Maximum Likelihood Estimation of a Random Coefficient Meat Demand System

By

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Abstract:

The paper demonstrates that random coefficient models can be estimated by maximum likelihood if they are specified as generalized least squares models. The paper uses maximum likelihood estimation on a random-coefficient, meat-demand system. Statistical tests show that price elasticities are random, but expenditure elasticities are not. The statistical tests allow one to count the number of factors that cause randomness without requiring one to know what they are. There appear to be only two factors that make the price elasticities random.

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This paper an extension to a model first presented in 1994 in which I estimated a random coefficient demand model for beef, pork, chicken, and turkey. The basic approach in the first paper was different from the most common approach that was developed by Swamy and Tinsley (1980). I treated random coefficients as a special case of heteroskedasticity. I used a three-step procedure to estimate my original random-coefficient demand system. However, I was able to show that my approach and the Swamy-Tinsley approach gave the same estimates of the mean parameter values given the same covariance matrix of random coefficients. The weakness of both the Swamy-Tinsley and the heteroskedasticity approaches is neither provides an efficient method of estimating the covariance matrix of random coefficients. The method that I present here can simultaneously estimate both mean parameter values and the covariance matrix of the random coefficients, if the covariance matrix meets some restrictions. This simultaneous estimation will provide efficient estimates of the mean-parameter vector and parameter-covariance matrix.

The first part of the paper is a justification for why consumer demand models might have random coefficients. There has been a great deal of work on meat demand; much of it has focused on testing whether meat demand has changed over time or not. My basic model assumes that demand may be fundamentally unstable because it may depend on unobserved factors consumers' environment with random components. These random components may cause price and income elasticities to vary randomly. One of the interesting things that the procedure can do is to count the number of environmental factors causing changes in demand elasticities or, at the very least, provide a lower bound on this number. While the procedure is not directly helpful for identifying what these factors might be, knowing that there are a limited number of factors is helpful for future research.

The use of full information, maximum likelihood (FIML) estimation makes it possible to test hypotheses about the random coefficient covariance matrix. Comparing the classic, linear model with the random-coefficient model is another test involving the covariance matrix. The classic linear model can be seen as a special case of the random-coefficient model. While the random-coefficient model has random slopes and intercept, the classic linear model only has random intercepts. The FIML procedure allows one to compare and test the classic and random-coefficient models by restricting the covariance matrix of the coefficients. To count the factors that cause random shifts in demand elasticities I test the rank[2] of the coefficient covariance matrix.

Reconciling Taste "Stability" and Random Econometric Equations

The U.S. demand for meat has been a very popular topic for investigation by economists. Much of the research has focused on whether or not U.S. consumers' underlying tastes for meat have been stable. Alston and Chalfant (1991) surveyed many of the econometric studies that attempted to measure/test the extent of changes in consumer tastes/demand for meat. They criticized this literature for being unable to distinguish between taste-shifts and misspecification bias. They noted that their earlier work (Chalfant and Alston, 1988) using non-parametric tests showed stable tastes.

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[2] Assuming that only the intercepts are random or that some of the slopes are not random also reduces the rank of the covariance matrix by eliminating blocks of terms. The rank tests reduce the rank of the covariance matrix without eliminating large blocks of terms.

In my previous random-coefficient demand paper, I noted that there was a fundamental conflict between the non-parametric approach's and the econometric approach's definitions of stable tastes. The non-parametric approach assumes that if prices, expenditures, and tastes do not change, then consumer's purchases will not change either. Econometric specifications always have one or more error terms. Error terms change from one time period to the next, implying changes in consumer purchases even if prices and expenditures do not change. All econometric specifications implicitly have unstable tastes. Econometric tests of taste stability are really tests of parameter stability.

