Prices versus Quantities versus Hybrids in the Presence of Co-pollutants

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Abstract: We investigate the optimal regulation of a pollutant given its interaction with another controlled pollutant under asymmetric information about firms' abatement costs. The co-pollutant is regulated, but perhaps not efficiently. Our focus is on optimal instrument choice in this setting, and we derive rules for determining whether a pollutant should be regulated with an emissions tax, tradable permits, or a hybrid price and quantity policy, given the regulation of its co-pollutant. The policy choices depend on the relative slopes of the damage functions for both pollutants and the aggregate marginal abatement cost function, including whether the pollutants are complements or substitutes in abatement and whether the co-pollutant is controlled with a tax or tradable permits.

Keywords: Emissions trading, emissions taxes, cap-and-trade, uncertainty, price controls, hybrid policies, prices vs. quantities

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1 Introduction

Concerns about how best to control greenhouse gases have generated intense interest in the co-benefits and adverse side-effects of climate policies. Perhaps the most well studied co-benefits of climate policy are the effects on flow pollutants like NOX, SO2 and PM that are emitted along with CO2 in combustion processes. Efforts to reduce CO2 emissions can reduce emissions of these pollutants providing a co-benefit of climate policy. The Intergovernmental Panel on Climate Change (IPCC) has reviewed many empirical studies of these co-benefits in Chapter 6 of IPCC (2014), and they have concluded that the benefits of reductions in emissions of CO2 co-pollutants can be substantial.[1] On the other hand, climate policy can also have adverse consequences, some of which come from increases in related pollutants. For example, Ren et al. (2011) suggest that increased use of biofuels as part of a policy to reduce CO2 emissions can result in greater water pollution from agricultural runoff.[2]

The presence of co-benefits or adverse side-effects presents challenges for efficient pollution regulation. The efficient regulation of one pollutant must account for how its control affects the abatement of its co-pollutants, and how the abatement interactions translate into changes in the damages associated with its co-pollutants. In addition, accounting for existing regulations of co-pollutants is critical for determining the net co-benefits or adverse consequences of pollution control.[3] Of course, full efficiency would require that the regulations of multiple interacting pollutants be determined jointly to maximize the net social benefits of a complex environmental regulatory system, but this is not realistic. Environmental regulations tend to focus on single pollutants, not joint regulations of multiple pollutants, and these single-pollutant regulations are likely to be inefficient for a host of reasons. At best, regulation of a particular pollutant may strive for efficiency, given the not-necessarily-efficient regulation of its co-pollutants.

[1] Nemet et al. (2010) surveyed empirical studies of air pollutant co-benefits of climate change mitigation and found a mean value of \$49 (2008 dollars) per ton of CO2 reduction. Similarly, Parry et al. (2014) calculated the average co-benefits for the top 20 CO2 emitting countries to be about \$57.5 for 2010 (in 2010 dollars). These values are about the same magnitude as estimates for the climate-related benefit per ton of CO2 reduction developed by the US Interagency Working Group on the Social Cost of Carbon. Using a 3% discount rate, the Interagency Working Group proposes a schedule for the social cost of carbon dioxide to be used in regulatory impact analysis that starts at \$33 (2007 dollars) per ton CO2 in 2010 and rises to \$71 per ton in 2050 (US Interagency Working Group on Social Cost of Carbon 2013).

[2] The IPCC considers many more ancillary consequences of climate policy besides those generated by co-pollutants. These include the effects of climate policy on other social goals like food security, preservation of biodiversity, energy access, and sustainable development. Many economists will be familiar with the effects of climate policy on the efficiency of broader tax policy (e.g., Goulder et al. 2010). In this paper we remain focused on regulation in the presence of co-pollutants.

[3] Accounting for existing co-pollutant regulation is not always present in analysis of the co-benefits of CO2 regulation (e.g., Nemet et al. 2010, Muller 2012, Boyce and Pastor 2013), but it is prominent in how the Intergovernmental Panel on Climate Change views the problem of how to account for co-benefits and adverse consequences in climate policy (see Chapter 3 of IPCC (2014)).

That is the situation we address in this paper. In particular, we investigate the optimal regulation of a pollutant given its interaction with another controlled pollutant under asymmetric information about firms' abatement costs. The co-pollutant is regulated, but perhaps not efficiently. Like most of the related literature we limit our analysis to pollutants that interact in terms of abatement, as opposed to multiple pollutants that have damage-related interactions. Consequently, we are concerned with pollutants that are either complements or substitutes in abatement. Our focus is on optimal instrument choice in this setting, and we derive rules for determining whether a pollutant should be regulated with an emissions tax, tradable permits, or a hybrid price and quantity policy, given the regulation of its co-pollutant.

Our work brings together two literatures, instrument choice under uncertainty and the regulation of multiple pollutants, although we are not the first to do so. Like the research interest in regulating multiple interacting pollutants, the challenge of climate change has also intensified research in policy design under the immense uncertainty in the benefits and costs of controlling greenhouse gas emissions. The seminal work of Weitzman (1974) is still relevant, because the marginal damage associated with carbon emissions is almost perfectly flat over a relatively short compliance period (e.g., Pizer 2002). Hence, uncertainty in the costs and benefits of controlling greenhouse gases suggest that a carbon tax is more efficient than carbon trading. However, the preference in some circles for emissions markets over emissions taxes has generated much interest and innovation in hybrid schemes. The most popular form of these hybrids, in the literature and in actual practice, involve tradable emissions permits with price controls. This is the form of hybrid policy that we model. The conceptual foundation for these policies originated with Roberts and Spence (1976), who demonstrated that since an emissions tax and a simple permit market are special cases of such a hybrid policy, emissions markets with price controls cannot be less efficient and will often be more efficient than either of the pure instruments. The performance of alternative hybrid policies has been examined theoretically (Grull and Taschini 2011), with simulations (Burtraw et al. 2010, Fell and Morgenstern 2010, Fell et al. 2012), and with laboratory experiments Stranlund et al. (2014). Recent theoretical work has also examined technology choices in emissions markets with price controls (Weber and Neuhoff 2010) and the enforcement of these policy schemes (Stranlund and Moffitt 2014). However, the literature on the design of hybrid policies and policy choice under uncertainty almost completely ignores the setting of multiple interacting pollutants.

While there is a substantial empirical literature on the co-benefits and adverse side effects of pollutant interactions in climate change policy, the theoretical literature on regulating multiple interacting pollutants is much smaller, and much of it focuses on integrating markets for co-pollutants. For example, Montero (2001) examines the welfare effects of integrating the policies for two pollutants under uncertainty about abatement costs and imperfect enforcement. Woodward

(2011) asks whether firms that undertake a single abatement activity that reduces two kinds of emissions should be able to sell emissions reduction credits for both pollutants. Under complete information, Caplan and Silva (2005) demonstrate that a global market for carbon with transfers across countries can be linked with markets to control more localized pollutants to produce an efficient outcome. In contrast, Caplan (2006) shows that an efficient outcome cannot be achieved with taxes. While most models in this literature are static, the climate change setting has led several authors to examine dynamically efficient paths for the multiple greenhouse gases that contribute to climate change. (e.g., Kuosman and Laukkanen 2011, Moslener and Requate 2007). None of these articles consider alternative policy instrument choices for multiple pollutants under uncertainty.

The only other work that we are aware that examines instrument choice in the presence of multiple interacting pollutant under uncertainty is Ambec and Coria (2013). The main difference between our work and theirs is that they consider optimal regulation of the two pollutants simultaneously, while we investigate the optimal regulation of a single pollutant, given regulation of its co-pollutant which may not be efficient. While the two approaches are obviously complementary, we feel that ours is of a more realistic policy environment given that pollutants are usually regulated separately. Moreover, our reading of IPCC (2014) suggests that they treat the problem of co-benefits or adverse side-effects of greenhouse gas regulation as one of choosing the appropriate climate policy, given existing policies for related activities.

Another major difference between our work and Ambec's and Coria's (2013) is in the modeling of hybrid policies. They take their hybrid regulation from Weitzman (1978), and propose a policy that includes an emissions tax, an emissions standard and a quadratic penalty function for exceeding the standard. This policy can be designed to achieve a first-best outcome, but we know of no examples of such a regulation in actual practice. In contrast, our hybrid model is from Roberts and Spence (1976) and consists of an emissions market with a price ceiling and price floor. Many recent proposed and implemented markets to control greenhouse gases include some form of price control—see Hood (2010) and Newell et al. (2013) for several examples—so our approach is more in line with current policy decisions.[4]

Our efforts produce several new results with important policy implications. Our main output are complete orderings of the expected social costs of regulating a pollutant with a tax, tradable permits, or a hybrid, given the regulation of its co-pollutant with either a tax or tradable permits. Like standard instrument choice problems, these orderings depend on the relative slopes of the damage functions for both pollutants and the aggregate marginal abatement cost function, including whether the pollutants are complements or substitutes in abatement and whether the co-pollutant

[4]There are other more minor but still important differences between our work and Ambec's and Coria's. In particular, they extend their base model to examine pollutant interactions in both damages and abatement costs, as well as uncertainty about whether two pollutants are substitutes or complements in abatement. We do not extend our work to these cases, but they are likely to be important extensions for the future.

is controlled with a tax or tradable permits. Interestingly, while these orderings depend on how the co-pollutants is regulated, they do not depend on whether the co-pollutant is regulated efficiently.

The dependence of instrument choice for a pollutant on how its-co-pollutant is regulated is due to how regulation affects the expected emissions of the co-pollutant. For example, if the co-pollutant is regulated with a fixed number of tradable permits, then the regulation of the main pollutant cannot affect its emissions. In this case, the rules for instrument choice are the same as in the single-pollutant case. However, if the co-pollutant is regulated with an emissions tax, a key component of the ordering of policy instruments is how regulation of the primary pollutant affects the variation in the marginal damage of the co-pollutant. This variation is higher under a pricing scheme when the two pollutants are complements in abatement, and it is higher under a quantity schemes when the two pollutants are substitutes. Moreover, the strength of these effects increase with the slope of the marginal damage function for the co-pollutant. Consequently, complementarity in abatement tends to favor emissions markets for the primary pollutant (with or without price controls) and the strength of this effect increases with the steepness of the co-pollutant's marginal damage function. On the other hand, substitutability in abatement tends to favor emissions pricing for the primary pollutant (either as part of a hybrid policy or a pure tax) and this effect is stronger as the marginal damage for the co-pollutant is steeper.

Since the rules for determining the optimal instrument for a pollutant depend on how the co-pollutant is regulated, many examples exist in which the optimal policy for the primary pollutant changes as the form of regulation of the co-pollutant is changed. For just one example, recall the conventional wisdom that the optimal instrument for carbon emissions is a tax because the marginal damage function is essentially flat over a compliance period that is not too long. This remains true in the multiple pollutant case as long as the co-pollutant is regulated with tradable permits. However, an emissions tax may not be the optimal choice if the co-pollutant is also regulated with an emissions tax. In fact, in this case we show that a tax for the primary pollutant is sub-optimal if the marginal damage function for the co-pollutant is upward sloping and the two pollutants are complements in abatement. Many such policy reversals are possible, so the intuition about instrument choice that environmental economists have developed over many years must be modified when policies must account for co-pollutants.

The remainder of the paper proceeds as follows. In the next section we lay out the fundamental abatement costs and pollution damages included in the regulatory choice model. In section 3 we characterize the optimal taxes, tradable permits, and hybrids for the primary pollutant, given that the co-pollutant is regulated with tradable permits or an emissions tax. This exercise is simplified somewhat because the optimal tax or emissions market are special cases of the optimal hybrid regulation. Therefore, we derive optimal hybrid policies and then derive the pure tax and pure trading programs as special cases. Section 4 contains the main result of the paper, which are the rules for instrument choice for a pollutant, given the regulation of its co-pollutant, along with an extended discussion of these results. In section 5 we specify the performance of the optimal policies in terms of expected aggregate emissions of the primary pollutant. Given the regulation of the co-pollutant, we demonstrate that expected emissions of the primary pollutant under the tax and hybrid policies are the same, and in turn these are the same as the optimal number of permits in the optimal emissions market. We also show how these emissions levels differ according to how the co-pollutant is regulated and whether it is regulated efficiently. Finally, since the recent literature on instrument choice under uncertainty and regulation in the presence of co-pollutants is motivated in large measure by debates about how best to design policies to control greenhouse gas emissions, we conclude in section 6 with a discussion about how our results inform these debates and how elements of the climate regulation problem suggest directions for future research.

2 Abatement costs and damages for multiple pollutants

The analysis throughout considers regulation of a fixed number of n heterogeneous, risk-neutral firms, each of which emits two pollutants. Both pollutants are uniformly mixed so that both cause damage that depends only on the aggregate amount emitted. In this section we specify firms' abatement costs and derive the aggregate abatement cost function.[5] We then specify damages from their emissions and derive the second-best quantities of the pollutants to use as benchmarks at a later point in the analysis. We take second-best to refer to aggregate emissions that minimize expected aggregate abatement costs and damages ex ante. [6]

2.1 Firms' abatement costs and emissions

Assume that there are two kinds of pollutants. A firm i emits q_{ij} units of j^{th} pollutant ($j = 1,2$). The firm's abatement cost function is:

$$C^i(q_{i1}, q_{i2}, u) = c_{i0} - (c_{i1} + u)(q_{i1} + q_{i2}) + \frac{c_{i2}}{2}\left(q_{i1}^2 + q_{i2}^2\right) - w_i q_{i1} q_{i2}, \tag{1}$$

[5] Ambec and Coria (2013) limit their analysis of instrument choice under uncertainty to policies for a single polluting firm. This seems to us to be unnecessarily restrictive, because it is straightforward to derive the aggregate abatement cost function from individual abatement cost functions, at least under the assumptions that individual abatement costs are quadratic and emissions permits are traded competitively.

[6] A first-best policy would minimize aggregate abatement costs and damage ex post, that is, after uncertainty about costs and damage is resolved. All the policies in this paper are determined before uncertainty is resolved, so they cannot result in the first-best outcome, except by accident. This is also true of the vast majority of the literature on instrument choice under uncertainty. Of course, there are other schemes that can achieve first-best, like those that motivate revelation of private cost information (e.g., Montero 2008) and Weitzman's (1978) model of hybrid control, but these schemes are not used in actual practice.

with constants $c_{i0} > 0$, $c_{i1} > 0$, and $c_{i2} > 0$. The constant w_i is positive or negative depending on whether the two pollutants are complements or substitutes in abatement, to be specified shortly. Random shocks that affect the abatement costs of all firms are captured by changes in u, which is a random variable distributed according to the density function $f(u)$ on support $[\underline{u}, \overline{u}]$ with zero expectation.[7] The Hessian of the total abatement cost function is positive definite so that $c_{i2} > 0$ and $c_{i2}^2 - w_i^2 = (c_{i2} + w_i)(c_{i2} - w_i) > 0$. This implies that each firm's abatement cost function is strictly convex and the abatement interaction term is limited by $c_{i2} + w_i > 0$ and $c_{i2} - w_i > 0$. Moreover, we assume that the minimum of a firm's abatement costs occurs at positive (and bounded) levels of emissions for every realization of u. These minimizing values are $q_{ij} = (c_{i1} + u)(c_{i2} + w_i)/(c_{i2}^2 - w_i^2)$, $j = 1, 2$, which given $c_{i2} + w_i > 0$ and $c_{i2}^2 - w_i^2 > 0$, are strictly positive if and only if $c_{i1} + u > 0$.

