Risking-sharing Efficiency of Hedging Strategies

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Abstract

Since agricultural production is significantly and directly influenced by weather, financial weather products based on temperature have been developed in recent decades. The crop producer can now hedge adverse temperature outcomes in either the exchange market or the over-the-corner (OTC) market. However, exchange-traded contracts invariably carry geographic basis risk because of differences in the market-quoted and local temperature outcomes. OTC option contracts, on the other hand, are at risk of possible default by the counterparty. Therefore, a portfolio combining OTC with exchange-traded contracts could potentially be used by crop producers to reduce overall income risk. In this paper, we examine the performances of these three alternative hedging strategies on the uncertainty of crop producer's income. Using a case study for western Canada, we find that a portfolio that combines OTC and exchange-traded contracts provides a most effective means of reducing potential risks, compared with stand-alone OTC contracts or exchange-market contracts because of their higher default and geographic basis risks, respectively.

Keywords: hedging agricultural risk; financial weather derivatives; default and basis risk; agricultural risk management

JEL categories: Q14, G32, G11, C54, C63

INTRODUCTION

Agriculture is affected by weather and crop yields are uncertain as a consequence. With pending climate change, economists have sought to establish a clear link between weather variables and crop yields. For example, Schlenker et al. (2006) used a large dataset and nonlinear regression to fine-tune a relation between U.S. corn yields and temperatures, as climate models are better at forecasting temperatures than precipitation. These authors determined the length of time during the growing season that crops were exposed to heat units constituting 1°C intervals between –5°C and +50°C; they regressed crop yield on these interval observations and a more general measure of precipitation. Robertson (2012) estimated relationships between temperatures and crop yields for the Canadian prairies. In regressing crop yields on temperature, she employed three different versions of the temperature variable: average daily temperatures, growing degree-days (GDD), and 1°C intervals from 0°C to 40°C.

While regression models predicting crop yields with such a fine 1° C interval scale might be useful for climate modeling, they are not always helpful in responding to crop producers' concerns about the yield risk associated with weather variability. In many cases, financial weather derivatives simply cannot deal with this degree of complexity. Rather, financial weather derivatives need to be based on a simple index whose impact on crop yields is significant but yet easy to determine. In that case, financial derivatives can be an efficient instrument for hedging weather uncertainty.

The most frequently exchange-traded weather indexes are heating degree days (HDD) and cooling degree days (CDD). HDD refers to the extent temperatures are below 18°C, and thereby constitute a measure of the energy that may be required for heating.

They are defined as $HDD = \sum_{d=1}^{D} \text{Max}(0,18 - \overline{T}_d)$ where $\overline{T}_d$ is the average temperature (measured in degrees of Celsius) on day *d* and there are *D* days in the period under consideration (one month, two or more months). Likewise, CDD is as an index of the need for energy for cooling purposes, and defined as $CDD = \sum_{d=1}^{D} \text{Max}(0, \overline{T}_d - 18)$. Initially, weather derivatives were only traded over-the-counter (OTC), but in 1999 the Chicago Mercantile Exchange (CME) launched trading of the first weather derivatives. Then, in 2005, it introduced HDD and CDD derivative trading for U.S cities, followed one year later by HDD and CDD trading for six Canadian cities.

Financial derivatives could be widely used in agriculture to hedge against adverse weather, particularly too few units or too low temperatures. A crop producer could use either exchange-traded weather derivatives or OTC weather products. Exchange-traded weather derivative contracts are standardized, easy to write and carry little risk of default. Nonetheless, they are affected by large geographic basis risk (Musshoff et al., 2011). In general, basis risk is defined to occur when the change in the price of a hedge does not match the change in the price of the asset. In agriculture, the specific basis risk is that the location of the farm does not match the site used to construct the weather index underlying the derivative contract – one of the CME cities where the weather index is measured and upon which the HDD or CDD derivative is based (Golden et al., 2007). By contrast, there is no geographic basis risk for an OTC weather derivative because the source of the weather index is very near or identical to the farm’s location, and this is completely specified in the contract.[1] Nevertheless, in the absence of efficient regulatory control, there is a risk that the counterparty does not settle the contract – the crop producer encounters default risk or nonpayment by the counterparty. There is then a trade-off between geographic basis risk and default risk.

The crop producer who wants to reduce her exposure to risk might purchase an exchange-traded CDD option contract and, in addition, an OTC options contract based on differences between the at-the-farm and exchange-market weather indexes. The exchange-traded contract carries almost no default risk, but high basis risk; an OTC contract is able to reduce the basis risk, but with high default risk if the OTC contract is written solely on the outcome of the local (on-farm) weather index. As a hedger, the crop producer aims to minimize the overall risks associated with such a financial weather portfolio. However, the choice of numbers of exchange-traded versus OTC contracts to include in the portfolio leads to a trade-off between expected returns and aggregate risks.

Other than standard OTC and exchanged-traded derivatives, Considine (2000) and Golden et al. (2007) set up an additional hedging instrument referred to as a basis derivative. The basis derivative is essentially an OTC contract based on the difference in the weather index measured at the farm's location and the exchanged-traded HDD or CDD contract at the nearest or most relevant CME city (or an index based on a combination of CME cities). The basis derivative is of necessity an OTC contract that hedges the difference between the outcome of the weather index at the farm and the outcome of the weather index at the relevant city. Thus, a crop producer might purchase an exchange-traded CDD options contract plus an OTC weather derivative that hedges

[1] There remains local (as opposed to geographic) basis risk, because yield variability is not perfectly correlated with the relevant local weather variable (Musshoff et al., 2011).

the difference in the weather index at the farm and that quoted by the exchange market.

