**Aquifer Depletion in the face of Climate Change andTechnical Progress.**

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**WORKING PAPER**

### Abstract

A dynamic optimization framework is applied to the problem where groundwater stock and the state of technology and the climate are the state variables and groundwater pumping is the control variable and the objective is to maximize the net present value of the stream of rents from irrigation over the life of the aquifer. Dynamical systems govern the evolution of the aquifer, the climate, and the rate of technical progress. These dynamical systems may be dependent upon periodic groundwater allocations, as in the case of the aquifer, or independent of the periodic allocations as in the case of climate and technical change. Alternative plans are considered where the planner ignores one or more of the state variables when prescribing an extraction path. The "information effect" in these plans is assessed by comparing extraction, depletion, and rents from irrigation paths. A well accepted formulation of hydrologic dynamics for the aquifer is employed and simple dynamic trends for climate and technology are developed. A simplified example of the model incorporating only deterministic aquifer and technical change dynamics is presented as a linear-quadratic optimal control problem. Numerical results from Sheridan County, KS, suggest that prescribing a pumping schedule ignoring the dynamics of climate change is most costly. Furthermore, once on the optimal path, relatively large savings in groundwater may be achieved with relatively small portions of profits foregone.

# 1 Introduction

The economies of large agricultural regions such as the Great Plains in the United States are dependent on groundwater availability, making aquifer depletion a much-discussed policy and research issue. Increasing temperatures and decreasing average precipitation raise the demand for irrigation groundwater in the future, which further intensifies the

stress on already depleting aquifers. This added stress may be offset to an unknown degree by improvements in crop varieties.

This paper explores the role of technical progress in mitigating the effects of climate on aquifer depletion. Technical progress in the form of crop varieties capable of producing larger yields per volume of pumped groundwater suggests that farmers are able to sustain crop production levels with decreasing rates of applied groundwater. However, the increased water productivity of the evolving crop varieties modify the incentives for farmers to allocate water. Higher future productivity provides the incentive to conserve groundwater in the present and apply it in the future when it is more valuable, but it also raises the incentive to apply more irrigation groundwater in subsequent periods. It is not clear if these competing effects would result in increasing or declining optimal rates of groundwater extraction (aquifer depletion).

Irrigated agriculture is among the largest users of groundwater in the world. Irrigators extract and apply groundwater to satisfy their crop water requirement in order to achieve yield levels so as to maximize farm profits. Groundwater extraction occurs as long as the value marginal product of groundwater equals or exceeds the marginal cost of applying it. Since groundwater is a fugitive resource, the stock available to an irrigator depends dynamically on the levels of extraction by neighboring irrigators. This constitutes the common pool resource property of groundwater.

Population and economic growth along with climate change intensify the stress on already depleting groundwater resources. Philosophers, thinkers, and rulers have raised the issue of providing for a growing population with finite resources millenia before the term "economy" acquired its modern meaning. Malthus posited in 1798 that food production would not be able to keep pace with the rate of population growth without imposing controls or limits on population and that, consequently, food production would be the ultimate limiting factor. Technical progress in agriculture and the expansion of the global agricultural frontier has so far allowed food production to grow much more rapidly than

population growth. However, agricultural output growth is fueled by the decline in the stocks of - essentially depletable - resources. Forrester (1971) suggested that depleting resources along with other environmental aspects play a similarly limiting role.

The question of depleting essential resources has attracted the interest of a number of prominent economists, most notably Hotelling (1931), in determining optimal paths of exploitation and possible policy instruments to induce agents to allocate the depleting resources as closely to optimal as possible. Although essential resources may be depleted in principle, it is not necessarily the case that output produced with the resource would decline to zero or even that the resource would be completely depleted (Dasgupta and Heal, 1974). Indeed, economists argue that market forces will drive the depleting resource's price up setting the incentives for economic agents to (i) develop or improve technologies to economize the resource and (ii) develop and utilize alternative inputs (Maddox, 1972; Nordhaus, 1973; and Kamien and Schwartz, 1978). In any case, the question of whether aquifer depletion should be a policy priority or not has not been settled (Peterson et al., 2003) and is not addressed here.

This paper explores the effect of technical progress on groundwater use and depletion in the presence of climate change using a model of inter-temporal common pool groundwater use, where pumping cost and stock externalities arise from the common property problem. Dynamic coefficients in a groundwater demand function conditional on climate variables represent technical change, so that technical progress causes the marginal value product of groundwater to increase. This is consistent with a demand curve derived from a production function in which irrigated yield levels can be achieved with lower levels of water being applied as time passes. Although technical progress makes it possible to produce at the same level with less water use, it also poses the incentive to expand irrigated acreage and intensify water application per acre, so the net effect is unclear in general.

This paper develops a model that incorporates time-varying parameters to capture changing climate and technology conditions to assess the potentially mitigating effects of

technical progress on aquifer depletion in the presence of the common-pool resource externality. The framework described in the next section focuses on revealing the nature of "information effects" of prescribing extraction plans ignoring climate change, technical progress, or both.

# 2    Conceptual Framework

The dynamics of technical progress and climate change and the way they affect the value marginal product of irrigation groundwater in the future, particularly with respect to the curvature of the reward function, is expected to determine the level of steady state groundwater stock in the aquifer. The findings from this inquiry will help inform what type of outcomes can be expected under different types of technical progress in the presence of declining precipitation and common-pool resource externalities.

A dynamic framework is essential to study groundwater use and management under changing conditions. This paper assumes that irrigated agriculture is the primary user of groundwater and the main driver of aquifer depletion. Given production technology and aquifer conditions, farmers decide how much groundwater to pump and apply to fields. A model based on the single-cell or "bathtub" aquifer framework is employed. The single-cell model considers an aquifer with a flat bottom, vertical sides, and water that flows laterally at an instantaneous rate so that withdrawals affect the water table height equally in all locations throughout the aquifer regardless of where it is pumped. A large number of users of water are assumed to be distributed across the (flat) overlying land surface, with identical technology and exogenous prices so that a representative, competitive user can be aggregated to reflect basin-level outcomes. Brozovic et al. (2010) indicate that the use of single-cell models may be adequate for small aquifers or a relatively small area within a much larger aquifer, as is the case here. The next four subsections detail the models of climate change, technical change, hydrology, and allocation decisions employed in this
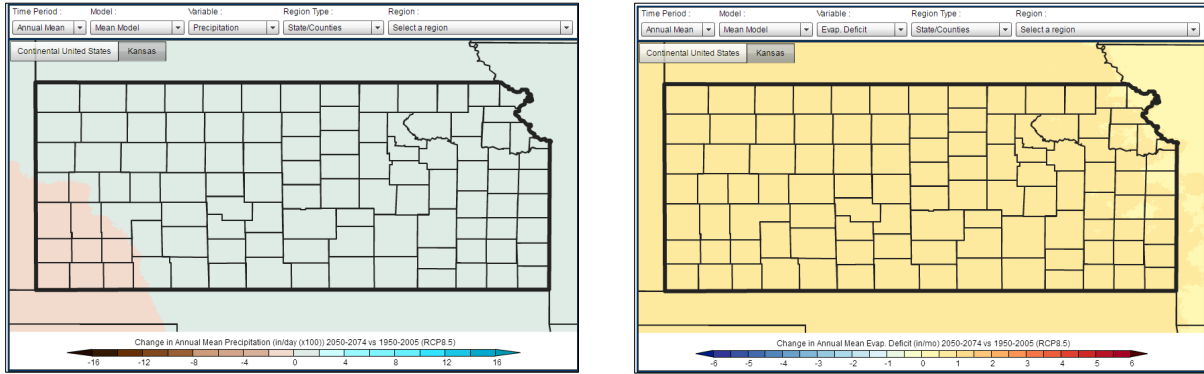
(a) Projected change in annual mean precipitation levels.