Throughout the analysis firms will face prices for emissions of each of the pollutants. These are competitive prices and they are uniform across firms. Let the price of pollutant 1 be p_1 and the price of pollutant 2 be p_2. Each firm will choose its emissions of the pollutants so that

$$-C_j^i(q_{i1}, q_{i2}, u) = p_j, \; j = 1, 2.$$

The solutions for these first order conditions are:

$$q_{i1}(p_1, p_2, u) = \frac{(c_{i1} + u)(c_{i2} + w_i) - c_{i2}p_1 - w_i p_2}{c_{i2}^2 - w_i^2}, \tag{2}$$

$$q_{i2}(p_1, p_2, u) = \frac{(c_{i1} + u)(c_{i2} + w_i) - c_{i2}p_2 - w_i p_1}{c_{i2}^2 - w_i^2}. \tag{3}$$

Note that $\partial q_{ij}/\partial p_j = -c_2^i/\left(c_{i2}^2 - w_i^2\right) < 0$ and $\partial q_{ij}/\partial p_k = -w_i/\left(c_{i2}^2 - w_i^2\right)$, for $j \neq k$. Thus, the own-price effect is always negative but the cross-price effect depends on the sign of w_i. If $w_i > 0$, then an increase in the price of emissions of one pollutant leads the firm to reduce emissions of both pollutants; hence, the two pollutants are complements in abatement if $w_i > 0$. In this case, the response of both pollutants to an increase in the price of one occurs in the following way. An increase in the price of a pollutant leads the firm to reduce its emissions of that pollutant. This, in turn, shifts the marginal abatement cost function for the co-pollutant down, which produces a decrease in emissions of that pollutant, given its price. On the other hand, if $w_i < 0$, then an increase in the price of one pollutant leads the firm to reduce its emissions of that pollutant, but to increase emissions of the co-pollutant because the marginal abatement costs of that pollutant is

[7] Introducing abatement cost uncertainty via a common random term is clearly a simplification. Yates (2012) shows how to aggregate idiosyncratic uncertainty in individual abatement costs to characterize uncertainty in an aggregate abatement cost function.

higher. In this case, the two pollutants are substitutes in abatement.

2.2 Aggregate abatement costs and emissions

Since policies in this paper will feature uniform prices for both pollutants, aggregate abatement costs will be minimized, given aggregate levels of the two pollutants and the realization of u. Let aggregate emissions of both pollutants be $Q_j = \sum_{i=1}^{n} q_{ij},\ j = 1,2$. The minimum aggregate abatement cost function for the industry is $C(Q_1, Q_2, u)$, which is the solution to:

$$\begin{aligned} \min_{\{q_{i1}\}_{i=1}^{n}, \{q_{i2}\}_{i=1}^{n},} \quad & \sum_{i=1}^{n} C^i (q_{i1}, q_{i2}, u) \\ \text{subject to} \quad & Q_j = \sum_{i=1}^{n} q_{ij},\ j = 1,2. \end{aligned} \tag{4}$$

(For simplicity we assume the emissions choices are strictly positive for every firm). The minimum aggregate abatement cost function has the following form:

$$C(Q_1, Q_2, u) = a_0 - (a_1 + u)(Q_1 + Q_2) + \frac{a_2}{2}\left(Q_1^2 + Q_2^2\right) - wQ_1Q_2, \tag{5}$$

with:

$$\begin{aligned} a_1 &= \frac{\sum_{i=1}^{n} \left(c_{i1} \prod_{k \neq i}^{n} (w_k - c_{k2})\right)}{\sum_{i=1}^{n} \left(\prod_{k \neq i}^{n} (w_k - c_{k2})\right)} > 0; \\ a_2 &= \frac{\sum_{i=1}^{n} \left(c_{i2} \prod_{k \neq i}^{n} \left(w_k^2 - c_{k2}^2\right)\right)}{\left[\sum_{i=1}^{n} \left(\prod_{k \neq i}^{n} (w_k - c_{k2})\right)\right]\left[\sum_{i=1}^{n} \left(\prod_{k \neq i}^{n} (w_k + c_{k2})\right)\right]} > 0; \\ w &= \frac{\sum_{i=1}^{n} \left(w_i \prod_{j \neq i}^{n} \left(w_k^2 - c_{k2}^2\right)\right)}{\left[\sum_{i=1}^{n} \left(\prod_{k \neq i}^{n} (w_k - c_{k2})\right)\right]\left[\sum_{i=1}^{n} \left(\prod_{k \neq i}^{n} (w_k + c_{k2})\right)\right]}; \end{aligned}$$

and a_0 is a constant. In addition, it is easy to demonstrate that $a_1 + u > 0$, $a_2 - w > 0$, $a_2 + w > 0$ and $a_2^2 - w^2 > 0$.[8] Therefore, the aggregate abatement cost function has the same basic structure as individual firms' abatement cost functions. It is quadratic with a positive definite Hessian matrix (hence, it is strictly convex), and changes in the random variable u produce parallel shifts of the aggregate marginal abatement cost functions. Given a realization of u, the minimum of the aggregate abatement cost function occurs at strictly positive emissions, $Q_j = (a_1 + u)(a_2 +$

[8]The proofs of the structure of the aggregate abatement cost function and our assertions about its characteristics are available upon request. They are omitted here to save space.

$w)/(a_2^2 - w^2)$, $j = 1,2$. The parameter w determines whether abatement of the two pollutants are complements or substitutes in abatement at the aggregate level: they are complements if $w > 0$ and they are substitutes if $w < 0$. Note that it is sufficient for complementarity (substitutability) at the aggregate level if the two pollutants are substitutes (complements) for every firm, although this is certainly not a necessary condition.[9]

2.3 Damages

Suppose the damage functions take the following quadratic forms:

$$D^1(Q_1) = d_{11}Q_1 + \frac{d_{12}}{2}Q_1^2; \tag{6}$$

$$D^2(Q_2) = d_{21}Q + \frac{d_{22}}{2}Q_2^2; \tag{7}$$

with constants $d_{11} > 0$, $d_{12} > 0$, $d_{21} \geq 0$, and $d_{22} \geq 0$. As noted in the introduction, we do not model a potential interaction between the two pollutants in the damage they cause. Both damage functions are convex, though perhaps weakly convex. We assume that it will never be optimal to chooses policies that produce zero emissions of either pollutant. In part, this requires that the intercept of the marginal abatement cost function will never be below either of the intercepts of the marginal damage functions; that is, $a_1 + u > d_{11}$ and $a_1 + u > d_{21}$. The damage functions are known with certainty. Alternatively, we could assume that they are imperfectly known, but that the uncertainty only affects the intercepts of the marginal damage functions and that this uncertainty is uncorrelated with the abatement cost uncertainty. In this case, it is well known that damage uncertainty has no bearing on the optimal choices of policy instruments.

2.4 Second-best emissions

It is useful to specify the second-best optimal aggregate emissions for the two pollutants, which are the emissions that minimize expected aggregate abatement costs plus damages ex ante. These values are useful as benchmarks for judging the environmental performance of the policies in this paper. The second-best levels of aggregate emissions are the solutions to:

$$\min_{Q_1,Q_2} E\left[C(Q_1,Q_2,u) + D^1(Q_1) + D^2(Q_2)\right]. \tag{8}$$

[9]Although we do not dwell on this feature of the model in this paper, it is interesting that two pollutants can be complements (or substitutes) at the aggregate level without having to be complements (or substitutes) for every firm. This feature may have important consequences for modeling aggregate abatement costs that include firms (that may belong to different industries) with different abatement or production technologies that produce differences in the abatement interactions of multiple pollutants.

Carrying out this optimization with the explicit forms of the aggregate abatement cost function and the damage functions produces:

$$\widehat{Q}_1 = \frac{(a_1 - d_{11})(a_2 + d_{22}) + w(a_1 - d_{21})}{(a_2 + d_{12})(a_2 + d_{22}) - w^2}; \tag{9}$$

$$\widehat{Q}_2 = \frac{(a_1 - d_{21})(a_2 + d_{12}) + w(a_1 - d_{11})}{(a_2 + d_{12})(a_2 + d_{22}) - w^2}. \tag{10}$$

To get a quick idea of how the damage caused by one pollutant affects the second-best quantity restrictions for both pollutants, consider the effects of a change in the intercept of the marginal damage of one pollutant on $\widehat{Q}_j,\ j = 1,2$:

$$\partial \widehat{Q}_j / \partial d_{j1} = -(a_2 + d_{k2}) / ((a_2 + d_{12})(a_2 + d_{22}) - w^2) < 0; \tag{11}$$

$$\partial \widehat{Q}_k / \partial d_{j1} = -w / ((a_2 + d_{12})(a_2 + d_{22}) - w^2),\ j,k = 1,2,\ j \neq k. \tag{12}$$

(11) indicates that a parallel shift of the marginal damage function of one pollutant decreases its second-best emissions. However, the effect on the other pollutant depends on the sign of $-w$. If the pollutants are complements in abatement so that $w > 0$, then an increase in the intercept of the marginal damage function of one pollutant leads to a reduction in the second-best emissions for both pollutants. If the pollutants are substitutes in abatement ($w < 0$), then an increase in the intercept of the marginal damage function of one pollutant leads to a decrease in the second-best emissions for that pollutant, but an increase in the second-best emissions of its co-pollutant.

3 Optimal policies, given the regulation of a co-pollutant

As noted in the introduction, however, joint second-best control of multiple pollutants is unlikely. We now turn to the main focus of the paper, which is the optimal control of one pollutant, given the not-necessarily-optimal control of its co-pollutant. In this section we specify optimal regulations for pollutant 1 given the regulation of pollutant 2. Control of pollutant 2 is exogenous and is either a tax $\bar{t}_2$ or competitively-traded permits $\overline{L}_2$. Pollutant 1 is controlled by a tax, tradable permits, or a hybrid. The hybrid is an emissions permit market with a price ceiling and a price floor that was first proposed by Roberts and Spence (1976), and studied extensively by, for example, Burtraw et al. (2010), Fell et al. (2012), Grull and Taschini (2011), Stranlund and Moffitt (2014), and others. A pure emissions tax and pure emissions trading are special cases of these sorts of hybrids, so we will specify optimal hybrid policies and then use them to specify the simple tax and simple trading regulations. [10]

To calculate the optimal policies for pollutant 1 given the regulation of pollutant 2, we need aggregate emissions responses for all policy combinations. If both pollutants are controlled by arbitrary prices, then the aggregate emissions responses are determined by equating the aggregate marginal abatement costs for each pollutant to these prices; that is:

$$p_j = -C_j\left(Q_1, Q_2, u\right),\ j = 1,2. \tag{13}$$

Solving these equation simultaneously for Q_1 and Q_2 yields the emissions responses:

$$Q_j\left(p_j, p_k, u\right),\ j = 1,2 \text{ and } j \neq k. \tag{14}$$

On the other hand, if one pollutant is controlled by a price and the other with a fixed number of tradable permits L_k, then the emissions response of the priced pollutant is the solution to:

$$p_j = -C_j\left(Q_j, L_k, u\right),\ j = 1,2, \text{ and } j \neq k, \tag{15}$$

resulting in:

$$Q_j\left(p_j, L_k, u\right),\ j = 1,2, \text{ and } j \neq k. \tag{16}$$

Of course, if the emissions of both pollutants are controlled with tradable permits, they are fixed at L_j, $j = 1,2$.

3.1 Optimal policies for pollutant 1, given a tax on pollutant 2

A hybrid policy for pollutant 1 has the following features: λ_1 permits are distributed to the firms (free-of-charge); the government commits to selling additional pollutant 1 permits at price τ_1, and it commits to buying permits from firms at price σ_1. Collectively, the hybrid policy is denoted $h_1 = (\lambda_1, \tau_1, \sigma_1)$. Note that τ_1 provides a price ceiling for pollutant 1 permits, while σ_1 provides the price floor. Clearly, these policy variables are restricted by $\tau_1 \geq \sigma_1$.

We first derive the optimal hybrid policy for pollutant 1, given that pollutant 2 is controlled with the tax $\bar{t}_2$. To specify the expected social cost function we must specify values of u where the permit supply and the price ceiling bind together, and where the permit supply and the price floor bind together. Denote these values as u^{τ_1} and u^{σ_1}, respectively, where $u^{\tau_1} \geq u^{\sigma_1}$. Using (15), these

[10] As noted in the introduction, a major difference between our work and Ambec's and Coria's (2013) is the form of hybrid policy employed. Their approach builds on Weitzman's (1978) combination of a price, a quantity standard and a nonlinear penalty for exceeding the standard. This sort of policy can be designed to achieve the first-best outcome, so it can never be less efficient than a pure tax or trading program. The approach we take–an emissions market with price controls–is common in practice, but it cannot achieve the first-best outcome. Because none of the policies we consider in this paper produce the first-best solution, no policy can dominate the others in every situation.

cut-off values are the solutions to $\tau_1 = -C_1(\lambda_1, Q_2, u^{\tau_1})$ and $\sigma_1 = -C_1(\lambda_1, Q_2, u^{\sigma_1})$. Of course, pollutant 2 emissions depend on the pollutant 1 policy, so at u^{τ_1} and u^{σ_1} we have:

$$z = -C_1(\lambda_1, Q_2(\lambda_1, \bar{t}_2, u^z), u^z),\ z = (\tau_1, \sigma_1), \tag{17}$$

which implicitly define the cut-off values as:

$$u^z = u^z(\lambda_1, z, \bar{t}_2),\ z = (\tau_1, \sigma_1). \tag{18}$$

For values of $u < u^{\sigma_1}$ the price floor binds and the pollutant 1 permit price is equal to σ_1. For values of u between u^{σ_1} and u^{τ_1}, the permit supply binds and the permit price is equal to $-C_1(\lambda_1, Q_2(\lambda_1, \bar{t}_2, u), u)$. Values of u above u^{τ_1}cause the price ceiling to bind so the permit price is equal to τ_1. Given this price schedule, equilibrium emissions of both pollutants are:

$$(Q_1, Q_2) = \begin{cases} (Q_1(\tau_1, \bar{t}_2, u), Q_2(\tau_1, \bar{t}_2, u)) \text{ for } u \in [u^{\tau_1}, \bar{u}] \\ (\lambda_1, Q_2(\lambda_1, \bar{t}_2, u)) \text{ for } u \in [u^{\sigma_1}, u^{\tau_1}] \\ (Q_1(\sigma_1, \bar{t}_2, u), Q_2(\sigma_1, \bar{t}_2, u)) \text{ for } u \in [\underline{u}, u^{\sigma_1}]. \end{cases} \tag{19}$$

