A common factor of stochastic volatilities between oil and commodity prices

Lee, Eunhee[1], Doo Bong Han[1], Shoichi Ito[2] and Rodolfo M. Nayga, Jr. [3]

[1]Department of Food and Resource Economics, Korea University,

[2]Faculty of Agriculture, Kyushu University

[3]Department of Agricultural Economics and Agribusiness, University of Arkansas and Department of Food and Resource Economics, Korea University and Norwegian Institute for Bioeconomy Research

Authors Emails: leeeunhee@korea.ac.kr, han@korea.ac.kr, sito@agr.kyushu-u.ac.jp, rnayga@uark.edu

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Preliminary and Incomplete

Abstract

This paper analyzes the multivariate stochastic volatilities with a common factor which is affecting both the volatilities of crude oil and agricultural commodity prices in both biofuel and non-biofuel use. We develop a stochastic volatility model which has a latent common volatility with two asymptotic regimes and a smooth transition between them. In contrast with conventional volatility models, stochastic volatilities in this study are generated by a logistic transformation of the latent factors, which consists of two components: the common volatility factor and the idiosyncratic component. In this study, we analyze the stochastic volatility model with a common factor for oil, corn and wheat from August 8, 2005 to October 10, 2014 using a Markov-Chain-Monte-Carlo (MCMC) method and estimate the stochastic volatilities and also extract the common factor. Our results suggest that the volatility of oil and grain markets are very persistent since the common factor generating the stochastic volatilities of oil and commodity markets is highly persistent. In addition, the volatilities of oil prices are more affected by a common factor while the volatilities of corn are more determined by the idiosyncratic component.

Keywords: Stochastic Volatility Model, Regime Switching with Smooth Transitions, Common Latent Factor, MCMC, Gibbs sampling, Oil and Grain Prices

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[1] Department of Food and Resource Economics, Korea University, leeeunhee@korea.ac.kr

[2] Department of Food and Resource Economics, Korea University

[3] Kyushu University

[4] University of Arkansas, Korea University, and Norwegian Institute for Bioeconomy Research

1. Introduction

Commodity and oil prices have shown a common factor of stochastic volatilities since the middle of 2000s. Both higher crude oil prices and regulations on carbon dioxide emissions have turned biofuels as an alternative to fossil fuels. As the demand and prices for corn and soybean increased in 2007 due to increased demand for biofuels, grain prices (e.g., wheat) for food and fuel uses have also increased. Since an increasing percentage of corn and soybean production is being used for alternative energy sources, both the mean and variance of other commodity prices have also increased because of the conversion of planted acreages of non-energy grain to biofuel production. Several studies have empirically investigated the relationship between the price of crude oil and the global grain prices for corn, wheat, etc. However, most studies have focused on the interrelationship and transmission of conditional mean of price levels and less attention has been paid to the examination of the volatility linkage among energy and agricultural markets. Recent studies by Wu, Guan and Myers (2011), Du, Yu and Hayes (2011) and Harri and Darren (2009) have found significant volatility linkage between crude oil and grain prices. However, these studies did not analyze the common factor of energy and agricultural commodity volatilities. In particular, shocks in either oil or biofuel market may spill over into the other commodity markets, especially after the ethanol mandates in the US and EU in the mid-2000s. Therefore, the higher volatility in the energy or grain market may have contributed to the increase in volatility of the other market.

The purpose of this study, therefore, is to analyze the multivariate stochastic volatilities with a common factor which is affecting both the volatilities of crude oil and agricultural commodity prices in both biofuel and non-biofuel use. Recently, stochastic volatility (SV) models have drawn much more attention than ARCH and GARCH-type processes, which have been commonly used to model volatility in many economic and financial time series data. SV models are more acceptable and interesting after the global financial crisis in 2008 because volatility is not entirely predictable and can be driven by a shock not perfectly correlated with the past values of the underlying process. The main characteristic of the SV models is that the volatility is modelled as an unobserved latent variable. Compared with the GARCH modes, SV models have more attractive properties which were often observed in high-frequency series of asset returns (see, for example, Jacquier *et al.* (1994), Danielsson (1994), Carnero *et al*.(2003), Malmsten and Terasvirta (2010) and Terasvirta and Zhao (2011)). More importantly, SV models have a wide range of applications, including option and other derivative pricing because they provide greater flexibility in describing stylized facts about returns and volatilities (Shephard, 2005). However, their empirical applications have been very limited mainly because volatility in SV models is latent and relatively difficult to estimate. Therefore, there are theoretical as well as empirical reasons to study multivariate SV models.

In our study, we developed a trivariate SV model which has a latent common volatility with two asymptotic regimes and a smooth transition between them, as suggested by Kim *et al.* (2010). In contrast with conventional volatility models, stochastic volatilities in this study are generated by a logistic transformation of the latent factors, which consists of two components: the common volatility factor and the idiosyncratic component. The common volatility factor affects all the stochastic volatilities for oil and agricultural commodity prices and captures the common trend behaviors of the volatilities, while the idiosyncratic component characterizes the underlying process. The common factor could be either a macroeconomic fundamental or global uncertainty in commodity and energy markets. Therefore, this study will analyze the linkage between the common factor and economic fundamentals in commodity and energy markets. This is the main contribution of this study in contrast with the conventional stochastic volatility model. In particular, the common factor is assumed to be AR(1) process whereas the idiosyncratic factor is set to be *i.i.d*.

In addition, the actual volatilities in this study are generated by the parametric logistic function. The logistic function has several desirable properties to be used in the volatility model. It may be interpreted as representing the volatility levels in two asymptotic regimes, i.e., the low and high volatility regimes, with smooth transition with them. Our model is different from the usual regime switching model, which presumes an exogenous and abrupt change in switching regimes. Moreover, there are two transition parameters in the logistic function characterizing the transition between two regimes, i.e., the location and speed of the transition. The transition speed is allowed to be faster and the actual volatilities are generated by one of the two asymptotic regimes. We also allow for cross-dependency in the returns of corn and wheat.

To extract the common factor and estimate unknown parameters in our model, we use the Bayesian approach by implementing a Markov-Chain-Monte-Carlo (MCMC) method (see, for example Chib *et al*. (2002, 2006), Jacquier *et al*. (1994, 2004) and Kim *et al*. (1998)). For our MCMC procedure, we use the Gibbs sampler and the Metropolis-Hasting (MH) algorithm within the Gibbs sampler. The procedure allows us to effectively deal with the multi-dimensionality of our latent factors and parameters and the difficulties in sampling from the complicated target distributions.

