Optimal Stormwater Policy in a Dynamic Setting

Olesya Savchenko and Amy W. Ando
University of Illinois at Urbana-Champaign
savchen2@illinois.edu; amyando@illinois.edu

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1. Introduction

Urban stormwater runoff is among the main sources of non-point pollution of surface water in the U.S (Jacobson 2011). Urban development yields growing accumulations of impervious surfaces such as streets, parking lots, and building roofs. While this stock of impervious surfaces provides many private and social benefits, it inhibits infiltration of rainwater and snowmelt and creates runoff during storms. Runoff carries contaminants that degrade water quality, threatens the health of aquatic species, disrupts hydrological stream functions, and contributes to flood problems (NRC 2008).

The problem of excess stormwater runoff is exacerbated by changing landscapes, aging infrastructures, more severe and frequent rainfall events associated with changing climate, and lack of economic policies aimed at optimal long-term reductions of stormwater runoff. Effective management of the growing stormwater runoff problem is a significant challenge facing cities throughout the US as they strive to protect water quality and reduce their vulnerability to flooding. Yet, to date, little research has been done to help decision-makers in choosing economic policies that achieve socially optimal stormwater runoff reductions both in the short- and long-run. To fill this gap, we develop a multi-period optimal dynamic model for stormwater runoff management that treats impervious surface as a stock pollutant with negative externality. We find how to identify the dynamically optimal tax on additions to impervious surfaces. We show how a simple Pigouvian tax set to optimize short-run well-being will deviate from the dynamic optimum. Finally, we explore how efficient stormwater policy will vary with features of the city that is trying to set that policy.

Stormwater runoff in most cities is managed by a command-and-control approach under the National Pollution Discharge Elimination System of the Clean Water Act that requires municipalities and industrial sites to control runoff during construction process. This policy does not address the negative impact of stormwater runoff beyond the construction stage. It also provides no incentives to private property owners to control runoff on their individual properties. At the same time, the existing centralized stormwater management systems prove to be ineffective in responding to flood events, while their expansion is prohibitively expensive (Thurston 2012).

Alternative decentralized strategies for stormwater runoff include low-impact development (LID) stormwater solutions such as permeable pavements, bioswales, green roofs, rain gardens, rain barrels, and infiltration trenches among others. These parcel level solutions allow for design of hydrologically functional sites that resemble nature in detaining, storing, treating and infiltrating runoff (USEPA 2000). A growing literature on decentralized stormwater management indicates that decentralized low-impact development (LID) style stormwater solutions such as permeable pavements, rain gardens, rain barrels, and infiltration trenches reduce stormwater runoff at much smaller costs than conventional large-scale management systems (Ando and Braden 2012), while also offering multiple environmental benefits such as reductions in downstream flooding and decreases in surface water pollution, erosion and sedimentation (Braden and Johnston 2004). Cities like Chicago, Philadelphia and Washington D.C. have already begun initiatives to promote the use of green roofs, rain gardens/barrels and green building construction (NRDC 2011). However, despite these efforts and the documented benefits of LIDs (Braden and Johnston 2004, Ando and Braden 2012), the scale of LID implementation remains small. Lack of effective private economic incentives is a major barrier to widespread implementation of LIDs at urban watershed scale (Roy et al. 2008), and stormwater policies have not been implemented to adequately supplement private incentives.

Theoretical and empirical evidence of how market- based mechanisms can be used to achieve socially optimal reductions in stormwater runoff is needed to help urban decision-makers nationwide to design stormwater management policies that are socially optimal and forward-looking. However, only a handful of studies have explored the potential of market-based mechanisms to help individuals to internalize stormwater runoff externality and to achieve cost-effective and socially optimal runoff reductions. Doll and Scodari (1998) provide examples of credits for onside stormwater retention used by stormwater utilities throughout the country. Parikh *et al.* (2005) highlight the economic, hydrological and legal issues that could be associated with the use of price and quantity instruments to reduce stormwater runoff. Based on a case study of a small watershed in Cincinnati, Ohio (Thurston *et al.* 2003) demonstrate the effectiveness of tradable allowances as a low-cost economic policy to reduce stormwater runoff, while Goddard (2012) provides both deterministic and stochastic frameworks for a stormwater trading mechanism. Thurston *et al.* (2010) analyze the potential of reverse auctions in promoting LID adoption. Most recently Lu *et al.* (2013) explore the interactions of policy-makers through an agent-based model as well as simulate the impact of fees imposed on developers for not using LIDs.

While valuable, these studies concentrate on static economic models that provide little guidance to policymakers on how to design a stormwater management policy that is optimal over time. Recently proposed economic models also do not account for the stock nature of impervious surface. The short-run nature of such an approach ignores accumulation of a stock of impervious surfaces over time and its cumulative impact on the watershed. Our work contributes to the literature on managing a stock like impervious surface that generates negative externalities by developing a multi-period optimization model that identifies the optimal path of additions to the stock itself and additions to a stock of infrastructure that can abate the negative effects of the stock. We contrast the outcome of a static optimization model with the results of optimal control models with and without the abatement technology. We show that a static Pigouvian tax as compared to a dynamically optimal policy leads to higher flows of impervious surface and less abatement.