Using (18) and (19), expected social costs are:

$$\begin{aligned} &W(\lambda_1, \tau_1, \sigma_1, \bar{t}_2) = \\ &\int_{u^{\tau_1}(\lambda_1, \tau_1, \bar{t}_2)}^{\bar{u}} \left[C(Q_1(\tau_1, \bar{t}_2, u), Q_2(\tau_1, \bar{t}_2, u), u) + D^1(Q_1(\tau_1, \bar{t}_2, u)) + D^2(Q_2(\tau_1, \bar{t}_2, u))\right] f(u)\, du \\ &+ \int_{u^{\sigma_1}(\lambda_1, \sigma_1, \bar{t}_2)}^{u^{\tau_1}(\lambda_1, \tau_1, \bar{t}_2)} \left[C(\lambda_1, Q_2(\lambda_1, \bar{t}_2, u), u) + D^1(\lambda_1) + D^2(Q_2(\lambda_1, \bar{t}_2, u))\right] f(u)\, du \\ &+ \int_{\underline{u}}^{u^{\sigma_1}(\lambda_1, \sigma_1, \bar{t}_2)} \left[C(Q_1(\sigma_1, \bar{t}_2, u), Q_2(\sigma_1, \bar{t}_2, u), u) + D^1(Q_1(\sigma_1, \bar{t}_2, u)) + D^2(Q_2(\sigma_1, \bar{t}_2, u))\right] f(u)\, du \end{aligned} \tag{20}$$

The optimal hybrid policy for pollutant 1 is the solution to:

$$\min_{\lambda_1, \tau_1, \sigma_1} W(\lambda_1, \tau_1, \sigma_1, \bar{t}_2),\ \text{s.t. } \tau_1 \geq \sigma_1,\ u^{\tau_1} \leq \bar{u},\ u^{\sigma_1} \geq \underline{u}. \tag{21}$$

The constraints in (21) reveal how the optimal policy may turn out to be a simple tax or a pure emissions market. If the solution to (21) produces $\tau_1 = \sigma_1$, then the optimal policy is a pure price instrument because there is no chance that the permit supply will be the binding instrument. In this case, the model cannot distinguish between a policy that effectively subsidizes firms for reducing their emissions at rate σ_1 and a policy that taxes their emissions at rate τ_1. This is because there are a fixed number of firms and tax receipts and subsidy payments are transfers with no real effects. However, since a tax would be superior to a subsidy in an extended model, we assume that if the optimal policy is a pure price scheme that it is implemented with a tax. In this case, no emissions permits are issued and the optimal policy is a pure tax, which we denote as $t_1^*(\bar{t}_2)$. Similarly, if the solution to (21) produces $u^{\tau_1} = \bar{u}$ and $u^{\sigma_1} = \underline{u}$, then there is no chance that either of the price controls will bind and the optimal policy is pure emissions market. In this case, the price controls are disabled and the optimal policy is simply $L_1^*(\bar{t}_2)$ tradable permits. If none of the constraints in (21) bind at its solution, then the optimal policy is the hybrid $h_1^*(\bar{t}_2) = (\lambda_1^*(\bar{t}_2), \tau_1^*(\bar{t}_2), \sigma_1^*(\bar{t}_2))$.

In the appendix we demonstrate that the optimal hybrid policy for pollutant 1 can be written as:

$$\begin{aligned}\lambda_1^*(\bar{t}_2) = &\left\{a_2^2(a_1 - d_{11}) + w(a_2(a_1 - d_{21}) - d_{22}(a_1 - \bar{t}_2))\right.\\ &\left.+(a_2(a_2 + w) - wd_{22})E\left[u|u^{\sigma_1^*} \le u \le u^{\tau_1^*}\right]\right\}/A; \quad (22)\\ \tau_1^*(\bar{t}_2) = &\left\{B - \left(a_2^2 - w^2 + a_2(d_{12} + d_{22})\right)w\bar{t}_2\right.\\ &\left.+(a_2 + w)(a_2 d_{12} + d_{22}w)E[u|u^{\tau_1^*} \le u \le \bar{u}]\right\}/A; \quad (23)\\ \sigma_1^*(\bar{t}_2) = &\left\{B - \left(a_2^2 - w^2 + a_2(d_{12} + d_{22})\right)w\bar{t}_2\right.\\ &\left.+(a_2 + w)(a_2 d_{12} + d_{22}w)E[u|\underline{u} \le u \le u^{\sigma_1^*}]\right\}/A, \quad (24)\end{aligned}$$

where

$$\begin{aligned}A &= a_2\left(a_2^2 - w^2\right) + a_2^2 d_{12} + d_{22}w^2;\\ B &= a_1(a_2 + w)(a_2 d_{12} + d_{22}w) + \left(a_2^2 - w^2\right)(a_2 d_{11} + d_{21}w),\end{aligned}$$

and $E\left[u|u^{\sigma_1^*} \le u \le u^{\tau_1^*}\right]$, $E[u|u^{\tau_1^*} \le u \le \bar{u}]$, and $E[u|\underline{u} \le u \le u^{\sigma_1^*}]$ are conditional expectations of u. ((23) through (24) are not exact 'solutions', because the optimal policy variables appear on the right sides of the equations in the conditional expectations).

Since a pure emissions tax and pure emissions markets are special cases of the hybrid policy, we can specify the optimal pure instruments from (22) through (24). For the emissions tax, we disable the permit supply above by setting $u^{\tau_1^*} \le \underline{u}$. Then, $E[u|u^{\tau_1^*} \le u \le \bar{u}] = 0$. Substitute this into (23) and simplify to find the optimal pure tax:

$$t_1^*(\bar{t}_2) = \left\{B - \left(a_2^2 - w^2 + a_2(d_{12} + d_{22})\right)w\bar{t}_2\right\}/A. \quad (25)$$

To find the optimal pure trading program, disable the price controls by setting $u^{\sigma_1^*} \le \underline{u}$ and $u^{\tau_1^*} \ge \bar{u}$, which implies $E\left[u|u^{\sigma_1^*} \le u \le u^{\tau_1^*}\right] = 0$ in (22). Therefore, the optimal supply of tradable permits in a pure emissions market is:

$$L_1^*(\bar{t}_2) = \left\{a_2^2(a_1 - d_{11}) + w(a_2(a_1 - d_{21}) - d_{22}(a_1 - \bar{t}_2))\right\}/A. \tag{26}$$

3.2 Optimal policies for pollutant 1, given tradable permits for pollutant 2

The specification of the optimal policy for pollutant 1, given that pollutant 2 is controlled with $\overline{L}_2$ tradable permits, proceeds in the same way as when pollutant 2 is controlled with a tax. The cut-off values in this case, u^{τ_1} and u^{σ_1}, are determined from:

$$z = -C_1\left(\lambda_1, \overline{L}_2, u^z\right),\; z = (\tau_1, \sigma_1), \tag{27}$$

with implicit solutions:

$$u^z = u^z(\lambda_1, z, \overline{L}_2),\; z = (\tau_1, \sigma_1). \tag{28}$$

Equilibrium emissions as a function of u are then:

$$(Q_1, Q_2) = \begin{cases} \left(Q_1\left(\tau_1, \overline{L}_2, u\right), \overline{L}_2\right) \text{ for } u \in [u^{\tau_1}, \overline{u}] \\ \left(\lambda_1, \overline{L}_2\right) \text{ for } u \in [u^{\sigma_1}, u^{\tau_1}] \\ \left(Q_1\left(\sigma_1, \overline{L}_2, u\right), \overline{L}_2\right) \text{ for } u \in [\underline{u}, u^{\sigma_1}], \end{cases} \tag{29}$$

and the expected social social cost function is:

$$\begin{aligned} &W\left(\lambda_1, \tau_1, \sigma_1, \overline{L}_2\right) = \\ &= \int_{u^{\tau_1}\left(\lambda_1, \tau_1, \overline{L}_2\right)}^{\overline{u}} \left[C\left(Q_1\left(\tau_1, \overline{L}_2, u\right), \overline{L}_2, u\right) + D^1\left(Q_1\left(\tau_1, \overline{L}_2, u\right)\right) + D^2\left(\overline{L}_2\right)\right] f(u)\, du \\ &\quad + \int_{u^{\sigma_1}\left(\lambda_1, \sigma_1, \overline{L}_2\right)}^{u^{\tau_1}\left(\lambda_1, \tau, \overline{L}_2\right)} \left[C\left(\lambda_1, \overline{L}_2, u\right) + D^1(\lambda_1) + D^2\left(\overline{L}_2\right)\right] f(u)\, du \\ &\quad + \int_{\underline{u}}^{u^{\sigma_1}\left(\lambda_1, \sigma_1, \overline{L}_2\right)} \left[C\left(Q_1\left(\sigma_1, \overline{L}_2, u\right), \overline{L}_2, u\right) + D^1\left(Q_1\left(\sigma_1, \overline{L}_2, u\right)\right) + D^2\left(\overline{L}_2\right)\right] f(u)\, du. \end{aligned} \tag{30}$$

The optimal policy for pollutant 1, given $\overline{L}_2$, is the solution to:

$$\min_{\lambda_1, \tau_1, \sigma_1} W\left(\lambda_1, \tau_1, \sigma_1, \overline{L}_2\right),\; \text{s.t.}\; \tau_1 \geq \sigma_1,\; u^{\tau_1} \leq \overline{u},\; u^{\sigma_1} \geq \underline{u}. \tag{31}$$

Again, binding constraints in this problem have the same interpretation of indicating the optimality of pure instruments. In particular, if the solution to (31) involves $\tau_1 = \sigma_1$, then the optimal policy is the tax $t_1^*(\overline{L}_2)$. If $u^{\tau_1} = \overline{u}$ and $u^{\sigma_1} = \underline{u}$, then the optimal policy is a pure trading policy with $L_1^*(\overline{L}_2)$ tradable permits. If none of the constraints bind, then the optimal policy is the hybrid

$h_1^*(\overline{L}_2) = (\lambda_1^*(\overline{L}_2), \tau_1^*(\overline{L}_2), \sigma_1^*(\overline{L}_2))$.

The optimal hybrid policy for pollutant 1 in this case is derived in the appendix:

$$\lambda_1^*\left(\overline{L}_2\right) = \frac{a_1 - d_{11} + w\overline{L}_2 + E\left[u|u^{\sigma_1^*} \leq u \leq u^{\tau_1^*}\right]}{a_2 + d_{12}}; \tag{32}$$

$$\tau_1^*\left(\overline{L}_2\right) = \frac{a_1 d_{12} + a_2 d_{11} + w d_{12}\overline{L}_2 + d_{12} E\left[u|u^{\tau_1^*} \leq u \leq \overline{u}\right]}{a_2 + d_{12}}; \tag{33}$$

$$\sigma_1^*\left(\overline{L}_2\right) = \frac{a_1 d_{12} + a_2 d_{11} + w d_{12}\overline{L}_2 + d_{12} E\left[u|\underline{u} \leq u \leq u^{\sigma_1^*}\right]}{a_2 + d_{12}}. \tag{34}$$

As when pollutant 2 is controlled with a tax, the optimal pure tax on pollutant 1 is determined from (33) with $E[u|u^{\tau_1^*} \leq u \leq \overline{u}] = 0$; that is:

$$t_1^*\left(\overline{L}_2\right) = \frac{a_1 d_{12} + a_2 d_{11} + w d_{12}\overline{L}_2}{a_2 + d_{12}}. \tag{35}$$

The optimal supply of permits in a pure trading program is found by setting $E\left[u|u^{\sigma_1^*} \leq u \leq u^{\tau_1^*}\right] = 0$ in (33), yielding:

$$L_1^*\left(\overline{L}_2\right) = \frac{a_1 - d_{11} + w\overline{L}_2}{a_2 + d_{12}}. \tag{36}$$

4 Instrument choice in the presence of co-pollutants

We are now ready to present the main results of this work, which are complete orderings of the three policy instruments in the terms of expected social costs, given the regulation of pollutant 2. These orderings provide simple rules involving the parameters of the aggregate abatement cost function and the damage functions for determining whether the optimal control policy for pollutant 1 is an emissions tax, tradable permits, or a hybrid. These rules are presented in the following proposition, which is proved in the appendix.

Proposition 1: If pollutant 2 is regulated with an emissions tax $\bar{t}_2$, then:

(1) The emissions tax $t_1^*(\bar{t}_2)$ is the optimal policy for pollutant 1 if and only if $wd_{22} + a_2 d_{12} \leq 0$;

(2) A pure trading scheme with $L_1^*(\bar{t}_2)$ tradable permits is the optimal policy for pollutant 1 if and only if $wd_{22} \geq a_2(a_2 + w)$;

(3) The hybrid policy $h_1^*(\bar{t}_2)$ is optimal for pollutant 1 if and only if $wd_{22} \in (a_2 d_{12}, a_2(a_2 + w))$.

(4) If the choice of policy for pollutant 1 is limited to a pure emissions tax or pure emissions trading, then the tax $t_1^*(\bar{t}_2)$ is the optimal policy if and only if

$$wd_{22} < \frac{a_2(a_2^2 - w^2 - a_2 d_{12})}{2a_2 - w}.$$

The trading program with $L_1^*(\bar{t}_2)$ tradable permits is the optimal policy if and only if the inequality is reversed.

If pollutant 2 is regulated with $\overline{L}_2$ tradable permits, then:

(5) The emissions tax $t_1^*(\overline{L}_2)$ is the optimal policy for pollutant 1 if and only if $d_{12} = 0$;

(6) A pure trading scheme for pollutant 1 is never optimal.

(7) The hybrid policy $h_1^*(\overline{L}_2)$ is optimal for pollutant 1 if and only if $d_{12} > 0$.

(8) If the choice of policy for pollutant 1 is limited to a pure emissions tax or pure emissions trading, then the tax $t_1^*(\overline{L}_2)$ is the optimal policy if and only if $d_{12} < a_2$. The trading program with $L_1^*(\overline{L}_2)$ tradable permits is the optimal policy if and only if the inequality is reversed.

We can visualize the orderings of the three policy instruments for pollutant 1 provided by Proposition 1 with Figures 1 and 2. In both figures we try to make the notation as parsimonious as possible by using the relation $\succ$ to indicate social preference among the policies for pollutant 1. This preference is in terms of lower expected social costs, so that $v^* \succ y^*$ means that policy v^* has strictly lower expected social costs than policy y^*. Figure 1 illustrates the ordering of the pollutant 1 policies, given that pollutant 2 is controlled with a tax. The three cut-off values from (1) through (4) of Proposition 1, are such that

$$-a_2 d_{12} < \frac{a_2(a_2^2 - w^2 - a_2 d_{12})}{2a_2 - w} < a_2(a_2 + w),$$

for permissible values of d_{12}, a_2, and w. What the figure does not indicate, but what is clear in Proposition 1, is that $t_1^* \succ h_1^* \succ L_1^*$ when $wd_{22} = -a_2 d_{12}$, and $L_1^* \succ h_1^* \succ t_1^*$ when $wd_{22} = a_2(a_2 + w)$. Moreover, at $wd_{22} = a_2(a_2^2 - w^2 - a_2 d_{12})/(2a_2 - w)$, the pure tax and the pure trading scheme produce the same expected social cost. Figure 2 illustrates the expected social cost ordering of the pollutant 1 policies when pollutant 2 is controlled with tradable permits. In this figure, as in Proposition 1, $d_{12} = 0$ implies $t_1^* \succ h_1^* \succ L_1^*$ and $d_{12} = a_2$ implies indifference between the pure tax and pure trading policies.

