# Multiple Choices, Strategic Interactions, and Market Effects in Livestock Disease Risk Management

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## 1. Introduction

Infectious livestock diseases have become major public health and economic concerns (The Economist 2014). These problems are generally characterized by infection risks along multiple pathways: spread from infected neighbors, as well as import risks. Import risks are increasingly important as the livestock production supply chain tends to be disaggregated by production process, resulting in the need to transport many live animals. Livestock markets therefore play a key role in management of disease risks.

Prior work on infectious disease transmission in livestock systems (e.g., Hennessy 2007, 2008; Reeling and Horan 2015) makes one or more of the following simplifying assumptions: only a single transmission pathway is modeled (imports or spread), all possible risk management choices are reduced into a single biosecurity variable that reduces risks along all modeled pathways, and role of import markets in pricing and allocating risks is generally ignored.

Much of the prior work focuses on strategic interactions among producers, as the biosecurity choices made to protect one herd generate positive spillover effects that enhance the protection of neighboring herds. Hennessy (2007) shows that protection to reduce the likelihood of pathogen spread across neighboring farms exhibits strategic substitutability among producers. Hennessy (2008) finds that actions to protect a farm from import infection exhibit strategic complementarities among producers. Reeling and Horan (2015) consider a single action that can protect a farm from importing infectious pathogen and from contacting with infected neighboring farms. They find that biosecurity exhibits strategic complementarities (substitutability) among neighbors when producers have less (more) control over their own risks via the spread pathway relative to their neighbors' impact over their risks. One of their main concerns is that multiple equilibria could arise in the case of strategic complementarities, in which case the resultant equilibrium depends on producers' beliefs about others' decisions. Suboptimal equilibria may arise if producers are unable to coordinate their beliefs on the Pareto-dominant outcome; this situation is also known as coordination failure (Krugman 1991, and Vives 2005).

Few studies have focused on trade, and those that do have not examined the impacts on biosecurity choices and any associated strategic behavior. For instance, Hennessy et al. (2005) model trade and the scale of production, but they do not model biosecurity. Rich and Nelson (2007) use a simulation approach to study foot and mouth disease in South America. They have trade in the model, but they simply take supply and demand as given rather than considering the underlying decision processes of producers. Understanding these decision processes can aid in the design of public policies to efficiently manage infection risks.

The present paper expands on prior work by examining how strategic risk management behaviors are affected by the availability of multiple biosecurity choices to affect transmission risks along multiple pathways, as well as the role of import markets in pricing and allocating risks. We also explore the role of government policies in risk management decisions made by private sector.

We begin in section 2 by providing details of the model, including the different disease transmission pathways, how neighboring producers' decisions affect a particular producer's disease risk, what a producer can do to manage this risk, and the economic climate within which producers operate. Next in section 3, we examine the decentralized problem where each producer optimizes based on his own self-interest. With potential spillover effects, we apply a game-theoretic approach to obtain the equilibrium. Our analysis begins by examining important properties of the equilibrium including uniqueness and stability. Then, we explore the presence of multiple choices, market effects and government policies and illustrate key findings via numerical examples in section 4.

## 2. Model

Consider a region that has multiple, homogeneous feedlot managers or producers indexed by $i \in \{1,2,\ldots,N,N+1\}$, where $N$ is the number of neighbors for a particular producer. The feedlots operate by importing young animals, feeding them to enhance their weight, and then selling them for slaughter. Imports may come from two regions: region 1 initially has no infected animals, whereas there are some infected animals in region 2. Producers are put at risk either when they import from region 2 or when their neighbors do.

We model the producers' decisions by a static game with two strategies. The first strategy is the import decision. We simplify matters by assuming each producer must import one unit of cattle in each period (e.g., to fill the feedlot to its fixed capacity, which has been normalized to one). Producer $i$'s choice in this matter is the proportion of imports from each region, with the proportion from region 1 denoted by $z_i \in [0,1]$ and the proportion from region 2 being $1-z_i$.[1] Because there are infected animals in region 2, we assume that the region 2's cattle price is less than the region 1's price. This means that producers face a trade-off between lower purchasing cost and a higher probability of importing infected animals. Hence, we could consider a producer's allocation of imports a form of risk management, with a larger $z_i$ reducing the probability of importing infected animals.

Now consider the second strategy. If a particular herd avoids becoming infected via

[1] Note that we could generalize $z_i$ to include any action attempting to reduce imported risk such as testing of incoming cattle; however, we use only import share for simplification.

imports, then it may still face infection risks from neighboring producers who have imported infected cattle. Specifically, neighboring herds may generate risks to others via either direct or indirect contact. Producer $i$ can protect his herd with biosecurity, denoted $b_i$. Without loss of generality, we normalize $b_i \in [0,1]$. Each producer makes both decisions simultaneously. The health status of the imported animals is not known when the import decision is made, but the probability of infection along this pathway is known. At the end of the game, the health status of each importer's herd is revealed and payoffs to each importing producer are realized.

The epidemiological and economic details of the model are presented in the subsequent sections. To further simplify our analysis, we will focus on symmetric Nash equilibria (SNE) since the game is symmetric.[2] We therefore follow Hefti (2013) and model all neighbors to behave in the same way, with $z_j = Z$ and $b_j = B$ $\forall j \neq i$. In equilibrium, $z_i = Z$ and $b_i = B$.

### 2.1 The probability of infection

Infection is analyzed with the herd being the primary unit of analysis. That is, we examine the probability that individual herds, rather than individual animals, become infected. There are two pathways along which infection can occur: imports and spread. First consider the import pathway. The probability of producer $i$'s herd becoming infected via imports is denoted by $p^m(z_i)$. We assume $p^m_{z_i} < 0$, which implies that the higher import share from the risk-free region, the less likely the producer will import infected animals (note that the subscripts indicate partial derivatives). Moreover, $p^m_{z_i z_i} \geq 0$ is assumed, which implies a larger $z_i$ yields diminishing returns

[2] See Hefti (2013) for a discussion of asymmetric Nash equilibria.

to reducing the probability of importing infected animals.[3]

The second pathway is disease spread across neighboring farms. The probability that the herd contracts infection from any particular neighbor, conditional on producer *i* having not been infected via her own imports and also conditional on the neighbor having already become infected, is denoted by $p^c(b_i, B)$. Note that the probability of herd *i* becoming infected via spread from the neighbor is zero when that neighbor's herd is healthy. We assume that both biosecurity actions reduce the conditional probability of contacting infection $p^c_{b_i} < 0, p^c_B < 0$ with decreasing rates, $p^c_{b_i b_i} > 0, p^c_{BB} > 0$. We further assume that the neighbor's biosecurity reduces the effectiveness of producer *i*'s biosecurity, $p^c_{b_i B} > 0$. The probability of becoming infected from any one neighbor, conditional only on producer *i* having not been infected via her own imports, is $p^c(b_i,B)p^m(Z) = p^c(b_i,B)p^m(Z)+0\cdot(1- p^m(Z))$. Since a particular producer has *N* neighbors, we must calculate the probability that a particular producer becomes infected from at least one neighbor, conditional on that producer having not imported infected animals. A Bernoulli process is used to derive this probability, which is one minus the probability of avoiding infection from each of the infected neighbors: $p^s(b_i; B, Z) = 1-(1- p^c(b_i, B)p^m(Z))^N$. We denote this expression as the probability of spread infection.

Given the specification for the probability of infection along each pathway, we can obtain further insight on the technical relationship between the two risk management choices. First, suppressing functional arguments, the first derivatives of the probability of spread infection are negative which imply that all risk management choices reduce the probability of spread

[3]Since the probability of infection is considered a "bad" to producers, then the notion of diminishing return is captured by the positivity of the associated second derivative. We could write the probability of a "good" outcome (healthy herd) as 1-$p^m$, in which case the second derivative is negative to indicate diminishing returns.

infection: $p^s_{b_i} = N(1-p^c p^M)^{N-1}(p^c_{b_i} p^M) < 0$, $p^s_B = N(1-p^c p^M)^{N-1}(p^c_B p^M) < 0$,

$p^s_Z = N(1-p^c p^M)^{N-1}(p^c p^M_Z) < 0$. However, the sign of second derivatives and cross derivatives are ambiguous. For example, consider

(1) $$p^s_{b_iB} = [Np^M(1-p^c p^M)^{N-2}][(1-p^c p^M)p^c_{b_iB} - (N-1)p^M p^c_{b_i} p^c_B].$$

Since the first bracketed right-hand-side (RHS) term is always positive, the sign of $p^s_{b_iB}$ depends on the sign of the second bracketed RHS term. As we can see that both terms in the second bracketed term are positive, the sign will be determined by the relative magnitude of these terms. For example, if there is one neighbor, $N = 1$, then the second bracketed term becomes $(1-p^c p^M)p^s_{b_iB} > 0$ and so $p^s_{b_iB} > 0$. The derivative remains positive as $N$ is increased as long as the expected number of contacts with infected neighbors is sufficiently small, as can be seen in Table 1. But if $N$ is sufficiently large, the second bracketed term will be negative and could change the sign of $p^s_{b_iB}$ from positive to negative.[4] Similar ambiguities also hold for $p^s_{b_ib_i}$ and $p^s_{b_iZ}$.