The rest of this paper is organized as follows. In the section 2 we present several economic models of stormwater runoff management. First, we introduce a static model in which a planner chooses an optimal quantity of additions to the stock of impervious surface, and we contrast this result with the outcome of actions taken by the private market. Then we characterize outcomes of a dynamic framework and compare those findings (and the associated policy that would be needed to implement it) to the results of the static model. Finally, the dynamically optimal model is expanded to include LID abatement of stormwater runoff. In section 3 we provide results of numerical simulations with specific functional forms and several sets of hypothetical parameters to show how the results depend on features of the city at hand.

2. Model

Consider a city developed with impervious surfaces such as buildings, houses, roads, driveways, and parking lots. We adopt an area of impervious surface as the basic unit of analysis in all of our models because this is a comprehensive and measurable indicator of the urban development that is the key contributing factor to the volume of stormwater runoff (Kertesz et al. 2014).

In what follows, we first look at the market behavior in absence of stormwater water policy. We then analyze and compare socially optimal stormwater policy in static and dynamic frameworks. Models without and with abatement are considered.

2.1. Stormwater Management Policy with No Abatement

2.1.1. Static Market Behavior

Assume a competitive free market with many agents and an absence of any policy to reduce stormwater runoff. Each period agents maximize their private net benefits of building impervious surface as shown in equation (1), where the benefits that agents derive from the initial stock of impervious surface I_t and from additions to that stock, λ_t, is $B(I_t, \lambda_t)$ and the cost of building them is $C(\lambda_t)$:

$$max_{\lambda_t} B(I_t, \lambda_t) - C(\lambda_t) \quad (1)$$

While firms could build low-impact impervious surfaces (e.g. permeable pavements) that allow for some degree of rain penetration into the soil, we assume that the marginal cost of building such surfaces is greater than the marginal cost of building conventional structures (that assumption is consistent with the minuscule investment in LID construction we actually observe as a fraction of total new construction in the U.S.). Thus, in absence of any policy, private agents build using cheaper conventional options. Similarly, abatement that could take the form of low-impact developments (LID) such as bioswales, rain gardens, rain barrels, green roofs, and others is omitted from equation (1) if no policy exists to give private land owners an incentive to spend money on such abatement.

Equation (1) leads to an equilibrium addition to the stock of impervious surface in time t, λ_t^* where

$$B'(I_t, \lambda_t^*) = C'(\lambda_t^*) \quad (2)$$

With no incentive to internalize stormwater runoff externality, the market accounts only for the private cost of building impervious surface, which yields a privately optimal allocation of impervious surface that ignores the stormwater externality that these surfaces impose on society.

2.1.2. Static Social Planner's Problem

A socially efficient allocation of impervious surface can be achieved by solving a social planner's problem that incorporates the runoff externality produced by the impervious surface and maximizes the net social benefits. We first consider a model only with a flow and stock of impervious surface but no abatement. In a myopic static model each period the social planner solves the following problem:

$$W(\lambda_t^*) \equiv max_{\lambda_t}\, B(I_t, \lambda_t) - C(\lambda_t) - D(I_t, \lambda_t) \qquad (3)$$

In equation (2), $D(I_t, \lambda_t)$ is the damage function which includes both the initial stock and the additional flow of impervious surface that result in storwmater runoff externality. The social planner takes the current stock of impervious surface in period t as given without considering the future benefits and damages produced by this stock and its additions in period t. Assuming a growing urban area, we focus on interior solution for a socially efficient allocation of impervious surface additions, λ_t^*, which is obtained from the first order conditions:

$$B'(I_t, \lambda_t^*) = C'(\lambda_t^*) + D'(I_t, \lambda_t^*) \qquad (4)$$

The external marginal cost of an additional unit of impervious surface is $D'(I_t, \lambda_t^*)$. Equation (4) demonstrates the standard case of negative externality. The marginal benefit derived from the additional unit of impervious surface is equal to the sum of the marginal cost of adding a unit of impervious surface and the social damage it causes. An optimal tax on the additional unit of impervious surface that would help internalize the stormwater runoff externality is $\tau_t^* = D'(I_t, \lambda_t^*)$; this is the standard Pigouvian result. Note that even in cities that do have taxes on impervious surfaces, the discussion about how to set the tax rates is usually in terms of raising revenue for stormwater abatement projects rather than finding the tax rate that gives land owners efficient incentives to reduce additions to the stock of impervious surface with the increased stormwater flows that go with them.

While the social planner's problem in (3) solves for a tax that maximizes net social welfare in the current period, this tax accounts only for the additions of impervious surface in the current period and the damage resulting from during that time. The regulator is not forward looking since future damages from increases in the stock of impervious surface are not included in the planner's problem.