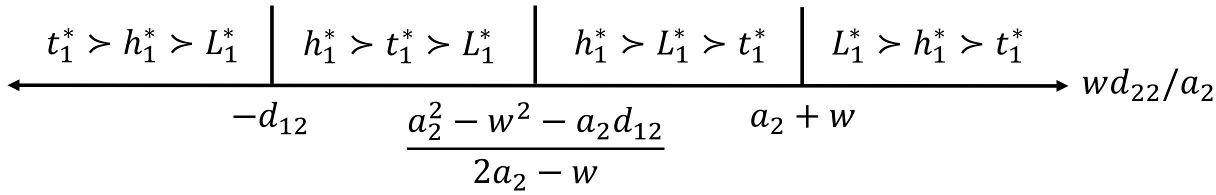

Figure 1: Instrument choice when a co-pollutant is controlled with a tax.

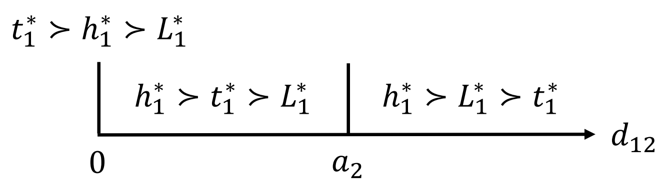

Figure 2: Instrument choice when a co-pollutant is controlled with tradable permits.

The specifications of the pollutant 1 policies in the previous section that the form and level of control of pollutant 2 affects the optimal policies of pollutant 1; that is, $\bar{t}_2$ or $\bar{L}_2$ affects the level of the pollutant 1 tax, the number of tradable permits, and the elements of a hybrid policy. And it is clear from Proposition 1 that the form of pollutant 2 regulation affects the instrument choice rules for pollutant 1. However, given the form of the co-pollutant regulation, the instrument choice rules for pollutant 1 do not depend on the stringency of the co-pollutant regulation. That is, the instrument choice rules for pollutant 1 are independent of the relative efficiency of the pollutant 2 regulation. This result may have important policy implications because it makes the instrument choice problem for the primary pollutant much simpler.

The instrument choice problem is also quite simple when the co-pollutant is regulated with a fixed number of tradable permits. Since the rules for determining the optimal instrument choice in this case (parts (5) through (8) of the Proposition) do not depend on the abatement interaction term w, they are the same as in the single pollutant case. These rules are familiar, at least when the choice is between a tax and a pure trading scheme. (8) recalls the result that a tax is preferred for pollutant 1 when the slope of the marginal abatement cost function is steeper than the slope of the marginal damage function; tradable permits are preferred when the inequality is reversed. When the instrument choice for pollutant 1 includes the hybrid policy, a pure trading scheme is never optimal and a pure tax is optimal if and only if the marginal damage function for pollutant 1 is flat.[11] In every other case the optimal policy is a hybrid policy. In the proof of the proposition we demonstrate that the difference between the price controls in this case is determined in part by the slope of the marginal damage function for pollutant 1, d_{12}. When this parameter is small, the difference between the price ceiling and the price floor of an optimal hybrid policy is also small. This suggests that the added complexity of constructing and implementing a hybrid policy may not be worth it when $d_{12}$is small.

The instrument choice rules for pollutant 1 do not differ from the single-pollutant case when pollutant 2 is regulated with a fixed number of tradable permits, because the pollutant 1 regulations cannot affect pollutant 2 emissions. The same is true if pollutant 2 is controlled with a tax but there is no abatement interaction between the two pollutants (i.e., $w = 0$). In this case it is straightforward to show that the instrument choice rules for pollutant 1–parts (1) through (4) of Proposition 1–are the same as when pollutant 2 is controlled with tradable permits, and consequently, the rules are the same as in the single-pollutant case.

However, when pollutant 2 is regulated with a tax and the pollutants are linked together in abatement, pollutant 1 regulations affect pollutant 2 emissions, and hence, the instrument choice rules for pollutant 1 are very different from the single-pollutant case. First focus on the situations in which an emissions tax is the optimal choice for pollutant 1 (part (1) of Proposition 1). In the single-pollutant case, or with multiple pollutants when the co-pollutant is controlled with fixed permits, a flat marginal damage function for the primary pollutant ($d_{12} = 0$) is necessary and sufficient for an emissions tax to be the preferred policy choice. However, when considering a co-pollutant that is controlled with a tax, an emissions tax for 1 is optimal if and only if $wd_{22} + a_2 d_{12} \leq 0$. Hence, a flat marginal damage function for the primary pollutant is neither necessary or sufficient for the tax to dominate alternative policies. Given $d_{12} = 0$, a tax on pollutant 1 is optimal if $wd_{22} \leq 0$, which occurs when the marginal damage function for the co-pollutant is also flat ($d_{22} = 0$), or if not, when the two pollutants are substitutes in abatement ($w < 0$). In fact, abatement substitutability of the two pollutants tends to favor a tax on pollutant 1. Conversely, the only way the tax can dominate the other policy alternatives when the two pollutants are complements is if the marginal damage function for both pollutants are flat ($d_{12} = d_{22} = 0$). If either marginal damage function is upward sloping, then a policy with emissions trading (either a hybrid or a simple market) will be preferred to the tax.

Turning to circumstances under which a pure permit market is the best choice for pollutant 1, from part (2) of Proposition 1, a pure trading scheme dominates when the co-pollutant is controlled with a tax if and only if $wd_{22} \geq a_2(a_2 + w)$. A necessary condition here is that $wd_{22} > 0$,

[11] There are cases in which pure trading program would be optimal if $a_2 = 0$, but we do not consider this possibility in this paper because we would not be able to guarantee that the aggregate abatement cost function is convex in all cases.

which implies that the optimal policy cannot be a pure trading scheme if either the pollutants are substitutes or if the marginal damage function for pollutant 2 is flat. Put differently, optimality of a simple emissions market requires that the two pollutants be complements in abatement and the marginal damage function for pollutant 2 be upward sloping. The stronger are these two effects, the more likely a pure emissions market dominates a policy with fixed prices (either a hybrid or a pure tax). [12]

An important conclusion emerges about instrument choice when a co-pollutant is controlled with a tax. That is, complementarity in abatement tends to favor emissions markets (with or without price controls) and the strength of this effect increases with the steepness of the co-pollutant's marginal damage function. On the other hand, substitutability tends to favor emissions pricing (either as part of a hybrid policy or a pure tax) and this effect is stronger as the marginal damage for the co-pollutant is steeper. Of course, the sign of w and the size of d_{22} does not determine everything about the instrument choice for pollutant 1, but it is clear that they play important roles so it is important to explore these roles further.

Instrument choice is always about balancing the variation in marginal damage of a pollutant against the variation in marginal abatement costs. In the canonical single-pollutant case, one can interpret the preference for prices over quantities when the slope of the marginal damage function is low compared to the slope of the marginal abatement cost function as being determined by the fact that the variance of marginal damage is lower under a tax than the variance of marginal abatement costs under tradable permits. The preference is reversed when the relative slopes of the marginal functions are reversed because the relative sizes of the variance of marginal damage and marginal abatement costs is reversed.

We can apply this same interpretation to the relative preference for prices and quantities when a co-pollutant is controlled with a tax. To keep matters simple so that the intuition is clear, let us focus on the choice between a simple tax and a simple emissions market. In Table 1 we present the standard deviations of marginal damages and marginal abatement costs for both pollutants under $t_1^*(\bar{t}_2)$ and $L_1^*(\bar{t}_2)$. These values include the standard deviation of the random variable u, denoted δ_u. Marginal abatement cost for pollutant 2 is fixed at $\bar{t}_2$, so its standard deviation is zero. Likewise, marginal abatement cost for pollutant 1 is fixed under $t_1^*(\bar{t}_2)$, so its standard deviation is zero. Under $L_1^*(\bar{t}_2)$ the standard deviation of marginal damage for pollutant 1 is zero because it is fixed at the emissions restriction. As a simple exercise, set $w = 0$ so that there is no abatement interaction between the two pollutants, and calculate the difference between the standard deviation of marginal abatement cost for pollutant 1 under $L_1^*(\bar{t}_2)$ and the standard deviation of marginal

[12] It is interesting that the slope of the marginal damage function for pollutant 1 (d_{12}) does not affect the choice between a hybrid and a simple emissions market. Of course, this parameter plays a key role in determining whether a simple tax should be the optimal pollutant 1 policy, but it plays no role in instrument choice, given that there will be a trading component of the policy.

		$t_1^*(\bar{t}_2)$	$L_1^*(\bar{t}_2)$
Pollutant 1	Marginal abatement cost	0	$(a_2+w)\delta_u/a_2$
	Marginal damage	$d_{12}\delta_u/(a_2-w)$	0
Pollutant 2	Marginal abatement cost	0	0
	Marginal damage	$d_{22}\delta_u/(a_2-w)$	$d_{22}\delta_u/a_2$

Table 1: Standard deviations of marginal abatement costs and marginal damages for a tax versus emissions trading, given a tax on the co-pollutant.

marginal damage of pollutant 1 under $t_1^*(\bar{t}_2)$ to be

$$\frac{(a_2-d_{12})\delta_u}{a_2}.$$

Thus, when the absolute value of the slope of the marginal abatement cost function is greater than the slope of the marginal damage function ($a_2 > d_{12}$), there is more variation in marginal abatement cost under $L_1^*(\bar{t}_2)$ than in marginal damage under $t_1^*(\bar{t}_2)$, and a pure tax is preferred to a pure trading program. Of course, the conclusion is reversed when $a_2 < d_{12}$. This exercise demonstrates how differences in the variation of marginal abatement costs and marginal damages play a role in determining policy instrument choices.

Now suppose that $w \neq 0$. The difference between the standard deviation of marginal abatement cost for pollutant 1 under $L_1^*(\bar{t}_2)$ and the standard deviation of marginal marginal damage of pollutant 1 under $t_1^*(\bar{t}_2)$ is now

$$\frac{(a_2^2-w^2-a_2d_{12})\delta_u}{a_2(a_2-w)}.$$

Note that the sign of this term is determined by the sign of $a_2^2-w^2-a_2d_{12}$, which also determines the sign of the cut-off value in part (4) of Proposition 1 (also see Figure 1) that helps determine the relative preference for $t_1^*(\bar{t}_2)$ and $L_1^*(\bar{t}_2)$. In fact, if the the marginal damage function for pollutant 2 is flat ($d_{22}=0$), then the sign of $a_2^2-w^2-a_2d_{12}$ completely determines the choice between $t_1^*(\bar{t}_2)$ and $L_1^*(\bar{t}_2)$.

But the most interesting aspect of this analysis is when the marginal damage function for pollutant 2 is not flat ($d_{22}>0$). From Table 1, note that the abatement interaction of the two pollutants does not affect the standard deviation of the marginal damage of pollutant 2 under $L_1^*(\bar{t}_2)$. However, the standard deviation is lower under $t_1^*(\bar{t}_2)$ if the two pollutants are substitutes in abatement and it is higher if the two pollutants are complements. The difference between the standard deviations of the marginal damage for pollutant 2 under $t_1^*(\bar{t}_2)$ and $L_1^*(\bar{t}_2)$ is

$$\frac{wd_{22}\delta_u}{a_2(a_2-w)}.$$

Hence the variation of pollutant 2 marginal damage is lower when the pollutants are substitutes and it is higher when they are complements. This is why the substitutability in abatement tends to favor emissions pricing while a complementary relationship tends favor quantity restrictions. Moreover, the magnitude of these effects depend in part on the slope of the pollutant 2 marginal damage function.[13]

We have focused on the choice between pure emissions trading and a pure emissions tax to simplify the intuition. However, the same principle applies when we consider hybrid policies. In fact, it is possible to show that the standard deviation of marginal damages for pollutant 2 under a hybrid policy lies strictly between the standard deviations under $t_1^*(\bar{t}_2)$ and $L_1^*(\bar{t}_2)$. (A demonstration is available upon request). This implies that the variation in the pollutant 2 marginal damage under a hybrid is greater than under the pure tax when the pollutants are complements, which tends to favor a policy involving an emissions market. On the other hand, the variation in pollutant 2 marginal damage is less under a hybrid than under simple emissions trading when the pollutants are substitutes, which tends to favor a policy involving fixed prices.

5 Environmental performance of pollutant 1 policies

We complete the analysis by examining how expected emissions of pollutant 1 depends on the form and stringency of the regulation of its co-pollutant. Our reference points for the relative leniency or stringency of regulation of both pollutants are the second-best emissions levels, $\widehat{Q}_1$ and $\widehat{Q}_2$, from (6) and (10) int subsection 2.4. The analysis begins with the following lemma.

Lemma 1: Given the regulation of pollutant 2, the optimal hybrid policy for pollutant 1 produces the same expected aggregate emissions as the optimal tax, which in turn is the same as the optimal number of pollutant 1 permits issued under a pure trading program; that is:

$$E\left(Q_1\left(h_1^*(\bar{x}),\bar{x},u\right)\right) = E\left(Q_1\left(t_1^*(\bar{x}),\bar{x},u\right)\right) = L_1^*(\bar{x}),\ \bar{x} = (\bar{t}_2,\overline{L}_2). \tag{37}$$

Moreover, if pollutant 2 is regulated with a tax, each of the optimal policies for pollutant 1 produce

[13]Unlike the case when $w=0$ (or the single pollutant case), differences in the standard deviations of marginal abatement costs and marginal damages cannot fully explain the choice between $t_1^*(\bar{t}_2)$ and $L_1^*(\bar{t}_2)$ when w is not equal to zero; in other words, it is not possible to derive part (4) of Proposition 1 from calculating differences in the standard deviations of marginal abatement costs and marginal damage. It is clear, however, that differences in the variation of marginal abatement costs and marginal damages play an important role in determining the optimal policy.

the same expected emissions of pollutant 2; that is:

$$E\left(Q_2\left(h_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) = E\left(Q_2\left(t_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) = E\left(Q_2\left(L_1^*(\bar{t}_2),\bar{t}_2,u\right)\right). \tag{38}$$

Lemma 1 may not be that surprising given that pure emissions trading and a pure tax are special cases of the optimal hybrid policy. However, it does offer an important insight: whatever the form and level of the regulation of its co-pollutant, all optimal regulations of a pollutant produce the same level of expected emissions. All that differs in terms of emissions is the variation of potential emissions around that expected outcome. In addition, if the co-pollutant is regulated with a tax, then all optimal policies of the primary pollutant produce the same expected emissions of that pollutant. While the regulations of of both pollutants differ in form and level, in terms of environmental performance, optimal policies for pollutant center expected emissions of both pollutants on the same values.

To simplify the notation from here on, let:

$$E\left(Q_1^*\left(\bar{x}\right)\right) = E\left(Q_1\left(h_1^*(\bar{x}),\bar{x},u\right)\right) = E\left(Q_1\left(t_1^*(\bar{x}),\bar{x},u\right)\right) = L_1^*(\bar{x}),\ \bar{x} = (\bar{t}_2,\overline{L}_2); \tag{39}$$

$$E\left(Q_2^*\left(\bar{t}_2\right)\right) = E\left(Q_2\left(h_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) = E\left(Q_2\left(t_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) = E\left(Q_2\left(L_1^*(\bar{t}_2),\bar{t}_2,u\right)\right). \tag{40}$$

We are now ready to specify how the form and relative stringency of the co-pollutant regulation affects the relative stringency of the regulation of the primary pollutant.