(b) Projected change in annual mean evaporative deficit levels.

Figure 1: Select climate change variables in Kansas counties. Source: USGS NCCV.

paper.

## 2.1  Climate Change

The United States Geological Survey (USGS) provides easy access to the output obtained from the 5th Climate Model Intercomparison Program (CMIP5) which provides climate information for the ongoing Fifth Assessment Report (AR5) of the Intergovernmental Panel on Climate Change (IPCC). The data is available through their National Climate Change Viewer (NCCV) and covers the historical period (1950-2005) and projections fro the 21st century (2006-2099) under four Representative Concentration Pathways (RCP) emission scenarios. The NCCV[1] allows to appreciate the change in climate at the county level. Figure 1 shows two screen captures from the NCCV.

Climate change is multifaceted and spatially heterogeneous. Different climate variables change in different directions in different regions. Although there is consensus on the way global average temperature and precipitation will change, this is not consistently true for every region of the world. Figure 1a shows that little to no change in mean annual precipitation can be expected across Kansas. However, very significant and meaningful

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[1]http://www.usgs.gov/climate_landuse/clu_rd/apps/nccv_viewer.asp
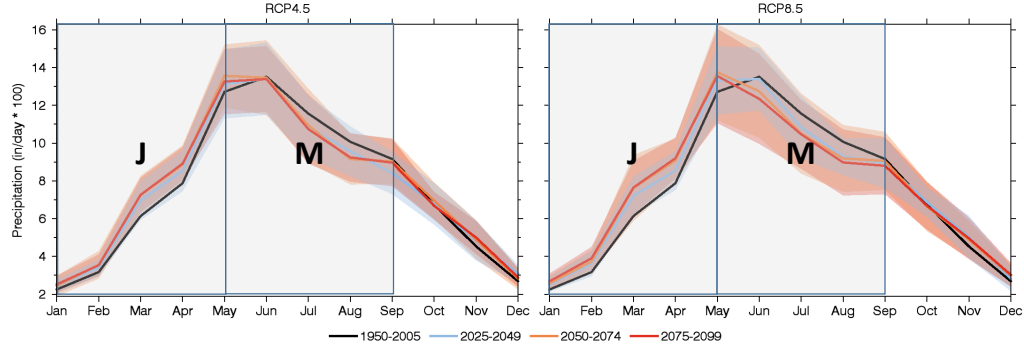
(a) Precipitation patterns.
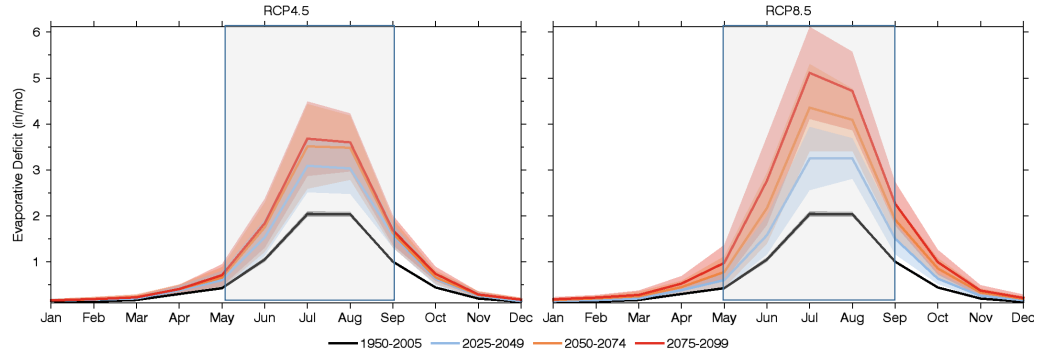


(b) Evaporative Deficit patterns.

Figure 2: Changing climate patterns in Kansas. Source: USGS NCCV.

changes in the timing of the precipitation occurs.

Figure 2a illustrates the changes in daily precipitation by month. Although annual mean precipitation exhibits little to no change, precipitation during the critical period between May and August is clearly declining, while temperature, and consequently the evaporative deficit (fig. 2b), show a drastic increase for those months. Consequently, models of climate change must account for the distinct ways in which climate patterns are expected to change over time.

In this paper, climate variables will enter as dynamic parameters that shift the (inverse) demand curve for groundwater. Climate is formulated as average daily precipitation between January and April ($J$) and between May and August ($M$) as well as evapotranspiration between May and August ($E$). Simple linear dynamics are devised for

these parameters following the equations of motion:

$$\dot{J} = a_0 - a_1 J \tag{1}$$

$$\dot{M} = a_2 - a_3 M \tag{2}$$

$$\dot{E} = a_4 - a_5 E \tag{3}$$

An explanation of how the parameters in these equations of motion are calibrated is offered in section 4.1. In the next two subsections, the dynamics of technical change and the aquifer are introduced.

## 2.2   Technical Change

The economics profession narrative with respect to depleting "irreplaceable natural resources, or of natural resources replaceable only with difficulty and long delay," (Hotelling, 1931) is that as a resource becomes more scarce it also becomes more valuable and, consequently, its price would increase providing the proper incentives for entrepreneurs to develop and supply substitutes (Maddox, 1971 and Nordhaus, 1972). There is an extensive literature suggesting this type of technical progress may emerge endogenously in economic systems. Induced innovation theory in agricultural development (Hayami and Ruttan, 1971) suggests that technical progress is determined by relative input and output prices. Specifically, increases in relative input prices induce input saving technologies while increases in output prices are related to research focus on the most valuable commodities (Quintana-Ashwell and Featherstone, 2014).