One way of merging the stable tastes of non-parametric tests with the unstable specification inherent in econometric specifications is to build randomness directly into our models of consumer tastes.[3] Suppose that consumer tastes were influenced by environmental factors, such as weather, that have substantial random components. One can model this in a utility-theoretic framework by making consumer utility depend on the environmental factors as well as the goods they consume.

Let Q_t and P_t be vectors of the goods and prices, and that x_t stands for the consumer's expenditures, all indexed over time. I will assume that the consumer's utility function is separable over time, so that the theoretical maximization problem is the same in all periods, t. In order to allow for changing tastes, I make the utility function depend on an additional vector of other factors by Z_t. The utility function is $U(Q_t,Z_t)$. The consumers' optimal demand will be given by $Q(P_t,x_t,Z_t)$.

The use of additional variables in the utility function or in a demand system is not unprecedented; the use of time trends as a measure of taste shifts is one that comes immediately

[3] Another way to build randomness into the model is to assume that tastes are truly stable and to include measurement errors between the observed and actual data.

to mind. Other “Z” variables that have been used in the meat-demand literature include measures of advertising and health information. This type of formulation allows the introduction of taste and demand shifts into a formal model. There is a fixed utility function, but the quantity demanded can change over time even if prices and expenditures do not because of changes in the Z variables.

One of the differences between my approach and others is that I assume that many or all of the elements in Z are not observed. I am, however, going to assume that the Z variables follow a stable, stationary stochastic process. Let Z_{μ} be the mean value of the Z vector. I can use a first order approximation to the demand function, $Q(P_t, x_t, Z_t)$, to write the quantity demanded in time t as:

(1) $$Q_t \approx Q(P_t, x_t, Z_{\mu}) + \frac{\partial Q}{\partial Z}\left[Z_t - Z_{\mu}\right]$$

Equation (1) shows how an unstable Z allows one to start with a “fixed” utility function and get random demands. Part of (1) is $Q(P_t, x_t, Z_{\mu})$. Because Z_{μ} does not change over time, this part of the demand function is stable. The fact that Z_{μ} does not change over time also allows me to ignore it when specifying the stable part of my model.

The second part of (1) is unstable. It is a function of the difference between the actual value of Z_t and its mean value. This second part serves as the random error term for the demand equations. The specification of the random component is (1) is more complex than usual. It is the quantities’ Z-derivative matrix times the deviation of the Z from their mean values. Equation (1) can be written in a more conventional format by replacing the complex error function with a single random error as in (1a), below.

(1a) $$Q_t \approx Q(P_t, x_t, Z_{\mu}) + e_t$$

So far my model explains how random, unobserved factors can produce econometric equations with random error terms. Why would this also produce random coefficients or elasticities of demand? One way of incorporating Z variables into a demand system is to make the demand system's parameters a function of the Z. Part of the randomness of the Z variables may translate into randomness of the coefficients.

One of the peculiar features of my first random-coefficient paper was the fact that I had trouble getting my second-stage estimate of the model's random coefficient covariance matrix to have full rank. If the covariance matrix has less than full rank then this has interesting implications about the Z_t vector. If there are as many Z_t as there are coefficients, then the covariance matrix of random coefficients will have a full rank. If there are fewer factors than coefficients, then the covariance matrix will not have full rank. Testing the rank of the random coefficient matrix allows one to put bounds on the number of factors causing shifts in price and income effects. In other words, it may be possible to count how many Z_t are missing from the "true" model.

Comparing the Heteroskedastic and Swamy-Tinsley Approaches to Random Coefficients

The random coefficient meat demand model will be estimated using a relatively simple specification for the random coefficients. The discussion that follows is generic to the problem of random-coefficient models, not only to demand models.

Assumed that the coefficients are independently and identically distributed over time[4]. Swamy and Tinsley would specify this type of random coefficient model using the following notation:

(2) $Y_t = X_t \mathrm{B}_t$

[4] Swamy and Tinsley allow the random coefficients to have complex time-series properties in their article.