Growing degree days (GDD) constitute an index used widely in agriculture to represent the exposure to heat (temperatures) needed for crop growth. GDD cumulates daily average temperatures exceeding 5^{o} C as follows: $GDD = \sum_{d=1}^{D} \text{Max}(0, \overline{T}_d - 5)$, where D usually refers to the number of days in a growing season and $\overline{T}_d$ is the average daily temperature as above. Despite the similarity in the definition between CDD and GDD, GDD is a more straightforward and a more precise measure of a weather index that might relate to crop yields, one that a crop producer is familiar with and more likely to choose as the underlying index upon which to base a financial weather derivative asset.

To hedge against the potential risk of loss associated with insufficient heat units (i.e., low temperatures) over the growing season, a crop producer has three available strategies. The first is to long (purchase) a GDD put option on the OTC market based on a local temperature index. This strategy possesses default risk and local basis risk (which we ignore in this study). The second strategy is to purchase a CDD put option contract in the exchange market, usually written on the closest city to minimize the basis risk. The third alternative is simultaneously to purchase a CDD put option on the exchange market and a complementary put option on the OTC market based on a comparison of the CDD at the city and GDD locally. Based on the closest city assigned by CME, a CDD put option is able to reduce default risk, and the complementary GDD put option written on the difference in GDD and CDD at two locations lessens the geographic basis risk. The three strategies are summarized as follows:

- <u>OTC Strategy</u>: Purchase a put option based on the local GDD index in OTC market;

- Exchange Strategy: Purchase a put option on the closest city in the exchange market;
- Multi Strategy: Create a portfolio that combines a CDD put option on the exchange market and a GDD put option on the OTC market based on the gap between GDD indexes at these two locations.

The purpose of the current research is to compare these hedging strategies and determine which strategy is most preferred because of its inherently lower risks. While a number of authors have examined the issue of basis risk (Rohrer, 2004; Paulson et al., 2006; Woodard et al., 2008), few papers have studied the simultaneous effects of default risk and basis risk. Two studies in this regard addressed a similar issue. Brockett et al. (2008) and Golden et al. (2007) examined the risk-sharing efficiency of combining CDD and HDD derivatives using a mean-variance utility framework, finding that a combination of exchange-traded and OTC contracts is preferred. The authors check the efficiency of financial weather derivatives based solely on the CDD index. In this paper, we extend the analysis to the GDD and CDD measures and conclude that a portfolio of financial weather derivatives based on different indexes performs most efficiently in mitigating risks.

We employ a mean-variance utility function approach similar to Golden et al. (2007) to examine the risk-sharing efficiency that occurs under the three available strategies. We begin by developing an analytical model to find the optimal hedging ratio under the alternative strategies, after which we calculate the risks measured by the variance in income associated with each strategy. Then, we test the hedging effectiveness of the strategies using an empirical example for Alberta. Our conclusions and suggestions for further research ensue.

METHODS

Before we compare the 'multi strategy' with the 'OTC strategy', it is necessary to reconcile the differences between the GDD and CDD weather indexes. For a growing season from May to August, we can first rewrite the two indexes as:

$$\text{GDD} = \sum_{d=1}^{D} \text{Max}(0, \overline{T_d} - 5) \approx \sum_{d=1}^{D} \overline{T_d} - 5D \text{ and} \tag{1}$$

$$\text{CDD} = \sum_{d=1}^{D} \text{Max}(0,\ \overline{T_d} - 18) = \sum_{d=1}^{D} \overline{T_d} - 18D + \sum_{d=1}^{D} \text{Max}(0,\ 18 - \overline{T_d}), \tag{2}$$

where D and $\overline{T}_d$ are defined above. Upon substituting (1) into (2),

$$\text{CDD} = \sum_{d=1}^{D} \overline{T_d} - 5D - 13D + \sum_{d=1}^{D} \text{Max}(0,\ 18 - \overline{T_d}) \approx \text{GDD} - 13D + \text{HDD}\ . \tag{3}$$

Using (3), it is possible to relate the exchange-traded weather index CDD with the OTC index GDD. Equation (1) follows from the fact that the average daily temperature in our study region generally exceeds 5°C across the entire growing season.

Following Doherty et al. (2002) and Golden et al. (2007), we employ mean-variance analysis to determine an optimal hedging ratio by maximizing the expected utility as opposed to net return of a portfolio. Mean-variance analysis considers both expected returns and the variance of returns to identify agents' optimal hedging portfolios, and thus risk-sharing efficiency. A risk-averse agent prefers the portfolio of assets that leads to the highest expected returns and the lowest variance of returns; in essence, there are two objectives – maximize expected returns and minimize risk. To create an efficient portfolio of income, a risk-averse agent needs to reallocate income between different financial derivatives, thereby finding the optimal hedging ratio.

Suppose the crop producer's initial income at the beginning of the year is I_0. She plants a crop and chooses a hedging strategy; there are positive or negative net returns to crop production and a positive or negative net payout from the hedge, including the premium payment. The crop producer's eventual income turns out to be I at the end of the year, and the mean-variance objective function is given by

$$u(I) = \mathrm{E}(I) - \lambda\, \sigma^2(I), \tag{4}$$

where $u(I)$ is crop producer's continuous and concave utility as a function of income, E is the expectation operator, $\sigma^2(I)$ refers to the variance of income, and λ is the risk aversion parameter. High expected income could improve the crop producer's income, while a large variance indicates greater volatility of income. The efficient portfolio of financial derivatives is the one that yields the highest utility level, which is found by maximizing the mean-variance objective function.

