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# **Endogenous Technical Change and Groundwater Management: Revisiting the Gisser-Sanchez Paradox**

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# Endogenous Technical Change and Groundwater Management: Revisiting the Gisser-Sanchez Paradox

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## **Abstract**

Traditional models of groundwater economics, as well as many current iterations of those models, assume that optimal aquifer depletion occurs with a fixed irrigation technology. As noted by Koundouri (2004), this assumption is one of several that contributes to the Gisser-Sanchez Effect (GSE), one of the most controversial theoretical/empirical results in groundwater management literature since it appeared in a seminal paper in 1980. The GSE states that economic benefits from managing the groundwater use for irrigation would be insignificant when the storage capacity of groundwater stock is relatively large and the demand for groundwater is highly inelastic. In this paper, we show that the elasticity of the groundwater demand curve decreases over time as increasing extraction costs drive movement to more efficient irrigation technologies. In addition, this shifting of the demand curve is even greater when incorporating a model of induced technical change through endogenous R&D expenditures. Using this model, we show that the GSE does not exist when the assumption of a fixed irrigation technology is relaxed.

*Key words:* The Gisser–Sanchez Paradox, consumptive water use, application rate, induced irrigation technology, optimal Government subsidy rate.

## **1. Introduction**

Following the wide-scale development of groundwater pumping for agriculture in the 1950's, a number of studies suggested that the open-access nature of groundwater meant that farmers were over-extracting the resource and would exhaust these renewable-but-depletable resources much sooner than is economically optimal (Milliman 1956, Bagley 1961, Burt 1967, Brown and Deacon 1972). These conclusions were called into question by Gisser and Sanchez (1980) in an influential paper which argued that the difference in producer surplus between the open-access and optimally managed cases was numerically insignificant for large aquifers subject to inelastic water demand. As noted by Koundouri

(2004), this Gisser-Sanchez Effect (GSE) has remained controversial and numerous studies have analyzed whether the GSE persists under a variety of specific conditions, including convex pumping costs (Worthington et al. 1985), shifting (non-constant) water demand (Brill and Burness 1994), adaptation by crop shifting (Kim et al. 1989), confined aquifers (Worthington et al 1985), heterogeneous users (Saak and Peterson 2007; Stratton 2008), strategic decision-making (Negri 1989; Rubio and Casino 2001), conjunctive management (Knapp and Olson 1995), risk aversion (Provencher and Burt 1993), and backstop water sources (Koundouri and Christos 2006). These studies generally find support for the GSE even under these conditions.

In this study, we identify some underlying limitations of the Gisser-Sanchez model and relax the key assumption of a fixed irrigation technology. We show that the GSE fails when irrigation technologies with different water use efficiency become available. Our results are robust and hold even when maintaining some of the very simple (and perhaps questionable) assumptions in the original model (such as constant marginal pumping costs per linear foot of lift). Moreover, the gains from optimal groundwater management become even more significant when irrigation technology is not only variable, but endogenous. By endogenous technical change we mean that higher water costs induce the development of technologies that improve water use efficiency.

Many refinements to models of optimal groundwater management exist. A number of studies look at the impact of water withdrawals on declining well yields and the impact on extraction costs (Sloggett and Mapp 1984, Brill and Burness 1994). Other studies look at complexities in aquifer structure, examining confined aquifers or multicell aquifers (Brozović et al 2006). Other studies examine the role of risk preferences (Knapp and Olson

1996), rising energy prices (Zilberman et al. 2008), or the role of competitive behavior and alternative Nash equilibria (Rubio and Casino 2001). We maintain the more simple assumptions of the original Gisser-Sanchez model as a point of departure in order to emphasize the critical role that technological innovation plays and also to facilitate the development of analytical solutions. Other studies that have examined the role of endogenous technology adoption decisions within the context of groundwater management include Shah et al (1995), Burness and Brill (2001), Koundouri and Christou (2006), Wang and Segerra (2011), and Ashwell and Peterson (2013).

The objective of our research is to reevaluate the validity of the GSE by reinvestigating the Gisser-Sanchez model of groundwater management. To achieve this goal, the remainder of our paper is organized as follows: We first identify the shortcomings of the Gisser-Sanchez model and discuss how these shortcomings can lead to their Paradox in Section 2. In Section 3, we discuss the effects of adopting an improved irrigation technology on groundwater use for irrigation. Section 4 extends the model to include induced irrigation technology where irrigation manufacturers respond to rising water pumping costs by allocating R&D toward systems with greater water use efficiency. We then present a dynamic model of groundwater use for irrigation in Section 5, which is based on consumptive-equivalent use of groundwater for irrigation, in the presence of an induced irrigation technology with uncertain dates of innovation and adoption. Economic properties of the optimal solutions for managing the groundwater use for irrigation are discussed. We investigate the GSE in Section 6 by discussing how the GSE fails when the assumption of a fixed irrigation technology is relaxed. Section 7 offers our concluding remarks.