In this study, we analyze the stochastic volatility model with a common factor for oil, corn and wheat from August 8, 2005 to October 10, 2014 by using the MCMC method and estimate the stochastic volatilities and also extract the common factor. Our results suggest that the low and high levels of stochastic volatilities for oil are 0.79% and 8.4% in a day while the low and high volatilities of the corn are 0.48% and 7.72%, respectively. In the wheat case, the low volatility level is 0.64% while the high volatility level is 5.41% in a day. Therefore, oil is more volatile than grains. The volatility generating process of oil is closer to the common factor in terms of the value of the loading parameter, which indicates that the volatility generating processes of oil, corn and wheat are scaled-down compared with the magnitude of the common factor, which is higher than other cases. Regarding the volatility of the idiosyncratic component, the variance of the idiosyncratic part for oil is higher than that of others. This implies that oil volatility is more affected by the common macroeconomic uncertainty factor while corn volatility is more explained by the idiosyncratic component. Based on the estimated common volatility factor, the high volatility periods in the common factor are well matched to the recession period from December 2007 to June 2009, as recorded by the National Bureau of Economic Research (NBER), and the financial crisis of 2007- 2008.

To the best of our knowledge, our study is the first to empirically examine the common factor in the volatilities of energy and agricultural commodity markets, especially oil, corn and wheat markets. We found that the main factors of common volatility extracted from oil, corn and wheat are related to the macroeconomic and financial uncertainties in the energy and grain market.

2. Preliminary Analysis

2.1 Data

Figure1 shows the log differences of oil, corn and wheat prices from January 2, 1986 to October 10, 2014. The movement of commodity returns, such as corn and wheat were independent of oil returns before the August 8, 2005 imposition of the first ethanol mandate. As can be seen from Figure 1, their movements look similar after August 8, 2005. Figure 2 clearly shows this phenomenon. Figure 2 indicates the absolute values of log differences for oil, corn and wheat. It is likely that they move together after the first ethanol mandate. Therefore, the impact of the ethanol mandates on oil and commodity markets may not be trivial. Recently, the dynamic interaction between grain and oil prices has been a subject under extensive study. In this vein, oil, corn and wheat daily prices from August 8, 2005 to October 10, 2014 are used in this paper.

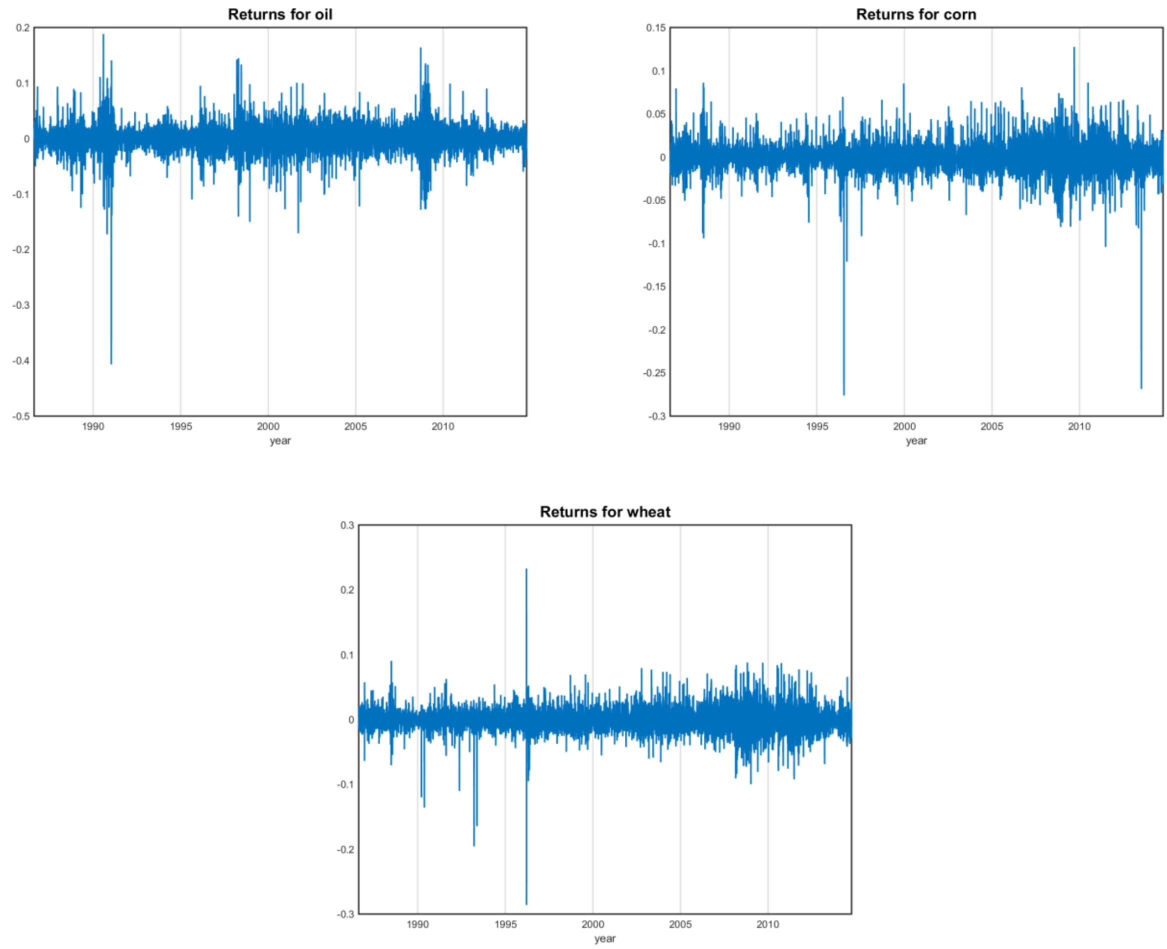
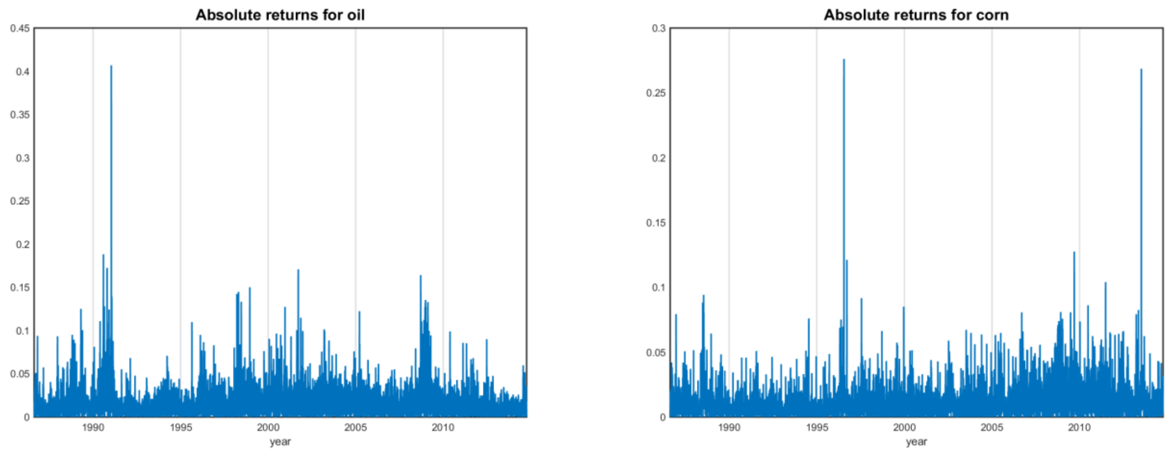