Optimal hedging without default risk

OTC market put options are written on the local GDD index. We first introduce the optimal portfolio without default risk. With the purchase of GDD put options at the beginning of the year, the eventual income (I) of the crop producer is

$$I = I_0 - \mathrm{E}(h \times r) + h \times r, \tag{5}$$

where $r = \max(0, K\text{–GDD})$ is the payoff of the put option and h is the optimal hedging ratio that leads to an efficient hedging portfolio. The expected payoff $\mathrm{E}(h \times r)$ is the premium paid by the crop producer to obtain the put option, while the actual indemnity for that year is the payout from the put option ($h \times r$).

Analogously, the crop producer's eventual income under the 'multi strategy' without default risk is given by:

$$I = I_0 - \mathrm{E}(h_e \times r_{e,m}) - \mathrm{E}(h_d \times r_{d,m}) + h_e \times r_{e,m} + h_d \times r_{d,m}, \tag{6}$$

where h_e and h_d are the optimal hedging ratios in the exchange and OTC markets, respectively. The subscript m denotes variables under the multi strategy without default risk. With the strike price fixed at $K_{e,m}$, the payoff of the exchange-traded CDD put option is $r_{e,m} = \max(0, K_{e,m} - CDD_{e,m})$; in OTC market, the payoff of the GDD put option with strike price $K_{d,m}$ is given by $r_{d,m} = \max(0, K_{d,m} - GDD_{d,m})$. Thus, the eventual income is determined as the initial income plus the option payoff after subtracting the premium.

Optimal hedging with default risk

To quantify default risk, a dummy variable θ is introduced and the probability of default is represented by $P(\theta=0) = 1-p$ (Golden et al., 2007). Under the 'OTC strategy', over-the-counter put options are solely written on the local GDD index. Taking default risk into consideration, the eventual income of the crop producer is:

$$I = I_0 - \mathrm{E}(\theta\, h_l\, r_l) + \theta\, h_l\, r_l, \tag{7}$$

where h_l is the hedging ratio that gives the optimal income invested in hedging financial derivatives and the payoff of the put option with strike price K_l given by $r_l = \max(0, K_l - GDD_l)$. After obtaining the expected value and the variance of the crop producer's eventual income, we can solve for the optimal hedging ratio that creates an efficient portfolio. Then, the crop producer's hedging problem becomes:

$$\underset{h_l}{Max}\,[\mathrm{E}(I) - \lambda\, \sigma^2(I)]. \tag{8}$$

It turns out that the first-order condition is necessary and sufficient. If $\mu_{r_1}^2$ and $\sigma_{r_1}^2$

denote the expectation and variance of the payoff to the local GDD put option, and the covariance between initial income and the put option's payoff is $V(I_0, r_l)$, the optimal hedging ratio h_l is then given by:[2]

$$h_l = \frac{V(I_0, r_1)}{\sigma_{r_1}^2 + (1-p)\mu_{r_1}^2}. \tag{9}$$

It can be shown that the optimal hedging ratio falls as the default risk (1–p) increases. The assumption here is that the hedging ratio is applied indefinitely to the purchase of OTC put options to hedge risks. Intuitively, a high default risk results in less use of OTC put options due to the associated high probability of loss. Further, the magnitude of the risk-aversion parameter has no effect on the optimal hedging ratio in equation (9). This is a result of the composition of eventual income, whose expectation merely relies on the initial income, as the historical expected payoff equals the price (i.e., premium) of the option. The risk-aversion parameter is eliminated in the first-order condition with respect to h_l.

Under the 'exchange strategy', the crop producer can directly purchase or long a CDD put option in the exchange market of the nearest CME city. With the existence of the clearing house, this exchange-traded option contract scarcely carries default risk, so default risk does not affect the eventual income or hedging ratio. With a long CDD put option at the beginning of the year, the eventual income of the crop producer is

$$I = I_0 - \mathrm{E}(h_{e,e} \times r_{e,e}) + h_{e,e} \times r_{e,e}, \tag{10}$$

where $r_{e,e}$ is the payoff of a put option and $h_{e,e}$ denotes the optimal hedging ratio on the

[2] The expectation is given by $\mathrm{E}(I_1) = \mathrm{E}(I_0) - \mathrm{E}(\theta\, h_p\, r_l) + \mathrm{E}\,(\theta\, h_p\, r_l) = \mathrm{E}(I_0)$, and variance by $V(I_1) = V[I_0 - \mathrm{E}(\theta\, h_p\, r\, p_l) + \theta\, h_p\, r_l] = V(I_0) + h_p^2\, p[\sigma^2\, r_l + (1-\mathrm{p})\mu^2\, r_l] + 2\, h_p p V(I_0, r_l)$. The first-order condition for maximizing $[\mathrm{E}(I) - \lambda\, \sigma^2(I)]$ is $\partial \mathrm{L}/\partial h_p = 2 h_p p[\sigma^2{}_{r_l} + (1-p)\mu^2{}_{r_l}] + 2pV(I_0, r_l) = 0$. Solving gives result (9).

exchange market. The expected payoff $E(h_{e,e} \times r_{e,e})$ is the premium paid by the crop producer to obtain the put option, while the actual payoff for that year is the actual payout of the exchange-market CDD put option. Likewise, the optional hedging ratio is derived by maximizing the mean-variance utility function. In equation (11), $\sigma^2_{r,e}$ and $V(I_0, r_{e,e})$ respectively denote the variance of the CDD put option's payoff and the covariance between initial income and the option's payoff. The optimal hedging ratio $h_{e,e}$ is:[3]

$$h_{e,e} = -\frac{V(I_0, r_{e,e})}{\sigma^2_{r_{e,e}}}. \tag{11}$$

If we rewrite the covariance as the product of the correlation of initial income and the option's payoff $\rho_{0,e}$, standard deviation of I_0 (denoted σ_{I0}), and $r_{e,e}$ ($\sigma_{r,e}$), equation (11) can be simplified as:

$$h_{e,e} = -\rho_{0,e} \frac{\sigma_{I0}}{\sigma_{r,e}}, \tag{12}$$

The third alternative is a 'multi strategy' that combines the CDD and GDD options. Similar to the 'OTC strategy', with the existence of default risk, the eventual income of the crop producer is given by:

$$I = I_0 - E(h_e\, r_e) - E(\theta\, h_d\, r_d) + h_e\, r_e + \theta\, h_d\, r_d, \tag{13}$$

where h_e and h_d are the hedging ratios on the exchange and OTC markets, respectively. If we use temperature indexes CDD and GDD, with an exchange-market strike price of K_e and OTC market strike price of K_d, the payoff of the exchange-traded CDD put option is $r_e = \max(0, K_e - \text{CDD}_e)$ and the payoff of the GDD put option on the OTC market is $r_d =$