## 2. Re-evaluation of the Gisser-Sanchez Dynamic Model of Groundwater Use

The Gisser-Sanchez model begins with an inverse linear irrigation water demand function derived from a quadratic crop production function based on application rate such that:

$$P_w = a - bW, \quad b > 0 \quad (1)$$

where  $P_w$  is the marginal benefits of water (or marginal cost of groundwater pumping) in dollars per acre-foot at time  $t$ ,  $a$  and  $b$  are parameters, both of which have the technology assumption embedded in them. The variable  $W$  measures acre-feet of groundwater withdrawn from the entire aquifer and applied to the crop. The marginal pumping cost ( $mc$ ) is represented as:

$$mc = k(SL - h(t)) \quad (2)$$

where,  $k$  is a constant pumping cost per acre-foot of groundwater per foot of lift,  $SL$  represents the elevation in feet of the irrigation surface level above sea level, and  $h(t)$  is water table level above sea level. Finally, the hydrologic differential equation was given by:

$$\dot{h}(t) = \frac{R + (\alpha - 1)W(t)}{AS} \quad (3)$$

where,  $R$  is the natural recharge to the aquifer.  $A$  is acreage above the aquifer,  $S$  is storativity coefficient, and  $\alpha$  is a return-flow coefficient representing the portion of the withdrawn water that recharges to the aquifer. As with the coefficients of the marginal revenue curve, the return flow coefficient implicitly contains the fixed technology assumption.

Using equations (1) through (3), the Gisser-Sanchez presented an optimal control model of groundwater use such that:

$$\text{Max} \int_0^{\infty} e^{-\delta t} [(a - k(SL - h(t)))W(t) - 0.5bW(t)^2] dt \quad (4)$$

subject to a hydrologic differential equation in (3) and the transversality condition that the user cost,  $\lambda(t)$ , at the terminal time equals zero, where  $\delta$  is the rate of discount.

The optimal time path of groundwater use ( $W^0$ ) is then given by:

$$W^0(t) = [h(t=0) - \frac{1}{k} (\frac{kR}{\delta(AS)} - (a - k(SL) + \frac{bR}{\alpha - 1}))] (\frac{AS}{\alpha - 1}) X \exp(X)t \quad (5-1)$$

where  $X = 0.5(\delta - \sqrt{\delta^2 - \frac{\delta k(\alpha - 1)}{b(AS)}})$ .

Meanwhile, the time path of water use under common property competition ( $\bar{W}$ ) is given by:

$$\bar{W}(t) = -\frac{R}{(\alpha - 1)} + [\frac{R}{(\alpha - 1)} + \frac{1}{b} (a - k(SL) + kh(t=0))] \exp[\frac{k(\alpha - 1)}{b(AS)}]t \quad (5-2)$$

From this producer-surplus optimization problem, Gisser-Sanchez find that, analytically, the difference in net present value between socially optimal groundwater management (i.e., equation (5-1)) and myopic groundwater management (i.e., equation (5-2)), which entirely ignores the marginal user cost ( $\lambda$ ), is insignificant. The marginal user cost is imposed through the impact of the hydrologic model on the marginal pumping cost given by a formula that depends up the elasticity of demand, the marginal pumping cost ( $k$ ), the return flow coefficient ( $\alpha$ ), and the size of the aquifer,  $(h_0 - h_L)AS$ , where  $h_0$  is the initial elevation of the aquifer's water table above sea level and  $h_L$  is the physical and economical lower bound of water table above sea level.

A key feature of water management is that consumptive use of water is less than the amount applied. Problems of this type are not uncommon with agricultural production

practices, and include both irrigation water (Caswell and Zilberman, 1986; Shah et al., 1995) and agricultural fertilizers (Lee and Kim, 2002). One shortcoming of the Gisser-Sanchez model is that it estimates economic benefits from irrigation water based on the application rate. Kim and Schaible (2000) demonstrated that this overestimates the economic benefits in proportional to the amount of irrigation water lost through leaching, runoff and evaporation. When equation (4) is corrected by specifying benefits in terms of consumptive water use, the objective function to be maximized becomes:

$$Max \int_0^{\infty} e^{-\delta t} [(\beta a - k(SL - h(t)))W(t) - 0.5\beta b W(t)^2] dt \quad (4')$$

where  $\beta$  represents irrigation efficiency such that  $\beta < (1-\alpha)$ .

A second shortcoming of the Gisser-Sanchez model is that it assumes that groundwater leached back into the soil recharges the aquifer without delay. The time for irrigation water to percolate back into underground aquifers depends upon the soil type, climate, current farming and irrigation technologies, and length of the vadoze zone. Previous work indicates that time lag for return-flow is approximately one foot per year on silt soil (Corps of Engineers, U.S. Army 1951).<sup>1</sup> The omission of this time lag results in an underestimation of the user-cost associated with groundwater depletion (Kim et al., 1993), and thus an underestimation of the benefits of groundwater management.

A third limitation of the Gisser-Sanchez model is that it assumes exogenously determined, fixed irrigation technology over an infinite planning horizon. When the imminent exhaustion of an irreplenishable natural resource like groundwater threatens to

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<sup>1</sup> Nitrogen fertilizer is highly water soluble. Time lags for nitrates leaching into the aquifers are approximately 30-60 years in Southern California (Pratt and Adriano 1973; Pratt 1984), 20 years in Buffalo County, Nebraska (Bentall 1975), and 30-60 years in Kansas (Nkonya and Featherstone 2000).



limit economic growth, this limitation may be offset by technical progress along with increasing capital accumulation and substitution (Kamien and Schwartz 1978). That is, the difference between the application rate and the rate of consumptive use of groundwater can be reduced by improving the efficiency of irrigation technology. As the energy and pumping costs increase, history has shown us that induced technologies, such as dropped nozzle, low energy precision application (LEPA) irrigation and subsurface drip irrigation technology, have been developed.

In the next section, we discuss how adopting improved irrigation technology affects the economic benefits from irrigation water use.