Figure 1 Returns for oil, corn and wheat



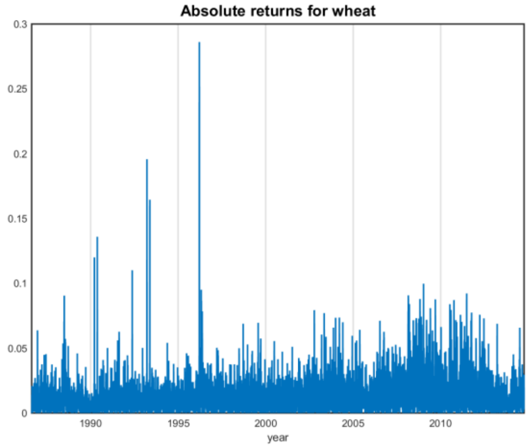


Figure 2 Absolute values of returns

Before we estimate our SV model, we need to empirically demonstrate that returns for oil and the commodity prices have a dominating common factor, and for this, we estimate a univariate stochastic volatility model for each of the returns. We then conduct a principal component (PC) analysis in order to check whether there is a leading common factor among the latent factors generating the stochastic volatilities for each of the returns. First, we introduce and estimate the univariate SV model for oil, corn and wheat returns and extract the latent factor, and then do the PC analysis for three latent volatility factors.

2.2 Principal Component Analysis

We consider the following stochastic volatility model introduced by Kim, Lee, and Park, (2008).

$$y_t = \sqrt{f_t(x_t)}\varepsilon_t , \qquad \varepsilon_t \sim iid(0,1) \qquad (1)$$

$$x_t = \alpha x_{t-1} + e_t , \qquad e_t \sim iid(0,1) \qquad (2)$$

where y_t is observable and demeaned return process and x_t is scalar latent volatility factor and $|\alpha| \leq 1$. The volatility function is given by the parametric logistic function[5]. The latent factor (x_t) and unknown parameters in the model can be estimated by the density-based Kalman filter, or by the Bayesian method using Gibbs sampling method. Figure 3 shows the extracted latent factors x_t for oil, corn and wheat. As can be seen from the picture, latent factors generating stochastic volatilities for oil, corn and wheat returns, have the common trend. Therefore in order to check whether there exists a dominating common factor among latent factors, we do the PC analysis with extracted latent factors and find the leading factor. The right picture of the bottom panel shows the leading factor with three estimated latent factors for oil, corn and wheat returns. Results indicate that the leading factor can explain around 75% of the variation of three latent factors generating the stochastic volatility for each of the returns. Hence, we have strong evidence that there exists a dominating common factor in the conditional variances of oil, corn and wheat return processes and this evidence supports our stochastic volatility mode with a common factor for oil and commodity returns, which is introduced in the next section.

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[5] More about the volatility function in next section