2.1.3 Dynamic Social Planner's Problem

While the social planner accounts for the damage from the flow of impervious surface as shown in equation (3), the stock nature of impervious surface is not taken into consideration. Here we develop a simple dynamic model that allows us to capture the stock nature of impervious surface that generates both benefits in the form of urban structures and damages in the form of stormwater runoff externality that persist across time. It allows us to include important intertemporal aspects into the design of stormwater management policy. The social planner's problem is now to maximize the discounted flow of net benefits to determine the optimal intertemporal allocation of impervious surface. The time horizon is infinite and the future benefits are discounted at a constant social discount rate, $r > 0$. We set up a dynamic version of the problem in equation (5):

$$max_{\lambda_t} \int_0^\infty e^{-rt}\left\{B(I_t, \lambda_t) - C(\lambda_t) - D(I_t, \lambda_t)\right\}\text{dt} \quad (5)$$

$$s.t. \quad \dot{I}_t = \lambda_t - \zeta I_t, \quad I_{t=0} = I_0 > 0, \quad (6)$$

where equation (6) represents the equation of motion governing the accumulation of the stock of impervious surface. The stock accumulates as a result of additional flow of impervious surface λ_t net of any depreciation in stock, ζI_t, with $0 < \zeta < 0$.

In setting up the social planner's optimal control model in equation (5) we build upon the established economic literature on the carbon stock externality (Keeler et al. 1971, Ulph and Ulph 1994, Farzin 1996). In the economics literature on carbon emissions and non-point source pollution control it is typical for the benefit or profit function to depend only on the flow of emissions with stock externality entering as damages in the social welfare function. In the case of stormwater runoff, the stock of impervious surface generates both benefits and damages that extend into the future. We, therefore, allow for the benefit function to depend directly on the stock of impervious surface as well as its flow. While agents derive benefits from building an additional unit of impervious surface, they also benefit from the stock of existing structures such as road, parking lots and other buildings. This stock of existing structures also results in urban runoff and thus is included in the social damage function.

The assumptions of the model are as follows. Note that we keep the model simple, emphasizing only those elements that are essential to the runoff problem. Thus, at this point we abstract away from the spatial, specific hydrological and stochastic elements in our model.

Assumption (1): Benefits are a function of the stock and flow of impervious surface. The benefit function is concave and continuously differentiable in both arguments, that is $B_I > 0$, $B_{II} < 0$ and $B_\lambda > 0$, $B_{\lambda\lambda} < 0$, where subscripts indicate first and second derivatives and the time subscripts on λ_t and I_t are dropped for brevity.

Assumption (2): The cost function of adding a unit of impervious, $\mathrm{C}(\lambda_t)$, is strictly increasing, twice differentiable and strictly convex with $C_\lambda(\lambda_t) > 0$, $C_{\lambda\lambda}(\lambda_t) > 0$ and $\mathrm{C}_0 = 0$.

Assumption (3): Damages depend on the stock and flow of impervious surface. The damage function is continuous, twice differentiable and strictly convex in both arguments, that is $D_I > 0$, $D_{II} > 0$ and $D_\lambda > 0$, $D_{\lambda\lambda} > 0$.

The current value Hamiltonian for the social planner's problem in equation (5) subject to equation (6) is given by:

$$H = B(I_t, \lambda_t) - C(\lambda_t) - D(I_t) - \mu_t(\lambda_t - \zeta I_t) \quad (7)$$

where μ_t is the co-state variable that represents the shadow cost associated with the stock of impervious surface. Note that the co-state variable has a negative sign to facilitate its interpretation.

Using Pontryagin's maximum principle, we obtain the following first-order conditions that are necessary and sufficient conditions for interior solution are as follows:

$$B_\lambda(I_t,\lambda_t) - C_\lambda(\lambda_t) - D_\lambda(I_t, \lambda_t) - \mu_t = 0 \quad (8)$$

$$\dot{\mu} = (r + \zeta)\mu_t + B_I(I_t,\lambda_t) - D_I(I_t, \lambda_t) \quad (9)$$

$$\dot{I}_t = \lambda_t - \zeta I_t \quad (10)$$

$$\lim_{t\to\infty} e^{-rt} I_t = 0 \quad (11)$$

Equation (8) has the usual intuitive economic interpretation – along the optimal path the marginal benefit of additional impervious surface is equal to the cost of building this unit of impervious surface plus the marginal external cost it inflicts on the society in the form of stormwater externality. However, unlike equation (4) in the static model, equation (9) also includes μ_t, the shadow cost of adding an additional unit of impervious surface to the existing stock. This distinction implies that a statically optimal Pigouvian tax will be smaller than a dynamically optimal tax. This is because the static tax accounts only for the social damage that results from the flow of impervious surface in the current period, while the dynamically optimal tax includes both the external damage from the flow and the stock damages that result from impervious surface.

To further illustrate the comparison with the static optimum, we follow (Leandri 2009) and denote λ_t^o and I_t^o be the optimal values of λ_t and I_t at time t along the optimal path. Let the function $G(I_t,\lambda_t)$ equal equation (8) such that

$$G(I_t,\lambda_t) = B_\lambda(I_t,\lambda_t) - C_\lambda(\lambda_t) - D_\lambda(I_t,\lambda_t) - \mu_t \qquad (12)$$

According to the assumptions imposed on the benefit, cost and damage functions, $G(\lambda_t, I_t)$ is decreasing in λ_t. At any point in time t, an optimum is described by equation (12):

$G(I_t, \lambda_t^o) = B_\lambda(I_t,\lambda_t^o) - C_\lambda(\lambda_t^o) - D_\lambda(I_t, \lambda_t^o) - \mu_t = 0$.