[3] The expectation is given by $E(I_1) = E(I_0)$, and variance by $V(I_1) = V[I_0 - E(\theta\, h_{ee}\, r_{ee}) + \theta\, h_p\, r_{ee}] = V(I_0) + h_{ee}^2\, \sigma^2\, r_{ee} + 2\, h_{ee} V(I_0, r_{ee})$. The first-order condition for maximizing $[E(I) - \lambda\, \sigma^2(I)]$ is $\partial L/\partial h_{ee} = -2h_{ee}\sigma^2_{r_{ee}} + 2V(I_0, r_{ee}) = 0$. Solving gives result (11).

max(0, K_d – GDD_d). The GDD put option is essentially a basis option (Considine, 2000, Golden et al., 2007) constructed on the difference in GDD between the two locations.

As before, the mean-variance objective function is:

$$\underset{h_e,h_d}{Max}[\mathrm{E(I)} - \lambda\sigma^2(\mathrm{I})], \tag{14}$$

and the optimal hedging ratios are derived as:[4]

$$h_e = \frac{(p-1)\mu_{r_d}^2 V(I_0, r_e) + pV(I_0, r_d)V(r_e, r_d) - \sigma_{r_d}^2 V(I_0, r_e)}{(1-p)\sigma_{r_e}^2 \mu_{r_d}^2 + \sigma_{r_e}^2 \sigma_{r_d}^2 - pV^2(r_e, r_d)} \tag{15}$$

$$h_d = \frac{V(I_0, r_e)V(r_e, r_d) - \sigma_{r_e}^2 V(I_0, r_d)}{(1-p)\sigma_{r_e}^2 \mu_{r_d}^2 + \sigma_{r_e}^2 \sigma_{r_d}^2 - pV^2(r_e, r_d)}, \tag{16}$$

where $V(r_d, r_e)$ is the covariance between r_e and r_d, while σ^2 and μ denote the respective variance and expectation of the put options' payoffs. Identical to the 'OTC strategy', the optimal hedging ratio h_d is negatively related to default risk (1–p). However, under the 'multi strategy', the optimal hedging ratio h_d is also strongly affected by the covariance in the payoffs between the put options on two markets. Further, the optimal hedging ratios are independent of the crop producer's risk-aversion parameter.

Hedging effectiveness

From the derived optimal hedging ratios, we can find the initial income (a proxy for wealth) that has a significant impact on the crop producer's choice of a hedging portfolio. The correlation between initial income I_0 and the payoffs of put options are

[4] Expected income is given by $\mathrm{E}(W) = \mathrm{E}(W_0) - \mathrm{E}(h_e r_e) - \mathrm{E}(\theta h_d r_d) + \mathrm{E}(h_e r_e) + \mathrm{E}(\theta h_d r_d)$, while its variance is $\mathrm{V}(W) = \mathrm{V}[W_0 - \mathrm{E}(h_e r_e) - \mathrm{E}(\theta h_d r_d) + h_e r_e + \theta h_d r_d] = \mathrm{V}(W_0) + \mathrm{V}(h_e r_e) + \mathrm{V}(\theta h_d r_d) + 2\mathrm{V}(W_0, h_e r_e) + 2\mathrm{V}(W_0, \theta h_d r_d) + 2\mathrm{V}(h_e r_e, \theta h_d r_d) = \mathrm{V}(W_0) + h_e^2\sigma^2 r_e + h_d^2 p[\sigma^2 r_d + (1-p)\mu^2 r_d] + 2h_e\mathrm{V}(W_0, r_e) + 2h_d p\mathrm{V}(W_0, r_d) + 2\, h_e h_d\, p\mathrm{V}(r_e, r_d)$. To maximize $\mathrm{L} = \mathrm{E}(W) - \lambda\sigma^2(W)$ with respect to h_e and h_d, where W is given by equation (10), find the first-order conditions: $\partial \mathrm{L}/\partial h_e = 2h_e\sigma^2 r_e + 2\mathrm{V}(W_0, r_e) + 2h_d p\mathrm{V}(r_e, r_d) = 0$; and $\partial \mathrm{L}/\partial h_d = 2h_d p[\sigma^2 r_d + (1-p)\mu^2 r_d] + 2p\mathrm{V}(W_0, r_d) + 2h_e p\mathrm{V}(r_e, r_d) = 0$. From these, we obtain results (15) and (16).

important in determining the crop producer's optimal hedging strategy. However, as the components of initial income are quite complicated, we assume that the producer does not borrow (at least in purchasing weather derivatives) but can self-finance. Thus, the initial income is the expected income from the previous years' crop sales, and primarily those of the previous year. Given a positive relationship between the local GDD index and crop yields, higher levels of growing-season degree days have a positive effect on the income from crop sales. If the average crop price is held to be constant over time, we then assume that initial income I_0 in year t is a linear function of local GDD in the previous year. That is, we assume a change in the GDD index results in a constant change in initial income regardless of the actual level of the index (see Nhemachena et al., 2007).