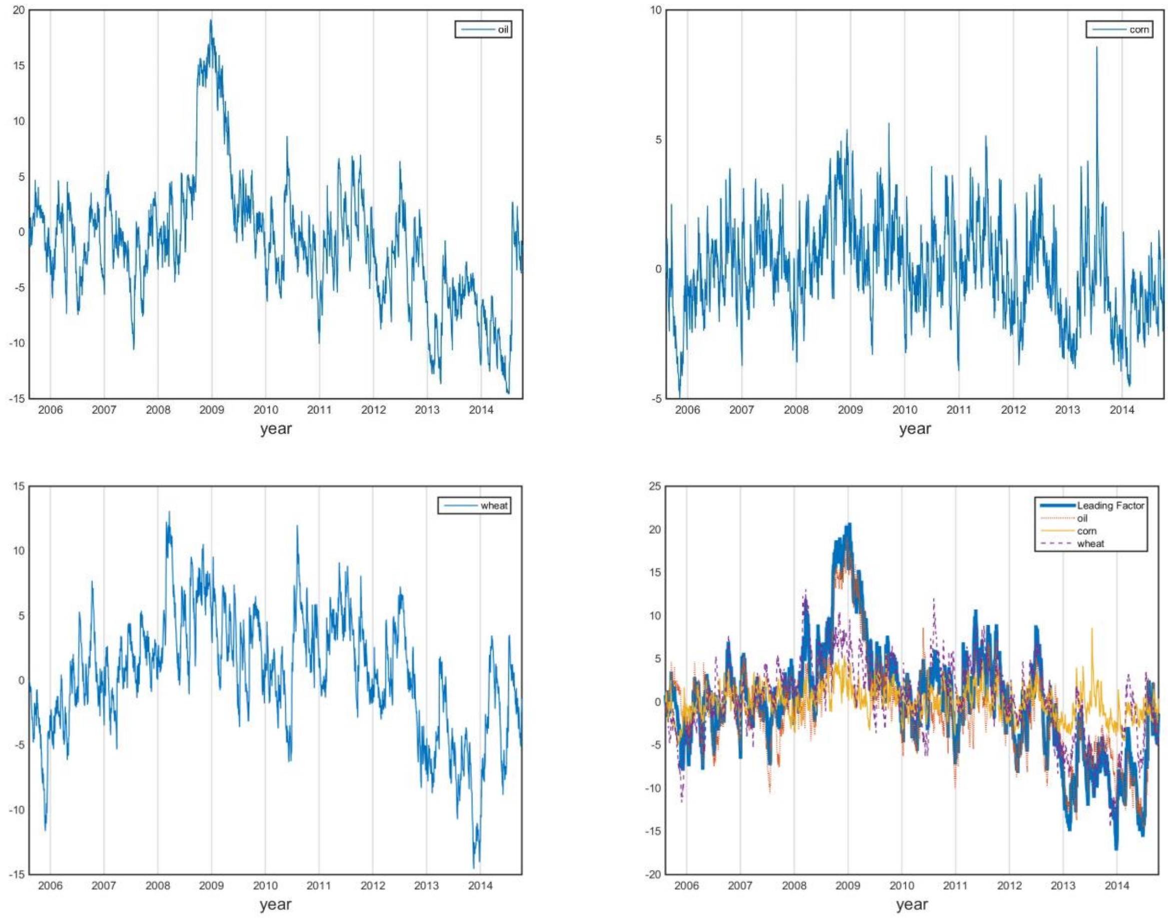

Figure 3 Estimated latent factors with a leading factor

3. Stochastic Volatility Model with a Common Factor

3.1 Model

In our study, we consider the multivariate SV model which has a latent common volatility with two asymptotic regimes and a smooth transition between them, as suggested by Kim *et al*. (2010). Let y_t^j the zero mean return process $for\ j = oil, corn\ and\ wheat\ or\ j = 1,2,3$.

The multivariate SV model with a common factor is specified as

$$y_t^j = \sqrt{f_t^j(x_t^j)}\varepsilon_t^j,\ \ \varepsilon_t = (\varepsilon_t^1, \varepsilon_t^2, \varepsilon_t^3)' \sim iid(0, \Sigma)$$

$$\text{where } \Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \rho \\ 0 & \rho & 1 \end{pmatrix}$$

$$x_t^j = \lambda_j\ w_t + e_t^j$$

$where \;\; e_t^j \sim iid(0, \sigma_j^2) \;\; for\, j$=1,..,N.

$w_t \;= \alpha\, w_{t-1} \; + u_t \quad where \;\; u_t \sim iid(0,1)$

While Σ describes the dependence in returns by the constant correlation coefficient (ρ), ε_t^j, e_t^j and u_t are independent. We only allow the dependence in returns between corn and wheat. Du, Yu, and Hayes (2011) estimated the correlation coefficient between oil and corn market and the correlation coefficient between oil and wheat with different periods. Interestingly, they found a significant correlation coefficient between corn and wheat market regardless of the sample period used. We also found a significant correlation between corn and wheat returns but insignificant correlation between oil/corn and oil/wheat. Hence, to simplify the model, we only allow the dependence in returns between corn and wheat

In addition, in our model we do not impose the leverage effect indicating the negative correlation between the return and the volatility. It is well known that volatility in equity markets is asymmetric. For instance, negative returns are associated with higher volatility than positive returns and a number of empirical studies for stock markets have found strong evidence of leverage effects. In many studies, however, in the oil or commodity markets, such as Schwartz and Trolle (2009), Larsson and Nossman (2011) and Vo (2009), the estimate of the correlation coefficient is negative but insignificant. It is often claimed that commodities exhibit an "inverse leverage effect" implying that increasing prices are associated with increasing volatility. The "inverse leverage effect" would be associated with a positive estimate of the correlation coefficient between the return and the volatility. Geman and Shih (2009) and Larsson and Nossman (2011) find the "inverse leverage effect" in crude oil prices. None of them however support any strong conclusions regarding the sign of this correlation. Therefore, we do not consider the leverage effect and so we specify a zero correlation between ε_t and e_t , the errors of the mean and variance equation in our model.

We assume that the latent factor ($x_t^j \; for\; j = oil, corn\; and\; wheat$), which generates the stochastic volatilities of oil, corn and wheat, have two components, a common volatility factor (w_t) and idiosyncratic components (e_t^j). This assumption for our model is simple but very clear and well demonstrated by our preliminary analysis. The common volatility factor affects all the stochastic volatilities for oil and agricultural commodity prices and captures the common trend behaviors of the volatilities for oil and commodity returns, while the idiosyncratic component characterizes the underlying process specific information. The common factor could be either a macroeconomic fundamental or global uncertainty in commodity and energy markets. Therefore, this will analyze the linkage between the common factor and economic fundamentals in commodity and energy markets. This is the main contribution of this study in contrast with the conventional stochastic volatility model. In particular, the common factor is assumed to be AR(1) process whereas the idiosyncratic factor is set to be *i.i.d*. When AR(1) coefficient of the common factor, $\alpha \approx 1$ is obtained, the return process can be considered to have persistence so that it can generate highly autocorrelated volatility or volatility clustering, which is the well-known stylized fact about returns; meaning that high volatility is followed by another high volatility, or the other way around.

The actual volatilities in this study are generated by the parametric logistic function, which is given by

$$f(x_t^j) = \mu_j \frac{\beta_j}{1 + exp(-(x_t^j - \kappa_j))}$$

where $\mu_j > 0$, $\beta_j > 0$, and $\kappa_j > 0$

According to Park (2002), the stochastic volatility with logistic function can capture the volatility clustering and fat-tail features[6]. The logistic function has several desirable properties to be used in the volatility model. It may be interpreted as representing the volatility levels in two asymptotic regimes, i.e., the low and high volatility regimes, with smooth transition with them. Namely, the parameters μ and $\mu + \beta$ dictate the asymptotic low and asymptotic high volatility regimes, respectively and the parameter κ specifies the transition between two regimes, i.e., the reflection point of the transition. This model is different from the usual regime switching model, which presumes an exogenous and abrupt change in the switching regimes.