We can now compare this to the static optimal result $B'(I_t, \lambda_t^*) = C'(\lambda_t^*) + D'(I_t, \lambda_t^*)$ described in equation (4), where λ_t^* is the static optimal value of additions to impervious surface. Comparing the result from (4) and (11) using $G(\lambda_t, I_t)$ we get

$$G(I_t,\lambda_t^*) = B_\lambda(I_t,\lambda_t^*) - C_\lambda(\lambda_t^*) - D_\lambda(I_t,\lambda_t^*) - \mu_t \qquad (13)$$

$$G(I_t,\lambda_t^*) = -\mu_t \quad \text{(because } B'(I_t, \lambda_t^*) = C'(\lambda_t^*) + D'(I_t, \lambda_t^*)\text{)}$$

Since $G(I_t,\lambda_t^*)$ is a decreasing function in λ_t, the above result implies that $\lambda_t^o < \lambda_t^*$ for any I, which shows that the dynamically optimal addition to impervious surface that accounts for the external damages from the stock of impervious surface is less than that which emerges in the static optimum. This an important result that demonstrates the need to use a dynamic model to develop an optimal policy for stormwater runoff externality that results not only from the addition to the impervious surfaces in this period but also from the existing and growing stock of these surfaces.

To facilitate economic interpretation of equation (9) and applying the transversality conditions in equation (11) lead to equation (14):

$$\mu_t = \int_t^{\infty} e^{-(r+\zeta)(s-t)}[B_I(s) - D_I(s)]\ ds \qquad (14)$$

Equation (14) has important economic interpretation. It shows that the shadow price of additions to the stock of impervious surface at time t equals a discounted sum of the difference between all the marginal stock benefits and marginal stock damages in period t and all future periods by increasing cumulative levels of impervious surface.

2.2. Stormwater Management Policy with Abatement

The models in Section 2 demonstrate the need to use a dynamic framework to formulate an optimal stormwater policy that includes both the flow and the stock damages resulting from impervious surface. We now introduce stormwater runoff abatement in the form of on-site low-impact development (LID) solutions such as vegetated swales, infiltration trenches, permeable pavements, rain barrels and rain gardens, and green roofs into the model. Our objective is to find optimal levels of impervious surface and an optimal mix of conventional forms of stormwater abatement such as sewers and LIDs.

Similar to the previous section, we first start with a static model, followed by a dynamic model and comparison of the two. In addition to the model assumptions (1)-(3) we add the following assumptions:

Assumption (4): Abatement costs of adding additional conventional and LID, $C(\gamma_t)$ and $C(\alpha_t)$, respectively, are strictly increasing, twice differentiable and convex with $C_\gamma(\gamma_t) > 0$, $C_{\gamma\gamma}(\gamma_t) > 0$ and $C_\alpha(\alpha_t) > 0$, $C_{\alpha\alpha}(\alpha_t) > 0$.

Assumption (5): Damages depend on the stock and flow of impervious surface and stock of abatement, A_t. The damage function $D(I_t, \lambda_t, A_t)$ is continuous, twice differentiable and strictly convex in both arguments, $D_I > 0$, $D_{II} > 0$ and $D_\lambda > 0$, $D_{\lambda\lambda} > 0$, $D_A < 0$, $D_{AA} > 0$.

2.2.1. *Static Social Planner's Problem*

A social planner solves the maximization in equation (15), which now includes γ_t and α_t that represent conventional ("grey") infrastructure used to mitigate stormwater runoff and LID such as bioswales, green roofs, rain barrels, rain gardens and others.

$$W(\lambda^*, \alpha^*, \gamma^*) \equiv max_{\lambda_t, \alpha_t, \gamma_t} B(I_t, \lambda_t) - C(\lambda_t, \alpha_t, \gamma_t) - D(I_t, \lambda_t, A_t), \qquad (15)$$

where the stock of abatement is $A_t = A_{t-1} + \alpha_t + \gamma_t$, with A_{t-1} representing the stock of abatement that has accumulated from previous period.

Equation (16) shows the first order conditions corresponding to equation (15).

$$B_\lambda = C_\lambda + D_\lambda \qquad (16)$$

$$C_\alpha = D_\alpha \qquad (17)$$

$$C_\gamma = D_\gamma \qquad (18)$$

From equation (16) we see that the marginal benefits of additions to impervious surface are equal to their marginal costs plus the marginal damage they create. Equations (17) and (18) show that the marginal costs of abatement are equal to the reduction in marginal damage from this abatement.

2.2.2. *Static Social Planner's Problem*

$$max_{\lambda_t, \alpha_t, \gamma_t} \int_0^\infty e^{-rt} \{B(I_t, \lambda_t) - C(\lambda_t, \alpha_t, \gamma_t) - D(I_t, \lambda_t, A_t)\} \mathrm{dt} \qquad (19)$$

$$s.t. \quad \dot{I}_t = \lambda_t - \zeta I_t \qquad (costate\ \mu_t) \qquad (20)$$

$$\dot{A}_t = \alpha_t + \gamma_t - \varphi A_t \qquad (costate\ \eta_t) \qquad (21)$$

$$I_{t=0} = I_0 > 0; A_{t=0} = A_0 > 0 \qquad (22)$$

Equation (19) describes a dynamic optimization problem that now includes additions to the impervious stock and two types of abatement. This dynamic model includes two equations of motion to represent the evolution of stock of impervious surfaces I_t and the stock of abatement

A_t. As in the equation (6), the stock of impervious surface grows due to additions of impervious surface net of stock depreciation, ζI_t, where $0 < \zeta < 1$. The stock of abatement increases due to the additions of conventional and LID abatement net of stock depreciation represented by φA_t, where $0 < \varphi < 1$.