Combining the two probabilities of infection (import and spread), the total probability producer $i$ becomes infected is

(2) $$p(b_i, z_i, B, Z) = p^m(z_i) + (1-p^m(z_i))p^s(b_i, B, Z).$$

Equation (2) can be used to derive the marginal impact of each risk control measure on the total probability of infection as follows. First, $p_{z_i} = (1-p^s)p^m_{z_i} < 0$, which implies that importing a

[4] It is important to note that Hennessy (2007) shows that biosecurity preventing spread infection is a technical substitute, while the technical relationship is ambiguous in our model. The difference can be explained from the fact that Hennessy assumes disease transmits across only the nearest neighbor, so effectively $N$ is 1 in his specification of $p^s$. According to equation (1), when $N = 1$, then biosecurity is unambiguously a technical substitute across producers. So, we could argue that the approach of modelling the probability of spreading disease used in this paper is more generalized.

larger proportion of animals from safe sources reduces one's total probability of infection via the import pathway, although the effect is diminished by a greater probability of infection via the spread pathway. Note that $p_{z_i z_i} = (1 - p^s) p^m_{z_i z_i} > 0$, so that there are diminishing returns to this effect. Second, $p_{b_i} = (1 - p^m) p^s_{b_i} < 0$, which implies that one's biosecurity reduces the total probability of infection via the spread pathway, although the effect is diminished by a greater probability of infection via the import pathway. The second derivative, $p_{b_i b_i} = (1 - p^m) p^s_{b_i b_i}$, is ambiguous in sign, as we have discussed earlier that $p^s_{b_i b_i}$ is ambiguous in sign. Third, $p_{b_i z_i} = p_{z_i b_i} = -p^m_{z_i} p^s_{b_i} < 0$ implying that the marginal effect of biosecurity $b_i$ in reducing the total probability of infection is increasing with higher level of $z_i$ and vice versa.[5] Intuitively, as producer *i* increases one risk management choice, it increases the probability of not being infected through that pathway, thereby increasing the marginal effectiveness of the other risk management choice (i.e., $b_i$ and $z_i$ are technical complements). Fourth, $p_{z_i B} = -p^s_B p^m_z < 0$ and $p_{z_i Z} = -p^s_Z p^m_z < 0$.[6] neighbors' risk management choices are technical complements to $z_i$. The reasoning is analogous to that for our third point above: more risk management by neighbors increase producer *i*'s probability of not being infected through the spread pathway, putting more weight on the import pathway and increasing the marginal effectiveness of reducing risk by importing from the disease-free region. Finally, $p_{b_i B} = (1 - p^m) p^s_{b_i B}$, $p_{b_i Z} = (1 - p^m) p^s_{b_i Z}$ have ambiguous signs due to the ambiguous signs of $p^s_{b_i B}$ and $p^s_{b_i Z}$, as discussed earlier. Please see Table 1 for a summary of the second derivatives of the total probability function.

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[5] Analogy to the traditional Cobb-Douglas's production function where the output is a function of two inputs, labor and capital. It can be shown that the marginal product of labor is increasing when capital increases, and vice versa.
[6] In general, actions $s_1$ and $s_2$ are technical complements (substitutes) in managing the probability of infection if $\partial(\partial(1 - p_i)/\partial s_1)/\partial s_2 = -p_{s_{1i} s_2} > 0$ (< 0) where $s_1$ and $s_2$ are any risk management choice, which is equivalent to the expression in the text.

## 2.2 Economic model

After the animals are fed to the target weight, they are sold to slaughterhouses. If the lot is healthy, the net return (i.e., revenue less feeding costs, excluding biosecurity and purchasing cost) for the lot equals $R > 0$. There is a loss associated with the infection, so the return of infected should be less than $R$. Without loss of generality, we assume the infected lot receives zero net return in order to simplify the calculation.

The cost of biosecurity is assumed to be a convex function represented by $C(b_i)$ with $C' > 0, C'' > 0$. We assume cattle are purchased in competitive, region-specific spot markets. The cost of procurement is $z_i w^1(Z) + (1 - z_i) w^2(1 - Z)$, where $w^1(Z)$, and $w^2(1-Z)$ are inverse excess supply functions for feeder cattle from regions 1 and 2, respectively. By the law of supply, we assume $w_Z^1 > 0$ and $w_Z^2 < 0$. Note that these supply relations depend only on aggregate import decisions of producers; individual producers operate in competitive markets and therefore take these prices as given. Procurement costs are rewritten as $z_i w^1(Z) + (1 - z_i)w^2(1 - Z)$, where $W(Z) = w^1(Z) - w^2(1-Z)$ is the difference in supply prices, with $w_Z > 0$ so that $W(Z)$ is an upward sloping relative price function.

It is also interesting to investigate an impact of government policy on the decentralized outcome. We select three policies including (i) a biosecurity subsidy, $sb_i$ where $s$ is the subsidy rate, (ii) a tax on risky imports, $t(1 - z_i)$, where $t$ is the tax rate, and (iii) an indemnity payment, $v$ where $v$ is a fixed payment to infected producers regardless of their risk management choices.

We complete the economic specification by assuming each producer is risk-neutral, so that producer $i$'s objective is to maximize his expected profit,

$$\pi_i = R - p(b_i, z_i, B, Z)(R - v) - C(b_i) + s\, b_i - W(Z)\, z_i - t(1 - z_i) - w^2(1 - Z) .$$

## 3. Decentralized Strategic Behavior

In this section, we analytically discuss the decentralized equilibrium of the disease risk management game described in previous section. We start with a derivation of the SNE solution and discuss its associated properties including existence, uniqueness and stability. Since the conclusions about uniqueness and stability are ambiguous due to the complexity of interactions between producers, we will impose certain assumptions based upon insights from prior works. Then, we explore some interesting implications stemming from (i) multiple choices, (ii) market effects, and (iii) government policies.

### 3.1 SNE solution and its existence

Recall that the strategy set is already assumed to be compact and convex. If we also assume the profit function is twice differentiable concave in own strategies, then at least one SNE is guaranteed to exist (Debreu 1952, Cachon and Netessine, 2004).[7] The concavity of the profit function is not too restrictive; for instance, multiple SNE could still arise despite the assumption. Unfortunately, uniqueness and stability of SNE are more difficult to verify. Before we are able to discuss those properties, we need to derive the SNE. Consider the representative producer's optimization problem (which allows us to drop the subscript *i*), defined as

(3) $$\max_{b,z} \pi(b, z; B, Z) \text{ subject to } 0 \le b \le 1, \text{ and } 0 \le z \le 1$$

The associated first-order conditions (FOCs) of an interior solution are

(4) $$\nabla \boldsymbol{\pi}(b, z, B, Z) = 0$$

[7] $\pi_{z_i z_i} < 0$ is already satisfied by definition given in section 2. The additional assumptions for concavity of the profit function are $\pi_{b_i b_i} < 0$ and $\pi_{b_i b_i} \pi_{z_i z_i} - \pi_{b_i z}^2 > 0$. In Table 2, we explore the conditions under which the profit function is likely to be concave.

where $\nabla\boldsymbol{\pi}(b,z,B,Z)=0$ is the gradient vector of the profit function with respect to the producer's own strategies, $\nabla\boldsymbol{\pi}(b,z,B,Z)\equiv\begin{bmatrix}\pi_b\\ \pi_z\end{bmatrix}=\begin{bmatrix}-p_b(R-v)-C'-s\\ -p_z(R-v)-W(Z)+t\end{bmatrix}$.

The simplest way to solve for the SNE is to impose symmetry, *b* = *B* and *z* = *Z*, so that relation (4) becomes

$$\text{(5)}\qquad \nabla\boldsymbol{\pi}^*(B,Z)\equiv\begin{bmatrix}\pi_b^*(B,Z)\\ \pi_z^*(B,Z)\end{bmatrix}\equiv\begin{bmatrix}\pi_b(b,z,B,Z)\\ \pi_z(b,z,B,Z)\end{bmatrix}\Bigg|_{b=B,z=Z}=0$$

The solution to relation (5) is the SNE.

The conditions for uniqueness and stability of the SNE depend on the first derivatives of relation (5) with respect to *B* and *Z* (please see section 3.2 for uniqueness and 3.3 for stability discussion). To aid in our analysis of these properties, it is helpful to introduce notation for the first derivatives of the FOCs, or the Jacobian matrices of relation (4). This is because the derivatives of relation (5) can be written as a function of the Jacobian matrices of relation (4) with respect to one's own strategies and neighboring strategies.

Denote the Jacobian matrices of the FOCs with respect to one's own strategies, and neighboring strategies as follows

$$\mathbf{F}_1=\begin{bmatrix}\pi_{bb} & \pi_{bz}\\ \pi_{zb} & \pi_{zz}\end{bmatrix}=\begin{bmatrix}-p_{bb}(R-v)-C'' & -p_{bz}(R-v)\\ -p_{bz}(R-v) & -p_{zz}(R-v)\end{bmatrix}, \text{ and}$$

$$\mathbf{F}_2=\begin{bmatrix}\pi_{bB} & \pi_{bZ}\\ \pi_{zB} & \pi_{zZ}\end{bmatrix}=\begin{bmatrix}-p_{bB}(R-v) & -p_{bZ}(R-v)\\ -p_{bB}(R-v) & -p_{zZ}(R-v)-W'\end{bmatrix}, \text{ respectively.}$$

Note that $\mathbf{F}_1$ is a standard Hessian matrix with respect to own strategies. Since we assume the profit function is concave in own strategies, $\mathbf{F}_1$ is negative semi-definite. $\mathbf{F}_2$ represents the marginal impact of neighboring strategies on marginal profit of representative producer's strategies which indicates the strategic relationship among producers. The positive sign of an element in $\mathbf{F}_2$, say $\pi_{mn}$, implies strategic complementarity between one's own choice *m* and others' choice *n* (i.e., one's marginal net benefits of choice *m* are increasing in other's choice *n*), while a negative sign implies strategic substitutability between these choices (i.e., one's marginal net benefits of choice *m* are decreasing in other's choice *n*). Note that in the absence of market effects $(w' = 0)$ and the indemnity ($v = 0$), the RHS of the expression for $\mathbf{F}_2$ indicates the strategic relationships are implied by the technical probabilistic relationship involving the variables of interest.