In (2) Y_t, X_t, and B_t, are the vector of endogenous variables, the vector of exogenous variables, and the vector of random coefficients. It is assumed that the B_t are independently and identically distributed over time with mean B and covariance matrix Σ. Swamy and Tinsley's estimation procedure estimates B_t and B by minimizing:

(3) $$\sum_t \left(\overline{B}_t - \overline{B}\right)' \Sigma^{-1} \left(\overline{B}_t - \overline{B}\right)$$

subject to:

(4) $$Y_t = X_t \overline{\mathrm{B}}_t$$

The terms in (3) and (4) with over-bars are the estimated parameters.

Swamy and Tinsley demonstrated that their procedure provided efficient estimates of the mean parameter vector, B given the true covariance matrix, Σ. The problem with their approach is that it requires the true Σ matrix or, at the least, a good estimate. The estimates of B_t are less efficient. In particular, they noted that the estimated B_t vector is much more tightly distributed around the mean than the true B_t. This "tightness" of the B_t estimates prevents one from being able to start with an arbitrary covariance matrix, estimate the B_t, then use that estimate to get a consistent estimate of Σ.

Since this kind of sequential estimation will not produce a consistent estimate of Σ, what about simultaneously estimating all the model's parameters? Swamy and Tinsley used the following likelihood function, (5), to demonstrate that FIML estimates would not exist either:

(5) $$-\tfrac{1}{2}\sum_t \left(\overline{\mathrm{B}}_t - \overline{\mathrm{B}}\right)' \overline{\Sigma}^{-1} \left(\overline{\mathrm{B}}_t - \overline{\mathrm{B}}\right) - \tfrac{T}{2}\log\left(\det(\overline{\Sigma})\right)$$

Equation (5) would be maximized subject to (4). The Σ with an over-bar in (5) is the estimated value of Σ. The problem with maximizing (5) is that its value becomes indefinitely large when the estimate of Σ becomes singular. It is very easy to make this estimate singular. The classic linear model with only random intercepts has a singular Σ. The more restricted model, the classic linear model, can have a larger likelihood than the more general model.

The heteroskedastic formulation takes the Swamy-Tinsley model and specifies it as if it were the classic, linear model:

(6) $Y_t = X_t \mathrm{B} + e_t$

where e_t is defined as:

(7) $e_t = X_t\left(\overline{\mathrm{B}}_t - \overline{\mathrm{B}}\right)$

The covariance matrix of e_t will be denoted by σ_t and is defined by:

(8) $s_t = X_t'\Sigma X_t$

My approach in the first paper was to estimate the mean parameter vector in the first stage. In the second stage I got an estimate of Σ by correcting these errors for heteroskedasticity. The third stage used the estimated Σ to generate an estimate of σ_t That I used in a generalized least squares, GLS, procedure.

The GLS problem can be converted to a FIML by maximizing the following likelihood function:

(9) $-\frac{1}{2}\sum_t \left[\left(Y_t - X_t\overline{\mathrm{B}}\right)'\bar{s}^{-1}\left(Y_t - X_t\overline{\mathrm{B}}\right) + \log\left(\det(\bar{s}_t)\right)\right]$

subject to:

(10) $\bar{s}_t = X_t'\overline{\Sigma}X_t$

The FIML problem defined in (9) and (10) does not fully solve the problem of the singular covariance matrix. The likelihood function defined in (9) still exists if the estimate of Σ is singular. Only the estimate of σ_t has to have full rank. On the other, it is still possible to make the likelihood defined in (9) infinitely large by making the estimate of σ_t singular. To do this, select any period, t, in the sample. Calculate a mean parameter vector that perfectly predicts the value of Y in that period. Then set up the coefficient covariance matrix so that:

(11) $$0 = \bar{\Sigma} X_t$$

The estimated covariance matrix in (11) is singular, which need not generally be a problem, except that in this case it make the σ_t estimate singular also. This makes the likelihood in (9) infinitely large by making the determinant of the estimate of σ_t zero.