Under the self-finance assumption, the optimal hedging ratios (9), (11), (15) and (16) can be rewritten as follows:

$$h_1 = -\frac{V(T_{t-1}, r_1)}{\sigma_{r_1}^2 + (1-p)\mu_{r_1}^2} \tag{17}$$

$$h_{e,e} = -\frac{V(T_{t-1}, r_{e,e})}{\sigma_{r_{e,e}}^2}, \tag{18}$$

$$h_e = \frac{(p-1)\mu_{r_d}^2 V(T_{t-1}, r_e) + pV(T_{t-1}, r_d)V(r_e, r_d) - \sigma_{r_d}^2 V(T_{t-1}, r_e)}{(1-p)\sigma_{r_e}^2\mu_{r_d}^2 + \sigma_{r_e}^2\sigma_{r_d}^2 - pV^2(r_e, r_d)} \tag{19}$$

$$h_d = \frac{V(T_{t-1}, r_e)V(r_e, r_d) - \sigma_{r_e}^2 V(T_{t-1}, r_d)}{(1-p)\sigma_{r_e}^2\mu_{r_d}^2 + \sigma_{r_e}^2\sigma_{r_d}^2 - pV^2(r_e, r_d)}, \tag{20}$$

where $V(T_{t-1}, r_1)$, $V(T_{t-1}, r_{e,e})$, $V(T_{t-1}, r_e)$ and $V(T_{t-1}, r_d)$ represent the respective co-variances of the temperature index of the previous year with the payoff of the current year GDD put option in the OTC market, the CDD put option in the exchange market, the put option in the exchange market (under the 'multi strategy'), and the basis option in the OTC market.

Once we determine the optimal hedging ratios, it is quite straightforward to construct the efficient hedging portfolios.

The aggregate risk of a hedging portfolio could be measured by the variance of eventual income. The variances of eventual income under each of the ‘OTC’, ‘exchange’ and ‘multi’ strategies are given as follows:

$$Var_{OTC}(I) = \sigma^2_{T_t-1} + ph_l^2\sigma^2_{r_1} + p(1-p)h_l^2\mu^2_{r_1} + 2ph_lV(T_{t-1}, r_1) \tag{21}$$

$$Var_{exchange}(I) = \sigma^2_{T_{t-1}} + h^2_{e,e}\sigma^2_{r_{ee}} + 2h_{e,e}V(T_{t-1}, r_{e,e}) \tag{22}$$

$$\begin{aligned} Var_{Multi}(I) = {} & \sigma^2_{T_{t-1}} + \sigma^2_{\varepsilon} + h_e^2\sigma^2_{r_e} + ph_d^2\sigma^2_{r_d} + p(1-p)h_d^2\mu^2_{r_d} + 2h_eV(T_{t-1}, r_e) \\ & + 2ph_dV(T_{t-1}, r_d) + 2ph_eh_dV(r_e, r_d), \end{aligned} \tag{23}$$

where $\sigma^2_{T_{t-1}}$ is the variance of the weather (temperature) index.

The criterion we use to identify a most preferred strategy is the hedging effectiveness (HE) parameter (Kumar et al., 2008). The hedging effectiveness ratio expresses the extent to which the uncertainty in eventual income is reduced by one strategy compared to an alternative strategy. It is used to determine whether the application of an exchange-traded put option associated with basis risk, for example, could mitigate the default risk attached to an OTC-traded GDD put option under the ‘OTC strategy’. HE parameters for each pairwise comparison of the three strategies are derived using the variances (21) through (23) as follows:

$$\text{HE}_1 = \frac{Var_{OTC} - Var_{Multi}}{Var_{OTC}} \tag{24}$$

$$\text{HE}_2 = \frac{Var_{exchange} - Var_{Multi}}{Var_{exchange}} \tag{25}$$

$$\text{HE}_3 = \frac{Var_{exchange} - Var_{OTC}}{Var_{exchange}}. \tag{26}$$

A positive value for HE_1 indicates that the ‘multi strategy’ of a CDD put option plus GDD put option has lower uncertainty than under the ‘OTC strategy’. Similarly, a positive value for HE_2 reveals that the ‘multi strategy’ can reduce the basis risk compared to a single CDD put option contract as under the ‘exchange strategy’. Larger values of HE_1 or HE_2 indicate the extent to which the ‘multi strategy’ is preferred compared to other available strategies. That is, if HE_1 and HE_2 are both positive, the ‘multi strategy’ is preferred to the other two alternatives considered here. Finally, a positive value for HE_3 indicates a preference for an over-the-counter put option based on the local GDD measure (‘OTC strategy’) is preferred to the purchase of a CDD put option in the CME market (‘exchange strategy’).

AN APPLICATION TO ALBERTA

To illustrate the potential effectiveness of the ‘multi strategy’, we consider a representative farm and an exchange-traded financial weather product in Alberta, Canada. The study region consists of Mountain View County in central Alberta (Figure 1), where agricultural producers plant significant areas to spring wheat and for which good weather data are available. Weather data for the period 2003-2013 are available for the Town of Olds situated within Mountain View County and the weather station located at Calgary International Airport to the northeast of the City of Calgary and some 75-80 km from Olds. Chicago Mercantile Exchange CDD and HDD financial products for Calgary are based on temperature data from the weather station at the international airport.

A crop producer located near Calgary’s airport can long a CDD put option directly with little basis risk, but a farmer adjacent to the Town of Olds bears considerable basis risk if she relies solely on Calgary-based weather derivatives. We employ daily temperature data for the growing season (May through August) for Calgary International Airport and the local weather station located in the Town of Olds to construct the GDD and CDD indexes for 2003-2013. Only contracts that cover the growing season are considered. As indicated in Figure 2, temperatures at Olds are somewhat lower than at Calgary. Thus it would be appropriate to long a put option on the difference between the two locations. Descriptive statistics for the temperature indexes are provided in Table 1.