The Hamiltonian and the necessary and sufficient conditions for interior solution are presented in equations (24)-(30):

$$H = B(I_t, \lambda_t) - C(\lambda_t, \alpha_t, \gamma_t) - D(I_t, \lambda_t, A_t) - \mu_t(\lambda_t - \zeta I_t) + \eta_t(\alpha_t + \gamma_t - \varphi A_t) \quad (23)$$

$$B_\lambda - C_\lambda - D_\lambda + \mu_t = 0 \quad (24)$$

$$-C_\alpha - D_\alpha + \eta_t = 0 \quad (25)$$

$$- C_\gamma - D_\gamma + \eta_t = 0 \quad (26)$$

$$-(\dot{\mu}_t - r\mu_t) = B_I - D_I + \zeta\mu_t = 0 \quad (27)$$

$$-(\dot{\eta}_t - r\eta_t) = D_A - \theta\eta_t = 0 \quad (28)$$

$$\dot{I}_t = \lambda_t - \zeta I_t \quad (29)$$

$$\dot{A}_t = \alpha_t + \gamma_t - \varphi A_t \quad (30)$$

$$\lim_{t\to\infty} e^{-rt}\mu_t I_t = 0; \quad \lim_{t\to\infty} e^{-rt}\eta_t A_t = 0 \quad (31)$$

Economic interpretation of equation (24) is the same as equation (8) of model with no abatement - along the optimal path the marginal benefit of flow of impervious surface is equal to its the marginal cost and the marginal damage it causes and the shadow cost of increasing the stock of impervious surface. Equations (25) and (26) have important economic interpretation: the cost of additional unit of conventional (LID) abatement is equal to the reduction in marginal damage that this unit of conventional (LID produces plus the shadow value of increasing the stock of abatement.

Equations (25) and (26) along with equations (27) and (28) characterize the optimal paths of the co-state variables and stocks. Using equations (25) and (26) along with the transversality conditions in (31) we can derive the following equations for the co-state variables:

$$\mu_t = \int_t^{\infty} e^{-(r+\zeta)(s-t)}\{B_I(s) - D_I(s)\}ds \tag{32}$$

$$\eta_t = -\int_t^{\infty} e^{-(r+\theta)(s-t)}\{D_A(s)\}ds \tag{33}$$

Equation (32) has the same economic interpretation as equation (14) - the shadow price of adding one more unit to the stock of impervious surface at time t is the discounted sum of the difference between all the marginal benefits and marginal damages caused by the stock of impervious surface in that period and all future periods. The economic interpretation of equation (33) suggests that the shadow price of adding one more unit of abatement at time t equals the discounted sum of the reduction in marginal damage in that period and in all future periods.

Comparison of the static and dynamic models with respect to impervious surface remains the same as described in section 2.1.3. In terms of abatement, however, the static model does not account for the accumulations of abatement stock and only considers the flows of conventional and LID abatement taking the level of accumulated from previous periods as given. Unlike equations (17) and (18) of the static social planner's problem, equations (25) and (26) in the dynamic framework account for the shadow price of increasing the stock of abatement, that is in this formulation the social planner takes into consideration not only the current reductions in stormwater runoff due to flows of abatement but also the benefit of increasing the stock of this abatement. This represents the true social marginal benefit from abatement of stormwater runoff. This also implies that the static result will lead to less abatement because the shadow price of stock of abatement is not accounted for in the static model.

2.3 *Summary of Analytical Results*

Before introducing specific functional forms and numerical analysis for both static and dynamic frameworks, it is important to summarize the analytical results thus far (Table 1). The comparison of the static and dynamic models with and without abatement demonstrated that in the context of stormwater runoff problem, it is essential to take into consideration the stock of impervious surface and stock of abatement. Optimal control problems allow us to account for the shadow prices of these stocks in the derivation of optimal solutions. These results, in turn, have important policy implications. Using a static approach to stormwater policy can result in the suboptimal use of policy instruments such as taxes. As demonstrated in the analytical models, static Pigouvian tax is less than a dynamically optimal tax and, therefore, would be less effective at helping private market to internalize the stormwater externality and will lead to less than optimal levels of abatement. This result follows from the fact that in a static model only the external damage from the flows of impervious surface and the benefits from flows of abatement is included. A dynamically optimal model, however, allows the decision-maker to incorporate current and future benefits and damages from the stock of impervious surface as well as current and future benefits of increasing the stock of abatement.