Now we derive the first derivatives of relation (5). Let **G** be the Jacobian matrix of $\nabla\boldsymbol{\pi}^*(B,Z)$ with respect to *B* and *Z*, which we can write as the summation of $\mathbf{F}_1$ and $\mathbf{F}_2$ evaluated at $b = B$ and $z = Z$:

$$\mathbf{G}(B,Z) \equiv \begin{bmatrix} \frac{\partial \pi_b^*(B,Z)}{\partial B} & \frac{\partial \pi_b^*(B,Z)}{\partial Z} \\ \frac{\partial \pi_z^*(B,Z)}{\partial B} & \frac{\partial \pi_z^*(B,Z)}{\partial Z} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial [\pi_b(b,z,B,Z)]\big|_{b=B,z=Z}}{\partial B} & \frac{\partial [\pi_b(b,z,B,Z)]\big|_{b=B,z=Z}}{\partial Z} \\ \frac{\partial [\pi_z(b,z,B,Z)]\big|_{b=B,z=Z}}{\partial B} & \frac{\partial [\pi_z(b,z,B,Z)]\big|_{b=B,z=Z}}{\partial Z} \end{bmatrix}$$

$$= \begin{bmatrix} \pi_{bb} + \pi_{bB} & \pi_{bz} + \pi_{bZ} \\ \pi_{zb} + \pi_{zB} & \pi_{zz} + \pi_{zZ} \end{bmatrix} = (\mathbf{F}_1 + \mathbf{F}_2)\big|_{b=B,z=Z}.$$

### 3.2 Uniqueness of SNE

There are several ways to verify uniqueness of SNE including (i) the diagonal dominance approach, (ii) the univalence approach, (iii) the index theorem approach, and (iv) the algebraic approach (see Vives (1999), Cachon and Netessine (2004), and Hefti (2013) for detailed discussions). We adopt the index theorem approach (Vives 1999; Cachon and Netessine 2004; Hefti 2013) because it is more general than the diagonal dominance and univalence approaches. Vives (1999) mentions that the diagonal dominance approach is a specific case of the univalence approach, while we find that the univalence approach is a specific case of the index theorem approach in case of two-dimensional strategies (see appendix A1 for the proof). The sufficient condition for a unique SNE is indicated in Theorem 1, which is a re-statement of the Poincare-Hopf index theorem (Vives 1999, and Cachon and Netessine 2004)

**Theorem 1 (sufficiency for uniqueness).** *If* $\left|-\mathbf{G}(B,Z)\right|>0$ *whenever* $\nabla\boldsymbol{\pi}^{*}(B,Z)=0$ *(i.e.,* $\left|-\mathbf{G}(B,Z)\right|>0$ *at each SNE), then there is only one interior SNE. Specifically, the sufficient condition for a unique SNE is*[8]

(6) $$\left(\frac{\partial\pi_b^{*}(B,Z)}{\partial B}\right)\left(\frac{\partial\pi_z^{*}(B,Z)}{\partial Z}\right)-\left(\frac{\partial\pi_z^{*}(B,Z)}{\partial B}\right)\left(\frac{\partial\pi_b^{*}(B,Z)}{\partial Z}\right)>0 \text{ at each SNE.}$$

Condition (6) can be verified numerically. Multiple SNE and/or corner solutions may arise if condition (6) does not hold.

### 3.3 Stability of SNE

Dixit's (1986) pseudo-dynamic approach for modeling a tâtonnement process (e.g., see Krugman 1987) is adopted to examine the local stability of an SNE. This approach essentially models a myopic tâtonnement process (rather than a real dynamic optimization process) that indicates what would happen if the system did not begin at the SNE. Specifically, the representative producer adjusts myopically to neighboring decisions, *B* and *Z,* which we assume are the average decisions in the region. The adjustment process is specified to be proportional to the marginal

[8] Vives (1999) uses $\left|-\mathbf{G}(B,Z)\right|>0$ which is same as our representation, whereas Cachon and Netessine (2004) use $(-1)^n\left|\mathbf{G}(B,Z)\right|>0$ where *n* is the dimension of **G.** Both representations are equivalent since $\left|-\mathbf{G}(B,Z)\right|=\left|-\mathbf{I}\right|\left|\mathbf{G}(B,Z)\right|$ and $\left|-\mathbf{I}\right|=(-1)^n$. Vives (1999) also notes that this condition is "almost" necessary condition except for vanishing $\left|-\mathbf{G}(B,Z)\right|$ at the equilibrium point.

profits associated with an activity:

$$\begin{bmatrix} \dot{b} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \delta_B \, \pi_b(b,z,B,Z) \\ \delta_Z \, \pi_z(b,z,B,Z) \end{bmatrix}$$

For instance, if $\pi_b(b,z,B,Z) > 0$, then the representative producer would increase his level of biosecurity. All producers would do the same due to the homogeneity of producers; the average biosecurity therefore increases. The same argument applies to the adjustment process of the average import decision. Hence, after imposing symmetry, the adjustment process becomes

$$\begin{bmatrix} \dot{B} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} \delta_B \, \pi_b^*(B,Z) \\ \delta_Z \, \pi_z^*(B,Z) \end{bmatrix} \tag{7}$$

where $\delta_B, \delta_Z > 0$ are parameters defining the speed of adjustment of *B*, and *Z*, respectively. Notice that a steady state for system (7) means that condition (5) is satisfied. Hence, the steady state(s) of system (7) are, by definition, the SNE. In subsequent numerical examples, as well as in our discussion of government policy, we use the dynamic system (7) to construct phase planes to provide a graphical aid in the analysis of stability and uniqueness.

Analytically, the local stability properties of a potential steady-state equilibrium associated with system (7) are derived by linearizing the adjustment process (i.e., the right hand side of (7)) around that the equilibrium. The Jacobian matrix associated with this linearization is

$$\mathbf{J} = \begin{bmatrix} \delta_B \dfrac{\partial \pi_b^*(B,Z)}{\partial B} & \delta_B \dfrac{\partial \pi_b^*(B,Z)}{\partial Z} \\ \delta_Z \dfrac{\partial \pi_z^*(B,Z)}{\partial B} & \delta_Z \dfrac{\partial \pi_z^*(B,Z)}{\partial Z} \end{bmatrix}.$$

Define the trace of **J** as *tr* = $\delta_B \, \partial \pi_b^*(B,Z) / \partial B + \delta_Z \, \partial \pi_z^*(B,Z) / \partial Z$, and the determinant of **J** as $det = \delta_B \delta_Z [(\partial \pi_b^* / \partial B)(\partial \pi_z^* / \partial Z) - (\partial \pi_z^* / \partial B)(\partial \pi_b^* / \partial Z)]$. Following Dixit (1986) (see Conrad and Clark 1987 for a complete derivation), a SNE is locally stable if all eigenvalues contain a negative real part, which implies certain restrictions on *tr* and *det*, as summarized in Theorem 2.

**Theorem 2 (local stability)**. *A SNE is locally stable (either node or focus) if and only if the following are both satisfied:* [9]

(8a) *det > 0*

(8b) *tr < 0.*

Note that *det* > 0 is also required in condition (6) in Theorem 1 (because $\delta_B, \delta_Z > 0$), except that here *det* > 0 is not required for all SNE. This means stability does not imply uniqueness. Assume there is more than one SNE. Then, if we apply Theorem 1 to the results in Theorem 2, we find that *det > 0* cannot hold for all SNE as Theorem 1 indicates there should only be one SNE in this case. This means we must have *det* < 0 for at least one SNE in the case of multiple SNE. In other words, not all SNE can be stable. This situation could create the potential for multiple, locally stable SNE separated by unstable SNE. Hefti (2013) also shows that this result holds for a symmetric game with more than two strategies. Theorem 2 indicates we need an additional requirement for a SNE to be stable: *tr* < 0 or $\delta_B (\partial \pi_b^*(B,Z) / \partial B) + \delta_Z (\partial \pi_z^*(B,Z) / \partial Z) < 0$. This condition holds for any value of $\delta_B, \delta_Z > 0$ when $\partial \pi_b^*(B,Z) / \partial B < 0$ and $\partial \pi_z^*(B,Z) / \partial Z < 0$. This result implies that not all unique SNE are stable. Finally, global stability is ensured if conditions (8a) and (8b) hold whenever $\nabla \boldsymbol{\pi}^*(B,Z) = 0$ (i.e., for all interior SNE), in which case the interior SNE is unique. If condition (8a) is satisfied for all interior SNE, while condition (8b) is not, then the interior SNE is unstable (a saddle). In this case, the corners would be stable SNE.

[9] In general case with *n* strategies, the condition (8a) changes to $(-1)^n$ det **J** >0 which is same as condition (6). See Dixit (1986) page 117. Note that stability is an intertemporal concept and depends on the nature of the underlying adjustment processes. The conditions presented here are based on the assumption of continuous-time adjustment.

## 4 Implications

We will now attempt to understand the influence of three features of the model: behavioral feedbacks arising from multiple choices, market effects, and government policies. To facilitate the discussion, we first establish a baseline case with two choices, no market effects, and no government policy. We then examine several modifications of the baseline model: (i) modeling one choice, (ii) adding market effects, and (iii) adding government policies. We draw insights by comparing results between the modified case and the baseline case.

*Case 0: The baseline case, two choices, no market effect and no policy*

Our baseline case is where each producer has two choices for protecting his herd, no market effects $(w' = 0)$, and no policy $(s = t = v = 0)$. This case is most closely related to Reeling and Horan (2015), with the only difference being two risk management choices. The uniqueness and stability condition depends on sign and magnitude of the second derivatives represented in **G**, which remain ambiguous in this case. However, we can obtain insights by comparing the uniqueness condition between the two-choice model and the one-choice model.

*Case 1: "One choice", no market effect and no policy.*

Consider a special case of the model where there is a single individual choice, *x*, and a single neighboring choice, *X*. The relation between *x* and *b* and *z* (and *X* and *B* and *Z*) are described below. The producer's optimization problem can be rewritten as

(9) $$\max_x \pi(x, X) \text{ subject to } 0 \le x \le 1$$

We use problem (9) to relate the single-choice models of prior work to the two-choice model defined by problem (3). Note that if we apply Theorems 1 and 2 to the single-choice model, the uniqueness and stability conditions become identical:

(10) $$\pi_{xx} + \pi_{xX} < 0 \text{ for all interior SNE.}$$

We examine three prior studies in the context of the single-choice model. First, Hennessy (2007) models the single choice as a protection against spread risk (*b*) whereas protection against import risk (*z*) is effectively held constant i.e. $x=b$, $X=B$ and $z=Z=Z_0$, where $Z_0$ is fixed. In his model, the number of effective neighbors is sufficiently low ($N$=1), he argues that *b* and *B* exhibit technical and strategic substitution ($\pi_{bB} < 0$) which we will also assume. The SNE is unique and stable if

(10a) $$\overbrace{\pi_{bb}}^{(-)} + \overbrace{\pi_{bB}}^{(-)} < 0 \text{ for all interior SNE.}$$

Obviously, $\pi_{bb} + \pi_{bB} < 0$ for any value of *B* because $\pi_{bb} < 0$ due to concavity of the profit function. This implies that any SNE for the single-choice model of *b* is unique and stable since the source of instability from the two-choice model, *Z*, is essentially held fixed in this case.