One way to avoid this problem is to make the random coefficients' covariance matrix block diagonal. The intercepts will be in their own block, and the slopes in theirs. As long as the intercept's block has full rank, the estimated σ_t will also have full rank because the "X" matrix always includes intercept.

It is still possible to impose singularity of the estimate of σ_t by making the covariance block for the intercepts singular as well. This also could lead to an indefinitely large value for (9) likelihood for one or more periods. However, the conditions for the validity of FIML only require a local optimum in the neighborhood of the true parameter values. For all the models, I estimated the covariance matrix converged to a local optimum with a full-rank intercept block.

The CBS Meat Demand Model

The meat demand system is specified using Keller and Van Driel's (1985) CBS model. "CBS" stands for the Central Bureau of Statistics in the Netherlands, where Keller and Van Driel worked. The CBS model is a differential model of consumer demand. Keller and Van Driel showed how the CBS is similar to the Rotterdam model and the differential version of the Almost Ideal Demand System, (AIDS). (In their 1989 article, Barten and Bettendorf show the inverse versions of the Rotterdam, CBS, and differential AIDS model.) The CBS model starts with a set of partial differential equations that take the form:

$$\textbf{(12)} \qquad w_i \cdot \left[\partial Lnq_i - \sum_j w_j \partial Lnq_j \right] = \sum c_{i,j} \partial Lnp_j + b_j \left(\partial Lnx - \sum_j w_j \partial Lnp_j \right)$$

where:

$$\textbf{(13)} \qquad w_i = \frac{p_i q_i}{x}$$

In (12) and (13) above, q_i is the quantity of good "i," pj is the price of good "j," and x the total expenditure. The terms ∂lnq_j, ∂lnp_j, and ∂lnx are the derivatives of the logarithms of the quantity, price, and expenditures, and the $c_{i,j}$ and b_i are coefficients. The w_i are budget shares.

In order to be consistent with utility maximization, the coefficients have to meet the following restrictions:

$$\textbf{(14)} \qquad \sum_i c_{i,j} = \sum_j c_{i,j} = \sum_i b_i = 0,$$

$$\textbf{(15)} \qquad c_{i,j} = c_{j,i}, \forall i, j$$

Further, the matrix formed by the $c_{i,j}$ has to be negative, semi-definite. This negative, semi-definite restriction implies among other things that the compensated demands slope downward. It is not usually imposed in estimation of the CBS and related systems.

The system specified by equations (12-15) is a set of partial differential equations. It is not directly useful for estimating a model, as we observe prices and quantities, not derivatives. Differential demand systems are estimated using the assumption that the differential system is well approximated by a difference system. Usually, these models are specified using first differences.

The Empirical Model

Monthly data on the prices and per-capita consumption of beef, pork, chicken, and turkey were calculated and released by the USDA's Economic Research Service until the end of 1996. The model is based on the assumption that meat demand is separable from other goods. This is a common assumption in meat demand analysis, and research by Moschini, Mora, and Green (1994) suggests that this assumption is valid.

The data included observations for all months in the years 1979-1996 inclusive. There is considerable seasonal variation in the meat demand, particularly for turkey. I decided to handle this seasonal variation by using the year-to-year changes in prices, quantities, and expenditures rather than first differences. I took the difference between January 1980's and January 1979's values, etc. rather than between January 1980's and December 1979's. The heteroskedastic formulation of my meat-demand CBS model can be written:

(16)
$$s_{i,t}\left(\ln\left[\frac{q_{i,t}}{q_{i,t-12}}\right]-\sum_j s_{j,t}\ln\left[\frac{q_{j,t}}{q_{j,t-12}}\right]\right)=$$
$$a_i+\sum_j c_{i,j}\ln\left[\frac{p_{j,t}}{p_{j,t-12}}\right]+b_i\left(\ln\left[\frac{x_t}{x_{t-12}}\right]-\sum_j s_{j,t}\ln\left[\frac{p_{j,t}}{p_{j,t-12}}\right]\right)+e_{i,t}$$

