum likelihood estimation techniques and parameters are estimated by maximizing a joint likelihood function. First, we estimate Gaussian copula parameters and Kendall's tau value to have the glimpse of dependence structure that is captured by models in current literature. Estimates of parameters and Kendall's tau from Gaussian copulas are presented in Table 5. Second, we choose a copula family from wide variety of copulas based on Akaike Information Criterion (AIC) to model pairs of price series. Results of copula models based on chosen copula family are presented in Table 6. Parameters presented in Table 5 and Table 6 represent copula model estimates of the joint distribution of  $\Delta(p_t^i - p_{t-1}^i)$  and  $\Delta(p_t^j - p_{t-1}^j)$ .

Parameter estimates from Gaussian copulas (Table 5) have been found to be statistically significant for most pairs of the price changes. Table 5 also presents the values of Kendall's tau statistics. Most values of Kendall's tau are very small, especially for the pairs of prices with rice prices.

These values suggest a small degree of dependency and, thus, do not support strongly a general notion of market integration among international food grain markets. However, the question remains, if the dependency is such low, then why do prices for rice and wheat increase so sharply in response to a demand shock in corn markets due to increased ethanol production? Use of copulas that capture distributions other than the Gaussian and also capture tail dependence answer this question.

**Table 5: Parameter Estimates and Kendall's Tau of Gaussian Copula**

Market Pairs	Parameters	Standard. Errors	Cramér von Mises	Kendall's Tau
R1R2	0.6266*	0.0782	6.598*	0.4311
R1W1	0.0417	0.0820	14.33*	0.0266
R1W2	0.0216	0.0833	15.42*	0.0137
R1C1	0.0914	0.0858	11.86*	0.0583
R1C2	0.0743	0.0801	23.69*	0.0473
R2W1	0.0838	0.0794	49.25*	0.0534
R2W2	0.0575	0.0801	21.88*	0.0366
R2C1	0.1101	0.0802	20.99*	0.0703
R2C2	0.0743	0.0801	36.60*	0.0473
W1W2	0.8273*	0.0466	0.353*	0.6203
W1C1	0.4901*	0.0700	5.249*	0.3261
W1C2	0.4682*	0.0715	5.195*	0.3102
W2C1	0.5209*	0.0706	4.683*	0.3488
W2C2	0.4968*	0.0721	2.091*	0.3310
C1C2	0.7893*	0.0539	0.441*	0.5791

Note: \* indicates significance of the coefficients at 10 percent or lower level.

Then maximum likelihood estimation techniques were used to estimate the copula specification that relax the assumptions of zero tail dependence and symmetric dependence that minimized the AIC across a range of copula specifications that allow for tail dependency and asymmetric dependency between market pairs<sup>8</sup>. The resulting estimates and statistics are presented in Table 6.

<sup>8</sup> We also exclude student t copula from consideration as student t copula, when degrees of freedom parameter is less than 2, cannot be estimated; while few market pairs are truly ended up

There are two tests, Kolmogorov–Smirnov (KS) and general Cramer von Mises (CvM), to measure goodness-of-fit statistics for copula models. However, the KS statistic tends to be sensitive around the median of the distribution and less sensitive to deviations in the tails, while the CvM statistic tends to be stable across the distribution, including deviations in the tails (Berg and Bakken , 2006). We have, thus, applied the Cramér von Mises (CvM) statistics with 100 bootstraps to test the goodness-of-fit of the estimates.

Cramér von Mises statistics strongly support the selected copula family over a Gaussian copula. Cramer von Mises statistics indicates that parameter estimates from the selected copula family for twelve market pairs out of fifteen, are favorable than parameter estimates of Gaussian copula. Estimates of upper and lower tail dependencies for the estimated copula for each pair of market prices are also presented for the in Table 6. In many cases, significant dependency has been revealed in one or the other tail, which imply the fact that the Archimedean copulas impose zero dependency in one tail. In most cases, the dependence is tighter in one tail, indicating more adjustment for extreme values of the price differentials.

The estimates of Kendall's tau for the estimated copula of each pairs of market prices are also presented in Table 6. We see that Kendall's tau values improve significantly when we choose a copula family that captures tail dependence and allows for asymmetric dependence. Dependencies are generally large and the values of Kendall's tau tend to be high for most price pairs. We find very high dependence between all the market pairs except between 100% broken rice prices in

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with degrees of freedom parameter less than 2. However, it should not weaken the results as we have estimated Gaussian copula separately to compare with the selected copula family.

Thailand and the price of winter hard wheat in the USA<sup>9</sup>. Excluding this exception, we see high dependence among prices for all three cereal grains. Especially A1 Super rice in Thailand is well integrated with the wheat and corn markets in USA and this strong dependency explains the quick shock transmission from corn market in the USA to Rice markets in the Asia region during 2008 price boom due demand shock in corn market from increased ethanol production. We also find some tail dependence are high for some pairs of markets. The results imply that improvements in the accuracy of price forecasts could be possible from considering an alternative copula to the Gaussian copula currently applied in most research.