Proposition 2: If pollutant 2 is controlled with a tax, the relationship between expected emissions of pollutant 1 and expected emissions of pollutant 2 can be characterized as:

$$E\left(Q_1^*\left(\bar{t}_2\right)\right) - \hat{Q}_1 = \frac{-d_{22}w}{a_2^2 - w^2 + a_2 d_{12}}\left[E\left(Q_2^*\left(\bar{t}_2\right)\right) - \hat{Q}_2\right]. \tag{41}$$

On the other hand, if pollutant 2 is controlled with tradable permits, the relationship between expected emissions of pollutant 1 and emissions of pollutant 2 can be characterized as:

$$E\left(Q_1^*\left(\overline{L}_2\right)\right) - \hat{Q}_1 = \frac{w}{a_2 + d_{12}}\left(\overline{L}_2 - \hat{Q}_2\right). \tag{42}$$

As one might expect, if regulation of pollutant 2 produces its second-best level of emissions (in expectation if pollutant 2 is controlled with a tax), then the optimal policies for pollutant 1 produce its second-best level of emissions. More interestingly, however, if the regulation of pollutant 2 does not produce its second best emissions, then the deviation of pollutant 1 expected emissions from its second-best level depends on: (1) the deviation of pollutant 2 emissions from its second-best; (2) whether the pollutants are complements or substitutes in abatement, and (3) whether the co-pollutant is regulated with a fixed number of tradable permits or an emissions tax.

To illustrate these effects, suppose at first that pollutant 2 is controlled with tradable permits and that the pollutants are substitutes in abatement. Then,

$$sgn(E\left(Q_1^*\left(\overline{L}_2\right)\right)-\widehat{Q}_1)=sgn(\overline{L}_2-\widehat{Q}_2).$$

So, for example, if regulation of pollutant 2 is too lenient relative to its second-best emissions $(\overline{L}_2>\widehat{Q}_2)$, then the optimal regulation of pollutant 1 produces expected emissions that are also higher than its second-best emissions. The reason is that if the two pollutants are complements and emissions of pollutant 2 are too high, then the expected marginal abatement cost for pollutant 1 is also higher, which calls for a policy that results in higher expected emissions. On the other hand, if the two pollutants are substitutes, $sgn(E\left(Q_1^*\left(\overline{L}_2\right)\right)-\widehat{Q}_1)=-sgn(\overline{L}_2-\widehat{Q}_2)$. In this case, if emissions of pollutant 2 are too high relative to second-best, then the expected marginal abatement cost for pollutant 1 is low and, consequently, regulation of pollutant 1 should produce expected emissions of the pollutant that are lower than the second-best level.

Matters are very different if pollutant 2 is controlled with a tax. If the two pollutants are complements in this case,

$$sgn(E\left(Q_1^*\left(\bar{t}_2\right)\right)-\hat{Q}_1)=-sgn[d_{22}(E\left(Q_2^*\left(\bar{t}_2\right)\right)-\hat{Q}_2)].$$

Therefore, if the tax on pollutant 2 is too low so that its expected emissions are higher than the second-best level, then the optimal regulation of pollutant 1 should work in the opposite direction and produce expected emissions that are lower than second-best control, at least as long as the damage function for pollutant 2 is strictly convex. The intuition here is as follows. If the tax on pollutant 2 is too low resulting in expected emissions that are too high, then the expected marginal abatement cost for pollutant 1 is too high. In the case of tradable permits for pollutant 2, higher expected marginal abatement costs for pollutant 1 implied that emissions of the pollutant should be higher than second-best. However, if regulation of pollutant 1 produces higher expected emissions when pollutant 2 is controlled with a tax, then the expected marginal abatement cost for pollutant 2 would also be higher, which, given the fixed pollutant 2 tax, would result in even higher expected emissions of pollutant 2. Thus, a pollutant 1 regulation that increases expected emissions in response to a pollutant 2 tax that is too low would lead to an even larger deviation of pollutant 2 expected emissions from the second-best level. This is costly when the marginal damage function for 2 is upward sloping because it increases the wedge between marginal damage for the co-pollutant and expected marginal abatement costs. To minimize the consequences of the low tax on pollutant 2, optimal regulations of pollutant 1 will be stricter than required to reach its second-best level of emissions.

The opposite it true if the two pollutants are substitutes in abatement. In this case, a pollutant 2 tax that is too low calls for a more lenient pollutant 1 policy that produces expected emissions above the second-best level to limit the deviation of expected pollutant 2 emissions from its second-best level. Interestingly, when the marginal damage function for pollutant 2 is flat, optimal pollutant 1 policies adjust to a suboptimal tax on pollutant 2 so that expected emissions of pollutant 1 equal the second-best level. This is due to the fact that the wedge between pollutant 2 marginal damage and its expected marginal abatement cost is constant.

6 Lessons for climate policy

Since the issues associated with regulation under uncertainty with co-pollutants is motivated in large part by debates about how best to design policies to control greenhouse gas emissions, it is worthwhile to examine how our results inform these debates.

An important characteristic of the control of greenhouse gases is that the expected marginal damage function in a relatively short compliance period is estimated to be almost perfectly flat. This, coupled with other common assumptions, implies that the optimal control policy is a pure emission tax—adding a quantity restriction to form a hybrid regulation or a pure trading scheme necessarily reduces expected welfare.[14] This conclusion has to be revisited if carbon regulation is to take account of its co-pollutants, at least when changes in carbon emissions cause changes in the emissions of co-pollutants. We show in the two pollutant case that the instrument choice rules are the same as in the single-pollutant case if the co-pollutant is regulated with a fixed number of emissions permits. Again, the reason for this is that regulation of the primary pollutant cannot cause changes in the emissions of the co-pollutant. However, as noted in the introduction the most-studied carbon co-pollutants are flow pollutants like NOX, SO2 and PM that are emitted along with carbon. Efforts to reduce carbon can also reduce emissions of these pollutants, suggesting that carbon and these co-pollutants are complements in abatement. Given the flat marginal damage for carbon and a co-pollutant that is a complement in abatement, our results suggest that the only way a carbon tax remains the optimal policy choice is if the marginal damage function for the co-pollutant is also flat. Otherwise, the optimal carbon policy must have a trading component, most likely as part of a hybrid scheme. Part of the reason for this is that when the marginal damage function for the co-pollutant is increasing and the two pollutants are complements, the variation in marginal damage of the co-pollutant is highest under a simple carbon tax.

We also show that the deviation of expected emissions of the primary pollutant from its second-best value depends on how the co-pollutant is regulated, whether it is regulated efficiently, and

[14] However, Stranlund (2014) shows how a positive correlation between abatement costs and damage can call for a hybrid regulation even when the marginal damage function is perfectly flat.

whether the pollutants are substitutes or complements in abatement. Efficient regulation of carbon will produce the second-best value if the co-pollutants are also regulated efficiently. However, this is unlikely so regulation and expected emissions of carbon are likely to deviate from the second-best value if the regulation accounts for its co-pollutants. An interesting special case is when the marginal damage of the co-pollutant is also flat. In this case, the optimal carbon policy is a tax set so that expected carbon emissions equal the second-best value. However, this does not imply that the carbon tax will equal marginal damage; instead, the tax will deviate from marginal damage so that expected carbon emissions are equal to the second-best level.

While our work has important implications for optimal carbon regulation, this context also points out fruitful areas for future research. Important elements of our results depend on how regulation of the primary pollutant changes emissions of the co-pollutant. We have examined two possible cases, one in which the co-pollutant doesn't change because it is controlled with an exogenous number of fixed permits and the other in which emissions of the co-pollutant are variable because it is controlled with a tax. However, other regulations of the co-pollutant will allow it vary as emissions of the primary pollutant vary. One example, among many, is when the co-pollutant is controlled with a performance standard. In the climate context, emissions of carbon co-pollutants are controlled with many different policies that are unlike the pure trading and pure tax modeled of our model in important ways. An interesting area for future work is to determine how the rules for instrument choice change with different regulations of the co-pollutant than those we considered in this paper.

Other elements to consider in future research include examining the consequences of multiple co-pollutants with spatially differentiated damages. While we focus on the regulation of a pollutant with one co-pollutant, carbon has several co-pollutants. In fact, some of these co-pollutants are complements while others may be substitutes. Future work can address how the combination of heterogeneous abatement interactions of multiple co-pollutants affects the design of greenhouse gas policies, including instrument choice. Moreover, we have assumed that the two pollutants in our model are uniformly mixed pollutants. While carbon is a uniformly mixed pollutant, NOX, SO2 and PM produce spatially heterogeneous damages. This suggests that efficient regulation of carbon that accounts for co-pollutants may have a spatial component. These and other characteristics of carbon co-pollutants are important factors to consider in designing efficient greenhouse gas regulation.

7 Appendix

The results in this paper are derived with a large number of fairly involved calculations. Here, we tend to provide only sketches of derivations or proofs. The complete set of derivations is available upon request.

7.1 Derivation of second-best emissions (9) and (10).

To derive the second-best emissions, first substitute the aggregate abatement cost function (5) and total damages for both pollutants (6) and (7) into the objective (8) to obtain

$$\min_{Q_1,Q_2} E\left[a_0-(a_1+u)(Q_1+Q_2)+\frac{a_2}{2}\left(Q_1^2+Q_2^2\right)-wQ_1Q_2+d_{11}Q_1+\frac{d_{12}}{2}Q_1^2+d_{21}Q_2+\frac{d_{22}}{2}Q_2^2\right].$$

The first order conditions are:

$$-a_1+a_2Q_1-wQ_2+d_{11}+d_{12}Q_1=0;$$
$$-a_1+a_2Q_2-wQ_1+d_{21}+d_{22}Q_2=0.$$

Solving this system of equations yields the desired results, (9) and (10).□

7.2 Derivations of the optimal hybrid policies, equations (22) through (24), and equations (32) through (34).

The derivation of the optimal hybrid policies requires the emissions responses when both pollutants are controlled by price. First, using the aggregate abatement cost function (5), note that the explicit form of (13) is

$$p_j=(a_1+u)-a_2Q_j+wQ_k,\ j=1,2 \text{ and } j\neq k. \tag{43}$$

Solving these equation simultaneously for Q_1 and Q_2 produces the explicit forms of (14):

$$Q_j\left(p_j,p_k,u\right)=\frac{(a_1+u)(a_2+w)-a_2p_j-wp_k}{a_2^2-w^2},\ j=1,2 \text{ and } j\neq k. \tag{44}$$

On the other hand, if one pollutant is controlled by a price and the other by fixed quantities, use (43)to specify the emissions response of the priced pollutant:

$$Q_j\left(p_j,Q_k,u\right)=\frac{a_1+u-p_j+wQ_k}{a_2}. \tag{45}$$

To derive the characterization of the optimal hybrid policy when pollutant 2 is controlled with a tax, that is, equations (22) through (24), first use the definitions of the cut-off values u^{τ_1} and u^{σ_1} in (17) to specify the following relationships:

$$Q_1\left(\tau_1, \bar{t}_2, u^{\tau_1}\right) = Q_1\left(\sigma_1, \bar{t}_2, u^{\sigma_1}\right) = \lambda_1; \tag{46}$$

$$Q_2\left(\tau_1, \bar{t}_2, u^{\tau_1}\right) = Q_2\left(\lambda_1, \bar{t}_2, u^{\tau_1}\right); \tag{47}$$

$$Q_2\left(\sigma_1, \bar{t}_2, u^{\sigma_1}\right) = Q_2\left(\lambda_1, \bar{t}_2, u^{\sigma_1}\right). \tag{48}$$

We also have

$$-C_1\left(Q_1\left(z, \bar{t}_2, u\right), Q_2\left(z, \bar{t}_2, u\right), u\right) = z,\ z = (\tau_1, \sigma_1); \tag{49}$$

$$-C_2\left(Q_1\left(z, \bar{t}_2, u\right), Q_2\left(z, \bar{t}_2, u\right), u\right) = \bar{t}_2. \tag{50}$$

With (46) through (50), the first order conditions for the unconstrained version of (21) are:

$$\frac{\partial W\left(\lambda_1, \tau_1, \sigma_1, \bar{t}_2\right)}{\partial \lambda_1} = \int_{u^{\sigma_1}}^{u^{\tau_1}} \left\{C_1\left(\lambda_1, Q_2\left(\lambda_1, \bar{t}_2, u\right), u\right) + D_1^1\left(\lambda_1\right) + \right.$$
$$\left. + \left[-\bar{t}_2 + D_2^2\left(Q_2\left(\lambda_1, \bar{t}_2, u\right)\right)\right] \frac{\partial Q_2\left(\lambda_1, \bar{t}_2, u\right)}{\partial \lambda_1} \right\} f(u)\, du = 0;$$

$$\frac{\partial W\left(\lambda_1, \tau_1, \sigma_1, \bar{t}_2\right)}{\partial \tau_1} = \int_{u^{\tau_1}}^{\bar{u}} \left\{ \left[-\tau_1 + D_1^1\left(Q_1\left(\tau_1, \bar{t}_2, u\right)\right)\right] \frac{\partial Q_1\left(\tau_1, \bar{t}_2, u\right)}{\partial \tau_1} \right.$$
$$\left. + \left[-\bar{t}_2 + D_2^2\left(Q_2\left(\tau_1, \bar{t}_2, u\right)\right)\right] \frac{\partial Q_2\left(\tau_1, \bar{t}_2, u\right)}{\partial \tau_1} \right\} f(u)\, du = 0;$$

$$\frac{\partial W\left(\lambda_1, \tau_1, \sigma_1, \bar{t}_2\right)}{\partial \sigma_1} = \int_{\underline{u}}^{u^{\sigma_1}} \left\{ \left[-\sigma_1 + D_1^1\left(Q_1\left(\sigma_1, \bar{t}_2, u\right)\right)\right] \frac{\partial Q_1\left(\sigma_1, \bar{t}_2, u\right)}{\partial \sigma_1} \right.$$
$$\left. + \left[-\bar{t}_2 + D_2^2\left(Q_2\left(\sigma_1, \bar{t}_2, u\right)\right)\right] \frac{\partial Q_2\left(\sigma_1, \bar{t}_2, u\right)}{\partial \sigma_1} \right\} f(u)\, du = 0. \tag{51}$$

Given the emissions responses in terms of prices from (44) we have:

$$Q_1\left(z, \bar{t}_2, u\right) = \frac{\left(a_1 + u\right)\left(a_2 + w\right) - a_2 z - w \bar{t}_2}{a_2^2 - w^2},\ z = (\tau_1, \sigma_1); \tag{52}$$

$$Q_2\left(z, \bar{t}_2, u\right) = \frac{\left(a_1 + u\right)\left(a_2 + w\right) - a_2 \bar{t}_2 - w z}{a_2^2 - w^2},\ z = (\tau_1, \sigma_1), \tag{53}$$

and given the emissions responses to a price when the co-pollutant is fixed with a quantity standard