As GDDs at the two locations track quite closely, it facilitates the practical use of exchange-traded contracts. Compared with Calgary, however, temperatures at Olds are not only lower but also fluctuate to a greater extent. The higher aggregate temperature at Calgary suggests that there also exists geographic basis risk for the farmer at Olds. To hedge the uncertainty in sales from crops, the crop producer near Olds can choose from three strategies: (1) long an OTC put option based on GDD measured at the local weather station ('OTC strategy'), although this carries a certain amount of default risk; (2) long an exchange-traded CDD put option written on Calgary directly on the CME market ('exchange strategy'); and (3) purchase a hedging portfolio that combines an exchange-traded CDD put option based on Calgary and an OTC put option based on the difference in temperature indexes between the two locations ('multi strategy'). We use our weather data to compute the variances in eventual income under each strategy to deduce the most effective one.

The optimal hedging portfolio is computed from the 2003-2013 historical data for different strike prices, where each strike price represents a certain percent deviation from the historic mean of the temperature index. The probability of default (default risk) is given by (1–p), and it is difficult to predict before contracts are settled, although there exists a well-established credit rating system that offers reliable estimates of the probability of default. We apply a credit rating system that ranks creditworthiness from AAA to CCC, with the corresponding probability of default increasing from 0.005% to 30.000% as indicated in Table 2 (see Murphy, 2013 pp.66).

Then, using equations (17) to (20), we obtain the optimal hedging ratios under different strategies. We find that the optimal hedging ratio h_l under the 'OTC strategy' shows no significant variation for any given strike price. Not surprisingly, the optimal hedging ratio decreases in default risk under the 'OTC strategy'. The crop producer will reduce purchases of OTC-traded GDD put options to avoid the high probability of default. The optimal hedging ratios under the 'multi strategy' reveal the same pattern. The hedging ratio for OTC-traded contracts h_d declines with default risk, while the hedging ratio of exchange-traded contracts h_e increases with default risk. Exposure to high default risk induces the crop producer to allocate more of her hedging resources to exchange-traded contracts to avert potential default. Since the CME market does not carry default risk, the hedging ratio $h_{e,e}$ under the 'exchange strategy' is solely affected by the choice of strike price, and not by the credit risk.

Strike prices

The strike price is a fundamental component of an option contract, although there is a tradeoff between the premium and the strike price. Generally, for a put option, the premium rises with the strike price; and the premium of a call option falls as the strike price goes up. The choice of the strike price level in option contracts is affected by the farmer's risk attitude and her forecast of the weather conditions. Thus, we initially set the strike prices at the historical average of the temperature index as a benchmark and restrict the strike price to be within a certain range. This is common in empirical analyses of weather derivatives (e.g., Leggio 2007; Roustant et al. 2003).

The average GDD for the town of Olds is 1033.35 with a standard deviation of 85.24 (Table 1). Under the OTC strategy, the strike price for a GDD put option contract is 1,033.35. In Table 3, we indicate how the hedging parameter H1 changes as a result of increasing risk and a strike price that declines from 1,033.35 to 893.85 (which are determined by the standard deviations from the mean).

When the farmer applies the exchange strategy, she will purchase an exchange-traded CDD put option on the basis of Calgary weather. The average historical CDD in Calgary is 45.17 (Table 1) and its standard deviation is 18.19. The farmer will set the strike price of a CDD put option contract at 45.17. Other scenarios where the strike price ranges from 45.17 to 28.80 are provided in Table 4.

When the multi strategy is applied, both an exchange-traded CDD put option contract and an OTC-traded GDD put option contract are purchased. Just as with the OTC strategy, the strike price of OTC-traded GDD put option is based on the GDD index for Olds. Using the mean GDD and its standard deviation for Olds, we can employ equation (3) to derive equivalent local CDDs associated with various strike prices for GDD. The difference between the derived local CDD and CDD based on Calgary is the strike price of exchange-traded CDD put option. For example, the farmer purchases an OTC-traded GDD put option with a strike price of 1033.35. On the basis of equation (3), the equivalent local CDD is 36.55.[5] But the average historical CDD in Calgary is 45.17, which is greater than 36.55 from the OTC-traded contract. To eliminate the difference, the farmer then purchases an exchange-traded CDD put option with a strike price of 8.62.[6] Thus, under the multi strategy, the farmer purchases an OTC-traded GDD put option with a strike price of 1,033.35 and simultaneously an exchange-traded put option with a strike price of 8.62.

Results

When the default risk rises from 0.005% to 30.000%, the hedging effectiveness parameters HE_1 provided in Table 3 depict the extent to which the 'multi strategy' has reduced or increased income uncertainty compared to the 'OTC strategy'. Analogously, the hedging effectiveness parameters HE_2 in Table 4 compare the variances of the 'multi strategy' against the 'exchange strategy', while the hedging effectiveness parameters HE_3 in Table 5 compare those of the 'OTC strategy' against the 'exchange strategy'. The tables clearly indicate how the hedging effectiveness parameters vary under different combinations of the risk of default risk and the strike price.

