Effects of restrictions on parameter estimates of US agricultural production

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1 - Overview

The economic theory of producer behavior requires certain conditions to hold in order for a functional form to be representative of a production technology.

Agricultural production studies are usually conducted using classical econometrics that do not allow for the imposition of curvature conditions in flexible functional forms. Therefore, some conditions required by economic theory do not hold globally in estimation.

Some studies report the proportion of the sample for which curvature conditions do not hold, and the reader is warned about the unknown distorting effects that those data points might have on their final results.

Bayesian methods allow for the imposition of first- and second-order restrictions in the estimation of flexible functional forms.

We estimate a flexible representation of the US agricultural production technology using Bayesian econometrics under alternative sets of restrictions, and elaborate on the effects of the restrictions on the pdfs of the parameter estimates.

2 - Objective

To show how the pdfs of the parameter estimates change when curvature restrictions are imposed to guarantee that the theoretical properties of production functions hold.

3 - Data

USDA data on agricultural production by State for 1960 through 2004 is used in estimation: 48 states, 11 regions, 1 aggregate agricultural output, 3 variable inputs (capital, labor, materials). Data details provided in Ball et al. (2004).

4 - Methodology

The proposed production function $y = f(X,D,t)$ relating output, y, with inputs, $X=x_1, x_2, x_3$, time, t, and regional dummies, $D=D_1,\ldots,D_{11}$, consists of:

$$f(X,D,t) = \sum_{r=1}^{11}\beta_{0r}D_r + \sum_{i=1}^{3}\beta_i x_i + \beta_t t + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\beta_{ij}x_i x_j + \sum_{i=1}^{3}\beta_{ti}x_i t + \frac{1}{2}\beta_{tt}t^2$$

where $\beta_{ij}=\beta_{ji}$ by Young's theorem.

- Monotonicity requires that the output elasticity with respect to each input be positive:

$$\frac{\partial f(X,D,t)}{\partial x_i} = \beta_i + \sum_{j=1}^{3}\beta_{ij}x_j + \beta_{ti}t \geq 0$$

- Concavity requires that for any conformable vector A and the Hessian H:

$$A'HA \leq 0;\ H = \begin{pmatrix} \beta_{11} & \cdots & \beta_{13} \\ \vdots & \ddots & \vdots \\ \beta_{13} & \cdots & \beta_{33} \end{pmatrix}$$

6 Models are estimated using R:

- Model 1: Unrestricted estimation
- Model 2: Concavity imposed in estimation
- Model 3: Monotonicity at data means imposed in estimation
- Model 4: Both monotonicity at means and concavity imposed in estimation
- Model 5: Monotonicity at all data points imposed in estimation
- Model 6: Both monotonicity at all points and concavity imposed in estimation

Model estimated using Monte Carlo Markov Chain methods, with 4 chains consisting of 5 million draws per chain. The first half of each chain was discarded for burn in purposes, yielding a total of 10 million simulated values for each parameter.

5 - Results

	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev
β_K	1.218	0.331 ***	-0.638	0.197 ***	1.622	0.279 ***	0.956	0.195 ***	2.918	0.308 ***	0.146	0.071 **
β_L	0.122	0.094	0.757	0.072 ***	0.076	0.088	0.641	0.078 ***	0.055	0.049	0.734	0.061 ***
β_M	1.047	0.111 ***	1.201	0.097 ***	0.972	0.106 ***	1.015	0.105 ***	0.063	0.089	0.946	0.076 ***
β_{KK}	1.615	0.223 ***	-0.088	0.049 *	1.415	0.207 ***	-0.660	0.103 ***	-0.382	0.068 ***	-0.032	0.019 *
β_{LL}	0.089	0.034 ***	-0.198	0.024 ***	0.099	0.033 ***	-0.180	0.026 ***	-0.021	0.020	-0.136	0.012 ***
β_{MM}	-0.302	0.047 ***	-0.338	0.039 ***	-0.295	0.046 ***	-0.278	0.040 ***	-0.286	0.039 ***	-0.217	0.021 ***
β_{KL}	-0.933	0.074 ***	-0.078	0.034 **	-0.946	0.072 ***	-0.035	0.044	-0.253	0.033 ***	-0.003	0.008
β_{KM}	-0.051	0.094	0.106	0.047 **	-0.032	0.094	0.076	0.057	0.056	0.048	0.006	0.010
β_{LM}	0.355	0.029 ***	0.257	0.029 ***	0.356	0.029 ***	0.222	0.030 ***	0.322	0.021 ***	0.171	0.014 ***
β_t	0.008	0.007	0.030	0.008 ***	0.005	0.007	0.011	0.008	-0.016	0.007 **	0.014	0.008 *
β_{tK}	-0.023	0.010 **	-0.024	0.007 ***	-0.023	0.010 **	-0.025	0.008 ***	-0.021	0.009 **	-0.001	0.002
β_{tL}	0.006	0.003 *	0.003	0.003	0.006	0.003 *	0.003	0.003	-0.001	0.001	0.001	0.002
β_{tM}	0.031	0.004 ***	0.037	0.003 ***	0.031	0.004 ***	0.037	0.004 ***	0.048	0.003 ***	0.036	0.003 ***
β_{tt}	-0.0006	0.0003 *	-0.0015	0.0003 ***	-0.0004	0.0003	-0.0006	0.0003 *	0.0003	0.0003	-0.0007	0.0003 **
σ^2	0.498	0.015	0.627	0.019	0.498	0.015	0.662	0.020	0.628	0.021	0.679	0.020
Likel.	-314.3	3.8	-563.6	3.9	-315.3	3.7	-620.7	4.1	-563.8	12.2	-648.7	5.2