We can illustrate this result graphically in terms of the best response function, $\tilde{b}(B, Z_0)$, that solves $\pi_b(b, Z_0, B, Z_0) = 0$. The graph is presented in Fig.1, with *B* on the horizontal axis and the vertical axis indicating $\tilde{b}(B, Z_0)$. The slope of $\tilde{b}(B, Z_0)$ is $-\pi_{bB}/\pi_{bb}$, which is negative. By definition, the SNE is the solution to $B^* = \tilde{b}(B^*, Z_0)$, this means that the SNE is the intersection of $\tilde{b}(B, Z_0)$ and the 45° line. We can conclude that the SNE is unique because the intersection of positively-sloped and negatively-sloped lines is unique.

To analyze stability, consider a reduction in both *b* and *B*. We know that $\pi_{bb} + \pi_{bB} < 0$ which implies that $\pi_b(B, Z_0, B, Z_0) > 0, \forall B < B^*$. The second order condition implies that the producer will increase his biosecurity choice. Other producers will do the same thing, due to symmetry, so the biosecurity choice will converge to the SNE level. The opposite adjustment is true $\forall B > B^*$. This adjustment process applies to all symmetric choices, which implies the SNE is unique and globally stable SNE (see arrows). If this adjustment process does not hold for one SNE, then a small deviation from this SNE will cause a divergence to another SNE.

The second type of single-choice model is where $z$ is chosen, holding $B$ fixed. For instance, Hennessy (2008) considers a decision that reduces import risk, $x=z$, $X=Z$, where spread risk is effectively assumed to be exogenously determined: $b=B=B_0$, where $B_0$ is fixed. Without market effects, he shows $z$ and $Z$ exhibit technical and strategic complementarities ($\pi_{zZ} > 0$). According to condition (10), the SNE is unique and stable if

(10b) $$\overbrace{\pi_{zz}}^{(-)} + \overbrace{\pi_{zZ}}^{(+)} < 0 \text{ for all interior SNE.}$$

The sign of $\pi_{zz} + \pi_{zZ}$ is generally ambiguous due to the strategic complementarity term $\pi_{zZ} > 0$. This implies that multiple SNE could arise when the degree of complementarity is sufficiently large i.e. $\pi_{zZ} > -\pi_{zz}$. This result is shown graphically in Fig. 2 using a best response function, denoted $z(Z,B_0)$, that solves $\pi_z(B_0, z, B_0, Z) = 0$. In contrast to Fig. 1, the best response function in Fig. 2 is upward sloping due to the complementarity. Accordingly, $z(Z,B_0)$ may intersect the 45° curve multiple times. Figure 2 illustrates an example with three SNE, represented at points $E_0$, $E_1$, and $E_2$. It is not difficult to see that $E_0$ and $E_2$ are stable whereas $E_1$ is an unstable SNE (see arrows).

We can compare condition (10b) to condition (6) to determine when the two-choice model SNE is more likely to be unique than the single-choice model SNE. Using the notation defined in **G**, we rewrite condition (6) which states that the two-choice SNE is unique if

(6a) $$\overbrace{\frac{\partial \pi_z^*(B,Z)}{\partial Z}}^{(?)} + \overbrace{\left(\frac{\partial \pi_z^*(B,Z)/\partial B}{-\partial \pi_b^*(B,Z)/\partial B}\right)}^{(+)} \overbrace{\frac{\partial \pi_b^*(B,Z)}{\partial Z}}^{(?)} < 0 \text{ or}$$

$$\frac{\partial \pi_z^*(B,Z)}{\partial Z} + A\frac{\partial \pi_b^*(B,Z)}{\partial Z} < 0 \text{ for all interior SNE}$$

where $A = \frac{\partial \pi_z^*(B,Z)/\partial B}{-\partial \pi_b^*(B,Z)/\partial B} > 0$.

The first term of condition (6a) is the same as the RHS of condition (10b), which we refer to as the "direct" effect. Recall from (10b) that this effect is the potential source of multiple SNEs: multiple SNEs may arise when the degree of complementarity between $z$ and $Z$ is sufficiently strong. The second term in condition (6a) is an adjustment, which we refer to as the "indirect" effect, that can enhance or offset the direct effect. In particular, the direct effect may be (partially or fully) offset if the indirect effect is negative, such that

(11) $$\frac{\partial \pi_b^*(B,Z)}{\partial Z} < 0,$$

so that condition (6a) is less stringent than condition (10b). In this case, the SNE of the two-choice model is more likely to be unique than the single-choice model SNE. Condition (11) also promotes stability by ensuring (8b) is satisfied. When we discuss equation (6b) below, we show that the indirect effect represents the biosecurity adjustment made in response to the change in $Z$. This interpretation means the ability to adjust biosecurity can increase the likelihood that an SNE is unique and stable.

The interpretation of the indirect effect as an adjustment emerges from an analysis of dynamics as described in equation (7). We depict the dynamic system (7) using the two-dimensional phase plane in Fig. 3, where the SNE is the intersection(s) of the $\dot{B} = 0$ and $\dot{Z} = 0$ isoclines (see Appendix B for a description of the numerical example). All points along the

$\dot{B}=0$ isocline represent the SNE of the single-choice model when *Z* is held constant. Similarly, the $\dot{Z}=0$ isocline represents the SNE of the single-choice model when *B* is held constant. The phase arrows are derived as follows. First, we derive $d\dot{B}/dB\,|_{\dot{B}=0} = \delta_B(\partial\pi_b^*(B,Z)/\partial b) < 0$, which means $\dot{B}<0$ above the $\dot{B}=0$ isocline, and vice versa below the isocline. Similarly, $d\dot{Z}/dB\,|_{\dot{Z}=0} = \delta_Z(\partial\pi_z^*(B,Z)/\partial b) > 0$ implies $\dot{Z}>0$ above the $\dot{Z}=0$ isocline, and vice versa below the isocline.

The slopes of the isoclines are derived by taking the total derivative of each isocline with respect to the state variables to obtain

(12a) $$\frac{dB}{dZ}\Big|_{\dot{B}=0} = -\frac{\partial\pi_b^*(B,Z)/\partial Z}{\partial\pi_b^*(B,Z)/\partial B} = \begin{cases} <0 & \text{,if } \partial\pi_b^*(B,Z)/\partial Z<0 \\ >0 & \text{,otherwise} \end{cases}$$ , and

(12b) $$\frac{dB}{dZ}\Big|_{\dot{Z}=0} = -\frac{\partial\pi_z^*(B,Z)/\partial Z}{\partial\pi_z^*(B,Z)/\partial B} = \begin{cases} >0 & \text{,if } \partial\pi_z^*(B,Z)/\partial Z<0 \\ <0 & \text{,otherwise} \end{cases}.$$

Notice that the $\dot{B}=0$ isocline is negatively sloped if $\partial\pi_b^*(B,Z)/\partial Z<0$, which is the same as condition (11). Moreover, we use expression (12a) to rewrite condition (6a) as

(6b) $$\frac{\partial\pi_z^*(B,Z)}{\partial Z} + \frac{\partial\pi_z^*(B,Z)}{\partial B}\frac{dB}{dZ}\Big|_{\dot{B}=0} < 0 .$$

Similar to condition (6a), condition (6b) decomposes the marginal impact of *Z* on the marginal incentives for *z*, $\pi_z^*$, into a direct effect (first term) and indirect effect (second term). However, here the indirect effect clearly represents an adjustment of *B* in response to a change in *Z*. The adjustment process occurs because, when *Z* changes, *B* adjusts along the $\dot{B}=0$ isocline. The term $\partial\pi_z^*(B,Z)/\partial B$ is positive (Table 2), with *z* and *B* being complementary. Hence, the indirect effect offsets the direct effect when *B* is a substitute for *Z* along the $\dot{B}=0$ isocline, so that this isocline has a negative slope. Note that the steeper the $\dot{B}=0$ isocline, the more likely for SNE to be unique and stable.

The slope of $\dot{Z} = 0$ isocline is generally ambiguous and depends on the sign of $\partial\pi_z^*(B,Z)/\partial Z$ , which determines uniqueness and stability in the single-choice model for *z* (from condition (10b)). If $\partial\pi_z^*(B,Z)/\partial Z < 0$ for all *Z*, then the $\dot{Z} = 0$ isocline has an unambiguously positive slope and there is a unique value of *Z* for each *B* (i.e., the SNE of the single-choice model for *z* is unique for all values of *B*). However, if the $\dot{Z} = 0$ isocline is U-shaped, then points on the downward-sloping portion of the isocline (with $\partial\pi_z^*(B,Z)/\partial Z > 0$ ) will be unstable while points on the upward-sloping portion (with $\partial\pi_z^*(B,Z)/\partial Z < 0$ ) will be stable. According to table 2, a SNE with $\partial\pi_b^*(B,Z)/\partial Z < 0$ arises if and only if

$$Np^c p^M < 1 - \frac{(1-p^c p^M) p^M p_z^m}{(1-p^m) p_Z^M}.$$

This inequality is likely to hold when the number of expected infected neighbors is sufficiently low, or the risk of being infected is not too high, in the sense that biosecurity is effectively able to reduce spread risk.