From Table 3, it is clear that, as the default risk increases, the hedging effectiveness parameter HE_1 gradually rises. When default risk is as low as 0.005%, the 'OTC strategy' is greatly preferred as it is associated with a smaller variance in the crop producer's income. As the probability of default rises, the higher default risk tends increasingly to augment the variation and uncertainty in a crop producer's income, implying that the 'OTC strategy' is less effective. From the table we find that, when the probability of default is as high as 30%, the 'multi strategy' carries a much smaller

[5] CDD ≈ GDD – 13° + HDD = 1033.35 − 13×123 +602.20=36.55.
[6] 8.62 = 45.17 − 36.55.

variance than an 'OTC strategy'. The reason is that the exchange-traded contracts under the 'multi strategy' mitigate part of the high default risk, so that the aggregate risk under the 'multi strategy' is not as high as that under the 'OTC strategy' despite the presence of basis risk. Also of interest is the finding that, as the strike price departs from the mean of the temperature index, hedging effectiveness declines quite quickly. One possible reason is the increase in basis risk that occurs when there is a large deviation from the historic mean. Thus, the aggregate risk associated with the 'multi strategy' exceeds the single default risk associated with the 'OTC strategy'. This suggests that the choice of strike price is vital for the crop producer's decision regarding an appropriate risk strategy.

Table 4 provides the values of the hedging effectiveness parameter HE_2 under various risk default and strike price combinations. In contrast to OTC-traded contracts, the exchange-traded contracts are scarcely affected by the default risk. As the default risk rises, the hedging effectiveness parameter HE_2 varies little. However, as the strike price deviates from the mean value of the temperature index, HE_2 increases gradually. From the table we see that exchange-traded contracts show a lower variance when the strike price is around the mean value of the temperature index. As it moves away from the mean value, the potential for high geographic basis risk decreases the hedging effectiveness of exchange-traded contracts. This suggests that a 'multi strategy' may be preferred when the strike price is quite different from the historic mean.

We also calculated the hedging effectiveness parameter HE_3 (Table 5), which compares variances under the 'OTC strategy' against those under the 'exchange strategy'. A positive value indicates that the variance is smaller under the 'OTC strategy'. When there exists a lower probability of default, there is no doubt that OTC-traded contracts written on local GDD outcomes are more effective. The basis risk from using the exchange market leads to a higher variance in income, although when default risk increases to 30%, exchange-traded contracts are preferred because there is then an increase in the variance of income from OTC-traded contracts.

CONCLUSION

In this paper, we investigated the effectiveness of three potential hedging strategies that crop producers can use to mitigate risks of too little heat during the growing season – over-the-counter GDD contracts based on local weather outcomes, CME-traded CDD contracts, and a combination of exchange-traded CDD contracts and OTC-traded GDD contracts. Each strategy incorporates distinct basis risk and/or default risk. Our results indicate that the extent to which the uncertainty in eventual income is reduced by the 'multi strategy' compared to the purchase of an over-the-counter hedge based on local temperatures during the growing season, with the effectiveness parameter HE_1 increasing with default risk because default risk is inherent in OTC contracts. We demonstrated that, theoretically, the introduction of a CDD exchange-traded contract in a 'multi strategy' could eliminate some of the default risk and reduce variance in farmers' incomes. Therefore, it is possible to use a 'multi strategy' to mitigate the aggregate risks imposed on crop producers – that is, a combination of exchange-traded CDD options and OTC-traded GDD options could effectively improve hedging effectiveness and reduce a farmer's aggregate risk from too few heat units for crop production.

In our application, we find that strike price in weather contracts is a crucial factor in choosing a hedging strategy, as differences in strike prices drive the choice of the most effective hedging strategy. Further, when the strike price is close to the historic index value as determined from growing-season temperatures, the ‘exchange strategy’ is more effective than the ‘multi strategy’ for high levels of default risk, although the ‘OTC strategy’ is always preferred for low levels of default risk. However, when the strike price deviates from the mean value, it is much harder to determine which strategy is preferred.

Future research needs to extend our results in several directions. First, we focused only on one region in Canada where the location of the representative farm (and weather station) at Olds was only a relatively short distance (< 90 km) from the weather station used to construct the weather index for the CME weather derivative options. It is necessary, therefore, to examine cases where the farm and the exchange-based product are much farther apart as is the case for most of the Canadian prairies. It is also necessary to consider different cropping regimes and determine an empirical relationship between the weather indexes used in the analysis and crop yields, and thus farm incomes. Finally, in some regions, including perhaps the current study region, growing-season temperatures are likely less of an issue than precipitation. Weather indexes based on precipitation might pose a significant challenge for the types of analysis considered here (see Martin et al. 2001).

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Table 1: the Statistics of Temperature Indexes from 2003 to 2013

Location→	Calgary International Airport			Town of Olds		
Temperature Index	GDD	CDD	HDD	GDD	CDD	HDD
Mean	1,136.15	45.17	520.94	1,033.35	24.15	602.20
Variance	5516.70	330.84	4945.52	7427.18	251.71	6108.47
Std. Dev.	74.27	18.19	70.32	85.24	15.87	78.16

Source: Environment Canada.

Table 2: One Year Probabilities of Default for a Given Credit Rating

Credit Rating	Probability of Default (=1–p)
AAA	0.005%
AA	0.020%
A	0.060%
BBB	0.400%
BB	1.500%
B	10.000%
CCC	30.000%

Table 3: Hedging Effectiveness Parameter HE_1 under Different Combinations of Default Risk and Strike Price

Default risk[b] Strike Price[c]	0.005%	0.020%	0.060%	0.400%	1.500%	10.000%	30.000%
1,033.35	-54.65%	-54.58%	-54.39%	-52.77%	-47.79%	-19.27%	14.28%
1,017.85	-41.98%	-41.93%	-41.79%	-40.61%	-36.95%	-14.89%	13.73%
1,002.35	-50.48%	-50.42%	-50.28%	-49.05%	-45.25%	-22.18%	8.22%
986.85	-10.97%	-10.96%	-10.91%	-10.51%	-9.25%	-0.61%	14.18%
971.35	-6.87%	-6.86%	-6.82%	-6.53%	-5.60%	0.98%	13.02%
955.85	4.58%	4.59%	4.60%	4.75%	5.24%	8.80%	16.06%
940.35	13.03%	13.03%	13.04%	13.12%	13.37%	15.33%	19.72%
924.85	11.45%	11.45%	11.46%	11.53%	11.77%	13.64%	17.96%
909.35	8.88%	8.89%	8.89%	8.96%	9.19%	11.00%	15.32%
893.85	4.76%	4.77%	4.77%	4.84%	5.06%	6.86%	11.28%

[a] $HE_1 = \frac{Var_{OTC} - Var_{Multi}}{Var_{OTC}}$, which compares OTC strategy with multi strategy.