Levels of significance: ***1%, **5%, *10%

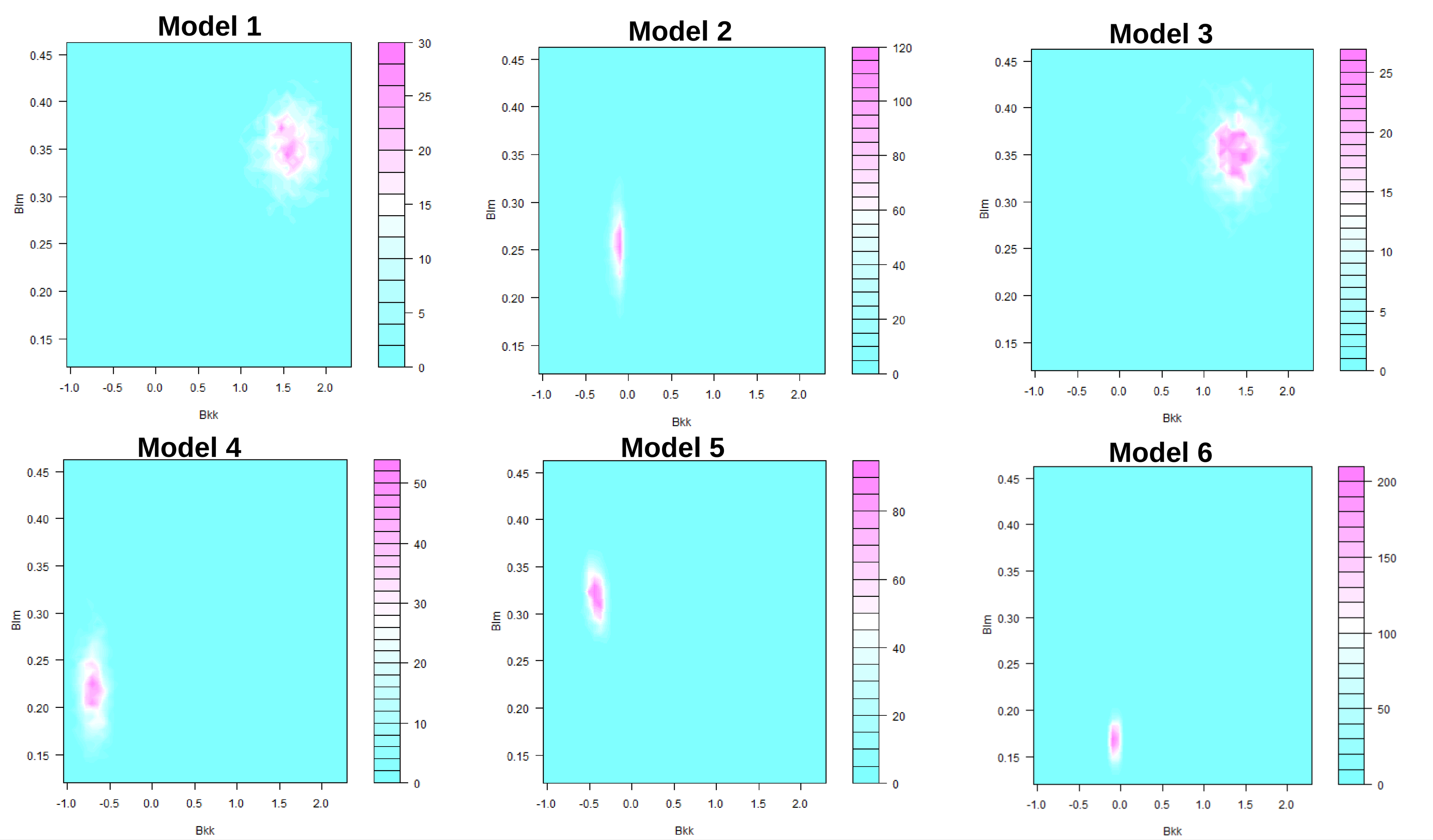
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Elasticity wrt K	-0.1657	-1.2169	0.0969	0.0203	1.7790	0.0942
Elasticity wrt L	0.4403	0.8433	0.4040	0.7219	0.3992	0.7861
Elasticity wrt M	1.8906	2.0367	1.8470	1.8717	1.3370	1.7314
Concavity at data means	Not concave	Concave	Not concave	Concave	Concave	Concave
Elasticity of Scale	2.1652	1.6631	2.3478	2.6139	3.5151	2.6117

Monotonicity violated

Concavity violated

All values computed at data means with mean parameter estimates

Example: Bivariate pdfs of β_{KK}, β_{LM}



6 - Key Findings

- **The technology recovered from this data set is neither monotonic nor concave in primal space. Therefore, conditions must be imposed to perform economic analysis.**
- **Imposing concavity (Model 2) reduces the goodness of fit of the model significantly more than imposing monotonicity at data means (Model 3).**
- **The output elasticities obtained after imposing monotonicity at data means and concavity (Model 4) are similar to those obtained imposing monotonicity at all data points and concavity (Model 6).**
- **The concavity condition calculated using mean parameter estimates obtained after imposing monotonicity at all data points (Model 5) is satisfied at the data means, but the implied output elasticities are substantially different from those obtained imposing both monotonicity and concavity (Models 4 and 6).**
- **US agricultural production showed increasing returns to scale over 1960-2004**

7 – Next Steps

- Analyze effects of imposing restrictions on agricultural technology in dual space, and joint primal-dual models.

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