### 3. Effects of an Induced Irrigation Technology on Irrigation Water Use

To correctly measure economic benefits from irrigation water use, following Caswell and Zilberman (1986) we consider a quadratic crop production function to derive a linear irrigation water demand, based on consumptive irrigation water use,  $W^*$ :

$$Y(W^*) = a_0 + a_1 W^* - \frac{a_2}{2} (W^*)^2 \quad (6)$$

where  $Y$  is output, and  $a_0, a_1, a_2$  are positive parameters. The consumptive irrigation demand function is given by:

$$P_{w^*} = P_y (a_1 - a_2 W^*) \quad (6-1)$$

where  $P_{w^*}$  is the marginal benefits of consumptive irrigation water use. Economic benefits from consumptive irrigation water use are then represented by:

$$B(W^*; P_{w^*}) = P_y (a_1 W^* - \frac{a_2}{2} (W^*)^2) \quad (6-2)$$

Now, let  $W^* = \beta W_0$ , where  $\beta$  is a coefficient of irrigation efficiency and  $W_0$  is the amount of irrigation water applied with the current irrigation technology. Using equation (6), the quadratic crop production function based on consumptive-equivalent irrigation water application is then represented as (see Ashwell and Peterson 2013):

$$Y(\beta W_0) = a_0 + a_1 \beta W_0 - \frac{a_2}{2} (\beta W_0)^2 \quad (7)$$

The irrigation water demand based on the application rate is given by:<sup>2</sup>

$$P_{w_0} = P_y \beta (a_1 - a_2 \beta W_0) \quad (7-1)$$

The economic benefits associated with irrigation water use estimated using equations (7-1), which is based on application rate, is presented in equation (7-2).

$$B(W_0 : P_{w_0}) = \int_0^{W_0} [P_y \beta (a_1 - a_2 \beta x)] dx = P_y \beta [a_1 W_0 - \frac{a_2}{2} \beta W_0^2] \quad (7-2)$$

Kim and Schaible (2000) showed that equation (7-2) overestimates the benefits of irrigation water, and the magnitude of overestimation is proportional to the rate of irrigation inefficiency. Inserting  $W^* = \beta W_0$  into equation (6-2) gives:

$$B(\beta W_0 : P_{w^*}) = P_y \beta (a_1 W_0 - \frac{a_2}{2} \beta W_0^2) \quad (8)$$

where  $\beta W_0$  is “consumptive-equivalent” irrigation water use. While the right-hand sides of equations (7-2) and (8) are mathematically identical (see Gisser and Johnson 1983; Kim and Schaible 2000), the economic benefits in equation (7-2) are valued at  $P_{w_0}$ , while those in equation (8) are valued at  $P_{w^*}$  (where  $\beta P_{w^*} = P_{w_0}$ ). Therefore, the economic benefit presented in equation (8) is rewritten as:

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<sup>2</sup> Equation (7-1) should be correctly represented by  $\frac{P_{w_0}}{\beta} = P_y \beta (a_1 - a_2 \beta W_0)$ , where  $P_{w^*} = \frac{P_{w_0}}{\beta}$ .

$$\begin{aligned}\tilde{B}(\beta W_0 : \beta P_{w^*} = P_{w_0}) &= \beta [P_y \beta (a_1 W_0 - \frac{a_2}{2} \beta W_0^2)] \\ &= \beta B(W_0 : P_{w_0})\end{aligned}\tag{9}$$

Comparing equations (6-2) and (9) with equation (7-2), one can conclude that:

$$B(W^* : P_{w^*}) = \beta B(W_0 : P_{w_0})\tag{10}$$

Results in equations (9) and (10) are illustrated in Figure 1. The the curve AD represents the irrigation water demand based on application rate valued at  $P_{w_0}$ , the curve BC represents the irrigation water demand based on consumptive use valued at  $P_{w^*}$ , and the curve AC represents the consumptive-equivalent irrigation water demand valued at  $P_{w_0}$ . As producers adopt a more efficient irrigation technology, the amount of groundwater pumped, for each unit pumping cost, declines from the curve AD to the curve AC. Furthermore, the correct measure of the economic benefits resulting from groundwater use for irrigation is represented by the area  $\Delta 0AC$ , while the objective function in (4) measures the economic benefits with the area  $\Delta 0AD$ , resulting overestimation of the economic benefits by the area  $\Delta ACD$  in Figure 1.

A number of studies have claimed that an increase in the efficiency of irrigation technology would increase the marginal benefits resulting from irrigation water use, and, as a consequence farmers would increase not only water use per acre (by switching to more water intensive crops) but also irrigate more acres (Huffaker and Whittlesey, 2003; Pfeiffer and Lin, 2010; Ward and Pulido, 2008). However, our analysis, which is solely based on

the crop-(consumptive) water production function, indicates otherwise: that water use would decline as more efficient irrigation technology is adopted.<sup>3</sup>

#### 4. Induced Innovation and Adoption of Irrigation Technologies

##### (a). Development of an induced irrigation technology with uncertainty

Whether or not economic damages from overexploitation of groundwater from an aquifer can be manageable largely depends upon the irrigation technologies available for managing groundwater use for irrigation. Technical change could then respond to economic and policy incentives for groundwater management. While endogenous technological change is classified into two broad and disjoint categories of invention and learning-by-doing (Romer, 1990, 1994; Young, 1993), for the more specific category of improving irrigation technology our focus is the outcome of deliberate research by government, academia, and business enterprises. As a factor of production becomes increasingly scarce (more costly), these institutions allocate more R&D resources to developing technologies that save or substitute for this scarce resource (Hayami and Ruttan 1985; Kim et al. 2010). We do not distinguish between public and private research, recognizing that these efforts are often collaborative. For instance, the Irrigation Technology Center (ITC) at Texas A & M University, whose missions include, among many others, the development of new and improved irrigation technologies, methods, and management practices, is jointly funded by government and corporate enterprises. Also, the Center for Irrigation Technology (CIT) at the California State University, Fresno has been the leading independent testing laboratory