To estimate the time-varying conditional mean component of the returns process for oil, corn and wheat, we use the local linear estimation. Therefore, the estimated y_t^j can be obtained by subtracting the estimated moving conditional mean component by using nonparametric method from log difference sequences of oil, corn and wheat prices. When doing nonparametric method, we search bandwidth such that y_t^j can be a martingale difference sequence, which means the autocorrelation of y_t^j can be close to zero.

3.2 Bayesian Algorithm

In the paper, we use the Bayesian approach to estimate our model.

Let T and N be the sample size and the number of individual units respectively, and define the observed samples, $Y = (y_1, \ldots\ldots, y_T)$ with $y_t = (y_t^1, y_t^2, y_t^3)'$ and the latent factors, $L = (X, W)$ with $X = (x_1, \ldots, x_T)$, $x_t = (x_t^1, x_t^2, x_t^3)$ and $W = (w_1, \ldots, w_T)$. And we define $\theta_j = (\mu_j, \beta_j, \kappa_j)$ for j=1,2,3. Moreover, we denote unknown parameters as $\Psi = (\theta, \alpha, \lambda, \sigma^2, \rho)$ with $\theta = (\theta_1, \theta_2, \theta_3)$, $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, and $\sigma = (\sigma_1^2, \sigma_2^2, \sigma_3^2)$. We let $D_t = (\text{diag} \sqrt{f_1(x_t^1)}, \sqrt{f_2(x_t^2)}, \sqrt{f_3(x_t^3)})$.

Now, we may easily deduce that the joint posterior density of the latent factors and unknown parameters is given by

$$p(L, \Psi|Y) \propto P(L, Y|\Psi)p(\Psi)$$

$$\propto \left(\prod_{t=1}^{T} p(y_t|x_t, \Psi)\, p(x_t^1|w_t, \Psi)\, p(x_t^2|w_t, \Psi)\, p(x_t^3|w_t, \Psi)\, p(w_t|w_{t-1}, \Psi)\right) p(\Psi)$$

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[6] Park (2002) shows that the model with asymptotically homogeneous functions of an integrated process has several nice statistical properties. First, the sample autocorrelations of the squared processes have the same random limit for all lags i.e. strong persistence. Secondly, the sample kurtosis has supports truncated on the left by the kurtosis of the innovations i.e., leptokurtosis. Since the logistic function belongs to the class of asymptotically homogeneous function, this model can capture the volatility clustering and fat-tail features of financial and economic time series.

where

$$p(y_t|x_t,\Psi)=\frac{1}{(\sqrt{2\pi})}det(D_t\Sigma D_t)^{\frac{1}{2}}\exp\left(-\frac{(y_t'\,(D_t\Sigma D_t)^{-1}\,y_t)}{2}\right)$$

$$p\left(x_t^j\middle|w_t,\Psi\right)=\frac{1}{\sqrt{2\pi\sigma_j^2}}\,exp\,\left(-\frac{(x_t^j-\lambda_j w_t)^2}{2\sigma_j^2}\right)$$

$$p(w_t|w_{t-1},\Psi)=\frac{1}{\sqrt{2\pi}}\,exp\,\left(-\frac{(w_t\ -\alpha w_{t-1})^2}{2}\right)$$

Priors are assumed to be independent and we employ the convenient conjugate and proper priors for parameters.

$$p(\alpha)\sim Beta$$

$$p(\mu)\sim Gamma$$

$$p(\beta)\sim Gamma$$

$$p(\kappa)\sim Normal$$

$$p(\lambda)\sim Truncated\ Normal$$

$$p\left(\sigma^2\right)\sim Inverse\ Gamma$$

$$p(\rho)\sim Truncated\ Normal$$

Following the usual Bayesian procedure, we will implement a MCMC method to sample $p(L,\Psi)$ from the joint posterior density $p(L,\Psi|Y)$. For our MCMC procedure, we use the Gibbs sampler and the Metropolis-Hastings (MH) algorithm within the Gibbs sampler.

Now we derive the conditional posteriors for latent factors and unknown parameters to implement the Gibbs sampler and MH algorithm. First, the conditional posterior distribution for the common latent factor w_t is given by

$$p(w_t|X,W_{-t},\Psi)\ \propto\ \prod_{j=1}p(x_t^j|W,\Psi)p(w_{t+1}|w_t)p(w_t|w_{t-1})$$

$$\propto\ \sim N(BA^{-1},A^{-1})$$

$$where\ A=\sum_j\frac{\lambda_j^2}{\sigma_j^2}+\alpha^2+1,\quad B=\alpha(w_{t+1}+w_{t-1})+\sum_j\frac{\lambda_j x_t^j}{\sigma_j^2}$$

where W_{-t} denotes W with w_t deleted.

The conditional posterior for x_t^j is also derived and it is given by

$$p\left(x_t^j\middle|X_{-t},W_t,\Psi\right)\propto\ p(y_t|x_t,\Psi)p\left(x_t^j\middle|\,w_t,\Psi\right)$$

$$\propto det(D_t \Sigma D_t)^{\frac{1}{2}} \exp\left(-\frac{(y_t' \, (D_t \Sigma D_t)^{-1} \, y_t)}{2}\right) \frac{1}{\sqrt{2\pi\sigma_j^2}} \, exp\left(-\frac{(x_t^j - \lambda_j w_t)^2}{2\sigma_j^2}\right)$$

The conditional posterior for parameters, α, λ_j $and\ \sigma_j^2$ are also given by

$$p(\alpha|X, W, \Psi_{-\alpha}) \propto \prod_t p(w_t|w_{t-1}) p(\alpha)$$

$$Let\ \alpha = 2\alpha^* - 1 \quad where\ \ p(\alpha^*) \sim B(\overline{\alpha_1}, \overline{\alpha_2})$$

$$Then, \qquad p(\alpha) \propto \left(\frac{1+\alpha}{2}\right)^{\overline{\alpha_1}-1} \left(\frac{1-\alpha}{2}\right)^{\overline{\alpha_2}-1}$$

$$\propto \sim N(DC^{-1}, C^{-1}) \, p(\alpha) \qquad where\ C = \sum_t w_{t-1}^2 \ , \ \ D = \sum_t w_t w_{t-1}$$

$$p\left(\lambda_j \middle| X, W, \Psi_{-\lambda_j}\right) \propto \prod_t p(x_t^j \,|\, w_t, \Psi) p(\lambda_j)$$