We can use Fig. 3 to compare the two-choice model and the single-choice model for *z*. The SNE of the two-choice model is the intersection of the isoclines, and it is unique at point $E_0$ where the equilibrium outcome is when $b=B=B^*$ and $z=Z=Z^*$. The ambiguity of slope of the $\dot{Z} = 0$ isocline means multiple SNE occur when biosecurity is fixed at certain levels, as in the single-choice model for *z*. Indeed, the single-choice model for *z* can be represented by fixing biosecurity at the SNE level, $B^*$, at which point there are three associated SNE for *Z*: $E_0$, $E_1$, and $E_2$. These points correspond to the SNE depicted in Fig. 2, with the same stability properties.

With multiple SNE, the choice of equilibria will be driven by producers' expectations about the average import decision. The zero import protection outcome, $E_2$, could be the equilibrium if each producer expects the others to put low effort (< $E_1$) into import protection, and $E_0$ arises otherwise.

Now consider what happens if *B* adjusts. In Fig.3, we can see that points $E_1$ and $E_2$ cannot be SNE because $B^*$ is not the optimal biosecurity level for producers given the values of *Z* at these points. Specifically, the negatively sloped $\dot{B} = 0$ isocline implies that the optimal biosecurity choice will exceed $B^*$ for any Z<Z*. Hence, $\dot{B} > 0$ in this region. An increase in biosecurity enhances the marginal productivity of individual import protection that would induce producers to raise the import protection effort, $\dot{Z} > 0$. As a result, the adjustment process will move the system towards the equilibrium point $E_0$. When producers are more flexible in terms of having more choices to management risks, the SNE outcome becomes more stable in this case.

The third type of single-choice model involves a single-choice that protects a herd from disease risks through both import and spread pathways, as in Reeling and Horan (2015). Specifically, they effectively assume *x*=*b*=*z* and *X*=*B*=*Z*. A SNE is the solution to the associated FOC evaluated at the symmetric decisions: $\pi_b(X,X) + \pi_z(X,X) = 0$. An SNE in this setting is unique and stable when the following condition holds for all interior SNE (where the signs of the partials follow from our assumptions above)

$$\text{(10c)} \qquad \overbrace{\frac{\partial \pi_z^*(B,Z)}{\partial Z}}^{(?)} + \overbrace{\frac{\partial \pi_b^*(B,Z)}{\partial Z}}^{(-)} + \overbrace{\frac{\partial \pi_z^*(B,Z)}{\partial B}}^{(+)} + \overbrace{\frac{\partial \pi_b^*(B,Z)}{\partial B}}^{(-)} < 0 .$$

Note that the uniqueness condition depends on strategic relationships along each risk pathway (importation, $\pi_{zZ}$, and spread, $\pi_{bB}$) and also across pathways $(\pi_{bZ}, \pi_{zB})$. Recalling that *x*=*b*=*z* and *X*=*B*=*Z* in the single-choice model, we can effectively rewrite condition (10c) as

$$\text{(13)} \qquad \frac{\partial(\pi_z + \pi_b)}{\partial z} + \frac{\partial(\pi_z + \pi_b)}{\partial b} + \frac{\partial(\pi_z + \pi_b)}{\partial Z} + \frac{\partial(\pi_z + \pi_b)}{\partial B} = \overbrace{\frac{\partial \pi_x}{\partial x}}^{(-)} + \overbrace{\frac{\partial \pi_x}{\partial X}}^{(?)} < 0 .$$

Note that $\partial \pi_x / \partial x < 0$ in a single-choice setting if we assume concavity of the expected profit function. Condition (13) suggests that the SNE is unique when *x* and *X* are strategic substitutes ( $\partial \pi_x / \partial X < 0$ ) or weak strategic complements ( $\partial \pi_x / \partial X > 0$ but relatively small), which is the same conclusion stated in Reeling and Horan (2015).

Condition (10c) can be compared with condition (6a). The first LHS terms of (10c) and (6a) are identical. Upon comparing the remaining three LHS terms in condition (10c) with the indirect effect of condition (6a), we find that condition (6a) is less stringent than (10c) – meaning the two-choice model is more likely to have a unique (and stable) SNE – when the following condition holds:

$$\frac{\partial \pi_b^*(B,Z)}{\partial Z} + \frac{\partial \pi_b^*(B,Z)}{\partial B} + \frac{\partial \pi_z^*(B,Z)}{\partial B} > A \frac{\partial \pi_b^*(B,Z)}{\partial Z}$$

$$\frac{\partial \pi_b^*(B,Z)}{\partial B} + \frac{\partial \pi_z^*(B,Z)}{\partial B} > (A-1) \frac{\partial \pi_b^*(B,Z)}{\partial Z}$$

$$(A-1) > (A-1) \frac{\partial \pi_b^*(B,Z) / \partial Z}{-\partial \pi_b^*(B,Z) / \partial B}$$

$$(A-1) \overbrace{\left(1 + \frac{\partial \pi_b^*(B,Z) / \partial Z}{\partial \pi_b^*(B,Z) / \partial B}\right)}^{(+)} > 0 \text{ or } A > 1$$

Note that the second line above stems from the fact that the numerator (denominator) of *A* is the same as the third (negative of the second) LHS terms in the first line.

The relation $A > 1$ is satisfied when $(\pi_{zb} + \pi_{bb}) + (\pi_{zB} + \pi_{bB}) > 0$ or, using the notation of condition (13), when

(14) $$-\frac{\partial \pi_x}{\partial b} < \frac{\partial \pi_x}{\partial B}.$$

Condition (14) indicates that a unique SNE is more likely to arise under the two-choice model than the single choice model when $\partial\pi_x / \partial B$ is relatively large. This means the two-choice model SNE is more likely unique when a producer's choices are either a sufficiently strong strategic complement or a sufficiently weak strategic substitute with others' biosecurity. The intuition is that there is less reliance on strategic substitutability between choices in the two-choice model because producers have more flexibility to substitute among inputs to manage risks. This increased management ability makes a unique outcome more likely. In contrast, under the single-choice model, producers have less ability to manage towards a unique outcome and therefore rely much more on the nature of strategic relations to ensure uniqueness.

*Case 2: Two choices with market effects*

We now capture market effects by applying assuming an increase in aggregate imports from the safe region will raise the relative import price $(W' > 0)$. Note that $\pi_{zZ}$ is the only second derivative term represented in **G** that is different with market effects: $\pi_{zZ}\,|_{W'>0} = -p_{zZ}R - w'$ with market effects, whereas $\pi_{zZ}\,|_{W'=0} = -p_{zZ}R$ without market effects, so that $\pi_{zZ}\,|_{W'=0} > \pi_{zZ}\,|_{W'>0}$, other things equal.

The uniqueness condition (6a) under market effects becomes

(15)
$$\begin{aligned}&\frac{\partial\pi_z^*(B,Z)}{\partial Z}\Big|_{W'>0} + \frac{\partial\pi_z^*(B,Z)/\partial Z}{-\partial\pi_b^*(B,Z)/\partial B}\frac{\partial\pi_b^*(B,Z)}{\partial Z}\\ &= \frac{\partial\pi_z^*(B,Z)}{\partial Z}\Big|_{W'=0} + \frac{\partial\pi_z^*(B,Z)/\partial Z}{-\partial\pi_b^*(B,Z)/\partial B}\frac{\partial\pi_z^*(B,Z)}{\partial Z} - W' < 0\end{aligned}$$

Notice that the first two terms after the equality sign in (15) are the same as the two LHS terms in condition (6a). The final term in condition (15), which arises due to the market effects, is negative since $W' > 0$. This implies that the uniqueness condition with market effects is less stringent than the one without market effects. Intuitively, market effects essentially reduce the degree of complementarity between $z$ and $Z$ because, as more producers purchase risky animals, the price of risky imports increases and this market effect helps to limit the amount of risky imports.

Fig. 4 illustrates the SNE generated from the same numerical example used to draw Fig. 3, except Fig. 4 includes the market effects. According to Fig. 3, the uniqueness of the SNE requires the $\dot{B}=0$ isocline to lie above the upward sloping portion of the $\dot{Z}=0$ isocline. For instance, multiple SNE could arise if the $\dot{B}=0$ isocline were to shift downward and intersect the $\dot{Z}=0$ isocline in two places. In contrast, the $\dot{Z}=0$ isocline is unambiguously positively sloped in Fig. 4. This means that the market effects now eliminate the chance of multiplicity of SNE. Put differently, the uniqueness of SNE is not sensitive to the location of $\dot{B}=0$ isocline in Fig. 4.

*Case 3: Two choices, market effects, and policies*

We now investigate how the uniqueness and stability of the SNE are impacted by the policies described above: (i) a biosecurity subsidy ($sb$), (ii) a tax on risky imports ($t[1-z]$), and (iii) an indemnity payment ($v$). Recall that, by Theorems 1 and 2, the uniqueness and stability of SNEs is determined by the properties of **G** and **J** evaluated at the SNEs. None of the policy variables indicated above affect **G** or **J** for given values of $B$ and $Z$. However, the policies do affect the incentives to invest in $B$ and $Z$ and hence the location of any SNEs. This means the values of **G** or **J** at the SNEs are affected, and so uniqueness and stability could change due to policy interventions.

We motivate potential policy effects on SNE numerically by considering a case where multiple SNE arise even in the presence of two choices and market effects. Hence, our first numerical example, in which a unique and stable outcome arose with two choices and market effects, no longer applies and must be modified. Please see Appendix B for a detailed description of the new numerical model. Numerical results are presented in Table 3.

Fig. 5 illustrates the SNE of the two-choice model with market effects and no government policies, for comparison to the case with policies. There are three SNE in Fig. 5, represented by points $E_3$ (a stable focus or node depending on the speed of adjustment of $B$ and $Z$, $\delta_B$ and $\delta_Z$, from (7)), $E_4$ (a saddle point), and $E_5$ (a stable node). This outcome reflects Theorem 2's implication that there must be an unstable SNE when there are multiple SNE. Moreover, the basins of attraction for the stable SNE are separated by the saddle path associated with the unstable SNE $E_4$.[10] Any initial point from the RHS (LHS) basin will converge to $E_3$ ($E_5$). Note that we can Pareto rank the SNE where point $E_3$ Pareto dominates $E_4$ and $E_5$ (see Table 3).