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<sup>3</sup> From equation (6-1) and  $W^* = \beta W_0$ , total differential of equation (6-1) is given by:

$$dP_w^* = \frac{\partial P_w^*}{\partial \beta} d\beta + \frac{\partial P_w^*}{\partial W_0} dW_0, \text{ which leads to } \frac{dW_0}{d\beta} = -\frac{W_0}{\beta} < 0, \text{ where } dP_w^* = 0.$$

and applied research facility *for* the irrigation industry. That is, both ITC and CIT work with the public and private sector to advance irrigation technology, among many other missions.

Within our dynamic framework of groundwater management, induced technology enters the model two ways: first in terms of when a new technology that improves irrigation efficiency is introduced, and second, when the new technology is adopted.<sup>4</sup> We assume that the probability of developing an improved irrigation technology at a given point in time, given that the technical innovation has not occurred yet, is an increasing function of (i) the capital stock ( $K(t|k, g_i)$ ) of irrigation system manufacturers<sup>5</sup>, which depends on the energy costs of pumping per acre-foot of groundwater per foot of lift ( $k$ ) and the rate of government subsidy to producers to adopt improved irrigation technologies ( $g_i$ ), and (ii) the cost to producers of adopting the innovated irrigation systems ( $z_i(t)$ ).<sup>6</sup>

Let  $M_i(t)$  be the probability that a new irrigation technology with efficiency improvement  $\theta_i$  is developed by time  $t$ , where  $M_i(0) = 0$  and  $0 \leq M_i(t) \leq 1$ .  $\theta_i$  is the increase in water use efficiency over the current irrigation technology efficiency  $\beta$ , such that the new irrigation system has water use efficiency  $\beta(1+\theta_i)$ . The conditional probability of developing a technical innovation at time  $t$ ,  $m_i[z_i(t), K_i(t|k, g_i)]$ , is the probability that the

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<sup>4</sup> Application of a hazard function while considering the uncertainty of innovation timing derived from research investments was suggested by Kieffer (1988) and Rose and Joskow (1990) considered the uncertainty of adoption timing of an improved technology, while Kim et al. (2010) considered both the development and adoption of an induced technology under uncertainties.

<sup>5</sup> See Appendix A for a description of the investment decision by an irrigation manufacturer to develop a more efficient irrigation technology. In our study, capital investment includes R&D.

<sup>6</sup> The Natural Resources Conservation Service (NRCS) of USDA currently provides subsidies (cost-shares) to producers to adopt efficient irrigation systems, such as subsurface drip irrigations in Texas. We assume the irrigation industry responds to higher pumping costs and government subsidies as an indication of society's willingness to pay to improve farm income and avoid over-exploitation of groundwater from aquifers.

development of such an innovation will occur during the next time period,  $t+\Delta t$ , given that a new technology has not been developed at time  $t$ .

We assume that the time to develop innovation is uncertain and that the likelihood of developing a new irrigation technology during the next time period is expressed as follows:

$$m_i[z_i(t), K_i(t|k, g_i)] = \frac{\left(\frac{\partial M_i(t)}{\partial t}\right)}{1 - M_i(t)}, \quad (11)$$

where  $m_i(t=0) = 0$ ,  $\frac{\partial m_i}{\partial z_i} > 0$ ,  $\frac{\partial m_i}{\partial K_i} > 0$ , and  $\frac{\partial M_i(t)}{\partial t}$  is the probability density function for

the induced irrigation technology development. The probability that the development of a technical innovation occurring by time  $t$  is then represented as follows:

$$M_i(t) = 1 - \exp[-\alpha_i m_i(z_i(t), K_i(t|k, g_i))]t, \quad (12)$$

where  $\alpha_i = \frac{1}{1+\varepsilon}$  and  $\varepsilon$  is the time elasticity of hazard rate.<sup>7</sup> Equation (11) can be rewritten

as a state equation as follows:

$$\frac{\partial M_i(t)}{\partial t} = m_i[z_i(t), K_i(t|k, g_i)](1 - M_i(t)) \quad (13)$$

Once a technical innovation occurs at time  $t^*$ , economic benefits associated with groundwater use for irrigation depend largely on whether the induced irrigation technology is adopted.

#### (b). Adoption of an induced irrigation technology with uncertainties.

Even after an induced technology is developed, the adoption of this new technology may be delayed if the net economic benefits resulting from the adoption are less than those

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<sup>7</sup> The time elasticity of hazard rate,  $\varepsilon$ , is an unknown parameter, but it can be derived from estimated hazard function depending upon the distribution of hazard function.

from the conventional irrigation technology. Burness and Brill (2001) and Ashwell and Peterson (2013) considered models with endogenous capital investment to improve irrigation efficiency by producers, without new irrigation technology development. Therefore, the welfare gains from more efficient water use are offset to some extent by inefficiencies in investment (Koundouri, 2004).<sup>8</sup> Meanwhile, Shah et al. (1995) considered the adoption process of irrigation technology without capital investment.