Although advances in agricultural biotechnology, equipment, and machinery may well occur in response to market signals, these occur at aggregation levels that are distant from the relevant decision unit: the irrigator. Consequently, these technical changes are exogenous to farmers. This paper assumes a linear groundwater demand function that is conditional on climate conditions and where the intercept term represents the state of
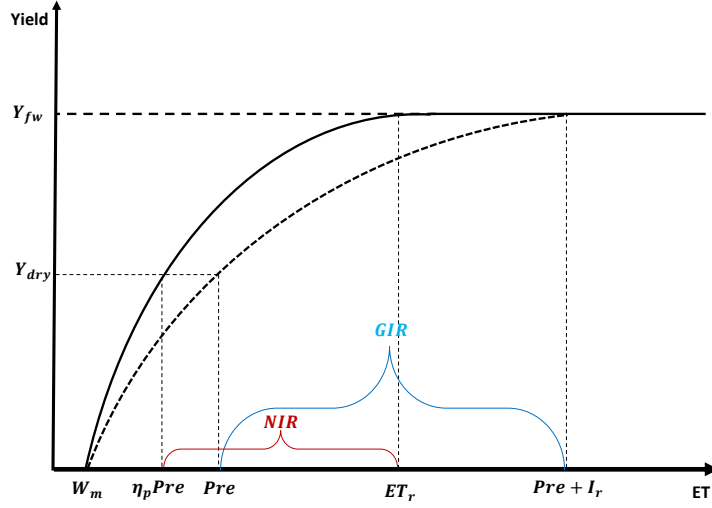
Figure 3: Yield response to water.

technology. For the purposes of this paper, both technology and climate variables act as shifters of the value marginal product (VMP) of groundwater in crop production.

This formulation is consistent with a production function for a composite irrigated crop that depends on the volume of irrigation water applied to it. In particular, the crop yield response to irrigation in such a production function is consistent with the model following the agronomy-based model and production functions by Martin et al. (1984). Figure 3 illustrates the agronomic model from which the concept of technical change in this paper is inspired.

Yield is a monotonic function of evapotranspiration (ET). There is a wilting point or lower threshold, $W_m$, below which no yield is possible and after an upper threshold $ET_r$ is evapotranspirated, the crop achieves its maximum "fully-watered yield", $Y_{fw}$. Irrigation, $I_r$, plays a supplemental role to precipitation, $Pre$. If no irrigation is applied, a minimum yield level, $Y_{dry}$, is achieved given precipitation levels, $Pre$, and effectiveness of precipitation, $\eta_p$. The net irrigation requirements ($NIR$) is the additional amount of evapotranspiration required in addition to the effective precipitation to achieve $Y_{fw}$, i.e.

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$NIR = ET_r - \eta_p Pre$. However, irrigation water is not applied with perfect efficiency and not all applied groundwater is available for evapotranspiration. The gross irrigation requirement is then an amount greater or equal to the net irrigation requirement depending on the degree of application efficiency of the irrigation systems employed. The relationship between net and gross irrigation requirements is $GIR = \frac{NIR}{\eta_w}$, where $\eta_w$ is the irrigation application efficiency.

Technical change may occur in several ways. Advances in biotechnology may result in one or several of the following changes: (i) the wilting point, $W_m$, may be reduced; (ii) the fully-watered yield, $Y_{fw}$, may be increased; (iii) the required evapotranspiration, $ET_r$, to achieve $Y_{fw}$ may decrease or increase; and (iv) the shape of the yield water response function between $W_m$ and $ET_r$ may change. Advances in equipment, machinery, and farming practices may result in improved precipitation effectiveness, $\eta_p$, or improved application efficiency, $\eta_w$. All these changes modify the incentives of farmers to pump groundwater.

Figure 4 illustrates the adapted agronomical model where yield is plotted against applied irrigation water. The solid black line represents the yield response function to evapotranspiration and the dashed line represents the actual yield response function to applied water. The red lines represent a change in minimum yield, $Y'_d$, without a change in maximum yield. The blue line represent a change in application efficiency without any change in yields. Notice that any time there is a change in the difference between minimum and maximum yields, $\Delta Y = Y_f - Y_d$, this will require a change in the coefficients of the yield response function, $a$ and $b$. This paper will examine the effects of improvements in crop varieties that result in increasing "dryland" and "fully watered" yields which are controlled by parameters $a$ and $b$ in this particular formulation.

Technical change may be modeled as the evolution of parameter $a$ in figure 4 over time. Deriving the marginal value product from the production functions in figure 4 would yield a linear inverse demand function for groundwater, which has an intercept of which will be a
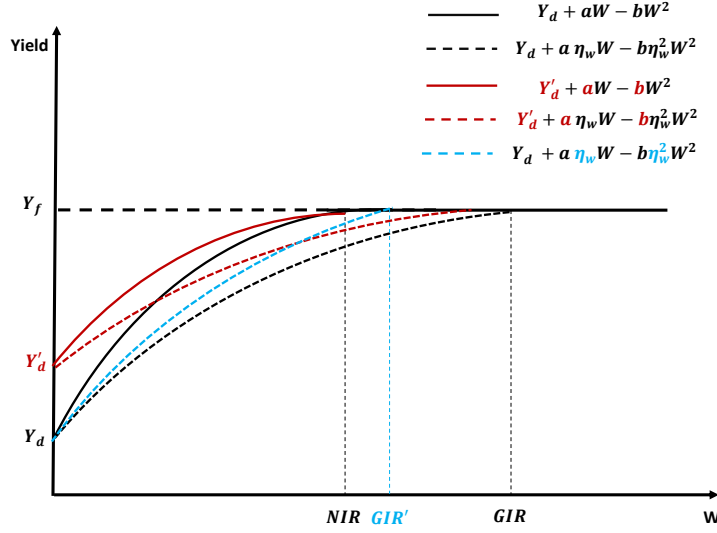
Figure 4: Yield response to irrigation.

function of $a$. [2] In section 2.4, this intercept is the parameter $\beta_0$. The equation of motion for this parameter is again specified as a linear approximation and determines the rate of technical progress:

$$\dot{\beta_0} = b_0 - b_1\beta_0 \tag{4}$$

Although holding curvature constant, by fixing the parameter $b$ we are implicitly accounting for the relationship between dryland yield, represented by $Y_d$, and maximum irrigated yield. In the next subsection, the hydrologic model is described. Different crop varieties show different patterns of yield increase between dryland and irrigated varieties. In the case of a composite crop as in this paper, it will depend on the mix of crops being considered.

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[2]Although not pursued here, a natural extension of this work would be to also model parameters $Y_d$ and $b$ dynamically. By fixing the parameter $b$ we are holding curvature of the production function constant so that a change in $a$ implicitly accounts for a changing relationship between dryland yield, represented by $Y_d$, and maximum irrigated yield.

## 2.3 Hydrology

The single cell framework has been a workhorse of the groundwater management literature since its inception (Gisser and Sanchez, 1980; Fienerman and Knapp, 1983). It can be criticized for its strong assumptions, which do not accord with the spatial heterogeneity and the slow rates of lateral flow observed in many aquifers (Saak and Peterson, 2012). Recent literature has relaxed the assumptions of instantaneous lateral flow and spatial uniformity (Gaudet et al. 2001, Xabadia et al. 2004, Saak and Peterson 2007, Brozovic et al. 2010, Pfeiffer and Lin 2012, Suter et al. 2012, Guilfoos et al. 2013, Peterson and Saak 2013,). Brozovic et al. (2010) indicate that the use of single-cell models may be adequate for small aquifers or a relatively small area within a much larger aquifer, as is the case here. The analysis in this paper is based on the single cell framework for several reasons. First, in spite of its limiting assumptions, it has been a productive research tool, which underlies a number of key insights on the nature of the common pool externalities. Second, results will be directly comparable with earlier insights obtained from the single-cell studies. Finally, it allows to study the research question - the effects of time-varying parameters on common pool externalities - in isolation from the effects brought about by spatial heterogeneity and varying lateral flows.

