The Impact of Data Frequency On Stationarity Tests
Of Commodity Futures Prices

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Introduction

➢Unit root testing is one of the most important procedures when performing time series analysis and it is crucial to test the stationarity of the time series in hand accurately and efficiently. Two paths can lead researchers to achieve this goal: obtaining more data, or improving the unit root test. This paper addresses both approaches.

➢For collecting more data, one can either collect data from a longer time period or use higher frequency data while keeping the same time span. Some research has said increasing the frequency while keeping the sample period constant does not change the mean reversion within the sample (Boswijk and Klaassen 2012). However this is not the case if the low frequency data is constructed by systematic sampling, i.e., skipping certain intermediate observations from high frequency sample, and this type of sampling is usually seen in stock market or asset market variables. For example, researchers sometimes pick the price of one day each week to construct weekly data from daily data.

➢Choi (1992) demonstrated by simulation that this kind of data aggregation will lower the power of augmented Dickey-Fuller and Phillips-Perron tests, although Chambers (2004) showed that this is a finite sample effect and asymptotically it is still possible to consistently test for a unit root when sampling frequency varies. Recently, Boswijk and Klaassen (2012) proved that the effects of systematic sampling on unit root testing is not negligible when a high-frequency sample has volatility clustering with fat-tailed innovations, which are the typical characteristics of financial market data. They simulated data sets and using likelihood ratio-based tests and conclude that these tests can have more power than the traditional ADF test on data processes holding the aforementioned behavior characteristics.

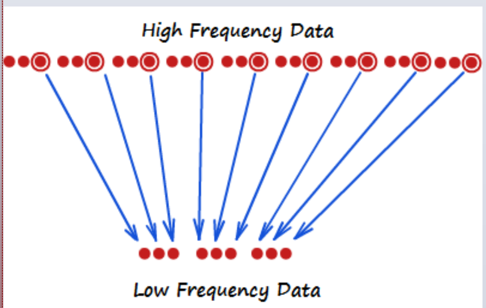

*Figure 1. Systematic Sampling***

➢Although these tests increased the power when testing the financial data, one of the common issue for the existing testing methods is that they all require some specific model specification assumption, either for the functional form (e.g., the ADF test requires the number of lags to be specified in the model) or the error term distribution (Gaussian distribution, GARCH, etc).

➢However, model misspecification may lead to erroneous conclusions since the unit root test results may well depend on the particular model considered (Moral-Benito 2013).

Objective

➢In this paper, we will devote efforts in the two aforementioned directions in hopes of improving unit root test results.

➢Using data on 5 commodity futures prices (corn, soybean, cotton, live cattle and lean hog), which all display typical financial series characteristics, we first show that systematic sampling does have effects on the results of unit root testing by testing three different frequency samples: daily, weekly, and monthly.

➢Then, more importantly, we will test the stationarity of these series by averaging 24 models using a Bayesian Model Averaging unit root test method derived in the previous chapter to confront the model specification uncertainty issue, and compare results with traditional unit root tests to show the performance of the BMA methods, as well as its ability to handle the model specification issue.

Data

➢5 commodity futures prices series are used to test and compare the unit root results: corn, soybean, cotton, live cattle and lean hog. To evaluate the effect of data frequency on the testing result, 3 different frequencies are used for each series: daily, weekly and monthly.

➢The high-frequency sample is the real daily settled price of each commodity from Chicago Board of Trade (corn, soybean, live cattle and lean hog) and Intercontinental Exchange (cotton).

➢Each daily data sample size is 2,000 which is from March, 2007 to March, 2015. The low-frequency sample is constructed from the daily data by what is usually referred to as systematic sampling.

➢Assume the daily sample is $\mathbf{Y}_t$, we skip certain observations to achieve the low-frequency data (Boswijk and Klaassen 2012):

$$\mathbf{Y}_j^* = \mathbf{Y}_{mj},\ j = 0,\ldots,n^* = \frac{n}{m}$$

➢For weekly data we take m=5 and m=20 for monthly data, which can be treated as end-of-week and end-of-month price given 5 trading days in a week and 20 days in a month. Since the daily data sample size is 2000, the constructed weekly sample size is 400 and the monthly data size is 100.

Methods

➢A robust numerical Bayesian unit root test for model uncertainty is adopted to analyze the data. Unlike the traditional unit root methods which all require a certain level of model specification, this newly developed approach allows us to fully consider model uncertainty through Bayesian model averaging technique.