$$= \prod_t \frac{1}{\sqrt{2\pi\sigma_j^2}} \, exp\left(-\frac{(x_t^j - \lambda_j w_t)^2}{2\sigma_j^2}\right) p(\lambda_j) \qquad where\ \ p(\lambda_j) \sim N(\overline{\lambda_j}, \overline{\lambda_j^2}) \mathrm{I}_{[0<\lambda_j]}$$

$$\propto \ \sim \ \mathrm{N}\,(\mathrm{KL}^{-1}, \mathrm{L}^{-1})\, \mathrm{I}_{[0<\lambda_j]}$$

$$where\ L = \frac{\sum_t w_t^2}{\sigma_j^2} + \frac{1}{\overline{\overline{\lambda_j^2}}} \quad , K = \frac{\sum_t w_t \, x_t^j}{\sigma_j^2} + \frac{\overline{\lambda_j}}{\overline{\overline{\lambda_j^2}}}$$

$$p\left(\sigma_j^2 \middle| X, W, \Psi_{-\sigma_j^2}\right) \propto \prod_t p(x_t^j \,|\, w_t, \Psi) p(\lambda_j)$$

$$= \prod_t \frac{1}{\sqrt{2\pi\sigma_j^2}} \, exp\left(-\frac{(x_t^j - \lambda_j w_t)^2}{2\sigma_j^2}\right) p(\sigma_j^2) \quad where\ \ p(\sigma_j^2) \sim IG(\overline{\nu_{J1}}, \overline{\nu_{J2}})$$

$$\propto \sim IG\left(\frac{n}{2} + \overline{\nu_{J1}}, W + \overline{\nu_{J2}}\right) \ where\ W = 1/2 \sum_t (x_t^j - \lambda_j w_t)^2$$

Finally, we derive the conditional posterior distribution of (θ, ρ). The conditional posterior distribution of $\theta_j = (\mu_j, \beta_j, \kappa_j)$ for *j=1,2,3* is

$$p\left(\theta_j | X, W, \Psi_{-\theta_j}\right) \propto \prod_t p(y_t|x_t, \Psi)\, p(\theta_j)$$

$$= \prod_t \frac{1}{\sqrt{2\pi}} det(D_t \Sigma D_t)^{\frac{1}{2}} \exp\left(-\frac{(y_t' \, (D_t \Sigma D_t)^{-1} \, y_t)}{2}\right) p(\theta_j)$$

$$where \;\; p(\mu_j) \sim G(\overline{\mu_{J1}}, \overline{\mu_{J2}}\,), \qquad p(\beta_j) \;\sim G(\overline{\beta_{J1}}, \overline{\beta_{J2}}\,), \qquad and\; p(\kappa_j) \sim N(\overline{\kappa_{J1}}, \overline{\kappa_{J2}})$$

The conditional posterior distribution of ρ is given by

$$p(\rho|X, W, \Psi_{-\rho}) \propto \prod_t p(y_t|x_t, \Psi)\, p(\rho)$$

$$where \;\; p(\rho) \sim N(\overline{\rho_1}, \overline{\rho_2}\,) I_{\{-1<\rho<1\}},$$

We apply the Metropolis-Hastings (MH) algorithm to draw sample from the conditional posterior distribution of (α, θ, ρ) and x_t. In the case of sampling from the conditional posterior of (θ, ρ), we use their prior distributions as the proposal densities when applying the MH algorithm. For the latent factors x_t, we use the transition equation of our model as the proposal density of x_t. For the candidate-generating density of α, $N(DC^{-1}, C^{-1}\,)$ is used in this paper.

4. Estimation Results

In this paper, oil, corn and wheat daily prices from August 8, 2005 to October 10, 2014 are used as mentioned before. We draw 200,000 samples for each parameter and latent variable using the Gibbs sampler and the MH algorithm, and discard the first 20,000 samples, which are considered as samples in the burn-in period. Table 1, 2 and 3 give the estimation results for oil, corn and wheat returns, respectively. The last column in Tables (1, 2, and 3) dictates the convergence diagnostics (CD) by Geweke (1992). As shown in Geweke (1992), CD converges to the standard normal distribution as the number of samples goes to infinity, if the sequence of Gibbs samples for a parameter is stationary. The last column in three tables (1, 2, and 3) dictates the convergence diagnostics (CD) by Geweke (1992). As shown in Geweke (1992), CD converges to the standard normal distribution as the number of samples goes to infinity, if the sequence of Gibbs samples for a parameter is stationary. Our results indicate relatively high convergence diagnostics, except for some of the parameter.

The AR coefficient of the common global factor is 0.986, which is persistent. It is well known that the return process has persistence so that it can generate highly autocorrelated volatility or volatility clustering.

Table 1 Estimation Results for Oil

	parameter	Posterior mean	Posterior Std	5%	95%	CD
oil	μ	0.000062	0.000018	0.000026	0.000099	0.1020
	β	0.0070	0.0015	0.0040	0.0100	-2.7234

	κ	3.4388	0.3275	2.7968	4.0807	-4.2082
	λ	0.1890	0.0314	0.1274	0.2506	0.2644
	σ	0.7847	0.1109	0.5674	1.0021	-0.6839
Common factor	α	0.9866	0.0049	0.9769	0.9962	-0.2094

Table 2 Estimation Results for Corn

	parameter	Posterior mean	Std	5%	95%	CD
corn	μ	0.000023	0.000011	0.000001	0.000045	-0.6767
	β	0.0059	0.0014	0.0033	0.0086	6.8366
	κ	3.0082	0.3420	2.3379	3.6785	2.5232
	λ	0.1176	0.0200	0.0784	0.1569	-0.6611
	σ	0.7217	0.0581	0.6079	0.8356	-0.7624
corn & wheat	ρ	0.6629	0.0121	0.6393	0.6866	-0.4598

Table 3 Estimation Results for Wheat

	parameter	Posterior mean	Std	5%	95%	CD
wheat	μ	0.000041	0.000020	0.000002	0.000079	1.0720
	β	0.0029	0.0011	0.0008	0.0050	-4.8949
	κ	1.9154	0.4390	1.0549	2.7759	-7.5918
	λ	0.1544	0.0346	0.0865	0.2223	2.8557
	σ	0.5750	0.0877	0.4031	0.7469	1.8102

The value of $\lambda_j$indicates that the volatility generating processes, x_t^j $for\ i = 1,2,3$ are scaled down compared with the magnitude of the macro global uncertainty w_t. The loading parameters, λ for oil, corn and wheat are 0.19, 0.12 and 0.15, respectively. This means that the stochastic volatilities of oil prices are more affected by the common factor than the other cases, while the stochastic volatilities of corm returns are more determined by the idiosyncratic component.