[b] Default risk equals 1–p.

[c] The strike price is determined by (u–cσ), where u and σ are the mean and standard deviation of local GDD, and c=0, 0.1, 0.2, 0.3… 0.9.

Table 4: Hedging Effectiveness Parameter HE_2 under Different Levels of Default Risk and Strike Price[b]

Default Risk Strike Price	0.005%	0.020%	0.060%	0.400%	1.500%	10.000%	30.000%
45.17	-3.07%	-3.07%	-3.07%	-3.07%	-3.09%	-3.20%	-3.48%
43.35	-1.68%	-1.68%	-1.68%	-1.69%	-1.72%	-1.99%	-2.55%
41.53	-0.61%	-0.61%	-0.61%	-0.63%	-0.67%	-1.02%	-1.74%
39.72	0.21%	0.21%	0.21%	0.19%	0.14%	-0.21%	-0.92%
37.90	0.30%	0.30%	0.30%	0.28%	0.22%	-0.19%	-0.95%
36.08	-0.19%	-0.19%	-0.19%	-0.21%	-0.28%	-0.74%	-1.56%
34.26	0.72%	0.72%	0.72%	0.69%	0.60%	0.00%	-1.02%
32.44	1.75%	1.75%	1.74%	1.71%	1.60%	0.85%	-0.34%
30.62	2.94%	2.94%	2.94%	2.90%	2.77%	1.91%	0.57%
28.80	4.37%	4.37%	4.36%	4.32%	4.17%	3.23%	1.80%

[a] $HE_2 = \frac{Var_{exchange} - Var_{Multi}}{Var_{exchange}}$, which compares exchange strategy with multi strategy.

[b] See notes in table 3.

Table 5: Hedging Effectiveness Parameter HE_3 under Different Level of Default Risk and Strike Price[b]

Default Risk Strike Price	0.005%	0.020%	0.060%	0.400%	1.500%	10.000%	30.000%
1033.35	33.36%	33.33%	33.24%	32.53%	30.25%	13.48%	-20.72%
1017.85	28.39%	28.36%	28.29%	27.68%	25.72%	11.23%	-18.86%
1002.35	33.14%	33.12%	33.05%	32.49%	30.69%	17.32%	-10.85%
986.85	10.08%	10.06%	10.02%	9.69%	8.60%	0.39%	-17.60%
971.35	6.71%	6.70%	6.67%	6.39%	5.51%	-1.19%	-16.05%
955.85	-4.99%	-5.00%	-5.02%	-5.21%	-5.82%	-10.47%	-20.99%
940.35	-14.14%	-14.15%	-14.17%	-14.30%	-14.74%	-18.11%	-25.82%
924.85	-10.95%	-10.96%	-10.98%	-11.11%	-11.53%	-14.81%	-22.31%
909.35	-6.52%	-6.52%	-6.54%	-6.66%	-7.07%	-10.21%	-17.42%
893.85	-0.41%	-0.42%	-0.43%	-0.55%	-0.94%	-3.90%	-10.69%

[a] $HE_3 = \frac{Var_{exchange} - Var_{OTC}}{Var_{exchange}}$, which compares OTC strategy with exchange strategy.

[b] See notes in table 3.

Figure 1: Location of the Study Region
Source: http://www.maptown.com/albertacounties.html

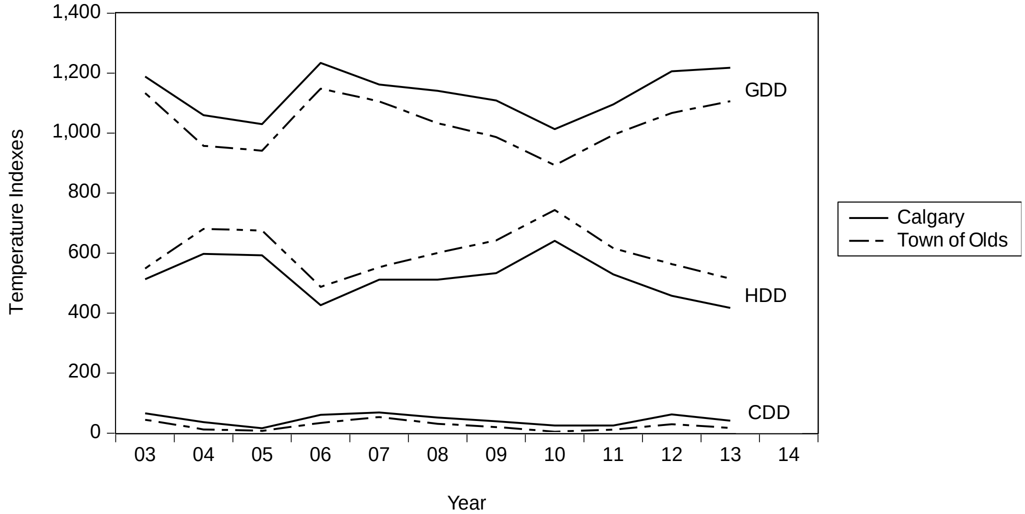

Figure 2: Temperature Indexes in Calgary and Town of Olds from 2003 to 2013
Source: Environment Canada