Let  $N_i(\tau)$  be the probability that the adoption of the  $i$ th induced irrigation technology developed at time  $\tau \geq t^*$  where  $N_i(\tau = t^*) = 0$  and  $0 \leq N_i(\tau) \leq 1$ . A producer's decision to adopt an improved irrigation system depends not only on the rising cost of pumping groundwater ( $mc(\tau)$ ), but also the amortized annual capital costs associated adoption and maintenance of the new system ( $z_i(\tau)$ ), the government subsidy rate ( $g_i$ ) and consultancy costs for learning how to operate the new system ( $E(t)$ ). Since the agricultural irrigation industry is largely unregulated and no independent design standards for irrigation systems (CIT, 2015), most agricultural irrigators do not have the expertise to determine if the system will perform as claimed. Therefore, it is necessary for agricultural irrigators to be educated on their use of new irrigation systems.

The conditional probability of adopting a new  $i$ th irrigation technology at time  $\tau$ ,  $n_i[E(\tau), mc(\tau), (1 - g_i)Z_i(\tau)]$ , is the probability that adoption of such an innovation will occur during the next time period,  $\tau + \Delta\tau$ , given that a new technology has not been adopted at time  $\tau$ . We also assume that the adoption time of innovation is uncertain and that the likelihood of adopting a new technology can be expressed as follows:

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<sup>8</sup> We do not consider the case where producers raise water use efficiency through capital investment in existing technology.

$$N_i(\tau) = 1 - \exp[-\sigma_i n_i(E(\tau), mc(\tau), (1 - g_i)z_i(\tau))] \tau \quad (14)$$

where  $\sigma_i = \frac{1}{1+\psi}$  and  $\psi$  is the time elasticity of the hazard rate.<sup>9</sup>

The probability density function,  $\frac{\partial N_i(\tau)}{\partial \tau}$ , which is obtained from equation (14), is represented as a state equation in our model as follows:

$$\frac{\partial N_i(\tau)}{\partial \tau} = n_i[E(\tau), mc(\tau), (1 - g_i)z_i(\tau)] [1 - N_i(\tau)], \quad (15)$$

where  $n_i(\tau = t^*) = 0$ ,  $\frac{\partial n_i}{\partial E} > 0$ ,  $\frac{\partial n_i}{\partial mc} > 0$ ,  $\frac{\partial n_i}{\partial z_i} < 0$ , and  $\frac{\partial n_i}{\partial g_i} > 0$ .

Equations (12) and (13) associated with the development of an induced irrigation technology and equations (14) and (15) associated with the adoption of an induced irrigation technology are then incorporated into an optimal control model of groundwater management in the following section.

## 5. A Dynamic Model of Groundwater Use under Uncertain Dates of the Development and Adoption of an Induced Irrigation Technology

Before we present our dynamic model of groundwater use for irrigation, the hydrologic equation must be revised to reflect the development and adoption of an induced irrigation technology. The hydrologic equation (3) is rewritten as:<sup>10</sup>

$$(AS)\dot{h}(t) = [R - (\beta W_0(t) + \beta(1 + \theta_i)W_i(t))] \quad (16)$$

where  $\beta$  represents efficiency of the current irrigation technology, while  $\beta(1 + \theta_i)$  represents efficiency of the  $i$ th induced irrigation technology, where  $\theta_i > 0$  and  $0 < \beta(1 + \theta_i) < 1$ .

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<sup>9</sup> See footnote 7.

<sup>10</sup> Delayed return-flow is not considered in our paper. Delayed response model requires the use of the current-value Hamiltonian, which would make us more difficult to investigate the Gisser-Sanchez Paradox.



The value function to be maximized for managing groundwater use for irrigation under uncertain dates of the development and adoption of an induced irrigation technology can be represented as a *nested* dynamic model as follows (Kim et al., 2010; Lewandrowski et al, 2014):

$$\begin{aligned}
& V(W_i(t), W_0(t), z(t), h(t), M(t), N(t), t_0) \\
& = \text{Sup} \int_0^{\infty} e^{-\delta t} \left\{ (1 - M_i(t)) [(P_y \beta^2 a_1 - k(SL - h(t))) W_0(t) - 0.5 P_y \beta^3 a_2 W_0^2] \right. \\
& \quad + M_i(t) [N_i(t) ((P_y \beta^2 (1 + \theta_i)^2 a_1 - k(SL - h(t))) W_i(t) - 0.5 P_y \beta^3 (1 + \theta_i)^3 a_2 W_i^2 - (1 - g_i) z_i(t))] \\
& \quad \left. + (1 - N_i(t)) ((P_y \beta^2 a_1 - k(SL - h(t))) W_0(t) - 0.5 P_y \beta^3 a_2 W_0^2) \right\} dt \tag{17}
\end{aligned}$$

where the economic benefits are based on the consumptive-equivalent irrigation water use as shown in equation (10),  $\delta$  is the rate of discount, and the subscripts “0” and “i” associated with  $W$  represent a conventional (current) irrigation technology and an induced irrigation technology, respectively. The value function in equation (17), which measures the present values of the expected net benefits from consumptive-equivalent irrigation water use, assumes that the induced technology is developed with the probability  $M_i(t)$ , and that this new technology would be adopted with the probability of  $N_i(t)$ . The probability of non-adoption,  $(1 - N_i(t))$ , implies the continued use of the conventional technology. Equation (17) can be rewritten as follows:

$$\begin{aligned}
V(W_i(t), q(t), h(t), M(t), N(t), t_0) & = \text{Sup} \int_0^{\infty} e^{-\delta t} \left\{ [(P_y \beta^2 a_1 - k(SL - h(t))) W_0(t) - 0.5 P_y \beta^3 a_2 W_0^2] \right. \\
& \quad + M_i(t) N_i(t) [((P_y \beta^2 (1 + \theta_i)^2 a_1 - k(SL - h(t))) W_i(t) - 0.5 P_y \beta^3 (1 + \theta_i)^3 a_2 W_i^2 - (1 - g_i) z_i) \\
& \quad \left. - (P_y \beta^2 a_1 - k(SL - h(t))) W_0(t) - 0.5 P_y \beta^3 a_2 W_0^2] \right\} dt \tag{18}
\end{aligned}$$