In (16) the term a_i is an intercept. The intercept terms in differential demand systems are interpreted as a taste-change parameter. It represents the general drift in demand over time. Because of the way that the endogenous variables are structured, they sum to 0. In order to be consistent with the budget constraint, the four intercept terms must also sum to 0. The $s_{i,t}$ are a weighted average of the $w_{i,t}$ and $w_{i,t-12}$, as in (16) below:

(17) $$s_{i,t}=q_t w_{i,t}+(1-q_t)w_{i,t-12}$$

In (17), θ_t is weighting parameter whose value changes over time.

As noted above, differential demand systems are estimated based on the assumption that the difference equations are a good approximation to the differential system. However, what is true for the differential equation is only approximately true for the difference equation. The weighted-average budget share improves the difference approximation by making it meet the same conditions as the differential equation. The following equation is used in setting up the CBS, Rotterdam, and differential AIDS models:

(18) $$\partial \ln x = \sum_i w_i \partial \ln p_i + \sum_i w_i \partial \ln q_i$$

Equation (18) is derived from the budget constraint. Equation (18) holds for the budget constraint's derivatives, but is only approximately true for budget constraint differences. The value of the weighting factor, θ_t, is calculated so the difference-equation version of (18) holds as a strict equality in each period. The values of θ_t range between 48-52% and average 50%.

I assumed that all the theoretical restrictions held for both the mean and time-varying parameter values. There are 4 a_i parameters, four b_i, and 16 $c_{i,j}$ terms. The equality constraints allow me to use elimination and substitution to reduce these 24 coefficients to 12: 3 a_i, 3 b_i and 6 $c_{i,j}$. Keller and Van Driel specified the endogenous variables so that they sum to 0, and are automatically consistent with the budget constraint. The full covariance matrix of error terms has a rank of 3 instead of 4. This type of system is estimated by dropping an equation; I dropped turkey. FIML estimates are independent of the equation dropped.

Tests, Results, and Estimates

I used a stepwise procedure in the estimation and testing of the model. The first allowed the b_i and $c_{i,j}$ to be random and checked to see if allowing them to be random improved the likelihood stastitic. The most general model that I estimated had a block-diagonal covariance matrix with three blocks: one block for the intercepts, the a_i, one for the expenditure terms, the b_i, and one for the $c_{i,j}$. The two additional blocks of the random-coefficient covariance matrix added 27 independent coefficients to the model. I compared the random coefficient model to one with only random intercepts using a likelihood ratio test.

Allowing for random slopes increased the value of the likelihood by 22.7. Assuming twice the difference in the likelihood is distributed as a chi-square with 27 degrees of freedom, this increase in the likelihood is significant at the 1.5% level.

Covariance matrices have to be positive, semi-definite; that is, they have sign constraints. Because the covariance matrix has sign constraints, there is a non-zero probability that the unconstrained and constrained models have the same likelihood. If the null hypothesis is true, very low values of the test statistic will be seen much more often than they would be if the test were truly distributed as a chi-square. It is likely that the true level of significance is even higher than the calculated level of 1.5%.

Statistical theory implies that negative variances or covariance-matrices that are not positive, definite are impossible. Mathematical optimization routines do not “know” statistical theory and can attempt to use these “impossible” values, which then generally causes the program to crash. To prevent this problem, I imposed the sign constraints by specifying the covariance matrix as the product of its Cholesky decomposition and its transpose. In the first phase, I restricted the lower bound of certain elements of the decomposition of the covariance matrix so that the covariance matrix would have full rank. The covariance block for the b_i had very small elements, and the restricted elements were all at their lower bounds. Eliminating the b_i covariance block in actually led to a trivial (0.0001) increase in the likelihood, because forcing this part of the covariance matrix to be positive actually decreased the likelihood slightly. For subsequent analysis, I accept the hypothesis that the b_i coefficients are not random. Recall that the bi coefficients multiply the expenditure terms and, hence, determine the expenditure elasticities. I can attribute all the increase in the likelihood statistic to the randomness of the $c_{i,j}$ coefficients. These terms determine price elasticities.