**Table 6: Parameter Estimates, Kendall's Tau and Tail Dependence Estimates of Selected Copula Family**

Market Pairs	Copula Family	Param. I	Std. Err. I	Param. II	Std. Err. II	Cramér von Mises	Kendall's Tau	Tail Dependence	
								L. Tail	U. Tail
R1R2	Survival Joe-Clayton	2.333*	0.117	4.722*	0.3986	10.41*	0.7093	0.654	0.863
R1W1	Joe	1.047*	0.075	0.994*	0.0047	26.39*	0.0266		0.062
R1W2	Joe-Frank	3.344*	0.241	0.965*	0.0188	<b>14.78*</b>	.5255		
R1C1	Joe-Frank	2.868*	0.193	0.977*	0.0133	10.17*	0.4793		
R1C2	Joe	1.049*	0.077				0.0277		
R2W1	Survival Joe-Gumbel	1.626*	0.002	2.444*	0.0893	<b>25.68*</b>	0.6968	0.809	
R2W2	Joe-Gumbel	1.628*	0.407	3.182*	0.5260	32.89*	0.7673		0.857
R2C1	Survival Joe-Frank	6.00*	0.329	0.927*	0.019	22.56*	0.6801		
R2C2	Survival Joe-Frank	3.141*	0.233	0.966*	0.0193	<b>13.07*</b>	0.5034		
W1W2	Survival Joe-Gumbel	1.626*	0.002	3.649*	0.1313	1.95*	0.7969	0.876	
W1C1	Joe-Gumbel	1.626*	0.002	3.248*	0.1207	11.23*	0.7719		0.8597
W1C2	Survival Joe-Gumbel	1.626*	0.002	2.780*	0.1051	10.22*	0.7335	0.834	
W2C1	Joe-Gumbel	1.917*	0.518	3.524*	0.6717	11.98*	0.8116		0.8919
W2C2	Survival Joe-Frank	6.000*	3.022	0.741*	0.2255	3.363*	0.5554		
C1C2	Survival Joe-Gumbel	1.626*	0.002	2.315*	0.0816	1.107*	0.6800	0.798	

Notes: \* indicates significance of the coefficients at 10 percent or lower level. Bold Cramer von Mises imply lower value corresponding to that for Gaussian Copula.

<sup>9</sup> The first exception can be explained through low substitutability between 100% broken rice and winter hard wheat (best quality wheat), while the second exception may attributes the vast distance between these two markets.



## **V. Conclusion**

We examined whether spatially distinct agricultural commodity markets are well integrated by studying the dependence structure or co-movement between commodity prices. This analysis also enabled us to assess the tail dependence and so determine whether the global agricultural commodity markets are moving together in times of high volatility. We found strong and significant dependence for most price pairs among global food grain markets. Especially A1 Super rice in Thailand (R2) and 100% Broken in Thailand (R1) are well integrated with the wheat and corn markets in USA, which means that this strong dependence between markets may lead to a strong volatility transmission and skewness spillover across markets when any of the markets experience demand/supply shock. This strong asymmetric dependence between Thailand' rice markets and the food grain markets in the USA might played crucial role in global commodity price boom in the late 2000s. Thus, this high dependence among markets could help as risk management tool in future policy formulation and in price forecasting for both speculators in the commodity futures markets and policy-makers in the food importing countries.

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## Appendix

**Table A1: Comparison of GARCH (1,1) fit with Normal distributions and Skewed Student t distributions**

Variables	Normal Distribution		Skewed Student t distribution	
	Log Likelihood	AIC	Log Likelihood	AIC
Rice (100%B, Thailand)	1896.827	-5.151	1936.369	-5.253
Rice (A1 Super, Thailand)	1808.561	-4.910	1860.875	-5.047
Wheat (Hard, USA)	1448.665	-3.931	1455.209	-3.943
Wheat (Soft, USA)	1288.273	-3.495	1291.928	-3.499
Corn (USA)	1380.772	-3.746	1390.952	-3.769
Corn (Argentina)	1384.113	-3.755	1405.836	-3.809

**Table A2: Range of Parameters, Tail dependence, and Kendall's tau for some commonly used copula models**

Copula	Coefficient	$\lambda_L$	$\lambda_U$	Kendall's $\tau$
Gaussian	$\rho \in [-1, 1]$	0	0	$\frac{2}{\pi} \arcsin(\rho)$
Student-t	$\rho \in [-1, 1],$ $v \in (2, +\infty]$	$2T_{v+1}(x)$ where $x = -\sqrt{v-1} \sqrt{\frac{1-\rho}{1+\rho}}$	$= \lambda_L$	$\frac{2}{\pi} \arcsin(\rho)$
Clayton	$\theta \in (0, \infty)$	$2^{-1/\theta}$	0	$\frac{\theta}{2 + \theta}$
Gumbel	$\theta \in [1, \infty)$	$2 - 2^{1/\theta}$	0	$1 - \frac{1}{\theta}$
Rotated Gumbel	$\theta \in [1, \infty)$	0	$2 - 2^{1/\theta}$	$1 - \frac{1}{\theta}$

Source: Heinen and Valdesogo (2012).