3. Specific Functional Forms and Numerical Analysis

To gain additional economic intuition about the model's behavior in static and dynamic frameworks, we assume specific functional forms such that they have the desired properties as described in assumptions (1)-(3) and can reasonably approximate the stormwater runoff problem facing many cities throughout the U.S. Our objectives are to highlight how the optimal pattern of impervious surface additions various with the size of the existing stock at the beginning of the planning period (e.g. big cities with large stock of impervious surface vs. small city with low levels of accumulated stock) and show the differences between policy responses to this stock that are myopic (Pigouvian tax) and forward looking (shadow price of impervious surface). To these ends, we use the following function forms.

Benefit function: $\mathrm{B}(\mathrm{I_t}, \lambda_t) = \mathrm{b}(\mathrm{I_t} + \lambda_t)^{\alpha}$, where $b > 0$ is the parameter scales the benefit function and shows the level of benefits at which additional increases in impervious surface provide no further benefits, $\alpha < 1$ is a parameter ensures that the benefit function has the desired concavity properties and can be used to vary the shape of the function, and $\mathrm{I_t} = \bar{I}$ is the stock that has accumulated from the previous pervious up to time t.

Cost function: $C(\lambda_t) = \frac{c}{2}{\lambda_t}^2$, where $c > 0$ is the parameter representing slope of marginal cost.

Damage function: $D(\mathrm{I_t}, \lambda_t) = \mathrm{d}[\theta(\mathrm{I_t} + \frac{\mathrm{d}}{2}\lambda_t)^2]$, where g slope of marginal damage, θ is a parameter that shows the stormwater runoff resulting from $\mathrm{I_t}$ and $\mathrm{I_t} = \bar{I}$ is the stock that has accumulated from the previous pervious upto time t, and d is parameter represents damage from flow of impervious surface.

With the above functional forms, the market problem in the static framework becomes

$$\max_{\lambda_t > 0}\ \mathrm{b}(\mathrm{I_t} + \lambda_t)^{\alpha} - \frac{c}{2}{\lambda_t}^2 \qquad (11)$$

and the static social planner's problem is

$$\max_{\lambda_t>0} \mathrm{b}(\mathrm{I_t}+\lambda_t)^\alpha - \frac{\mathrm{c}}{2}{\lambda_t}^2 - \mathrm{d}[\theta(\mathrm{I_t}+\frac{\mathrm{d}}{2}\lambda_t)^2] \qquad (12)$$

while in the dynamic setting the social planner's problem is

$$max_{\lambda_t} \int_0^\infty e^{-rt}\left\{ \mathrm{b}(\mathrm{I_t}+\lambda_t)^\alpha - \frac{\mathrm{c}}{2}{\lambda_t}^2 - \mathrm{d}[\theta\left(\mathrm{I_t}+\frac{\mathrm{d}}{2}\lambda_t\right)]^2 \right\} \mathrm{dt} \qquad (13)$$

$$s.t. \quad \dot{I}_t = \lambda_t - \zeta I_t, \quad I_{t=0} = I_0 > 0. \qquad (14)$$

Due to the nonlinearities in these functional forms, it is difficult to obtain explicit analytical solutions to equations (11), (12) and (13). We therefore provide numerical simulations to gain economic insight and compare optimal free market allocations of impervious surface with socially optimal outcomes in static and dynamic frameworks. In this section of the paper we use hypothetical parameters to demonstrate the different optimal policy response based on the size of initial stock and whether the policy is myopic or forward looking (future versions of this work will develop sets of parameters that approximate conditions in several real cities).

Table 1 summarizes the values of hypothetical parameters used in the numerical analysis performed in MATLAB. Figure 1 plots benefit, cost and damage functions parameterized with values specified when stock of impervious surface $\bar{I}$ equals 1 in Table 1.

4. Preliminary Numerical Results

Using parameters specified in Table 1, we solve for privately and socially optimal additions of impervious surface. We vary the initial stock of accumulated impervious surfaces to determine the response of the optimal allocations of impervious surface by the market and social planner. The results are plotted in Figure 2. Given larger amounts of accumulated stock of impervious surface both the market and social planner choose smaller additions to impervious surface stocks to optimize their respective objective functions; ceteris paribus, a larger stock means that the marginal benefit of further additions is smaller and the marginal damage of additions is larger. As predicted by the static market and social optimization models described in equations (2) and (4), the market would choose greater additions to the stock of impervious surface than is socially optimal because the market fail to account for the stormwater runoff damage resulting from the impervious surface. In addition, the gap between the market outcome and the socially optimal additions to the stock gets bigger with the size of the initial stock because the marginal external damage of additional impervious surface increases with the initial stock.

Figure 3 demonstrates the corresponding Pigouvian tax imposed on additions to impervious surface to help the market internalize the stormwater externality. As seen from the plot, as the optimal amount of additional impervious surface allocation is higher, the optimal Pigouvian tax increases as well. These results are intuitive. A social planner of large city with already large stock of impervious surface built up would impose a higher static tax because the marginal damage of additional impervious surface is higher..