The aquifer is represented by the pumping lift from the aquifer at time $t$, $L(t)$. As the aquifer depletes, groundwater is pumped from deeper underground. The equation of motion for pumping lift is

$$\dot{L} = \frac{dL}{dt} = \frac{1}{A_S} \left[ (1 - \alpha) \, w - r \right] \tag{5}$$

where $A_S$ is the number of acres overlying the aquifer times the specific yield, $r$ is the rate of natural recharge of the aquifer and $\alpha$ is the return flow, the portion of water applied that returns to the aquifer.

## 2.4 Groundwater allocation

In this framework, farmers decide how much groundwater to allocate (pump) each period. The main periodic trade-off faced by farmers is between current versus future irrigation for food production. In this context, net farm benefits are a good approximation for social welfare (Quintana and Peterson, 2015a). In this model, because the starting point is a conditional groundwater demand, instead of farm profits the variable is rents from irrigation, i.e. additional profits in addition to what can be achieved from dryland. Farm rents from irrigation are a function of applied groundwater as well as observed technology, climate, and aquifer conditions:

$$R(w; \beta_0, J, M, E, L) = \beta_0 w - \beta_1 J w - \beta_2 M w + \beta_3 E w - \frac{1}{2}\beta_4 w^2 - \frac{1}{2}c_2 L^2 \qquad (6)$$

where $w$ is groundwater pumping and the parameter $c_2$ captures the irrigation timing effect of reduced stocks of water[3]. The parameter $c_2$ also captures accelerated cost increases derived from groundwater pumping from deeper wells and allows the Hessian of the reward function to conform with economic theory (Tomini, 2014)[4].

The cost of pumping depends on the stock of groundwater available in the aquifer. As the stock of water in the aquifer decreases it becomes more costly to extract because pumping lift distances, $L$, increase and well yields decrease, causing pumps to work harder and command larger operating costs, most notably pump energy consumption. The pumping costs as a function of aquifer conditions and water pumped is

$$C(w, L) = (c_0 + c_1 L)\,w \qquad (7)$$

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[3]Because of reduced groundwater well yields decline as the aquifer decline, farmers may not be able to pump sufficient groundwater during the growing season and may opt for pumping much earlier than ideal to start saturating the soil before time. Consequently two similar total groundwater extraction levels may result in dissimilar rents from irrigation because of this timing effect.

[4]For instance, ever increasing pumping lift commands upgrading of pumps and other equipment, resulting in additional expenses over time indirectly due to the decreasing groundwater stock level. This term is also added to ensure that the Hessian of the net benefit function is negative semi-definite, which was a shortcoming in the seminal work of Gisser and Sanchez (1980).

where $L$ is pumping lift and $(c_0, c_1)$ are parameters calibrated to reflect the marginal cost of pumping in the initial period when the initial pumping lift is $\tilde{L}$.

A nonlinear marginal pumping cost function is calibrated based on engineering formulas to capture the effect of declining yield of wells on pumping costs. A reward function is calibrated to reflect both past irrigation groundwater demand as well as increasing scarcity rents from aquifer depletion. This last aspect is lacking in a number of previous studies following the seminal work of Gisser and Sanchez.

The periodic rents from irrigation function is

$$B(w; \beta_0, J, M, E, L) = \left[ \beta_0 - \beta_1 J - \beta_2 M + \beta_3 E - \frac{1}{2}\beta_4 w \right] w - \frac{1}{2}c_2 L^2 - (c_0 + c_1 L) w \quad (8)$$

Given the state of the climate, technology and the aquifer $(\beta_0, J, M, E, L)$, the planning solution consists in maximizing the net present value of the stream of periodic rents from irrigation over a planning horizon (infinity, in this case). A Social Planner would then maximize[5]

$$NPV = \int_0^\infty e^{-\rho t} B(w(t); \beta_0(t), J(t), M(t), E(t), L(t), t) dt \quad (9)$$

subject to the equations of motion (1),(2),(3), (4) and (5).

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[5]Dependency on t implicit and "$t$" 's dropped from notation hereafter.

The current value Hamiltonian is

$$\tilde{H} = \underbrace{B(w; \beta_0, J, M, E, L)}_{\text{periodic rents from irrigation}}$$

$$+ \underbrace{\mu_1 \left(a_0 - a_1 J\right) + \mu_2 \left(a_2 - a_3 M\right) + \mu_3 \left(a_4 - a_5 E\right)}_{\text{climate conditions}} \tag{10}$$

$$+ \underbrace{\mu_4 \left(b_0 - b_1 \beta_0\right)}_{\text{technical progress}} + \underbrace{\mu_5 \frac{1}{A_s} \left[(1 - \alpha)w - r\right]}_{\text{aquifer evolution}}$$

and optimality conditions

$$\frac{\partial \tilde{H}}{\partial w} = \beta_0 - \beta_1 J - \beta_2 M + \beta_3 E - \beta_4 w - c_0 - c_1 L + \mu_5 \frac{(1 - \alpha)}{A_s} = 0 \tag{11}$$

$$\dot{\mu}_1 - \rho \mu_1 = -\frac{\partial \tilde{H}}{\partial J} = \beta_1 w + \mu_1 a_1 \tag{12}$$

$$\dot{\mu}_2 - \rho \mu_2 = -\frac{\partial \tilde{H}}{\partial M} = \beta_2 w + \mu_2 a_3 \tag{13}$$

$$\dot{\mu}_3 - \rho \mu_3 = -\frac{\partial \tilde{H}}{\partial E} = -\beta_3 w + \mu_3 a_5 \tag{14}$$

$$\dot{\mu}_4 - \rho \mu_4 = -\frac{\partial \tilde{H}}{\partial \beta_0} = -w + \mu_4 b_1 \tag{15}$$

$$\dot{\mu}_5 - \rho \mu_5 = -\frac{\partial \tilde{H}}{\partial L} = c_1 w + c_2 L \tag{16}$$

where $\mu_1$, $\mu_2$, $\mu_3$, $\mu_4$, and $\mu_5$ are the costate variables for (1),(2),(3), (4) and (5), respectively. In the next section, the general solution to the so-called linear-quadratic control problem is presented and the different planning solutions are described.