The dynamic optimization problem of maximizing the expected net social welfare of groundwater use for irrigation, based on consumptive-equivalent groundwater use, in the presence of irrigation technological change with uncertainty is expressed by maximizing the value function in equation (18) subject to the hydrologic state equation (16), and the probability density functions of developing an induced irrigation technology and then adoption of this new technology, as presented in equations (13) and (15), respectively.

The Hamiltonian equation is then presented as follows:

$$\begin{aligned}
H = e^{-\delta t} \{ & [(P_y \beta^2 a_1 - k(SL - h(t)))W_0(t) - 0.5P_y \beta^3 a_2 W_0^2] \\
& + M_i(t)N_i(t)[((P_y \beta^2 (1 + \theta_i)^2 a_1 - k(SL - h(t)))W_i(t) - 0.5P_y \beta^3 (1 + \theta_i)^3 a_2 W_i^2) - (1 - g_i)z_i \\
& - ((P_y \beta^2 a_1 - k(SL - h(t)))W_0(t) - 0.5P_y \beta^3 a_2 W_0^2)] \} \\
& + \frac{\lambda_1}{AS} [R - (\beta W_0(t) + \beta(1 + \theta_i)W_i(t))] \\
& + \lambda_2 m_i(z_i(t), K_i(t | k, g_i)) [1 - M_i(t)] + \lambda_3 n_i(E(t), mc(t), (1 - g_i)z_i(t)) [1 - N_i(t)]
\end{aligned} \tag{19}$$

where  $W_0$ ,  $W_i$  and  $g_i$  are control variables,  $h(t)$ ,  $M_i(t)$  and  $N_i(t)$  are state variables, and  $\lambda_1(t)$ ,  $\lambda_2(t)$  and  $\lambda_3(t)$  are adjoint variables associated with  $h(t)$ ,  $M_i(t)$ , and  $N_i(t)$ , respectively.

The necessary conditions for optimality are presented in Appendix (B). The economic interpretation of optimal conditions presented in equations (B.1) through (B10) is better served by first gaining information on the adjoint variables  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$ . Following Kamien and Schwartz (1971), Seierstad and Sydsæter (1977), and Takayama (1988), the adjoint variables  $\lambda_i(t_0)$ , where  $i = 1, 2, 3$  and  $0 < t_0 < \infty$ , approximate the marginal effects on the value function, equation (17) or (18), of the state variables,  $h$ ,  $M$ , and  $N$ , respectively, as follows:

$$\frac{\partial V_0}{\partial h(t_0)} \equiv \lambda_1(t_0) \geq 0; \quad (20-1)$$

$$\frac{\partial V_0}{\partial M(t_0)} \equiv \lambda_2(t_0) \geq 0; \quad (20-2)$$

$$\frac{\partial V_0}{\partial N(t_0)} \equiv \lambda_3(t_0) \geq 0 \quad (20-3)$$

The adjoint variable  $\lambda_1$  in equation (20-1) measures the marginal contribution of the water table level to the objective function of maximizing net economic benefits resulting from groundwater use. As water table level increases, pumping cost declines so that the net economic benefits from groundwater use for irrigation increase. Therefore, the adjoint variable  $\lambda_1$  is positive. Similarly, the net economic benefits resulting from the adoption of an induced irrigation technology must be greater than those from the use of the conventional irrigation technology. Therefore, the adjoint variable  $\lambda_3$  in equation (20-3), which measures the change in the net economic benefits associated with an increase in the probability of adopting an induced irrigation technology, must be positive, which requires that the adjoint variable  $\lambda_2$  in equation (20-2) be positive. That is, the change in the net economic benefits associated with an increase in the probability,  $M(t)$ , of developing a new irrigation technology must be positive, prior to adoption.

We can now explain the economic properties of the necessary conditions for optimality presented in equations (B.1) through (B.10). Equations (B.1) and (B.2) assure that water use with a particular irrigation technology equates the present value of the expected marginal net benefits to their marginal user cost. They are necessary for allocative efficiency of groundwater over time with a variety of irrigation technologies. Equation (B.3) illustrates that there are three costs to producers including (1) the costs of installation

and maintenance of new irrigation equipment, (2) the expected net economic benefits of adopting an induced irrigation technology forgone due to *no development* of an induced irrigation technology, and (3) the expected net economic benefits of adopting an induced irrigation technology foregone due to *no adoption* of a developed induced irrigation technology by producers. The optimal rate of government subsidy to farmers assures in equation (B.3) that the sum of the present values of the expected costs to producers of adopting improved irrigation technology, and the expected foregone benefits from failing to develop a new technology and, when developed, failing to adopt the technology, must equal the sum of the shadow values of failing to develop, and failing to adopt an induced irrigation technology.