It is clear in Fig. 5 that an upward shift in the $\dot{B}=0$ isocline and/or a downward shift in the $\dot{Z}=0$ isocline will move the unstable SNE, $E_4$, to the left. As a result, the basin of attraction for the (stable) Pareto dominated SNE, $E_5$, is reduced whereas the basin of attraction for the Pareto dominant SNE, $E_3$, is increased. In other words, it becomes less likely for the system to equilibrate at $E_5$, and more likely for the system to equilibrate at $E_3$. A sufficient shift in the isoclines can eliminate the possibility of being at the Pareto dominated SNE.

Given the knowledge of how shifts in the isoclines can affect the SNE, we now examine how government policies affect the isoclines. Specifically, we use the implicit function theorem to derive comparative static results for the isoclines:

$$(16)\qquad \frac{dB}{ds}\Big|_{\dot{B}=0} = -\frac{\partial \pi_b^*(B,Z)/\partial s}{\partial \pi_b^*(B,Z)/\partial B} > 0\,,\ \frac{dB}{ds}\Big|_{\dot{Z}=0} = -\frac{\partial \pi_z^*(B,Z)/\partial s}{\partial \pi_z^*(B,Z)/\partial B} = 0\,,$$

$$(17)\qquad \frac{dB}{dt}\Big|_{\dot{B}=0} = -\frac{\partial \pi_b^*(B,Z)/\partial t}{\partial \pi_b^*(B,Z)/\partial B} = 0\,,\ \frac{dB}{dt}\Big|_{\dot{Z}=0} = -\frac{\partial \pi_b^*(B,Z)/\partial t}{\partial \pi_z^*(B,Z)/\partial B} < 0\,,$$

[10] The shape of the saddle path depends on speed of adjustment coefficients specified in (7).

$$(18)\qquad \frac{dB}{dv}\Big|_{\dot{B}=0} = -\frac{\partial \pi_b^*(B,Z)/\partial v}{\partial \pi_b^*(B,Z)/\partial B} < 0,\ \frac{dB}{dv}\Big|_{\dot{Z}=0} = -\frac{\partial \pi_b^*(B,Z)/\partial v}{\partial \pi_z^*(B,Z)/\partial B} > 0$$

where $\begin{bmatrix} \partial \pi_b^*(B,Z)/\partial s & \partial \pi_b^*(B,Z)/\partial t & \partial \pi_b^*(B,Z)/\partial v \\ \partial \pi_z^*(B,Z)/\partial s & \partial \pi_z^*(B,Z)/\partial t & \partial \pi_z^*(B,Z)/\partial v \end{bmatrix} = \begin{bmatrix} \pi_{bs} & \pi_{bt} & \pi_{bv} \\ \pi_{zs} & \pi_{zt} & \pi_{zv} \end{bmatrix} = \begin{bmatrix} 1 & 0 & p_b \\ 0 & 1 & p_z \end{bmatrix}.$

First consider the impact of a biosecurity subsidy. The comparative statics results in (16) indicate that an increase in the biosecurity subsidy shifts the $\dot{B} = 0$ isocline upward, while the $\dot{Z} = 0$ isocline remains unchanged. As illustrated in Fig. 6, a sufficient subsidy results in a unique and stable SNE (point $E_6$) involving more biosecurity and more safe imports.[11] The intuition behind this result is that the subsidy increases producer's incentives to invest in biosecurity (shifting the $\dot{B} = 0$ isocline upward), so that producers no longer expect others to choose a low level of biosecurity. Moreover, the resulting increase in *B* causes *Z* to increase because *z* and *B* are complements. Hence, the biosecurity subsidy encourages producers to move away from a Pareto dominated SNE such as $E_5$ in Fig. 5, to the unique SNE $E_6$. Moreover, Table 3 indicates that $E_6$ increases social welfare relative to the Pareto dominant SNE from the pre-subsidy case ($E_3$ in Fig. 5) due to the reduction in risk under the subsidy.

Now consider the impact of a tax on risky imports, *t*. The comparative statics results in (17) indicate an increase in *t* shifts the $\dot{Z} = 0$ isocline downward, while the $\dot{B} = 0$ isocline remains unchanged. As illustrated in Fig. 7, a sufficient import tax results in a unique and stable SNE (point $E_7$) involving completely safe imports ($Z = 1$) but zero biosecurity. Intuitively, the tax reduces the incentives for risky imports. As *Z* is increased to one, the exposure to spread risks vanishes and so biosecurity becomes unnecessary. Table 3 indicates that the SNE $E_7$ increases social welfare relative to the Pareto dominant SNE from the pre-tax case ($E_3$ in Fig. 5) due to the elimination of risk under the tax.

[11] We represent the post-policy isoclines by a dashed line and label with a prime.

Finally, consider the impact of an indemnity payment. The comparative statics results in (18) indicate an increase in the indemnity payment $v$ shifts the $\dot{B}=0$ isocline downward, but it shifts the $\dot{Z}=0$ isocline upward. As illustrated in Fig. 8, an indemnity payment results in a unique and stable SNE (point $E_8$) involving zero safe imports and low biosecurity. Producers receive larger (smaller) expected profits at point $E_8$, as compared to the pre-indemnity high-risk SNE $E_5$ (low-risk SNE $E_3$) in Fig. 5, due to the payment; however, social welfare is lower compared to $E_3$ or even $E_5$ (see Table 3). The indemnity payment leads to a suboptimal outcome because the payment reduces the potential losses from being infected (the potential benefits from being healthy); hence, it reduces incentives to invest in import protection and/or biosecurity to prevent these losses. Similar results have been shown in prior works (Gramig et al 2009, and Reeling and Horan 2015).

## 5. Conclusion

One of the main challenges in livestock production is to manage disease risks. Producers can implement preventive measures to reduce the likelihood of becoming infected; however, such efforts exhibit positive spillover effects that generate strategic behaviors. In this setting, prior work has shown that multiple equilibria could arise, particularly when one's risk management choices and neighbors' choices are strategic complements. We expand on prior work to examine how the availability of multiple risk management choices, as well as the role of market price responses, may affect the uniqueness and stability of strategic outcomes.

We find that modeling multiple choices is important because the presence of these choices can significantly alter the predicted strategic interactions and outcomes; for instance, we can identify conditions under which having multiple choices is more likely to yield unique and stable Nash equilibria. This knowledge is particularly useful when constructing simulation models for policy purposes. Moreover, modeling multiple choices is important when considering policy design. This is because we can use policies to target the different choices to more efficiently manage infection risks as well as to better manage the different strategic interactions, thereby reducing the risk of coordination failure and improving social welfare. Similarly, our results show the importance of considering the role of market price responses, which can also facilitate risk management to promote stability and uniqueness.

A caveat to our model is that we assume producers know the probability of importing infected animals of each source. In practice, however, producers might have partial knowledge about the import risks they face. Therefore, improving market information about animal health risks, e.g. via an animal tracking system, would further enhance the efficiency of markets as a vehicle to managing disease risks.

## Appendix A

We claim that the uniqueness condition under the univalence approach is a special case of the index theorem approach, which means we can focus our attention on the index theorem approach. The following is a proof of this assertion.

The univalence approach states that if **G** is negative quasi-definite matrix, then the solution to (5) is unique where **G** is negative quasi-definite if and only if $\mathbf{G}+\mathbf{G}^{\mathbf{T}}$ is negative definite. Note that this condition has to hold for all *b*=*B* and *z*=*Z*. As

$$\mathbf{G}=\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \text{ so } \mathbf{G}+\mathbf{G}^{\mathbf{T}}=\begin{bmatrix} 2G_{11} & G_{12}+G_{21} \\ G_{12}+G_{21} & 2G_{22} \end{bmatrix}.$$

$\mathbf{G}+\mathbf{G}^{\mathbf{T}}$ is negative definite if and only if

$$G_{11}<0, G_{22}<0, \text{ and}$$

$$\begin{aligned} 4G_{11}G_{22} &> (G_{12}+G_{21})^2 \\ 4G_{11}G_{22} &> G_{12}^2+2G_{12}G_{21}+G_{21}^2 \\ 4(G_{11}G_{22}-G_{12}G_{21}) &> G_{12}^2-2G_{12}G_{21}+G_{21}^2 \\ G_{11}G_{22}-G_{12}G_{21} &> (\frac{G_{12}-G_{21}}{2})^2>0 \end{aligned}$$

Recall the uniqueness condition implied by the index theorem approach states that if $|-\mathbf{G}|>0$ whenever $\nabla\boldsymbol{\pi}(B,Z,B,Z)=0$, then the SNE is unique. $|-\mathbf{G}|=G_{11}G_{22}-G_{12}G_{21}>0$. Therefore, the sufficient condition implied by the univalence approach is stricter than the one implied by index theorem.