# 3  Planning Solutions

The linear evolution of the state variables and the quadratic form of the objective function allows for tractable expression of the optimal control problem in matrix notation. Let $\mathbf{x} = [J, M, E, \beta_0, L]'$ be the vector of control variables, $\dot{\mathbf{x}} = [\dot{J}, \dot{M}, \dot{E}, \dot{\beta}_0, \dot{L}]'$ be the associated vector of time derivatives, and $\mu = [\mu_1, \mu_2, \mu_3, \mu_4, \mu_5]'$ be the vector of current-value costate variables. Following Gandolfo (2009), the current-value Hamiltonian is then expressed as

$$\tilde{H}(x, w, \mu) = Cw + \frac{1}{2}x'Dx + \frac{1}{2}w'Ew + w'Gx + \mu'(K + Mx + Nw) \qquad (17)$$

with matrix maximum principle conditions

$$\frac{\partial \tilde{H}}{\partial w} = C + w'E + x'G' + \mu'N = 0 \qquad (18)$$

$$\dot{\mu} - \rho\mu = -\frac{\partial \tilde{H}}{\partial x} = -x'D - w'G - \mu'M \qquad (19)$$

$$\dot{x}' = \frac{\partial \tilde{H}}{\partial \mu} = K' + x'M' + w'N', \qquad (20)$$

where $C$, $D$, $E$, $G$, $K$, $M$, and $N$ are conformable matrices with elements such that (17) coincides with (10).

The so-called linear-quadratic control problems have a solution with a known form, the costate and control paths depend linearly on the state variables:

$$\mu = R + Qx \qquad (21)$$

$$w = V + Wx \qquad (22)$$

where $R, Q, V, W$ are matrices of constants and coefficients that depend on model parameters. The elements of these matrices can be found by first noting that equation (21)

implies

$$\dot{\mu} = Q\dot{x} = QK + QMx + QNw, \tag{23}$$

which together with (21) can be substituted into (11) to solve for $w$:

$$w = E^{-1}\left(C' + Gx + N'R + N'Qx\right). \tag{24}$$

All of the above results are then substituted into (19) to obtain the Ricatti matrix

equations:

$$R = \frac{1}{\rho}\left[QK + M'R - \left(QN + G'\right)E^{-1}\left(C' + N'R\right)\right]$$
$$Q = \frac{1}{\rho}\left[QM + D + M'Q - \left(QN + G'\right)E^{-1}\left(G + N'Q\right)\right], \tag{25}$$

which must be solved numerically for $Q$ and $R$. The numerical solutions are finally used in

(24) to obtain numerical results for $V$ and $W$ in (22):

$$V = E^{-1}\left(C' + N'R\right)$$
$$W = E^{-1}\left(G + N'Q\right) \tag{26}$$

From this setting it is straightforward to populate this solution with the conditions
established in (10) to (16) or any variation of them. The optimal solution to the problem
in the previous section is solved following this algorithm. Additionally, alternative plans
are compared to the optimal path. In the alternative scenarios solutions are found for the
linear-quadratic problems that omit accounting for the dynamics of climate change,
technical progress, or both. All plans are solved in the same manner by modifying matrices
$C$ through $N$.

## 3.1   Planning Scenarios

By solving for different planned solutions, this paper is able to assess the "information
effects" of prescribing a groundwater extraction plan without accounting for important

dynamic factors that influence the incentives at play in irrigated agriculture. Four different plans are solved for. The first plan is the "Optimal" and account for all state variables and their evolution. The second plan omits the dynamics of technical change only, i.e. equations (4) and (15) are not part of the optimality conditions. The third plan omits the dynamics of climate change only, i.e. equations (1),(2), (3) and (12) to (14) are not part of the optimality conditions. The last plan considers only the aquifer dynamics and ignores both climate and technical change in solving for the groundwater allocation plan.

The effects of climate change and technical change on the aquifer become apparent when comparing aquifer outcomes (pumping lift) over time. Because conservation is not a goal in the optimization problem it is not necessarily expected that the optimal will result in an aquifer depleted to a lesser degree. Actually, because what is being maximized is the net present value of the rents from irrigation (NPVs), it is not even the case that long term periodic rents would higher under the optimal plan.

An interesting point of comparison is the stream of rents from irrigation resulting from each plan and the NPV os the sum of these values. To calculate this values, climate and technology are allowed to evolve in the manner prescribed by their respective equations of motion, so the allocations of each plan are valued in the presence of these changes, even if the prescribed allocation ignores them. In the next section, the case study from which parameters and initial conditions are drawn from is presented.

# 4    Case Study: Sheridan County, Kansas.

Most of Western Kansas can be described as arid to semi-arid. Groundwater is primarily used in agricultural irrigation. Population growth and density is minimal. In that context a model of rents from irrigation such as the one in this paper is appropriate for approximating the regional welfare derived from groundwater extraction and the resultin aquifer depletion. Kansas itself is an interesting region for a case study in agricultural
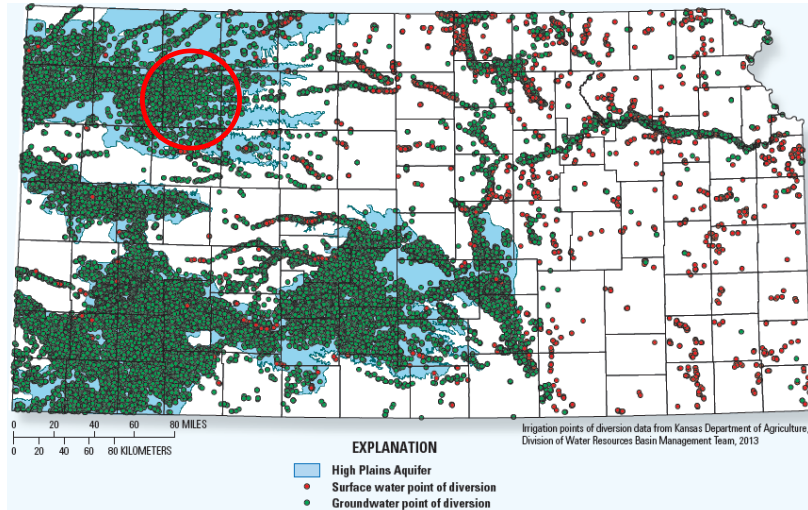
Figure 5: Location of High Plains aquifer and points of diversion in Kansas.
Source: USGS Kansas Water Science Center.

economics because of the variety of regional crops and climatic conditions that can be very similar to conditions across the mid-West. The common-pool resource externality becomes apparent in localities where high density of wells exists, inducing myopic groundwater use behavior. Figure 5 presents a map of Kansas where counties and the High Plains aquifer are outlined along with the registered points of diversion. This setting describes significant portions of agricultural western Kansas.

In terms of the Ogallala aquifer, the areas it underlies in Kansas are not uniform. For instance, net recharge in Kansas varies from 0.05 acre-inches per year to 6 acre-inches per year resulting in extremes of rapid aquifer depletion and places of positive changes in saturated thickness (Gutentag, 1984). Figures 6a and 6b illustrate levels of saturated thickness and aquifer depletion for the Ogallala aquifer in Kansas. Despite the heterogeneity of the region at large, the conditions become more uniform at more localized scales making the notion of a representative farmer more palatable at a county level scale (Quintana-Ashwell and Peterson, 2015ab).