(45) we have:

$$Q_2(\lambda_1, \bar{t}_2, u) = \frac{(a_1+u) - \bar{t}_2 + w\lambda_1}{a_2}. \quad (54)$$

Note that:

$$\frac{\partial Q_1(z, \bar{t}_2, u)}{\partial z} = -\frac{a_2}{a_2^2 - w^2},\ z = (\tau_1, \sigma_1); \quad (55)$$

$$\frac{\partial Q_2(z, \bar{t}_2, u)}{\partial z} = -\frac{w}{a_2^2 - w^2},\ z = (\tau_1, \sigma_1); \quad (56)$$

$$\frac{\partial Q_2(\lambda_1, \bar{t}_2, u)}{\partial \lambda_1} = \frac{w}{a_2}. \quad (57)$$

With (52), (53), and (54) and the functional forms of the marginal damage and marginal abatement cost functions we have:

$$C_1(\lambda_1, Q_2(\lambda_1, \bar{t}_2, u), u) = -(a_1+u) + a_2\lambda_1 - w\frac{(a_1+u) - \bar{t}_2 + w\lambda_1}{a_2}; \quad (58)$$

$$D_1^1(\lambda_1) = d_{11} + d_{12}\lambda_1; \quad (59)$$

$$D_2^2(Q_2(\lambda_1, \bar{t}_2, u)) = d_{21} + d_{22}\frac{(a_1+u) - \bar{t}_2 + w\lambda_1}{a_2}; \quad (60)$$

$$D_1^1(Q_1(\tau_1, \bar{t}_2, u)) = d_{11} + d_{12}\frac{(a_1+u)(a_2+w) - a_2 z - w\bar{t}_2}{a_2^2 - w^2}, z = (\tau_1, \sigma_1); \quad (61)$$

$$D_2^2(Q_2(\tau_1, \bar{t}_2, u)) = d_{21} + d_{22}\frac{(a_1+u)(a_2+w) - a_2\bar{t}_2 - wz}{a_2^2 - w^2}, z = (\tau_1, \sigma_1). \quad (62)$$

Substitute (52) through (57) into (51) to rewrite the first order conditions for the unconstrained version of (21) as:

$$\frac{\partial W(\lambda_1,\tau_1,\sigma_1,\bar{t}_2)}{\partial\lambda_1} = \int_{u^{\sigma_1}}^{u^{\tau_1}} \left[-(a_1+u)+a_2\lambda_1 - w\frac{(a_1+u)-\bar{t}_2+w\lambda_1}{a_2} + d_{11}+d_{12}\lambda_1 \right. \\ \left. + \left(-\bar{t}_2+d_{21}+d_{22}\frac{(a_1+u)-\bar{t}_2+w\lambda_1}{a_2}\right)\frac{w}{a_2}\right] f(u)\,du = 0;$$

$$\frac{\partial W(\lambda_1,\tau_1,\sigma_1,\bar{t}_2)}{\partial\tau_1} = \int_{u^{\tau_1}}^{\overline{u}} \left\{\left[-\tau_1+d_{11}+d_{12}\frac{(a_1+u)(a_2+w)-a_2\tau_1-w\bar{t}_2}{a_2^2-w^2}\right]\cdot\frac{-a_2}{a_2^2-w^2}\right. \\ \left. + \left[-\bar{t}_2+d_{21}+d_{22}\frac{(a_1+u)(a_2+w)-a_2\bar{t}_2-w\tau_1}{a_2^2-w^2}\right]\cdot\frac{-w}{a_2^2-w^2}\right\} f(u)\,du = 0;$$

$$\frac{\partial W(\lambda_1,\tau_1,\sigma_1,\bar{t}_2)}{\partial\sigma_1} = \int_{\underline{u}}^{u^{\sigma_1}} \left\{\left[-\sigma_1+d_{11}+d_{12}\frac{(a_1+u)(a_2+w)-a_2\sigma_1-w\bar{t}_2}{a_2^2-w^2}\right]\cdot\frac{-a_2}{a_2^2-w^2}\right. \\ \left. + \left[-\bar{t}_2+d_{21}+d_{22}\frac{(a_1+u)(a_2+w)-a_2\bar{t}_2-w\sigma_1}{a_2^2-w^2}\right]\cdot\frac{-w}{a_2^2-w^2}\right\} f(u)\,du = 0.$$

Derive (22) through (24) from these first order conditions by collecting common terms for u, λ_1, τ_1, σ_1 and $\bar{t}_2$ and rearranging them.

We take the same approach to derive the characterization of the hybrid policy when pollutant 2 is controlled with tradable permits, equations (32) through (34). Begin by noting the following relationships:

$$Q_1\left(\tau_1,\overline{L}_2,u^{\tau_1}\right) = Q_1\left(\sigma_1,\overline{L}_2,u^{\sigma_1}\right) = \lambda_1; \tag{63}$$

$$-C_1\left(Q_1\left(z,\overline{L}_2,u\right),\overline{L}_2,u\right) = z,\ z = (\tau_1,\sigma_1). \tag{64}$$

With these results we can write the first order conditions for the unconstrained version of (31) as

$$\begin{aligned}
\frac{\partial W\left(\lambda_1,\tau_1,\sigma_1,\overline{L}_2\right)}{\partial\lambda_1} &= \int_{u^{\sigma_1}}^{u^{\tau_1}} \left[C_1\left(\lambda_1,\overline{L}_2,u\right)+D_1^1(\lambda_1)\right] f(u)\,du = 0;\\
\frac{\partial W\left(\lambda_1,\tau_1,\sigma_1,\overline{L}_2\right)}{\partial\tau_1} &= \int_{u^{\tau_1}}^{\overline{u}} \left[-\tau_1+D_1^1\left(Q_1\left(\tau_1,\overline{L}_2,u\right)\right)\right]\frac{\partial Q_1\left(\tau_1,\overline{L}_2,u\right)}{\partial\tau_1} f(u)\,du = 0;\\
\frac{\partial W\left(\lambda_1,\tau_1,\sigma_1,\overline{L}_2\right)}{\partial\sigma_1} &= \int_{\underline{u}}^{u^{\sigma_1}} \left[-\sigma_1+D_1^1\left(Q_1\left(\sigma_1,\overline{L}_2,u\right)\right)\right]\frac{\partial Q_1\left(\sigma_1,\overline{L}_2,u\right)}{\partial\sigma_1} f(u)\,du = 0.
\end{aligned} \tag{65}$$

Next, with the emissions responses to a tax when the co-pollutant is fixed with tradable permits given by (45), note that

$$Q_1\left(z,\overline{L}_2,u\right)=\frac{a_1+u-z+w\overline{L}_2}{a_2},\ z=(\tau_1,\sigma_1) \tag{66}$$

with

$$\frac{\partial Q_1\left(z,\overline{L}_2,u\right)}{\partial z}=-\frac{1}{a_2},\ z=(\tau_1,\sigma_1). \tag{67}$$

Given the form of $Q_1\left(z,\overline{L}_2,u\right)$ and the marginal damage functions, we also have

$$D_1^1\left(Q_1\left(z,\overline{L}_2,u\right)\right)=d_{11}+d_{12}\frac{a_1+u-z+w\overline{L}_2}{a_2},\ z=(\tau_1,\sigma_1). \tag{68}$$

Moreover,

$$D_1^1(\lambda_1)=d_{11}+d_{12}\lambda_1, \tag{69}$$

and

$$C_1\left(\lambda_1,\overline{L}_2,u\right)=-(a_1+u)+a_2\lambda_1-w\overline{L}_2. \tag{70}$$

Substitute (66) through (70) into (65) to obtain:

$$\begin{aligned}
\frac{\partial W\left(\lambda_1,\tau_1,\sigma_1,\overline{L}_2\right)}{\partial\lambda_1} &= \int_{u^{\sigma_1}}^{u^{\tau_1}}\left[-a_1+d_{11}+(a_2+d_{12})\lambda_1-w\overline{L}_2-u\right]f(u)\,du=0;\\
\frac{\partial W\left(\lambda_1,\tau_1,\sigma_1,\overline{L}_2\right)}{\partial\tau_1} &= \int_{u^{\tau_1}}^{\overline{u}}\left(-\frac{a_1d_{12}+a_2d_{11}}{a_2^2}+\frac{a_2+d_{12}}{a_2^2}\tau_1-\frac{wd_{12}}{a_2^2}\overline{L}_2-\frac{d_{12}}{a_2^2}u\right)f(u)\,du=0;\\
\frac{\partial W\left(\lambda_1,\tau_1,\sigma_1,\overline{L}_2\right)}{\partial\sigma_1} &= \int_{\underline{u}}^{u^{\sigma_1}}\left(-\frac{a_1d_{12}+a_2d_{11}}{a_2^2}+\frac{a_2+d_{12}}{a_2^2}\sigma_1-\frac{wd_{12}}{a_2^2}\overline{L}_2-\frac{d_{12}}{a_2^2}u\right)f(u)\,du=0,
\end{aligned}$$

which can be rearranged to obtain equations (32) through (34). □

7.3 Proof of Proposition 1

Part (1). Recall from the discussion following (21) that the optimal policy is the tax $t_1^*(\bar{t}_2)$ if and only if the solution to (21) yields $\tau_1^*(\bar{t}_2)\le\sigma_1^*(\bar{t}_2)$. To determine the conditions under which this is true subtract (24) from (23) to obtain:

$$\tau_1^*(\bar{t}_2)-\sigma_1^*(\bar{t}_2)=\frac{(a_2+w)(a_2d_{12}+wd_{22})}{a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+w^2d_{22}}\left\{E\left[u|u^{\tau_1^*}\le u\le\overline{u}\right]-E\left[u|\underline{u}\le u\le u^{\sigma_1^*}\right]\right\}. \tag{71}$$

The denominator of (71) and a_2+w are strictly positive. Moreover, since $E(u)=0$, $E\left[u|\underline{u}\leq u\leq u^{\sigma_1^*}\right]\leq 0$ and $E\left[u|u^{\tau_1^*}\leq u\leq \bar{u}\right]\geq 0$, but they both cannot be zero simultaneously. Therefore, $E\left[u|u^{\tau_1^*}\leq u\leq \bar{u}\right]-E\left[u|\underline{u}\leq u\leq u^{\sigma_1^*}\right]>0$. Consequently:

$$\tau_1^*(\bar{t}_2)\leq\sigma_1^*(\bar{t}_2)\Longleftrightarrow a_2d_{12}+wd_{22}\leq 0,$$

which reveals that the tax $t_1^*(\bar{t}_2)$is the optimal policy if and only if $a_2d_{12}+wd_{22}\leq 0$, which is Part (1) of Proposition 3.

Part (2). To prove part (2) of Proposition 1, recall from the discussion following (21) that a simple emissions market is optimal if and only if:

$$u^{\tau_1^*}=u^{\tau_1}\left(\lambda_1^*(\bar{t}_2),\tau_1^*(\bar{t}_2),\bar{t}_2\right)\geq\bar{u}\text{ and }u^{\sigma_1^*}=u^{\sigma_1}\left(\lambda_1^*(\bar{t}_2),\sigma_1^*(\bar{t}_2),\bar{t}_2\right)\leq\underline{u}.$$

We begin the proof by specifying $u^{\tau_1^*}$ and $u^{\sigma_1^*}$. Recall from (17) that the cut-off values of u are implicitly defined by

$$z=-C_1\left(\lambda_1,Q_2\left(\lambda_1,\bar{t}_2,u^z\right),u^z\right),\ z=(\tau_1,\sigma_1).$$

Explicitly,

$$u^z=z-a_1+a_2\lambda_1-wQ_2,\ z=(\tau_1,\sigma_1).\tag{72}$$

Given that pollutant 2 is controlled with the tax $\bar{t}_2$, emissions of 2 at u^z satisfy $\bar{t}_2=-C_2\left(\lambda_1,Q_2,u^z\right)=a_1-a_2Q_2+w\lambda_1,\ z=(\tau_1,\sigma_1)$; that is,

$$Q_2\left(\lambda_1,\bar{t}_2,u^z\right)=\frac{a_1+u^z-\bar{t}_2+w\lambda_1}{a_2},\ z=(\tau_1,\sigma_1).$$

Substitute $Q_2\left(\lambda_1,\bar{t}_2,u^z\right)$ in for Q_2 in (72) to obtain

$$u^z\left(\lambda_1,t_1,\bar{t}_2\right)=-a_1+(a_2-w)\lambda_1+\frac{a_2z}{a_2+w}+\frac{w\bar{t}_2}{a_2+w},\ z=(\tau_1,\sigma_1).\tag{73}$$

At the unconstrained solution to (21):

$$u^{\tau_1}\left(\lambda_1^*(\bar{t}_2),\tau_1^*(\bar{t}_2),\bar{t}_2\right)=-a_1+(a_2-w)\lambda_1^*(\bar{t}_2)+\frac{a_2\tau_1^*(\bar{t}_2)}{a_2+w}+\frac{w\bar{t}_2}{a_2+w};\tag{74}$$

$$u^{\sigma_1}\left(\lambda_1^*(\bar{t}_2),\sigma_1^*(\bar{t}_2),\bar{t}_2\right)=-a_1+(a_2-w)\lambda_1^*(\bar{t}_2)+\frac{a_2\sigma_1^*(\bar{t}_2)}{a_2+w}+\frac{w\bar{t}_2}{a_2+w}.\tag{75}$$

After substituting (23) through (24) into (74) and (75), it is possible to show:

$$
\begin{aligned}
u^{\tau_1}\left(\lambda_1^*(\bar{t}_2),\tau_1^*(\bar{t}_2),\bar{t}_2\right) =& \frac{(a_2-w)\left(a_2^2+a_2w-wd_{22}\right)}{a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+d_{22}w^2}E\left[u|\,u^{\sigma_1^*}\leq u\leq u^{\tau_1^*}\right] \\
&+\frac{a_2\left(a_2d_{12}+d_{22}w\right)}{a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+d_{22}w^2}E\left[u|\,u^{\tau_1^*}\leq u\leq \overline{u}\right]; \qquad (76)
\end{aligned}
$$

$$
\begin{aligned}
u^{\sigma_1}\left(\lambda_1^*(\bar{t}_2),\sigma_1^*(\bar{t}_2),\bar{t}_2\right) =& \frac{(a_2-w)\left(a_2^2+a_2w-wd_{22}\right)}{a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+d_{22}w^2}E\left[u|\,u^{\sigma_1^*}\leq u\leq u^{\tau_1^*}\right] \\
&+\frac{a_2\left(a_2d_{12}+d_{22}w\right)}{a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+d_{22}w^2}E\left[u|\,\underline{u}\leq u\leq u^{\sigma_1 *}\right] \qquad (77)
\end{aligned}
$$