## Appendix B

We demonstrate our findings in section 4 with two numerical examples. The first numerical example illustrates the importance of multiple choices and market effects. The second numerical example illustrates government policy impacts when multiple SNEs arise prior to government intervention. We use the same functional forms for both examples; the only difference is with the parameter values. The relevant functional forms are specified as:

(B1) $$p^m(z;\bar{\theta},\underline{\theta},\kappa_m) = \frac{\bar{\theta}-(\bar{\theta}-(1+\kappa_m)\underline{\theta})z}{1+\kappa_m z},$$

(B2) $$p^c(b,B;\lambda,\kappa_c,\mu) = \frac{\lambda}{1+\kappa_c(\mu b+(1-\mu)B)},$$

(B3) $$C(b,\beta) = \beta b^2,$$

(B4) $$w^1(Z;\omega_{11},\omega_{12}) = \omega_{11}Z+\omega_{12}, \text{ and}$$

(B5) $$w^2(Z;\omega_{21},\omega_{22}) = \omega_{21}Z+\omega_{22}.$$

The probability functions $p^m(.)$ and $p^c(.)$ were specified to be decreasing in the choices, with diminishing returns, and to ensure the probabilities are bounded between 0 and 1. Specifically, $p^m(.)$ can be parameterized with three parameters: $\bar{\theta}$ ($\underline{\theta}$) is the probability of infection when the producer only imports from risky (low-risk) region, and $\kappa_m$ represents the effectiveness of $z$ where the higher $\kappa_m$ implies the higher effectiveness of $z$.[12] In contrast to $p^m(.)$, we let $p^c(.)$ approach to 0 as either $b$ or $B$ goes to infinity. However, we specify the biosecurity cost function to ensure the optimal biosecurity does not exceed 1. We use three parameters to represent important features of $p^c(.)$: (i) $\lambda$ is the probability of being infection when there is no use of biosecurity actions, (ii) $\kappa_c$ represents the effectiveness of biosecurity, where a larger $\kappa_c$ implies

[12] When $\underline{\theta}=0$, it implies that the low risk region has absolutely no infection risk.

greater effectiveness of *b*, and (iii) $\mu$ represents the relative influence of *b* and *B* on controlling $p^c(.)$ where $0 \le \mu \le 1$ and the larger is $\mu$, the greater the weight put on *b*. The cost of biosecurity takes a quadratic form as shown in equation (B3) and the inverse supply functions for both regions are assumed to be linear as in (B4) and (B5). The parameters of example 1 and 2 are specified as follows

| | $\overline{\theta}$ | $\underline{\theta}$ | $\kappa_m$ | $\kappa_c$ | $\lambda$ | $\mu$ | $\beta$ | $\omega_{11}$ | $\omega_{12}$ | $\omega_{21}$ | $\omega_{22}$ | $R$ | $N$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex.1 | 0.2 | 0 | 0.70 | 1 | 0.13 | 0.5 | 0.150 | 0.05 | 0.11 | 0.01 | 0.02 | 1 | 50 |
| Ex.2 | 0.2 | 0 | 0.45 | 1 | 0.13 | 0.5 | 0.125 | 0.01 | 0.14 | 0.01 | 0.01 | 1 | 50 |