Sheridan county is particularly attractive because it is relatively uniform in terms of its agricultural and hydrological conditions and because of recent policy innovations[6] that

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[6]The "Sheridan 6 Local Enhanced Management Area (LEMA6)" initiative was designated in April, 2013,
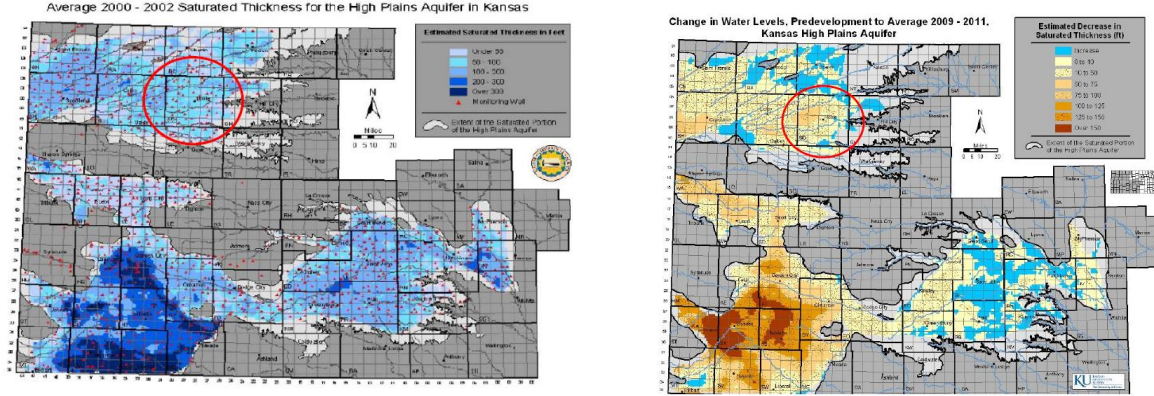
(a) Mean saturated thickness.　　　　　(b) Change in water levels.

Figure 6: Ogallala Aquifer conditions in Kansas. Source: Kansas Geological Survey.

signal a willingness from farmers in the area to accept groundwater management policies whereas the planned allocations presented in this paper could be thought of as such.

No less important in choosing a county in Kansas is the wealth of agricultural, agronomical, and hydrological data. Important resources include the Kansas Geological Survey (KGS), the Water Rights Information System (WRIS), and the Water Management and Analysis System (WIMAS) along with numerous technical publications from Kansas Board of Regents' universities.

A description of the parameters and variables populating the model is offered in the next subsection, followed by the results from the numerical analysis.

## 4.1 Model parameterization and initial values.

Initial values for average January to April ($J$) and May to August($M$) precipitation, and for May to August evapotranspiration ($E$) are obtained from Hendricks and Peterson (2012). The evolution of these variables over time is simplified so the periodic change in values occurs linearly depending on observed levels in the previous period. Two more

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and establishes a groundwater extraction cap of 55 acre-inches per irrigated acre for the period between 2013 and 2017, inclusive.
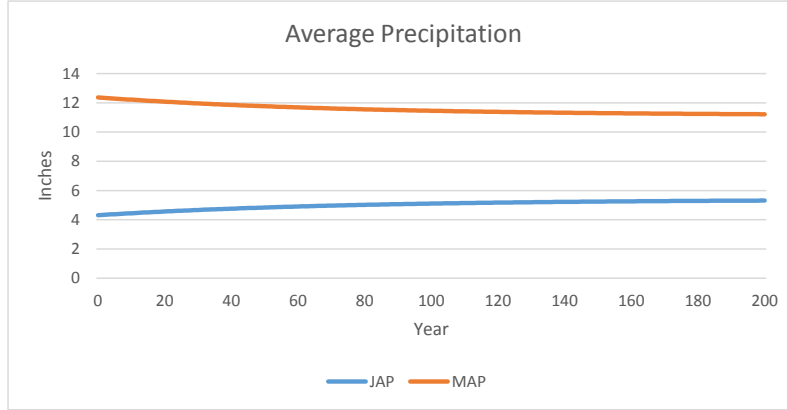
Figure 7: Evolution of average precipitation parameters.

points are needed to calculate the climate change equations of motion. The equations of motion are calibrated to reflect the change in levels expected in the USGS National Climate Change Viewer for average daily levels projected for the years 2075 to 2100, so that USGS projection for the averages constitute the asymptotic values for the climate variables in this model. The expected change[7] in average daily precipitation for April is used to calibrate $\dot{M}$, eq. (1); the expected change in average daily precipitation for August is used to calibrate $\dot{M}$, eq. (

Hydrological and agronomical initial conditions and parameter values for Sheridan County, KS as well as the calibrated parameters for climate and technical change are presented in table 1. Aquifer initial conditions and parameters were obtained from the Kansas Geological Survey (KSGS), the Water Rights Information System (WRIS), and the Water Information Management and Analysis System (WIMAS). The discount rate[8] considered is the average interest rate on farm loans as reported from the Kansas City Federal Reserve Bank (November, 2011).

Calibration of the rents from irrigation is based on a linear demand function for groundwater obtained from data and results from Hendricks and Peterson (2012). The

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[7]In all cases the changes are in percentage, not absolute value.

[8]There is a growing body of work in the literature that deals with the question of how are future outcomes to be discounted. A discussion of the topic is beyond the scope of this paper but a nice and recent one is Gollier and Hammitt(2014). In any case the 3.89% used here is within the range employed in the literature.
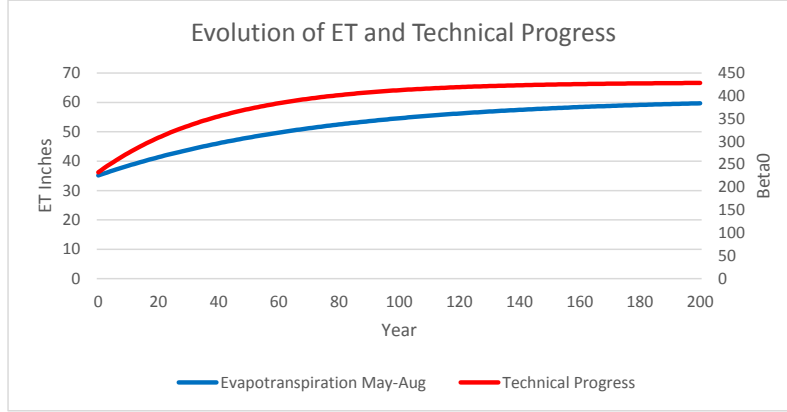
Figure 8: Evolution of Technical Progress and Evapotranspiration.