From the results of our analysis, the low level and high level of stochastic volatilities for oil are $\sqrt{\mu}$ =0.79% and $\sqrt{\mu+\beta}$ =8.4% in a day while the low and high volatilities of the corn are 0.48%

and 7.72%, respectively. In the wheat case, the low volatility level is 0.64% while the high volatility level is 5.41% in a day. Therefore, oil is more volatile than grains.

Table 4

	low asymptotic regime $\sqrt{\mu}$	high asymptotic regime $\sqrt{\mu+\beta}$
oil	0.0079	0.0840
corn	0.0048	0.0772
wheat	0.0064	0.0541

We also extract the latent common uncertainty process, w_t, and the volatility generating processes for the oil, corn and wheat growth rates, x_t. Figure 4 illustrates the extracted w_t and x_t for energy and the commodity prices. The dotted line displays x_t and the thick line displays w_t in Figure 4. As aforementioned, according to the values of λ^{oil}, λ^{corn} and λ^{wheat}, the volatility generating process x_t for oil is closer to the common macro volatility generating process, w_t than others. Based on the estimated common volatility factor, the high volatility periods in the common factor are well matched to the recession period from December 2007 to June 2009, as recorded by the National Bureau of Economic Research (NBER), and the financial crisis of 2007- 2008.

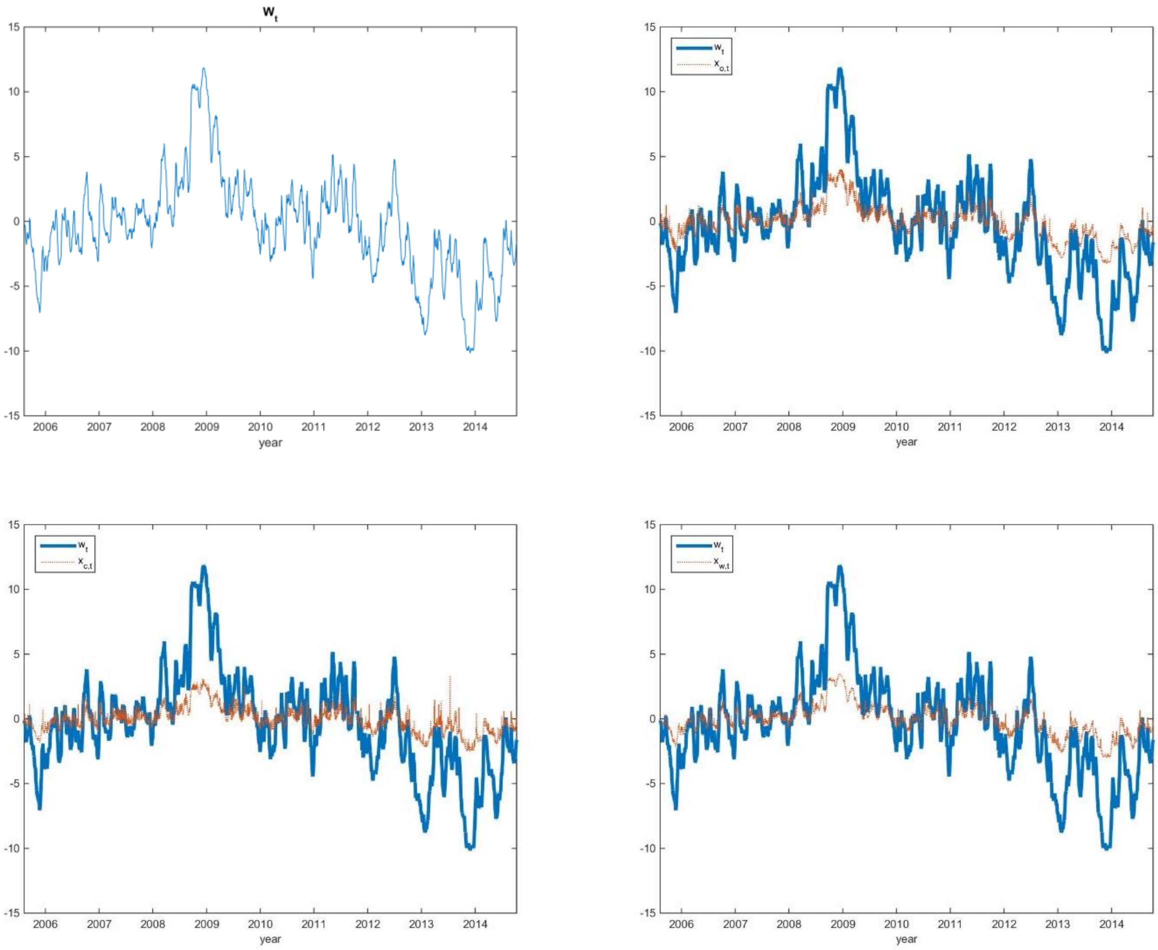

Figure 4 Common and volatility factors

Figures 5 plots the estimated volatility and the realized volatility for oil, corn and wheat, respectively. The dotted line and thick line display the absolute value of growth rates and the estimated volatilities, respectively. As can be seen from Figure 5, the estimated volatilities explain the realized volatilities quite well, particularly in light of their trend behaviors.