In the second phase, I tested the rank of the $c_{i,j}$ block. This block has a maximum rank of 6. Because I used the Cholesky decomposition to specify the $c_{i,j}$ block's covariance matrix, I could test the rank of the block by dropping its columns. The first column has 6 elements, the second 5, and so on. I could drop the last three columns without affecting the measured value of the likelihood at all. The third column, with 3 free terms, only added 0.9 to the likelihood. Dropping the last 2 columns decreased the likelihood by a statistically significant amount. Again, the "true" test distribution is likely to be skewed toward zero compared to the chi-square distribution, so the "true" significance level is higher than the calculated one.

The tests on the covariance matrix suggest that only the price-elasticity terms have random coefficients, and further that there are only two causes of randomness in price elasticities. These causes would be the unknown "Z_t" variables defined earlier in this paper. Presumably, if we could uncover what these two factors were, we could improve our analysis of meat demand.

All the estimated b_i were small. It might be the case that the b_i covariance-block is 0 because the expenditure terms in the CBS model are irrelevant[5]. In addition, the a_i introduce an element of drift in the demand equations. If these intercepts' mean terms are all 0, then tastes, corrected for environmental factors, are stable over the estimation period. The last phases of the estimation tested the intercept and expenditure terms. The test statistic for the intercepts was 99, while that for the expenditure terms was 3. Both of these tests have 3 degrees of freedom. The

[5] The smaller a b_i, the less meat i's budget share changes with meat expenditures. If all the b_i are 0, then all the meat expenditure elasticities are 1.

expenditure terms are not statistically significant, but the intercepts are at all conventional levels. These tests suggest that the all the meat-expenditure elasticities are 1 and that there is unexplained drift in the consumer tastes for meat.

I dropped the b_i, the expenditure terms, from the final model because they were not significant. Table 1 shows the mean parameter estimates and the elasticities associated with the mean parameters and the mean budget shares. Because all the b_i are 0, the expenditure elasticities are all 1. The a_i and c_{ij} were estimated with the equality constraints of utility theory imposed. The mean c_{ij} estimates are negative semi-definite. Table 2 shows the estimated covariance matrix. The covariance terms are rather small, so I multiplied their estimates by 10,000 to make them easier to read.

Implications and Directions for Further Research

Knowing more about the consumer demand for meat should improve our analysis of livestock and meat markets. What do the results of this study add to our knowledge of consumer demand for meats? Some of the results of this analysis are broadly consistent with previous studies. For instance, the fact that the intercepts are statistically significant supports the hypothesis that consumers' tastes have changed over time, which is a common result from previous analysis. The intercept values imply declining pork and beef consumption and rising poultry consumption even in the absence of price changes. Changes in meat expenditures and own-price effects are more important determinants of demand than cross-price effects.

I used the work by Moschini, Mora, and Green to justify the separating meat demand from other goods. As a group, all the meat expenditure elasticities are not significantly different from 1, and this implies that the demand for meat is homothetic and, consequently, strongly separable. This strong separability would simplify including meat demand in more complete demand systems. The four meat quantities and prices could be replaced with an aggregated meat quantity and price.