(inverse) demand function for groundwater is

$$p^w(w; \beta_0, J, M, E) = \beta_0 - 44.548J - 18.383M + 15.055E - 0.0031w. \tag{27}$$

The linear (in lift) marginal cost of pumping function is calculated in two stages from the nonlinear marginal pumping cost function calculated in Quintana and Peterson (2015a) following Rogers and Alam (2006). The periodic marginal cost function is linearly increasing in groundwater pumping and lift, i.e. $c_1 > 0$. A quadratic term in lift distance, $c_2 > 0$, is added to account for irrigation timing effects[9]. The non-linearity of the cost function with respect to aquifer levels is due to declining well yields as the aquifer is depleted and the quadratic form is merely a tractable approximation.

With regards to technical change, points of reference are harder to find.For the technical change parameters $b_0$ and $b_1$, the equation of motion is calibrated from the parameters and initial conditions according in table 1 so that water productivity asymptotically reaches levels approximately twice higher than in the initial period and a

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[9]When the aquifer reaches a certain level of depletion, well yields are such that it is not possible to pump adequate amounts during periods of peak water demand by crops. Many farmers respond by pumping at full capacity before the peak demand period in an effort to "bank soil moisture. Although total pumping may be similar compared to an irrigation schedule that perfectly matches the pattern of crop water demand, yields will be lower with the banking strategy because the soil profile has a storage limit and also loses water from seepage. The coefficient on $L$ accounts for these yield losses.

1.1% increase productivity is observed on the first period after the initial.

| Parameter | Value |
|---|---|
| **Aquifer** | |
| $A_S$ | 716,844.5363 |
| Irrigated area | 77,745 acres |
| Return flow ($\alpha$) | 0.086795 |
| Initial lift ($L_0$) | 111.5 ft. |
| Rate of natural recharge ($r$) | 28,747.08 AF/yr |
| Discount rate ($\rho$) | 0.0389 |
| **Irrigated crop coverage** | |
| Corn | 86.9% |
| Soybeans | 4.8% |
| Alfalfa | 4.8% |
| Wheat | 2.8% |
| Sorghum | 0.7% |
| Average Irrigation requirement($C_R$) | 0.897 AF/Acre |
| **Reward function** | |
| $\tilde{\beta}_0$ | 232.67 |
| $\beta_1$ | 44.548 |
| $\beta_2$ | 18.383 |
| $\beta_3$ | 15.055 |
| $\beta_4$ | 0.0031 |
| **Cost function** | |
| $c_0$ | -51.70 |
| $c_1$ | 0.6689 |
| $c_2$ | 64.324 |
| **Technical change** | |
| $\dot{\beta}_0 = 10.134 - 0.024\beta_0$ | |
| **Climate change** | |
| $\dot{J} = 0.071833 - 0.01333J$ | |
| $\dot{M} = 0.1484 - 0.01333M$ | |
| $\dot{E} = 0.8199 - 0.01333E$ | |

Table 1: Parameters and aquifer initial values for Sheridan Co.,KS

In the next section, parameter values are assigned to the analytical results presented earlier.

## 4.2 Simulation Results

This paper simulates the outcome of four different plans for groundwater extraction. In each scenario, the planner is forward looking and devices an optimal extraction plan based on what he knows about the future. Of the factors the planner knows affect the future, she has perfect foresight. The optimal path accounts for the dynamics of both technical and climatological variables, in addition to the evolution of pumping lift. The "information effects" we introduce, consists of forcing the planner to prescribe paths for groundwater extraction as if climate change, technical change, or both were unknown and were assumed to stay constant at initial values. Climate and technical progress evolve exogenously while the aquifer (pumping lift) is affected by the periodic extraction of groundwater. The rents from irrigation are calculated from the periodic reward function and all the state variables are allowed to update even in the outcomes where one or more of the state variables was ignored in the planning process, i.e. extraction occurs in the amount prescribed by each plan but the profits from that prescription are derived from updated technology and climate. As expected, there are significant differences between each of the planning scenarios.

The scarcity value of groundwater is represented by the value of the co-state variable for well lift. As groundwater becomes scarce (higher lift) it becomes more valuable (lift is more costly). Figure 9 illustrates the path of the scarcity value of groundwater under different planning scenarios. Notice that in all cases the cost of additional lift increases (co-state is decreasing) under all scenarios except the one in which the planned allocation of groundwater ignores both technical progress and climate change dynamics. The Optimal plan assigns the highest scarcity value to groundwater while the plan ignoring the evolution of technical progress corresponds to the lowest scarcity value of groundwater at the start of the simulation. This relationship continues over subsequent periods but the magnitude of the difference over time is also determined by the differing allocations under each plan.

As expected, periodic groundwater allocation differs under different planning scenarios.
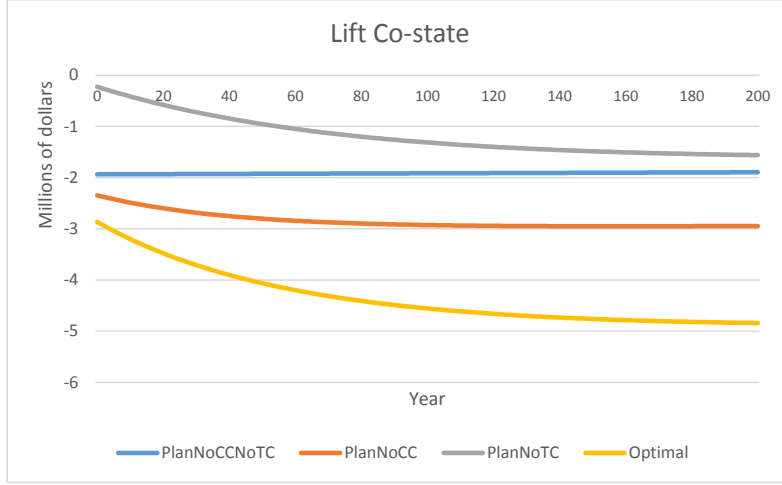
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Figure 9: Scarcity value of aquifer variable under different planning scenarios.

Because the optimal path of optimal control problems with a quadratic reward function and linear equations of motion can be expressed as linear functions of the state variables, it is expected that the evolution of the control variable, groundwater extraction, would tend to follow a path similar to the state variables. Figure 10 shows the time path of groundwater extraction prescribed under each plan. Notice that the plan that ignores the evolution of technology and climate has a strictly decreasing because only the decreasing stock of groundwater (increasing pumping lift) is relevant to that plan.

Figure 8 shows the paths of technical progress and evapotranspiration. In this formulation, because of the linear (in lift) marginal cost of pumping, evapotranspiration and technical change drive the optimal rate of extraction for about the first two hundred years. After that (not shown), the aquifer decline reaches levels in which it dominates the other two effects and force decreasing pumping thereafter. Steady states are not reached for lift or groundwater pumping even after one thousand years.