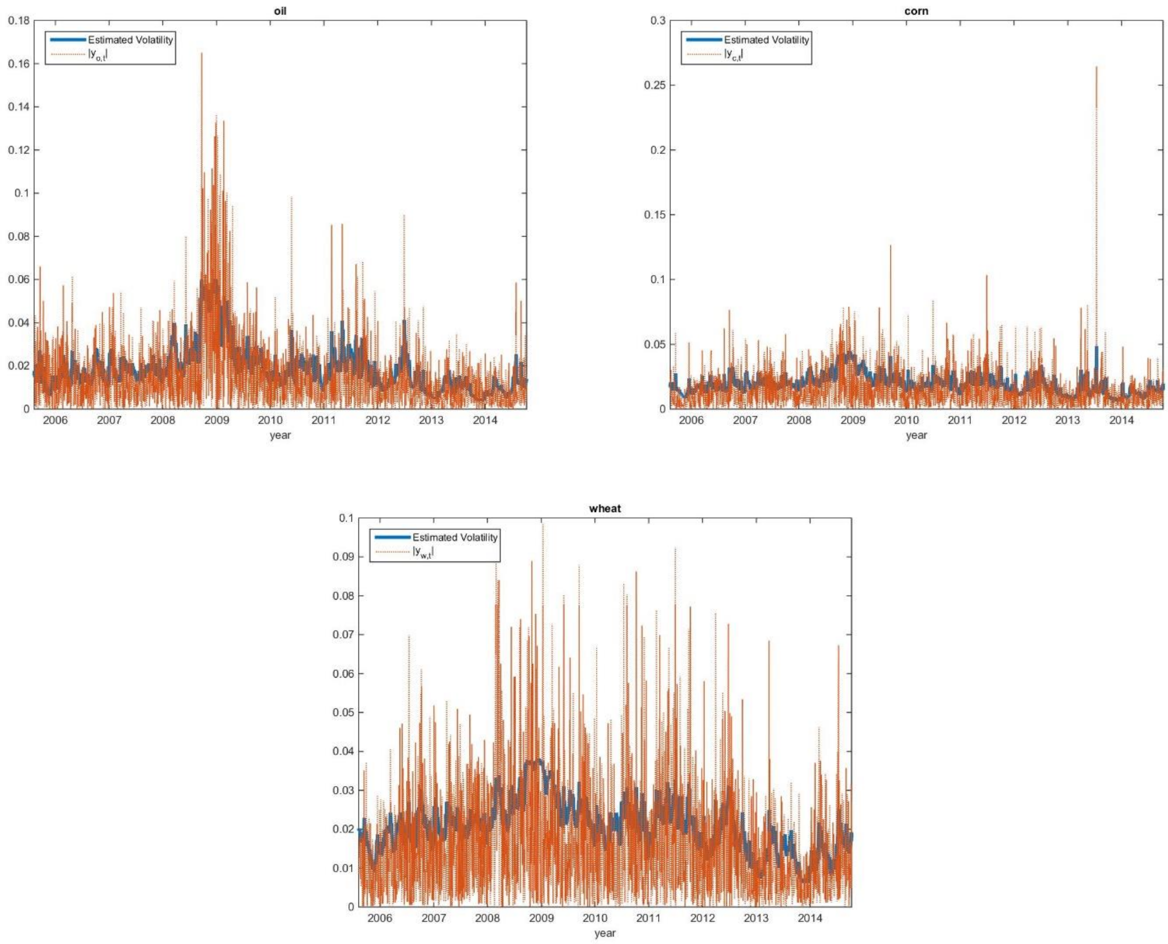

Figure 5 Realized and Estimated Volatility

We test whether the residual in the return equation, which is standardized by the corresponding volatility, is normal by using the Kolmogorov-Smirnov test. To test the normality for the residuals, the estimated residuals are obtained as

$$\hat{\varepsilon}_t^j = \frac{y_t^j}{\sqrt{\widehat{f_t^j}(\widehat{x_t^j})}} \sim N(0,1)$$

The estimated residuals therefore should be approximately $N(0,1)$. Table 5 presents the test results. According to Table 5, the residuals for oil, corn and wheat returns have means close to zero and standard deviations close to one. Moreover, skewness and kurtosis are approximately close to zero and 3, respectively, showing that the distributions of the estimated residuals are symmetric and do not have thick tails. As a result of the Kolmogorov-Smirnov test, the null hypothesis of normality cannot be rejected at the 5% level of significance for all cases. Therefore, our SV model with a common factor fits the data well.

Table 5 Diagnostic test on standardized residuals in the return equations in the SV model

	Oil	Corn	Wheat
Mean	0.0151	-0.0023	-0.0090
Standard deviation	0.9884	0.9127	0.9667
Skewness	-0.0010	-0.0604	0.0327
Kurtosis	2.5648	2.9439	2.7702
Kolmogorov-Smirnov (*p*-value)	0.0256 (0.0857)	0.0261 0.0758 (0.0758)	0.0221 (0.1913)

5. Conclusions

This paper analyzes the multivariate stochastic volatilities with a common factor which is affecting both the volatilities of crude oil and agricultural commodity prices in both biofuel and non-biofuel use. We developed a stochastic volatility model which has a latent common volatility with two asymptotic regimes and a smooth transition between them. In contrast with conventional volatility models, stochastic volatilities in this study are generated using a logistic transformation of the latent factors, which consists of two components: the common volatility factor and the idiosyncratic component.

In this study, we analyze the stochastic volatility model with a common factor for oil, corn and wheat from August 8, 2005 to October 10, 2014 using a MCMC method and estimate the stochastic volatilities and also extract the common factor. Our results indicate that the low level and high level of stochastic volatilities for oil are 0.79% and 8.4% in a day while the low and high volatilities of corn are 0.48% and 7.72%, respectively. In the wheat case, the low volatility level is 0.64% while the high volatility level is 5.41% in a day. Therefore, oil is more volatile than grains. The volatility generating process of oil is closer to the common factor in terms of the value of the loading parameter, which indicates that the volatility generating processes of oil, corn and wheat are scaled-down compared with the magnitude of the common factor, which is higher than other cases. Regarding the volatility of the idiosyncratic component, the variance of the idiosyncratic part for oil is higher than that of others. This implies that oil volatility is more affected by the common macroeconomic uncertainty factor while corn volatility is more explained by the idiosyncratic component. Based on the estimated common volatility factor, the high volatility periods in the common factor are well matched to the recession period from December 2007 to June 2009, as recorded by the National Bureau of Economic Research (NBER), and the financial crisis of 2007- 2008.

To the best of our knowledge, our study is the first to empirically examine the common factor in the volatilities of energy and agricultural commodity markets, especially oil, corn and wheat markets. We found that the main factors of common volatility extracted from oil, corn and wheat are related to the macroeconomic and financial uncertainties in the energy and grain market.

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