5. Conclusions and Ongoing Work

Stormwater runoff caused by the built environment is one of the main contributors to the pollution of water bodies across the U.S. (NRC 2008). The main objective of this paper is to develop an optimal forward-looking stormwater policy. To do this we develop several static and dynamic models and contrast their analytical results. An important feature of our models is that stock of impervious surface produces both the benefit in the form of urban environment such as buildings, roads, and other concrete infrastructure, and damages in the form of stormwater runoff. Though model comparisons we first demonstrate why it is necessary to develop a stormwater policy that is optimal over time. We demonstrate that the static optimal addition of impervious surface with no stormwater runoff regulation is higher than the socially optimal amount. We then contrast the social static optimum with a dynamic model that first only includes the stock of impervious surface and then also the stock of abatement in the form conventional grey infrastructure such as sewers and low-impact development (LIDs) solutions such as bioswales, rain gardens, green roofs, etc. This comparison demonstrates that a myopic policy such a static Pigouvian tax will result in higher optimal additions of impervious surface and lower levels of abatement.

In numerical simulations with specific functional forms we vary the size of the initial stock of impervious surface to determine the response of a myopic social planner choosing optimal additions of impervious surface in a static framework. The results of the numerical analysis demonstrate that the optimal additions of impervious surface decrease with higher initial stocks of impervious surface. This implies that it would be optimal for a city like Chicago that has a large stock of impervious surface to add smaller amounts of impervious surface in the period under analysis. The response of the market to higher aggregate levels of impervious surface in absence of any stormwater regulation is similar to the socially optimal outcome - the optimal allocation of flow of impervious surface decreases in response to higher levels of the initial stock of impervious surface. However, the market optimal allocation is higher than the socially optimal additions to impervious surface. Moreover, as the size of the stock increases the gap between the static market optimal and static social optimal allocations of impervious surface increases.

The ongoing work includes numerical simulations to demonstrate the features of optimal paths of additions to impervious surface and dynamically optimal taxes that would result in optimal stormwater policy overtime.

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Table 1: Summary of Analytical Results

	Market	**Static**	**Dynamic**
Optimal Allocation of flow of IS	$B'(I_t, \lambda_t^*)$=$C'(\lambda_t^*)$	$B'(I_t, \lambda_t^*)$= $C'(\lambda_t^*)$+$D'(I_t, \lambda_t^*)$	$B_\lambda(I_t,\lambda_t) = C_\lambda(\lambda_t)$+ $D_\lambda(I_t, \lambda_t)$+ μ_t
Economic Interpretation	Externality not internalized	Taking current stock of impervious surface as given, flow externality is internalized	Along the optimal path, the marginal benefit of additional impervious surface is equal to the cost of building this unit of impervious surface plus the marginal external cost and the shadow cost of increasing the stock of impervious surface
Optimal Abatement	none	$C_\alpha = D_\alpha$ and $C_\gamma = D_\gamma$	$D_\alpha = \eta_t - C_\alpha$ and $D_\gamma = \eta_t - C_\gamma$
Economic Interpretation	No abatement takes place	Runoff is abated at the point where marginal abatement cost equals marginal reductions in damages from this abatement	Along the optimal abatement path marginal cost of abatement is equal to the marginal reductions in damages and shadow cost of abatement capital
Policy Implications	Stormwater runoff persist as a problem	Damage imposed by the additions to the impervious surface stock are internalized. Externality tax is lower than a dynamically optimal tax. Level of abatement is lower than in dynamically optimal model.	Accounts for the benefits and damages of the stock of impervious surfaces. Also, accounts for the benefits of increasing the stock of abatement. A dynamically optimal policy will allow of internalization of true externalities imposed by the stock nature of impervious surfaces and the associated stormwater runoff pollution.

Table 2: Parameter Values for Numerical Analysis

Parameter	Description	Value
$\boldsymbol{\alpha}$	Exponent on the benefit function that determines the curvature of the function. Ensures that benefits are concave for $\alpha \in (0,1)$	0.25
$\boldsymbol{b}$	Scaling parameter for benefit function	5
$\boldsymbol{c}$	Slope of marginal cost	0.5
$\boldsymbol{d}$	Slope of marginal damage	0.6
$\boldsymbol{\theta}$	Relates stormwater runoff to the stock of impervious surface	0.7
$\boldsymbol{\zeta}$	Rate of decay of the stock of impervious surface	0.05
$\boldsymbol{r}$	Social discount rate	0.01
$\boldsymbol{\bar{I}}$	Level of impervious stock	Varied