The necessary condition (or conditions) for emissions trading to be the optimal policy is found by evaluating the conditions under which $u^{\tau_1^*}=u^{\tau_1}\left(\lambda_1^*(\bar{t}_2),\tau_1^*(\bar{t}_2),\bar{t}_2\right)\geq\overline{u}$ and $u^{\sigma_1^*}=u^{\sigma_1}\left(\lambda_1^*(\bar{t}_2),\sigma_1^*(\bar{t}_2),\bar{t}_2\right)\leq\underline{u}$. To do so, it is necessary to evaluate the conditional expectations in (76) and (77) under these conditions. First, since $E(u)=0$ and the support of u is $[\underline{u},\overline{u}]$, $u^{\tau_1^*}\geq\overline{u}$ and $u^{\sigma_1^*}\leq\underline{u}$ imply $E\left[u|\,u^{\sigma_1^*}\leq u\leq u^{\tau_1^*}\right]=0$. Next, we cannot directly evaluate

$$E[u|u^{\tau_1^*}\leq u\leq\overline{u}]=\frac{\int_{u^{\tau_1^*}}^{\overline{u}}uf(u)\,du}{\int_{u^{\tau_1^*}}^{\overline{u}}f(u)\,du},$$

given $u^{\tau_1^*}\geq\overline{u}$, but we can use l'Hopital's rule to determine

$$lim_{u^{\tau_1^*}\to\overline{u}}E[u|u^{\tau_1^*}\leq u\leq\overline{u}]=\overline{u}.$$

Similarly, we have

$$lim_{u^{\sigma_1^*}\to\underline{u}}E[u|\,\underline{u}\leq u\leq u^{\sigma_1 *}]=\underline{u}.$$

Substitute these limiting values and $E\left[u|\,u^{\sigma_1^*}\leq u\leq u^{\tau_1^*}\right]=0$ into (76) and (77) to obtain:

$$
\begin{aligned}
u^{\tau_1^*}=u^{\tau_1}\left(\lambda_1^*(\bar{t}_2),\tau_1^*(\bar{t}_2),\bar{t}_2\right)\geq\overline{u} &\Longrightarrow \frac{a_2\left(a_2d_{12}+d_{22}w\right)\overline{u}}{a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+d_{22}w^2}\geq\overline{u}; \\
u^{\sigma_1^*}=u^{\sigma_1}\left(\lambda_1^*(\bar{t}_2),\sigma_1^*(\bar{t}_2),\bar{t}_2\right)\leq\underline{u} &\Longrightarrow \frac{a_2\left(a_2d_{12}+d_{22}w\right)\underline{u}}{a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+d_{22}w^2}\leq\underline{u}.
\end{aligned}
$$

Since $a_2\left(a_2^2-w^2\right)+a_2^2d_{12}+d_{22}w^2$ and a_2 are strictly positive, these inequalities hold if and only if

$$\frac{a_2\left(a_2 d_{12}+w d_{22}\right)}{a_2\left(a_2^2-w^2\right)+a_2^2 d_{12}+w^2 d_{22}} \geq 1,$$

which simplifies to $wd_{22} \geq a_2(a_2+w)$: therefore, this is a necessary condition for an simple emission trading to be the optimal policy.

To prove that $wd_{22} \geq a_2(a_2+w)$ is also sufficient for emissions trading to be the optimal policy, suppose toward a contradiction that $wd_{22} \geq a_2(a_2+w)$, but that pure trading is not optimal; that is, $u^{\tau_1^*} = u^{\tau_1}\left(\lambda_1^*(\bar{t}_2), \tau_1^*(\bar{t}_2), \bar{t}_2\right) < \bar{u}$ or $u^{\sigma_1^*} = u^{\sigma_1}\left(\lambda_1^*(\bar{t}_2), \sigma_1^*(\bar{t}_2), \bar{t}_2\right) > \underline{u}$. Note first that since $u^{\tau_1^*} < \bar{u}$,

$$E\left[u|u^{\tau_1^*} \leq u \leq \bar{u}\right] > u^{\tau_1}\left(\lambda_1^*(\bar{t}_2), \tau_1^*(\bar{t}_2), \bar{t}_2\right).$$

Using (76),

$$\begin{aligned} E\left[u|u^{\tau_1^*} \leq u \leq \bar{u}\right] > & \frac{(a_2-w)\left(a_2^2+a_2 w-w d_{22}\right)}{a_2\left(a_2^2-w^2\right)+a_2^2 d_{12}+d_{22} w^2} E\left[u|u^{\sigma_1^*} \leq u \leq u^{\tau_1^*}\right] \\ & + \frac{a_2\left(a_2 d_{12}+d_{22} w\right)}{a_2\left(a_2^2-w^2\right)+a_2^2 d_{12}+d_{22} w^2} E\left[u|u^{\tau_1^*} \leq u \leq \bar{u}\right], \end{aligned}$$

which implies

$$\frac{(a_2-w)\left(a_2(a_2+w)-w d_{22}\right)}{a_2\left(a_2^2-w^2\right)+a_2^2 d_{12}+w^2 d_{22}}\left\{E\left[u|u^{\sigma_1^*} \leq u \leq u^{\tau_1^*}\right]-E\left[u|u^{\tau_1^*} \leq u \leq \bar{u}\right]\right\} < 0. \tag{78}$$

The first term of the left side of (78) is less than or equal to zero because $a_2\left(a_2^2-w^2\right)+a_2^2 d_{12}+w^2 d_{22} > 0$, $a_2-w > 0$, and $a_2(a_2+w)-wd_{22} \leq 0$ by assumption. The second term involving the conditional expectations is strictly negative because

$$E\left[u|u^{\sigma_1^*} \leq u \leq u^{\tau_1^*}\right] \leq u^{\tau_1^*} < E\left[u|u^{\tau_1^*} \leq u \leq \bar{u}\right].$$

Since the first term of (78) is weakly negative and the second is strictly negative, the inequality cannot hold and we have obtained our contradiction. Thus, $wd_{22} \geq a_2(a_2+w)$ is also a sufficient condition for the optimal policy to be a pure emissions market. As an aside, we could have obtained a similar contradiction by showing that $u^{\sigma_1^*} = u^{\sigma_1}\left(\lambda_1^*(\bar{t}_2), \sigma_1^*(\bar{t}_2), \bar{t}_2\right) > \underline{u}$ cannot hold if $wd_{22} \geq a_2(a_2+w)$.

Part (3). Part (3) follows directly from parts 1 and 2. Since the tax is the optimal policy pollutant 1 if and only if $a_2d_{12}+wd_{22} \leq 0$, the pure trading program is optimal if and only if $wd_{22} \geq a_2(a_2+w)$, then it must be the case that the hybrid policy is optimal if and only if $wd_{22} \in (-a_2d_{12}, a_2(a_2+$

$w)$).

Part (4). To begin the proof of Part (4) we calculate expected social costs under $t_1^*(\bar{t}_2)$ given by (25) and $L_1^*(\bar{t}_2)$ given by (26). With our specific functional forms, the expected social cost function is:

$$W = E\left[a_0 - (a_1 + u - d_{11})\,Q_1 - (a_1 + u - d_{21})\,Q_2 + \frac{a_2 + d_{12}}{2} Q_1^2 + \frac{a_2 + d_{22}}{2} Q_2^2 - wQ_1Q_2\right]. \quad (79)$$

Under the tax $t_1^*(\bar{t}_2)$, we have:

$$\begin{aligned} W(t_1^*(\bar{t}_2), \bar{t}_2) = E\,[&a_0 - (a_1 + u - d_{11})\,Q_1\,(t_1^*(\bar{t}_2), \bar{t}_2, u) - (a_1 + u - d_{21})\,Q_2\,(t_1^*(\bar{t}_2), \bar{t}_2, u) \\ &+ \frac{a_2 + d_{12}}{2}\left[Q_1\,(t_1^*(\bar{t}_2), \bar{t}_2, u)\right]^2 + \frac{a_2 + d_{22}}{2}\left[Q_2\,(t_1^*(\bar{t}_2), \bar{t}_2, u)\right]^2 - wQ_1\,(t_1^*(\bar{t}_2), \bar{t}_2, u)\,Q_2\,(t_1^*(\bar{t}_2), \bar{t}_2, u)\Big], \end{aligned} \quad (80)$$

where, using (44), we have:

$$Q_1\,(t_1^*(\bar{t}_2), \bar{t}_2, u) = \frac{(a_1 + u)\,(a_2 + w) - a_2 t_1^*(\bar{t}_2) - w\bar{t}_2}{a_2^2 - w^2};$$

$$Q_2\,(t_1^*(\bar{t}_2), \bar{t}_2, u) = \frac{(a_1 + u)\,(a_2 + w) - a_2\bar{t}_2 - wt_1^*(\bar{t}_2)}{a_2^2 - w^2},$$

and $t_1^*(\bar{t}_2)$ is given by (25). Under emissions trading with $L_1^*(\bar{t}_2)$ permits,

$$\begin{aligned} W(L_1^*(\bar{t}_2), \bar{t}_2) = E\Big[&a_0 - (a_1 + u - d_{11})\,L_1^*(\bar{t}_2) - (a_1 + u - d_{21})\,Q_2\,(L_1^*(\bar{t}_2), \bar{t}_2, u) + \frac{a_2 + d_{12}}{2}\left[L_1^*(\bar{t}_2)\right]^2 \\ &+ \frac{a_2 + d_{22}}{2}\left[Q_2\,(L_1^*(\bar{t}_2), \bar{t}_2, u)\right]^2 - wL_1^*(\bar{t}_2)Q_2\,(L_1^*(\bar{t}_2), \bar{t}_2, u)\Big], \end{aligned} \quad (81)$$

where, using (45),

$$Q_2\,(L_1^*(\bar{t}_2), \bar{t}_2, u) = \frac{a_1 + u - \bar{t}_2 + wL_1^*(\bar{t}_2)}{a_2},$$

and $L_1^*(\bar{t}_2)$ is given by (26). Subtracting $W(L_1^*(\bar{t}_2), \bar{t}_2)$ from $W(t_1^*(\bar{t}_2), \bar{t}_2)$ yields:

$$W(t_1^*(\bar{t}_2), \bar{t}_2) - W(L_1^*(\bar{t}_2), \bar{t}_2) = \left(\frac{E(u^2)}{2}\right)\left[\frac{wd_{22}(2a_2 - w) - a_2(a_2^2 - w^2 - a_2 d_{12})}{a_2^2\,(w - a_2)^2}\right].$$

Since $E(u^2)/2a_2^2(w-a_2)^2 > 0$:

$$
\begin{aligned}
sgn\left[W(t_1^*(\bar{t}_2),\bar{t}_2) - W(L_1^*(\bar{t}_2),\bar{t}_2)\right] &= sgn\left[wd_{22}(2a_2 - w) - a_2(a_2^2 - w^2 - a_2 d_{12})\right] \\
&= sgn\left[wd_{22} - \frac{a_2\left(a_2^2 - w^2 - a_2 d_{12}\right)}{(2a_2 - w)}\right].
\end{aligned}
$$

Part (4) of Proposition 1 follows directly from this equation.

Proving Parts (5) through (8) of Proposition 1 proceeds in the same way as the proofs to Parts 1 through 4.

Part (5). To prove Part (5) of Proposition 1 recall that, given that pollutant 2 is controlled by $\overline{L}_2$ tradable permits, the optimal policy is the tax $t_1^*\left(\overline{L}_2\right)$ if and only if the solution to (31) yields $\tau_1^*\left(\overline{L}_2\right) \leq \sigma_1^*\left(\overline{L}_2\right)$. To determine the conditions under which this is true subtract (34) from (33) to obtain

$$
\tau_1^*\left(\overline{L}_2\right) - \sigma_1^*\left(\overline{L}_2\right) = \frac{d_{12}}{a_2 + d_{12}}\left\{E\left[u|u^{\tau_1^*} \leq u \leq \overline{u}\right] - E\left[u|\underline{u} \leq u \leq u^{\sigma_1^*}\right]\right\} \tag{82}
$$

The denominator of (82) is strictly positive. Moreover, since $E(u) = 0$, $E\left[u|\underline{u} \leq u \leq u^{\sigma_1^*}\right] \leq 0$ and $E\left[u|u^{\tau_1^*} \leq u \leq \overline{u}\right] \geq 0$, but both cannot be zero simultaneously. Therefore, $E\left[u|u^{\tau_1^*} \leq u \leq \overline{u}\right] - E\left[u|\underline{u} \leq u \leq u^{\sigma_1^*}\right] > 0$. Consequently, $\tau_1^*\left(\overline{L}_2\right) \leq \sigma_1^*\left(\overline{L}_2\right) \Longleftrightarrow d_{12} \leq 0$. However, since $d_{12} \geq 0$,

$$
\tau_1^*\left(\overline{L}_2\right) \leq \sigma_1^*\left(\overline{L}_2\right) \Longleftrightarrow d_{12} = 0.
$$

Hence, the tax $t_1^*\left(\overline{L}_2\right)$is the optimal policy if and only if $d_{12} = 0$, which is the desired result.

Part (6). Recall that a simple emissions market is optimal if and only if

$$
u^{\tau_1^*} = u^{\tau_1}\left(\lambda_1^*\left(\overline{L}_2\right), \tau_1^*\left(\overline{L}_2\right), \overline{L}_2\right) \geq \overline{u} \text{ and } u^{\sigma_1^*} = u^{\sigma_1}\left(\lambda_1^*\left(\overline{L}_2\right), \sigma_1^*\left(\overline{L}_2\right), \overline{L}_2\right) \leq \underline{u}.
$$

Toward specifying $u^{\tau_1^*}$ and $u^{\sigma_1^*}$, recall from (27) that the cut-off values of u are implicitly defined by $z = -C_1\left(\lambda_1, \overline{L}_2, u^z\right)$, $z = (\tau_1, \sigma_1)$. Explicitly,

$$
u^z = z - a_1 + a_2\lambda_1 - w\overline{L}_2,\ z = (\tau_1, \sigma_1). \tag{83}
$$

At the unconstrained solution to (31):

$$
u^{\tau_1}\left(\lambda_1^*\left(\overline{L}_2\right), \tau_1^*\left(\overline{L}_2\right), \overline{L}_2\right) = \tau_1^*\left(\overline{L}_2\right) - a_1 + a_2\lambda_1^*\left(\overline{L}_2\right) - w\overline{L}_2; \tag{84}
$$

$$
u^{\sigma_1}\left(\lambda_1^*\left(\overline{L}_2\right), \tau_1^*\left(\overline{L}_2\right), \overline{L}_2\right) = \sigma_1^*\left(\overline{L}_2\right) - a_1 + a_2\lambda_1^*\left(\overline{L}_2\right) - w\overline{L}_2. \tag{85}
$$

After substituting (32) and (33) into (84), it is possible to write $u^{\tau_1}\left(\lambda_1^*\left(\overline{L}_2\right),\tau_1^*\left(\overline{L}_2\right),\overline{L}_2\right)$ as

$$u^{\tau_1}\left(\lambda_1^*\left(\overline{L}_2\right),\tau_1^*\left(\overline{L}_2\right),\overline{L}_2\right)=\frac{d_{12}}{a_2+d_{12}}E\left[u|u^{\tau_1^*}\leq u\leq\overline{u}\right]+\frac{a_2}{a_2+d_{12}}E\left[u|u^{\sigma_1^*}\leq u\leq u^{\tau_1^*}\right]. \quad (86)$$

Recall that

$$u^{\tau_1^*}=u^{\tau_1}\left(\lambda_1^*\left(\overline{L}_2\right),\tau_1^*\left(\overline{L}_2\right),\overline{L}_2\right)\geq\overline{u}\text{ and }u^{\sigma_1^*}=u^{\sigma_1}\left(\lambda_1^*\left(\overline{L}_2\right),\sigma_1^*\left(\overline{L}_2\right),\overline{L}_2\right)\leq\underline{u},$$

imply $E\left[u|u^{\sigma_1^*}\leq u\leq u^{\tau_1*}\right]=0$ and

$$lim_{u^{\tau_1^*}\to\overline{u}}E[u|u^{\tau_1^*}\leq u\leq\overline{u}]=\overline{u}.$$

Substitute these into (86) to obtain

$$u^{\tau_1}\left(\lambda_1^*\left(\overline{L}_2\right),\tau_1^*\left(\overline{L}_2\right),\overline{L}_2\right)\geq\overline{u}\Longrightarrow\frac{d_{12}\overline{u}}{a_2+d_{12}}\geq\overline{u}.$$

However, this is not possible since $d_{12}/(a_2+d_{12})<1$. This reveals that a pure trading scheme for pollutant 1 cannot be optimal. (We could have proved Part (6) of the Proposition by showing that $u^{\sigma_1}\left(\lambda_1^*\left(\overline{L}_2\right),\sigma_1^*\left(\overline{L}_2\right),\overline{L}_2\right)\leq\underline{u}$ is also not possible).