It may be fruitful to pursue improvements in the econometric technique. For one thing, the stochastic structure that I imposed on my random coefficients is very simple. Swamy and Tinsley considered auto-correlated random coefficients. Swamy and Tinsley were interested in the forecasting application of their random coefficient models. If there is autocorrelation in consumers' random meat demand elasticities, this can be used to improve the forecasting performance of the models. To incorporate very general autocorrelation into the heteroskedastic framework, you would expand the size of the covariance matrix to include cross-time correlation as well as cross-equation correlation. In this case, rather than inverting 204 (3 by 3) you would have to invert one (612 by 612) matrix. I am not sure that we have the hardware and software to reliably handle problems of this size. It might be possible to reduce the size of the problem by restricting the form of autocorrelation you consider.

I have argued that econometric demand equations are not consistent with the usual definitions of stable tastes. If we change the definition of stable tastes to include environmental or other factors whose values change randomly over time, then we can relate unstable econometric equations to stable tastes. The most intriguing result of this study is that the estimates suggest that only two of these factors cause random changes in the price elasticities of demand. The research cannot determine what these two are; further research that does identify these factors would be very helpful.

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Table 1—Mean parameter estimates and the implied elasticities of demand

Mean parameter estimates						
	Price coefficients				Meat expenditure[1]	intercepts
	beef	pork	chicken	turkey		
beef	-0.1554	0.1143	0.0216	0.0195	0.0000	-0.0056
pork	0.1143	-0.1229	0.0041	0.0044	0.0000	-0.0006
chicken	0.0216	0.0041	-0.0225	-0.0032	0.0000	0.0047
turkey	0.0195	0.0044	-0.0032	-0.0207	0.0000	0.0016
Implied elasticities						
	Own & Cross-price				Meat expenditure[1]	intercepts
	beef	pork	chicken	turkey		
beef	-0.827	-0.058	-0.110	-0.004	1.000	-0.010
pork	-0.116	-0.725	-0.135	-0.024	1.000	-0.002
chicken	-0.395	-0.243	-0.300	-0.062	1.000	0.031
turkey	-0.057	-0.160	-0.230	-0.553	1.000	0.039

[1] The meat expenditure terms were constrained to 0, making all the meat expenditure elasticities 1.

Table 2—Random-Coefficient Covariance Matrix Blocks, times 10,000

Intercept covariance matrix

	beef	pork	chicken	turkey
beef	0.82	-0.41	-0.28	-0.13
pork	-0.41	0.50	-0.13	0.03
chicken	-0.28	-0.13	0.40	0.01
turkey	-0.13	0.03	0.01	0.09

c_{ij} matrix covariance matrix, using symmetry to drop terms[1]

		beef beef	beef pork	beef chicken	beef turkey	pork pork	pork chicken	pork turkey	chicken chicken	chicken turkey	turkey turkey
beef	beef	60.36	-42.75	-17.84	0.23	18.71	16.67	7.36	-0.84	2.01	-9.61
beef	pork	-42.75	30.69	12.28	-0.22	-15.61	-11.52	-3.56	1.15	-1.91	5.69
beef	chicken	-17.84	12.28	5.58	-0.02	-3.50	-5.18	-3.61	-0.23	-0.17	3.80
beef	turkey	0.23	-0.22	-0.02	0.01	0.39	0.03	-0.20	-0.08	0.07	0.11
pork	pork	18.71	-15.61	-3.50	0.39	19.17	3.55	-7.11	-3.43	3.38	3.34
pork	chicken	16.67	-11.52	-5.18	0.03	3.55	4.80	3.17	0.15	0.22	-3.42
pork	turkey	7.36	-3.56	-3.61	-0.20	-7.11	3.17	7.50	2.13	-1.69	-5.61
chicken	chicken	-0.84	1.15	-0.23	-0.08	-3.43	0.15	2.13	0.76	-0.68	-1.36
chicken	turkey	2.01	-1.91	-0.17	0.07	3.38	0.22	-1.69	-0.68	0.64	0.98
turkey	turkey	-9.61	5.69	3.80	0.11	3.34	-3.42	-5.61	-1.36	0.98	4.52

[1] This matrix has a rank of 2. Its maximum rank is 6.