Equation (B.4), representing the adjoint equation, demonstrates that the expected groundwater pumping costs create the value associated with user cost. The adjoint equations (B.5) and (B.6) demonstrate that both the development of a new irrigation technology itself and the adoption of this new irrigation technology create the values associated with user costs (economic benefits), respectively. Equations (B.7) through (B.9) are the equations of motion, while equation (B.10) is the conventional transversality conditions.

In the following section, the optimality conditions in equations (B.1) and (B.2) are used to investigate the Gisser-Sanchez Paradox with and without the development and adoption of an induced irrigation technology.

## 6. Evaluation of the Gisser-Sanchez Paradox

We first show that our model reproduces the Gisser-Sanchez result under their assumptions. We then show that with induced irrigation technology, the Gisser-Sanchez paradox no longer holds.

In the Gisser-Sanchez model, irrigation technology is fixed, and therefore the probabilities associated with the development and adoption of improved irrigation technologies in our model are zero. The Hamilton equation in (19) above then becomes identical to the Gisser-Sanchez dynamic model of groundwater use for irrigation.

**Proposition 1** (Gisser-Sanchez Paradox): Under the assumption of fixed irrigation technology (so that  $M(t)=N(t)=0$  in the Hamiltonian equation (19)) and economic benefits based on the water application rate<sup>11</sup>, the marginal effects on the shadow value (user cost) of groundwater stock declines as the storage capacity of an aquifer increases, so that benefits from groundwater management are insignificant.

**Proof:** Integrate both sides of the hydrologic equation (16), where  $W_i(t)=0$ , to represent the physical relationship of groundwater as follows:<sup>12</sup>

$$(AS)h^0 \geq \int_0^{\infty} [\beta W_0(t) - R] dt, \quad \text{where } h^0 = h_0 - h_L \quad (21)$$

where  $h_0$  is the initial elevation of the aquifer's water table above sea level and  $h_L$  is the physical and economical lower bound of water table above sea level. Using equations (7-1) and (B.1), differential of equation (21) results in:

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<sup>11</sup> Since the value functions in (17) and (18) are based on the consumptive-equivalent irrigation water use, while the Gisser-Sanchez model is based on the application rate (see equation (8-5)), the first-order condition in equation (B.1) is converted into:  $e^{-\delta t} (1 - M_i N_i) [\beta^2 P_y (a_1 - k(SL - h(t)) - \beta^3 P_y a_0 W_0(t))] = \frac{\beta \lambda_1(t)}{AS}$ .

<sup>12</sup> Farzin (1986) provides a similar procedure for oil reserves, while Kim and Moore (1989) show this for the case of groundwater exploitation.

$$d(AS)h^0 = \int_0^\infty \left( \frac{\partial(AS)h^0}{\partial P_{w_0}} \right) \left( \frac{\partial P_{w_0}}{\partial \lambda_1} \right) d\lambda_1 dt = \int_0^\infty \left[ \frac{-\beta}{\beta^2 P_y a_2} \frac{\beta e^{\delta t}}{(AS)} d\lambda_1 \right] dt \quad (22)$$

Using the relationship that  $dt = \left[ \frac{(AS)e^{-\delta t}}{\beta \delta \lambda_1} \right] dP_{w_0}$  from equation (B.1), equation (22) is rewritten as:

$$\begin{aligned} d(AS)h^0 &= \int_{W_0(t=0)}^{W_0(t=\infty)} \frac{-\beta}{P_y \beta^2 a_2} \frac{\beta e^{\delta t}}{(AS)} \left[ \frac{(AS)e^{-\delta t}}{\delta \lambda_1 \beta} dP_w \right] d\lambda_1 \\ &= \int_{W_0(t=0)}^{W_0(t=\infty)} \frac{\beta}{\delta \lambda_1} \left( \frac{-1}{P_y \beta^2 a_2} dP_w \right) d\lambda_1 \\ &= \frac{\beta}{\delta \lambda_1} [W_0(t=\infty) - W_0(t=0)] d\lambda_1, \end{aligned} \quad (23)$$

The marginal effect of the storage capacity of the aquifer on the shadow values (user costs) of the groundwater stock is obtained from equation (23) as:

$$\frac{d\lambda_1}{d(AS)h^0} = \frac{\delta \lambda_1}{\beta [W_0(t=\infty) - W_0(t=0)]} < 0. \quad \text{Q.E.D.} \quad (24)$$

Equation (24) illustrates the Gisser–Sanchez Paradox (with a fixed irrigation technology) that the shadow values (user costs) of the groundwater stock declines as the storage capacity of the aquifer increases, a straightforward relationship between the shadow values (user costs) of groundwater stock and the storage capacity.

In Proposition 2, using our model, we demonstrate that the Gisser-Sanchez Paradox fails to hold when the assumption of a fixed irrigation technology is relaxed and the adoption rate of induced irrigation technologies is lower than almost perfect.

**Proposition 2:** The marginal effects of the storage capacity of aquifers on the shadow values (user costs) of the groundwater stock increase (decrease), if an induced irrigation

technology is developed, but the adoption rate of the induced irrigation technology is less (greater) than  $(\frac{\sigma_i n_i}{\delta + \sigma_i n_i})$ .

**Proof:** Under the assumption that an induced irrigation technology is developed so that  $M_i(t)=1$ , the irrigation water demand equation (7-1) is rewritten as:

$$P_{w_i} = P_y \beta (1 + \theta_i) [a_1 - \beta (1 + \theta_i) a_2 W_i(t)]. \quad (7-1')$$

Furthermore, equation (21) becomes:

$$(AS)h^1 \geq \int_{\tau}^{\infty} [\beta (1 + \theta_i) W_i(t) - R] dt, \quad \text{where } h^1 = h_{\tau} - h_L \quad (25)$$

where  $h_{\tau}$  is the water table above sea level at the time  $\tau$  of adopting an induced irrigation technology.