➢The basic idea is generalized from Dorfman (1993) who presented some early Bayesian unit root tests. We specify priors on the moduli of the eigenvalues of the following matrix which fundamentally drives the dynamic behavior of the system.

$$A = \begin{bmatrix} \rho_1 & 1 & 0 & 0 & \cdots & 0 \\ \rho_2 & 0 & 1 & 0 & \cdots & 0 \\ \rho_3 & 0 & 0 & \ddots & & \vdots \\ \vdots & 0 & 0 & \cdots & 0 & 1 \\ \rho_p & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

➢To incorporate model uncertainty in the mean function as well as the variance structure, 24 models are averaged to come to a final comprehensive conclusion, which can be categorized into 4 groups by variance structure:

1. GARCH (1, 1) with Student's t distribution
2. GARCH (1, 1) with Normal distribution
3. ARCH (1,1) with Student's t distribution
4. AR model with Student's t distribution

And for each of the error specifications, the mean function is specified as an autoregressive model with maximum lag varying from 1 to 6.

➢The priors are specified as follows:

1. Dominant root: Beta (30,2), all other root: Beta (1.1,1.1);
2. GARCH/ARCH Coefficients: N(0, 3) with indicator function to make sure positive variance structure;
3. Variance of normal likelihood: Inverse gamma
4. Degrees of freedom of Student's t distribution: truncated exponential distribution with following form:

$$p(\nu \mid \lambda,\delta) = \lambda \exp(-\lambda(\nu-\delta)) \bullet I(\nu > \delta)$$

Table 1. Test Results of Five Commodity Futures Prices Data

		BMA	DF 1	DF 2	DF 3	DF 4	DF 5	DF 6	PP
CORN	Day	0.503	0.576	0.538	0.475	0.339	0.390	0.315	0.54
	Week	0.717	0.530	0.535	0.497	0.577	0.648	0.622	0.55
	Mon	0.784	0.576	0.538	0.475	0.338	0.390	0.315	0.52
SOY BEAN	Day	0.372	0.271	0.312	0.282	0.332	0.359	0.357	0.30
	Week	0.406	0.346	0.381	0.319	0.296	0.277	0.256	0.32
	Mon	0.481	0.227	0.098	0.163	0.113	0.043	0.026	0.26
COT TON	Day	0.498	0.622	0.627	0.628	0.624	0.579	0.647	0.62
	Week	0.535	0.593	0.651	0.662	0.641	0.550	0.499	0.62
	Mon	0.685	0.489	0.320	0.216	0.468	0.463	0.402	0.52
LIVE CAT	Day	0.222	0.537	0.570	0.545	0.559	0.546	0.553	0.54
	Week	0.254	0.530	0.535	0.497	0.577	0.648	0.622	0.55
	Mon	0.408	0.576	0.538	0.475	0.338	0.390	0.315	0.52
LEAN HOG	Day	0.141	0.084	0.060	0.010	0.010	0.010	0.010	0.01
	Week	0.188	0.010	0.010	0.010	0.010	0.010	0.018	0.01
	Mon	0.396	0.010	0.010	0.010	0.010	0.010	0.026	0.01

Then the averaged probability of a unit root across the model space is:

$$pr(\phi_1 \ge 1 \mid \mathrm{x}) = \sum_{i=1}^{k} pr(\phi_1 \ge 1 \mid \mathrm{M}_k, \mathrm{x}) pr(\mathrm{M}_k \mid \mathrm{x})$$

The posterior density cannot be integrated directly so a numerical technique, so Gibbs sampling with a Metropolis–Hastings (MH) step is adopted to generate a sample from the posterior probability distribution of the dominant root.

Result

➢For each data series the robust test is based on averaging 24 models. For each model, we used 51,000 Monte Carlo iterations of the Gibbs with MH algorithm and discarded the first 21,000 draws to achieve better convergence and better posterior sample mixture. The Geweke test (Geweke 1992) is adopted to examine the convergence of each posterior sample.

➢Generally speaking the results vary across different commodities as well as different data frequencies, which is the focus of this paper. First, notice that with our the BMA result, although the probability of having a unit root varies for different frequency samples, the test conclusions are basically consistent except for cotton, which might be thought as "marginally stationary" and could be caused by sampling error.

➢Another result to notice is that for each commodity, the probability of a dominant root greater than 1 computed by averaging 24 models using BMA method increases as the frequency of tested data decreases. This indicates that more mean-reversion information is provided by using the high frequency data. To be more specific, high frequency samples carry more information through high volatile and fat-tail behavior which can be captured by GARCH and ARCH models with Student's t distributions in the BMA method. This information will be lost when constructing low frequency sample through systematic sampling and will be ignored by traditional methods like ADF or PP test.

➢The most desirable property of our BMA method is it can handle model specification uncertainty in a unit root test. Sometimes model uncertainty causes contradictory results which could lead to a misspecified model. Take soybean monthly data as an example. Using the ADF test on the monthly data and under a commonly used 10% significance level, it is confirmed nonstationary if the model specification is AR (1), AR (3) or AR (4). In contrast, for AR (2), AR (5) and AR (6) the test indicates stationarity of the data, opposite to the result using other lags as well as daily and weekly data. So the model specification uncertainty problem is important here since improper specification of the lag will lead to completely different results which will affect the following analysis. The BMA method confronts this problem by averaging all 6 possible lag specification (or more if the researcher needed) and reaching a final, more robust conclusion.

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