**Figures**

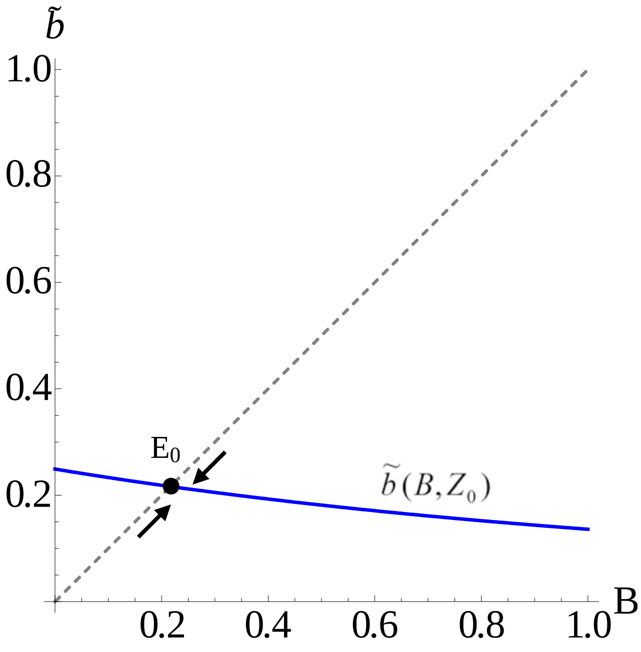


Figure 1: The single-choice model
when $x=b$, $X=B$, and $z=Z=Z_0$

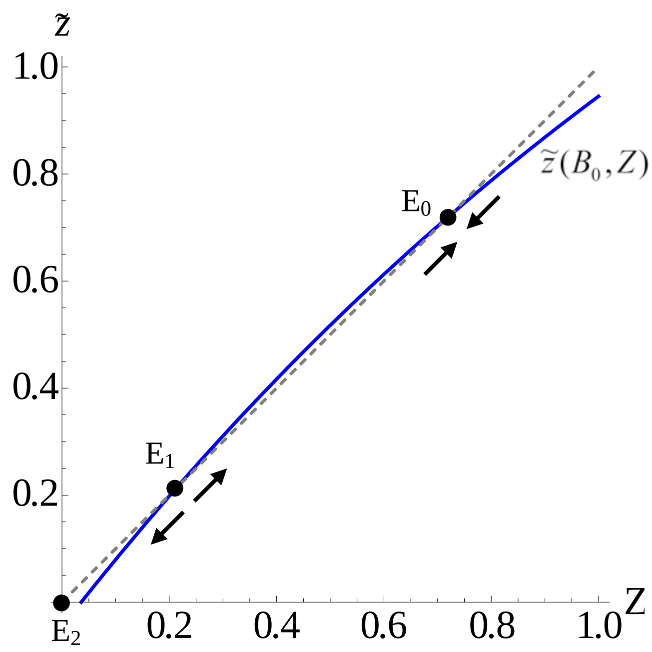


Figure 2: The single-choice model
when $x=z$, $X=Z$, and $b=B=B_0$

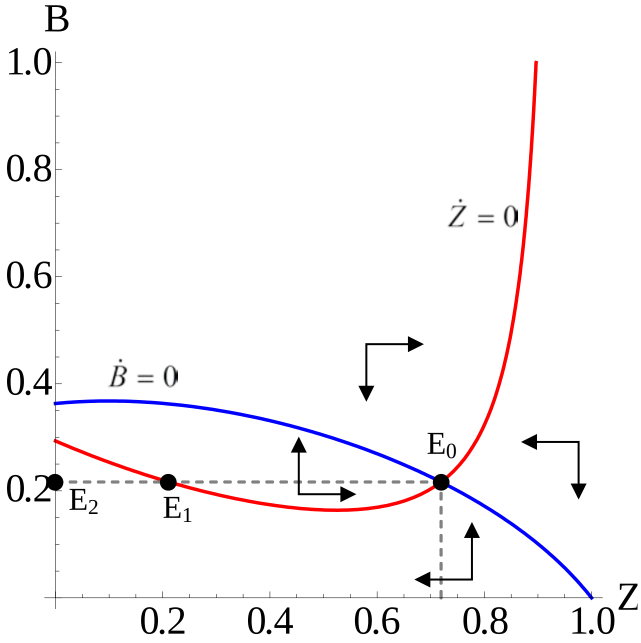


Figure 3: The two-choice model
without market effects

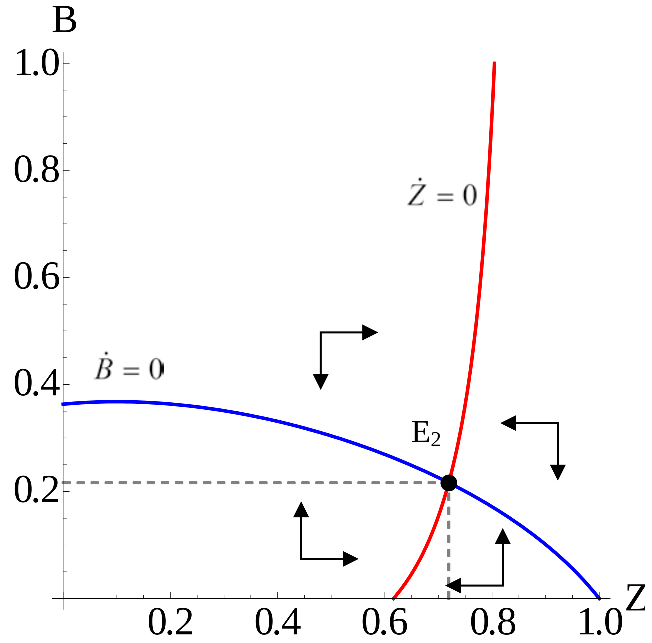


Figure 4: The two-choice model
with market effects

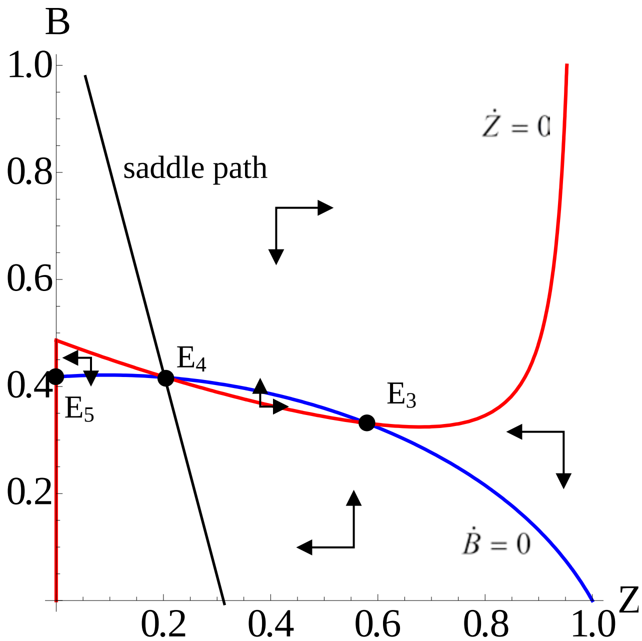


Figure 5: The two-choice model
without government policies

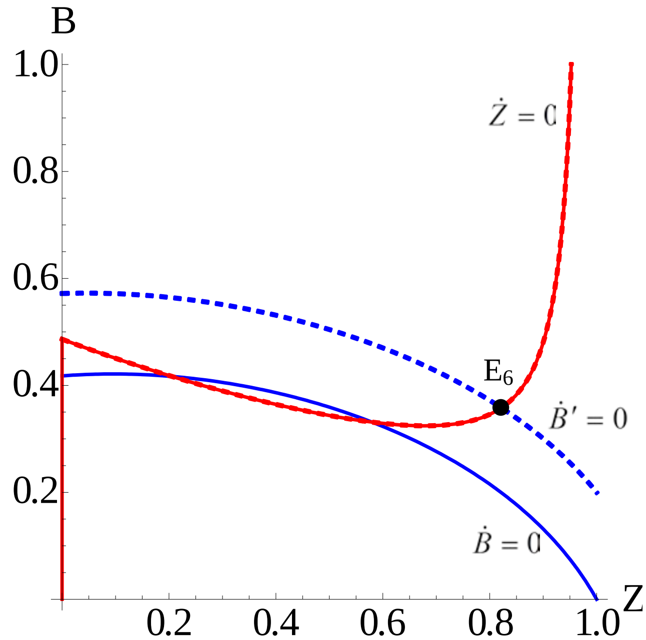


Figure 6: The two-choice model
with biosecurity subsidy

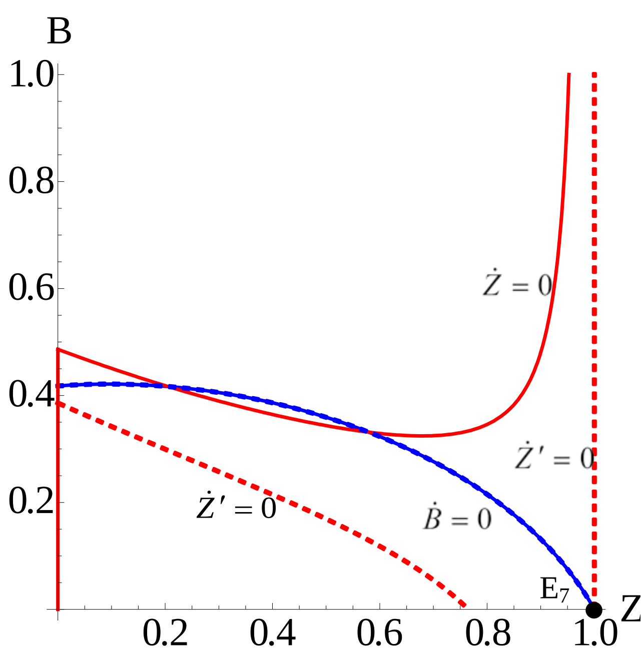


Figure 7: The two-choice model
with import tax

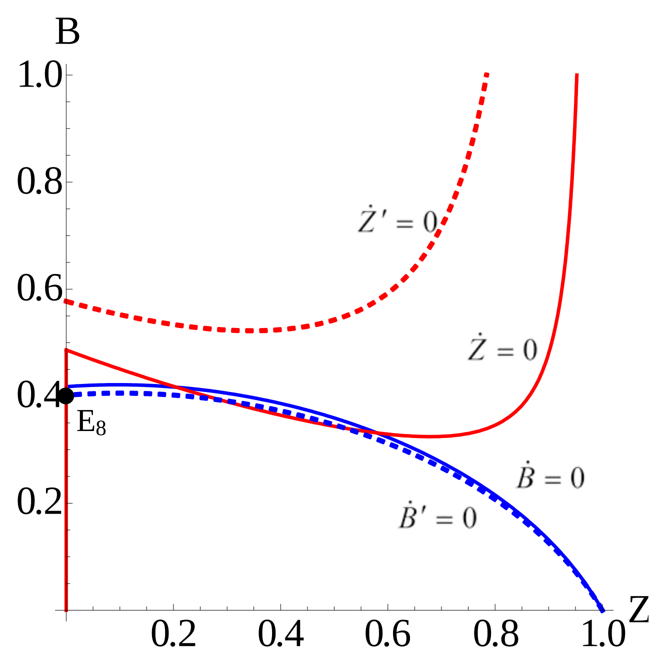


Figure 8: The two-choice model
with indemnity

**Tables**

Table 1: Table of second derivatives of the total probabilistic function of infection

| $p_{nm}$ | Specification | Sign | Remark[a,b] |
|---|---|---|---|
| $p_{bb}$ | $N(1-p^m)p^M(1-p^c p^M)^{N-2}$ $\left[(1-p^c p^M)p^c_{bb}-(N-1)p^M(p^c_b)^2\right]$ | ? | $p_{bb}>0$ if and only if $Np^c p^M < 1+(1-p^c p^M)(\Omega_b-1)$ |
| $p_{bz}$ | $-Np^M p^m_z p^c_b(1-p^c p^M)^{N-1}$ | – | |
| $p_{zz}$ | $(1-p^c p^M)^N p^m_{zz}$ | + | |
| $p_{bB}$ | $N(1-p^m)p^M(1-p^c p^M)^{N-2}$ $\left[(1-p^c p^M)p^c_{bB}-(N-1)p^M p^c_b p^c_B\right]$ | ? | $p_{bB}>0$ if and only if $Np^c p^M < 1+(1-p^c p^M)(\Omega_B-1)$ |
| $p_{bZ}$ | $N(1-p^m)p^M_Z p^c_b(1-p^c p^M)^{N-2}\left[1-Np^c p^M\right]$ | ? | $p_{bZ}>0$ if and only if $Np^c p^M < 1$ |
| $p_{zB}$ | $-Np^M p^m_z p^c_B(1-p^c p^M)^{N-1}$ | – | |
| $p_{zZ}$ | $-Np^c p^m_z p^M_Z(1-p^c p^M)^{N-1}$ | – | |

[a] $\Omega_b = \varepsilon_{p^c_b b}/\varepsilon_{p^c b}$, $\Omega_B = \varepsilon_{p^c_B b}/\varepsilon_{p^c b}$ . $\varepsilon_{p^c_b b} = (\partial p^c_b/\partial b)b/p^c_b$ is the elasticity of $p^c_b$ with respect to $b$ (with similar interpretations for the other elasticities).

[b] $Np^c p^M$ represents the expected number of effective contacts with infected neighbors. Note that $b$ will be technical substitute (complement) to $Z$ when $Np^c p^M < 1$ ($> 1$). A similar argument applies to the technical relationship between $b$ and $B$; however, it also depends on $\Omega_B$.

Table 2: Table of second derivatives of the expected profit function

| $\pi_{nm}$ | Specification | Sign | Remark[a] |
|---|---|---|---|
| $\pi_{bb}$ | $-(R-v)p_{bb} - C''$ | – | Assume $\pi_{bb} < 0$ due to concavity of profit function. Note that if $p_{bb} < 0$, then $\pi_{bb} < 0$ |
| $\pi_{bz}$ | $-(R-v)p_{bz}$ | + | |
| $\pi_{zz}$ | $-(R-v)p_{zz}$ | – | |
| $\pi_{bB}$ | $-(R-v)p_{bB}$ | – | Following Hennesy (2007), we assume $\pi_{bB} < 0$. |
| $\pi_{bZ}$ | $-(R-v)p_{bZ}$ | ? | $\pi_{bZ} < 0$ if and only if $p_{bZ} > 0$ |
| $\pi_{zB}$ | $-(R-v)p_{zB}$ | + | |
| $\pi_{zZ}$ | $-(R-v)p_{zZ} - W'$ | + with $W' = 0$<br>? with $W' > 0$ | $\pi_{zZ} > 0$ if and only if $W' < -(R-v)p_{zZ}$ |

[a] Note that when $Np^c p^M$ is large, it could create a non-convexities issue since $p_{bb}$ <0. In this paper, we assume convexity; however, this assumption is not too restrictive because multiple SNE are still possible.

Table 3: Outcomes of second numerical example with and without government policies

| | Without government policies | | | With government policies | | |
|---|---|---|---|---|---|---|
| | E3 | E4 | E5 | E6 ($s$ =.05) | E7 ($t$ = .0075) | E8 ($v$ = .025) |
| $B^*$ | 0.331 | 0.417 | 0.418 | 0.358 | 0.0 | 0.402 |
| $Z^*$ | 0.580 | 0.205 | 0.0 | 0.822 | 1.0 | 0.0 |
| Prob of infection | 0.327 | 0.564 | 0.683 | 0.140 | 0.0 | 0.686 |
| Expected profit | 0.569 | 0.371 | 0.275 | 0.738 | 0.858 | 0.308 |
| Expected government cost (+)/ revenue (-) | 0.0 | 0.0 | 0.0 | .018 | 0.0 | 0.034 |
| Social welfare[a] | 0.569 | 0.371 | 0.275 | 0.720 | 0.858 | 0.274 |

[a]Social welfare is expected producer's profit less expected government cost.

**References**

Cachon, Gerard P., and Serguei Netessine. 2004. "Game theory in supply chain analysis." In *Handbook of Quantitative Supply Chain Analysis*, pp. 13-65.

Conrad, Jon M., and Colin Whitcomb Clark. 1988. "Natural resource economics." *Cambridge Books*.

Dixit, Avinash. 1986. "Comparative statics for oligopoly." *International Economic Review*: 107-122.

Gramig, Benjamin M., Richard D. Horan, and Christopher A. Wolf. "Livestock disease indemnity design when moral hazard is followed by adverse selection." *American Journal of Agricultural Economics* 91, no. 3 (2009): 627-641.

Hefti, Andreas M. 2013. "Uniqueness and stability in symmetric games: Theory and Applications."

Hennessy, David A., Jutta Roosen, and Helen H. Jensen. 2005. "Infectious Disease, Productivity, and Scale in Open and Closed Animal Production Systems." *American Journal of Agricultural Economics* 87 (4): 900-917.

Hennessy, David A. 2007. "Biosecurity and spread of an infectious animal disease." *American Journal of Agricultural Economics* 89 (5): 1226-1231.

Hennessy, David A. 2008. "Biosecurity Incentives, Network Effects, and Entry of a Rapidly Spreading Pest." *Ecological Economics* 68 (1-2): 230-239.

Krugman, Paul. "History Versus Expectations." *The Quarterly Journal of Economics* 106, no. 2 (1991): 651-667.

Rich, Karl M. and Alex Winter-Nelson. 2007. "An Integrated Epidemiological-Economic Analysis of Foot and Mouth Disease: Applications to the Southern Cone of South

America." *American Journal of Agricultural Economics* 89 (3): 682-697.

Reeling, Carson J., and Richard D. Horan. 2015. "Self-protection, Strategic Interactions, and Relative Endogeneity of Disease Risks." *American Journal of Agricultural Economics 97(1).*

Vives, Xavier. 1999. "Oligopoly Pricing: Old Ideas and New Tools". MIT Press.

Vives, Xavier. 2005. "Complementarities and Games: New Developments." *Journal of Economic Literature* 43(2):437–479.