The periodic rents from irrigation is the variable that captures the "information effects" of prescribing allocations from plans including differing degrees of relevant information about the state and dynamics of the economy. The rents from irrigation accrued under each plan are calculated considering the groundwater allocation prescription under each
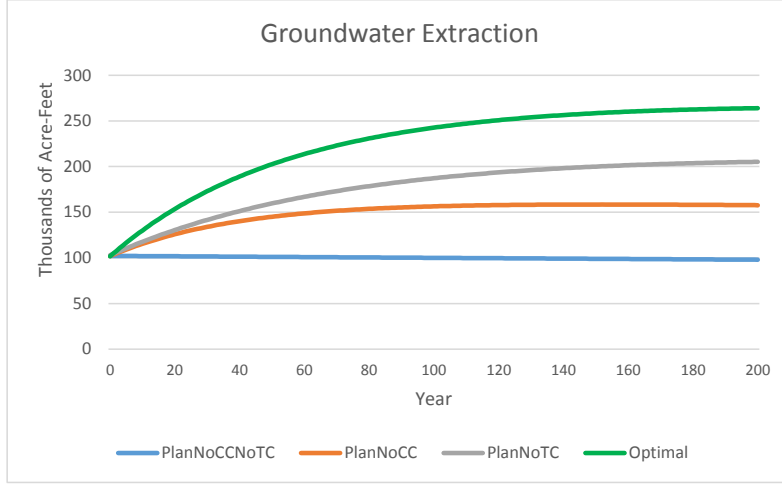
Figure 10: Periodic groundwater allocation under different planning scenarios.

plan but the technological and climate change paramters are actually allowed to evolve over time according to their respective equations of motion while the aquifer variable (lift) updates according to each plan's extraction schedule.

Figure 11 shows the time path of rents from irrigation under the different planning scenarios. Compared to the paths of groundwater extraction, the paths of rents from irrigation have the same ordering but the difference in magnitudes is much larger in groundwater extraction, i.e. it takes a lot more pumping to achieve a relatively small increase in rents. Notice that although the differences in extraction between the optimal plan and the plans ignoring technical change and climate change are relatively large, the difference in rents over time is small between the optimal plan and the plan that ignores technical change. In terms of rents from irrigation, these two plans are considerably superior to the plans which do not account for climate change. This indicates that it may be costly to craft a groundwater extraction plan ignoring the paths of climate change.

Table 2 presents the net present values of the rents from irrigation (NPVs), cumulative groundwater extraction for the first two hundred and the foregone profits and saved water associated with each alternative plan to the optimal. The NPVs are \$1.041 billion for the optimal plan, \$1.009 billion for the plan that ignores the dynamics of technical change but
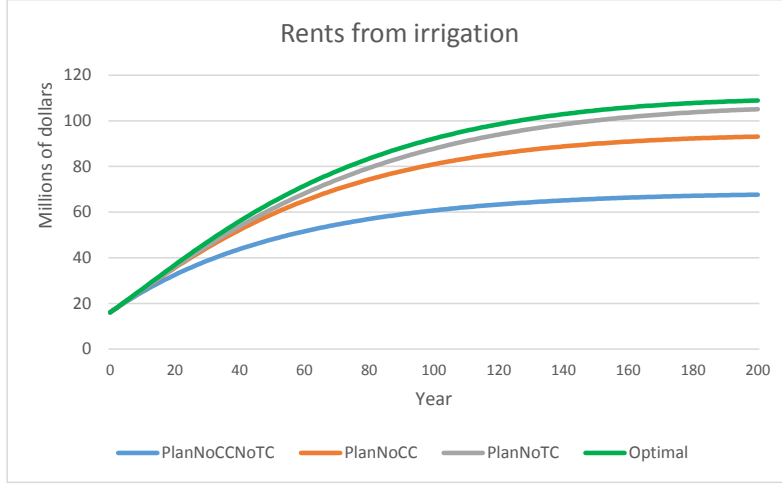
Figure 11: Periodic rents from irrigation under different planning scenarios.

accounts for climate change, $0.982 billion for the plan that ignores climate change but
accounts for technical change, and $0.860 billion for the plan that ignores both climate and
technical change. In relative terms, ignoring technical change comes at a cost of 3.06% of
NPV but results in 21.32% savings in total groundwater extraction. Ignoring climate
change commands a cost of 5.63% of NPV but results in 33.69% less groundwater extracted
over the period. Ignoring both climate and technical change is most costly at 17.35% of
potential NPV foregone but 55.45% less cumulative pumping.

| Plan | NPV (bil.) | Cost of Omission | Total GW (mil.) | GW Savings |
|---|---|---|---|---|
| **Optimal** | $1.041 | | 45.137 AF | |
| **Ignoring TC** | $1.009 | 3.06% | 35.514 AF | 21.32% |
| **Ignoring CC** | $0.982 | 5.63% | 29.931 AF | 33.69% |
| **Ignoring TC and CC** | $0.860 | 17.35% | 20.108 AF | 55.45% |

Table 2: Net present value of rents from irrigation and accumulated groundwater extraction.

The contrast between cumulative NPVs and cumulative groundwater extraction
highlights the implicit trade-off involved in the formulation of a resource management plan
in general and of irrigation groundwater in particular. The results from table refnpv can be
conversely read as relatively small costs in foregone profits may result in relatively large
savings in groundwater, i.e. conservation. However, if the goal is not zero extraction, the

problem is to identify the best path for depletion. In this paper the optimal path is determined by the maximization of the NPV of rents from irrigation over the life of the resource which, in this particular formulation, does not reach a steady state even after a thousand years.

# 5    Summary

This paper presents a simple framework to study the effects of omitting the dynamics of technical change, climate change, or both on the expected rents from irrigation in the context of a declining common-pool resource. The problem is formulated as a linear quadratic optimal control problem where groundwater extraction is the control variable and pumping lift represents the state of the aquifer in all cases. The "information effects" of ignoring relevant technological and climatological dynamics are teased-out by forcing planning solutions that ignore one or both of these changing states. We populate our model with agronomical and hydrological parameters from Sheridan county, KS and with linearized dynamics for technical and climate change. Climate change variables include periodic average precipitation between January and April ($J$), periodic average precipitation between May and August ($J$), and periodic average evapotranspiration between May and August ($E$, alfalfa-based).

Numerical results indicate that, in this formulation, groundwater pumping is driven more by technical change and the increasing evapotranspirative demands than by the decline of the aquifer during the first several decades of simulation and, consequently, higher extraction rates occur in plans that account for one or more of those components. The resulting increased rates of extraction also correspond to higher levels of periodic rents from irrigation so that both, faster decline and higher profitability could be expected in the next several decades of plan implementation. The results suggest that it is more costly to ignore climate change when prescribing an allocation plan that ignoring technical change.

These results are partially driven by the simple linear-quadratic formulation. More refined results can be obtained incorporating a non-linear (in lift) marginal pumping cost and estimating climate change dynamics from existing projection and technical change empirically.

When the optimal path of extraction is determined by the maximization of the net present value of the rents from irrigation, relatively large groundwater savings may be achieved at relatively small foregone profits. However, when limited alternative (valuable) uses are available for the resource, NPV maximization seems an adequate plan evaluation metric and the question that remains is how to discount the long-run which is beyond the scope of this paper.

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