Figure 1: Shapes of Parameterized Functions Used in Simulations

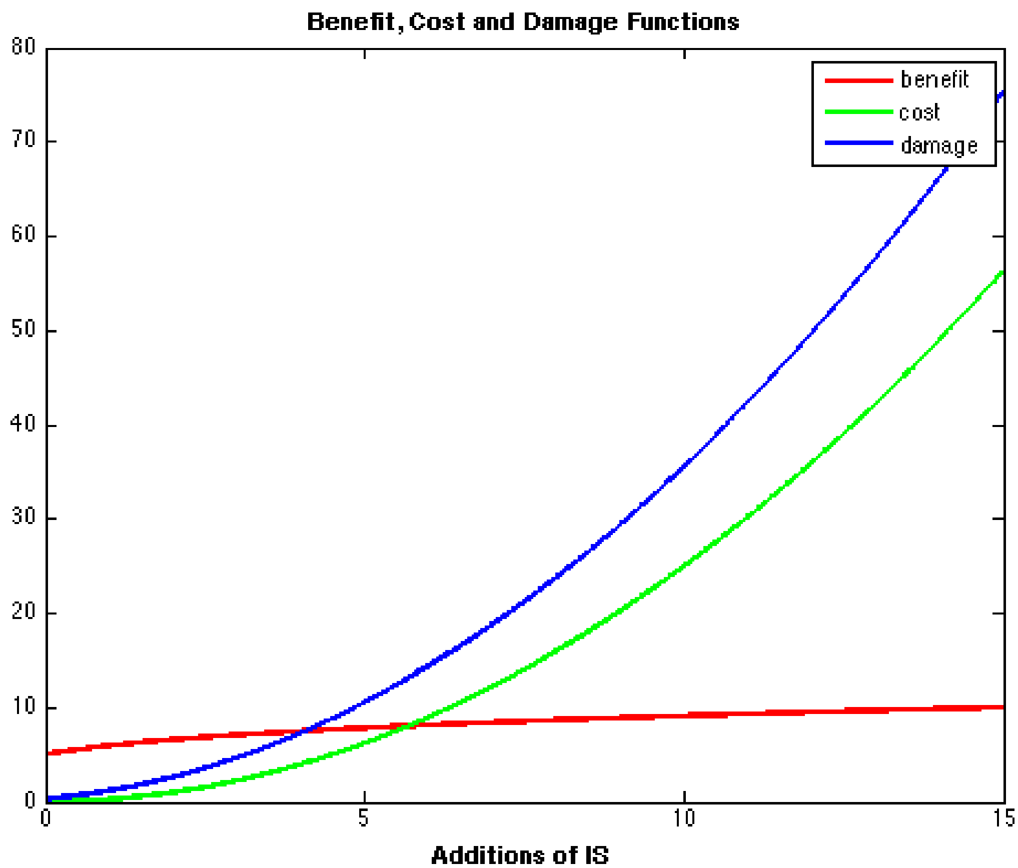

Figure 2: Optimal Private and Social Response to Different Stocks of Impervious Surface

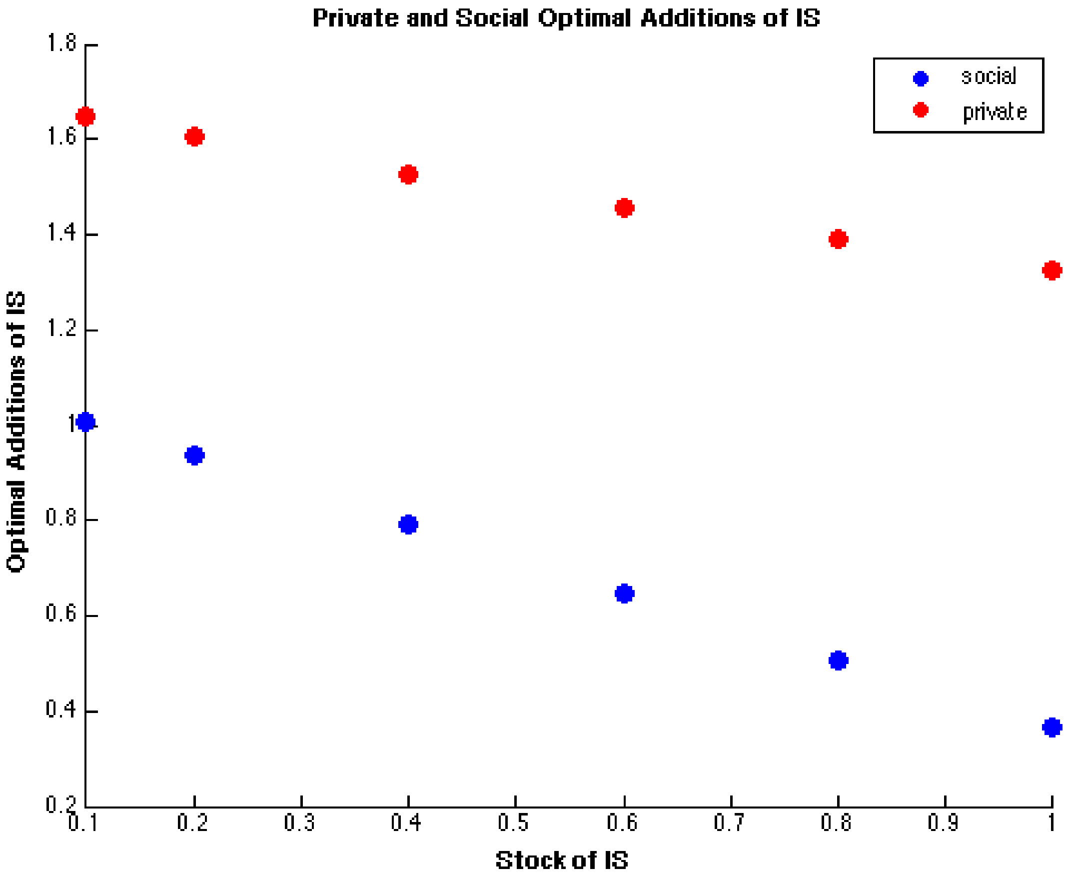

Figure 3: Optimal Tax