Part (7). This part follows directly from Parts (5) and (6). Since the tax is optimal if and only if $d_{12}=0$ and a pure trading schemes is never optimal, it must be true that the hybrid policy is optimal if and only if $d_{12}>0$.

Part (8). If pollutant 2 is controlled with $\overline{L}_2$ tradable permits, the social welfare function (79) under $t_1^*(\overline{L}_2)$ is

$$\begin{aligned}W(t_1^*(\overline{L}_2),\overline{L}_2)=E\Bigg[&a_0-(a_1+u-d_{11})\,Q_1\left(t_1^*(\overline{L}_2),\overline{L}_2,u\right)-(a_1+u-d_{21})\,\overline{L}_2+\frac{a_2+d_{12}}{2}\left[Q_1\left(t_1^*(\overline{L}_2),\overline{L}_2,u\right)\right]^2\\&+\frac{a_2+d_{22}}{2}\left[\overline{L}_2\right]^2-wQ_1\left(t_1^*(\overline{L}_2),\overline{L}_2,u\right)\overline{L}_2\Bigg],\end{aligned} \quad (87)$$

where, using (45),

$$Q_1\left(t_1^*(\overline{L}_2),\overline{L}_2,u\right)=\frac{(a_1+u)-t_1^*(\overline{L}_2)+w\overline{L}_2}{a_2}$$

and $t_1^*(\overline{L}_2)$ is given by (35). Under emissions trading with $L_1^*(\overline{L}_2)$ permits,

$$W(L_1^*(\overline{L}_2),\overline{L}_2) = E\left[a_0 - (a_1+u-d_{11})L_1^*(\overline{L}_2) - (a_1+u-d_{21})\overline{L}_2 + \frac{a_2+d_{12}}{2}\left[L_1^*(\overline{L}_2)\right]^2 + \frac{a_2+d_{22}}{2}\left[\overline{L}_2\right]^2 - wL_1^*(\overline{L}_2)\overline{L}_2\right], \tag{88}$$

where $L_1^*(\bar{L}_2)$ is given by (36). Subtracting $W(L_1^*(\overline{L}_2),\overline{L}_2)$ from $W(t_1^*(\overline{L}_2),\overline{L}_2)$ yields

$$W(t_1^*(\overline{L}_2),\overline{L}_2) - W(L_1^*(\overline{L}_2),\overline{L}_2) = \frac{E(u^2)(d_{12}-a_2)}{2a_2^2}.$$

Since $E(u^2)/2a_2^2 > 0$,

$$sgn\left[W(t_1^*(\overline{L}_2),\overline{L}_2) - W(L_1^*(\overline{L}_2),\overline{L}_2)\right] = sgn(d_{12}-a_2).$$

Part (8) of Proposition 1 follows directly from this equation. We have now completed the proof of Proposition 1. □

7.4 Proof of Lemma 1

The proof proceeds by first deriving expected emissions of pollutant 1 under a hybrid policy, given the regulation of pollutant 2. we use this derivation to demonstrate (37). For each case of pollutant 2 regulation we demonstrate (37). We then derive expected emissions of pollutant 2, given that it is regulated with a tax and pollutant 1 is regulated with the optimal hybrid policy. We use this derivation to demonstrate (38).

Given the regulation of pollutant 2, $\bar{x}$, expected aggregate emissions of pollutant 1 under the optimal hybrid policy $h_1^*(\bar{x}) = (\lambda_1^*(\bar{x}), \tau_1^*(\bar{x}), \sigma_1^*(\bar{x}))$ is:

$$E\left(Q_1\left(h_1^*(\bar{x}),\bar{x},u\right)\right) = \int_{\underline{u}}^{u^{\sigma_1^*}} Q_1\left(\sigma_1^*(\bar{x}),\bar{x},u\right) f(u)\,du + \int_{u^{\sigma_1^*}}^{u^{\tau_1^*}} \lambda_1^*(\bar{x}) f(u)\,du + \int_{u^{\tau_1^*}}^{\overline{u}} Q_1\left(\tau_1^*(\bar{x}),\bar{x},u\right) f(u)\,du. \tag{89}$$

Suppose that pollutant 2 is regulated with a tax $\bar{t}_2$. Using (52), (89) can be expressed as

$$E\left(Q_1\left(h_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) = \int_{\underline{u}}^{u^{\sigma_1^*}} \frac{(a_1+u)\left(a_2+w\right)-a_2\sigma_1^*\left(\bar{t}_2\right)-w\bar{t}_2}{a_2^2-w^2} f\left(u\right)du + \int_{u^{\sigma_1^*}}^{u^{\tau_1^*}} \lambda_1^*\left(\bar{t}_2\right)f\left(u\right)du.$$
$$+\int_{u^{\tau_1^*}}^{\overline{u}} \frac{(a_1+u)\left(a_2+w\right)-a_2\tau_1^*\left(\bar{t}_2\right)-w\bar{t}_2}{a_2^2-w^2} f\left(u\right)du. \tag{90}$$

Substituting the optimal hybrid components, (22) through (24) into (90) and manipulating terms produces

$$E\left(Q_1\left(h_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) = \frac{a_2^2(a_1-d_{11})+w(a_2\left(a_1-d_{21}\right)-d_{22}(a_1-\bar{t}_2))}{A} + \frac{a_2(a_2+w)-wd_{22}}{A}G,$$

where

$$G = \int_{\underline{u}}^{u^{\sigma_1^*}} uf\left(u\right)du + \int_{u^{\sigma_1^*}}^{u^{\tau_1^*}} uf\left(u\right)du + \int_{u^{\tau_1^*}}^{\overline{u}} uf\left(u\right)du.$$

Clearly, $G=0$ for a hybrid policy. Moreover, it is zero for the optimal tax and the optimal trading program as well. In the case of the tax, $u^{\sigma_1^*} = u^{\tau_1^*} \leq \underline{u}$, resulting in $G=0$. In the case of a pure trading program, $u^{\sigma_1^*} \leq \underline{u}$ and $u^{\tau_1^*} \geq \overline{u}$, so again $G=0$. Therefore, we have

$$\begin{aligned} E\left(Q_1\left(h_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) &= E\left(Q_1\left(t_1^*(\bar{t}_2),\bar{t}_2,u\right)\right) = L_1^*(\bar{t}_2) \\ &= \frac{a_2^2(a_1-d_{11})+w(a_2\left(a_1-d_{21}\right)-d_{22}(a_1-\bar{t}_2))}{A}. \end{aligned} \tag{91}$$

This is (37) for $\bar{x} = \bar{t}_2$.

Now suppose that pollutant 2 is regulated with $\overline{L}_2$ permits. Using (66), (89) can be expressed as

$$E\left(Q_1\left(h_1^*(\overline{L}_2),\bar{L}_2,u\right)\right) = \int_{\underline{u}}^{u^{\sigma_1^*}} \left(\frac{a_1+u-\sigma_1^*\left(\overline{L}_2\right)+w\overline{L}_2}{a_2}\right) f\left(u\right)du + \int_{u^{\sigma_1^*}}^{u^{\tau_1^*}} \lambda_1^*\left(\overline{L}_2\right)f\left(u\right)du$$
$$+\int_{u^{\tau_1^*}}^{\overline{u}} \left(\frac{a_1+u-\tau_1^*\left(\overline{L}_2\right)+w\overline{L}_2}{a_2}\right) f\left(u\right)du. \tag{92}$$

Substituting the optimal hybrid components, (32) through (34) into (92) and manipulating terms produces

$$E\left(Q_1\left(h_1^*(\overline{L}_2),\overline{L}_2,u\right)\right) = \frac{a_1-d_{11}+w\overline{L}_2+H}{a_2+d_{12}},$$

where

$$H = \int_{\underline{u}}^{u^{\sigma_1^*}} uf\left(u\right)du + \int_{u^{\sigma_1^*}}^{u^{\tau_1^*}} uf\left(u\right)du + \int_{u^{\tau_1^*}}^{\overline{u}} uf\left(u\right)du.$$

By similar arguments given in the case that pollutant 2 is regulated with a tax, $H = 0$ under $h_1^*(\overline{L}_2)$, $t_1^*(\overline{L}_2)$ and $L_1^*(\overline{L}_2)$. Therefore,

$$E\left(Q_1\left(h_1^*(\overline{L}_2), \overline{L}_2, u\right)\right) = E\left(Q_1\left(t_1^*(\overline{L}_2), \overline{L}_2, u\right)\right) = L_1^*(\overline{L}_2) = \frac{a_1 - d_{11} + w\overline{L}_2}{a_2 + d_{12}}. \tag{93}$$

This is (37) for $\overline{x} = \overline{L}_2$.

Deriving (38) for expected emissions of pollutant 2 follows the same approach. When pollutant 2 is regulated by a tax $\overline{t}_2$ and pollutant 1 is controlled with the optimal hybrid policy with given $\overline{t}_2$, the expected emissions of pollutant 2 is:

$$\begin{aligned} E\left(Q_2\left(h_1^*\left(\overline{t}_2\right), \overline{t}_2, u\right)\right) = & \int_{\underline{u}}^{u^{\sigma_1^*}} Q_2\left(\sigma_1^*\left(\overline{t}_2\right), \overline{t}_2, u\right) f(u)\, du + \int_{u^{\sigma_1^*}}^{u^{\tau_1^*}} Q_2\left(\lambda_1^*\left(\overline{t}_2\right), \overline{t}_2, u\right) f(u)\, du \\ & + \int_{u^{\tau_1^*}}^{\overline{u}} Q_2\left(\tau_1^*\left(\overline{t}_2\right), \overline{t}_2, u\right) f(u)\, du. \end{aligned} \tag{94}$$

By the same process used above it is possible to show

$$\begin{aligned} E\left(Q_2\left(h_1^*\left(\overline{t}_2\right), \overline{t}_2, u\right)\right) &= E\left(Q_2\left(t_1^*\left(\overline{t}_2\right), \overline{t}_2, u\right)\right) = E\left(Q_2\left(L_1^*\left(\overline{t}_2\right), \overline{t}_2, u\right)\right) \\ &= \frac{a_1 a_2\left(a_2 + d_{12} + w\right) - w\left(a_2 d_{11} + d_{21} w\right) - \left(\left(a_2^2 - w^2\right) + a_2 d_{12}\right)\overline{t}_2}{A}, \end{aligned} \tag{95}$$

which demonstrates (38). This completes the proof of Lemma 1. □

7.5 Proof of Proposition 2

Given the notation defined in (39) and (40), as well as our calculations (91), (93), and (95):

$$E\left(Q_1^*\left(\overline{t}_2\right)\right) = \frac{a_2^2(a_1 - d_{11}) + w(a_2\left(a_1 - d_{21}\right) - d_{22}(a_1 - \overline{t}_2))}{A}; \tag{96}$$

$$E\left(Q_1^*(\overline{L}_2)\right) = \frac{a_1 - d_{11} + w\overline{L}_2}{a_2 + d_{12}}; \tag{97}$$

$$E\left(Q_2^*\left(\overline{t}_2\right)\right) = \frac{a_1 a_2\left(a_2 + d_{12} + w\right) - w\left(a_2 d_{11} + d_{21} w\right) - \left(\left(a_2^2 - w^2\right) + a_2 d_{12}\right)\overline{t}_2}{A}. \tag{98}$$

To derive (41) of Proposition 2, first subtract (9) from (96) to obtain

$$E\left(Q_1^*\left(\overline{t}_2\right)\right) - \hat{Q}_1 = \frac{N_1}{A\left[\left(a_2 + d_{12}\right)\left(a_2 + d_{22}\right) - w^2\right]}, \tag{99}$$

where

$$N_1 = \left[a_2^2(a_1 - d_{11}) + w(a_2(a_1 - d_{21}) - d_{22}(a_1 - \bar{t}_2))\right]\left[(a_2 + d_{12})(a_2 + d_{22}) - w^2\right]$$
$$- A\left[w(a_1 - d_{21}) + (a_1 - d_{11})(a_2 + d_{22})\right].$$

Now subtract (10) from (98) to obtain

$$E\left(Q_2^*(\bar{t}_2)\right) - \hat{Q}_2 = \frac{N_2}{A\left[(a_2 + d_{12})(a_2 + d_{22}) - w^2\right]} \tag{100}$$

where

$$N_2 = \left[a_1 a_2(a_2 + d_{12} + w) - w(a_2 d_{11} + d_{21} w) - \left(\left(a_2^2 - w^2\right) + a_2 d_{12}\right)\bar{t}_2\right]\left[(a_2 + d_{12})(a_2 + d_{22}) - w^2\right]$$
$$- A\left[w(a_1 - d_{11}) + (a_1 - d_{21})(a_2 + d_{12})\right].$$

Dividing (99) by (100) and rearranging terms gives us

$$E\left(Q_1^*(\bar{t}_2)\right) - \hat{Q}_1 = \frac{N_1}{N_2}\left[E\left(Q_2^*(\bar{t}_2)\right) - \hat{Q}_2\right]. \tag{101}$$

It is straightforward to show that

$$\frac{N_1}{N_2} = \frac{-d_{22} w}{\left(a_2^2 - w^2\right) + a_2 d_{12}},$$

which upon substitution into (101) gives us our desired result (41).

To derive (42), take (98) and substitute $\overline{L}_2 = \overline{L}_2 - \hat{Q}_2 + \hat{Q}_2$ and collect terms to obtain

$$E\left(Q_1^*(\overline{L}_2)\right) = \frac{a_1 - d_{11} + w\hat{Q}_2}{a_2 + d_{12}} + \frac{w\left(\overline{L}_2 - \hat{Q}_2\right)}{a_2 + d_{12}}. \tag{102}$$

It is straightforward to show

$$\frac{a_1 - d_{11} + w\hat{Q}_2}{a_2 + d_{12}} = \hat{Q}_1,$$

which upon substitution into (102) gives us our desired result (42). This completes the proof of Proposition 2. □

8 References

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