Using equations (7-1') and (B.2), differential of equation (25) is written as:

$$\begin{aligned} d(AS)h^1 &= \int_{\tau}^{\infty} \left[ \frac{\partial(AS)h^1}{\partial P_{w_i}} \frac{\partial P_{w_i}}{\partial \lambda_1} \right] d\lambda_1 dt \\ &= \int_{\tau}^{\infty} \left[ \frac{-\beta^2 (1 + \theta_i)^2}{P_y \beta^2 (1 + \theta_i)^2 a_2} \frac{e^{\delta t}}{(AS)(1 - e^{-\sigma_i n_i t})} dt \right] d\lambda_1 \end{aligned} \quad (26)$$

Using the relationship that  $dt = \frac{(AS)}{\lambda_1} \left\{ \frac{(1 - e^{-\sigma_i n_i t})^2}{e^{\delta t} [\delta - e^{-\sigma_i n_i t} (\delta + \sigma_i n_i)]} \right\} dP_{w_i}$ , equation (26) is rewritten as follow:

$$\begin{aligned} d(AS)h^1 &= \int_{W_i(t=\tau)}^{W_i(t=\infty)} \frac{-\beta^2 (1 + \theta_i)^2}{P_y \beta^2 (1 + \theta_i)^2 a_2} \frac{e^{\delta t}}{AS(1 - e^{-\sigma_i n_i t})} \frac{(AS)}{\lambda_1} \left\{ \frac{(1 - e^{-\sigma_i n_i t})^2}{e^{\delta t} [\delta - e^{-\sigma_i n_i t} (\delta + \sigma_i n_i)]} \right\} dP_{w_i} d\lambda_1 \\ &= \left[ \frac{\beta^2 (1 + \theta_i)^2}{\lambda_1} \right] \left[ \frac{1 - e^{-\sigma_i n_i t}}{\delta - e^{-\sigma_i n_i t} (\delta + \sigma_i n_i)} \right] [W_i(t = \infty) - W_i(t = \tau)] d\lambda_1 \end{aligned} \quad (27)$$

When the induced irrigation technology is available for farmers to adopt, the marginal effect of the storage capacity of the aquifer on the shadow values (user costs) of

the groundwater stock is then obtained from equation (27) as follows:

$$\frac{d\lambda_1}{d(A^*S)h^1} = \frac{\lambda_1[\delta - e^{-\sigma_i n_i t}(\delta + \sigma_i n_i)]}{\beta^2(1+\theta_i)^2(1 - e^{-\sigma_i n_i t})[W_i(t=\infty) - W_i(t=\tau)]} \geq 0 \quad (28)$$

Using equation (14), the following conditions are derived from equation (28):

$$\frac{d\lambda_1}{d(A^*S)h^1} \begin{cases} > 0 & \text{if } N_i(t) < \frac{\sigma_i n_i}{\delta + \sigma_i n_i} \\ = 0 & \text{if } N_i(t) = \frac{\sigma_i n_i}{\delta + \sigma_i n_i} \\ < 0 & \text{if } N_i(t) > \frac{\sigma_i n_i}{\delta + \sigma_i n_i} \end{cases} \quad (29) \quad \text{Q.E.D.}$$

When the adoption rate of an induced irrigation technology is less than  $(\frac{\sigma_i n_i}{\delta + \sigma_i n_i})$ , equation

(29) shows that the shadow values of the groundwater stock increase as the storage capacity of groundwater stock increases. As the conditional probability of adopting an improved irrigation technology increases, it is more likely that the user costs associated with the groundwater stock increase, and therefore, the Gisser-Sanchez Paradox fails.

## 7. Summary

The GSE states that the economic benefits from managing groundwater use are insignificant when the storage capacity of the aquifer is relatively large. Since their influential paper, numerous researchers have investigated this paradox by modifying and/or extending the Gisser-Sanchez model without evaluating shortcomings of the Gisser-Sanchez model itself. Consequently, these modifications/extensions also contain the shortcomings of the Gisser-Sanchez model, and therefore, their empirical results have generally supported the Gisser-Sanchez Paradox.



There are at least two major shortcomings in the Gisser-Sanchez model that we correct for in our model. First, the Gisser–Sanchez model is based on the assumption that the economic benefits estimated using a consumptive irrigation water use and those estimated based upon irrigation application rate are commensurate (Gisser and Johnson, 1983). However, economic benefits based on application rate overestimate economic benefits of irrigation and the overestimation is proportional to the rate of irrigation water losses through leaching, runoff and evaporation (Kim and Schable, 2000). Second, the Gisser-Sanchez model assumes a fixed irrigation technology over the planning period.

We amended the Gisser-Sanchez model by measuring economic benefits based on consumptive-equivalent groundwater application in the presence of induced irrigation technology development and then adoption under uncertainty. Using our model, we demonstrated that the GSE no longer holds when the fixity of irrigation technology is relaxed.

Rapidly depleting aquifers in the U.S. High Plains and Central Valley are putting future agricultural production in these locations at risk (Scanlon et al. 2012). Our results imply there may be considerable scope for improving groundwater management, including regulating the rate of groundwater withdrawals. Our model provides a theoretical justification for developing socially optimal rates of groundwater extraction rather than leaving this resource to the tragedy of the commons.

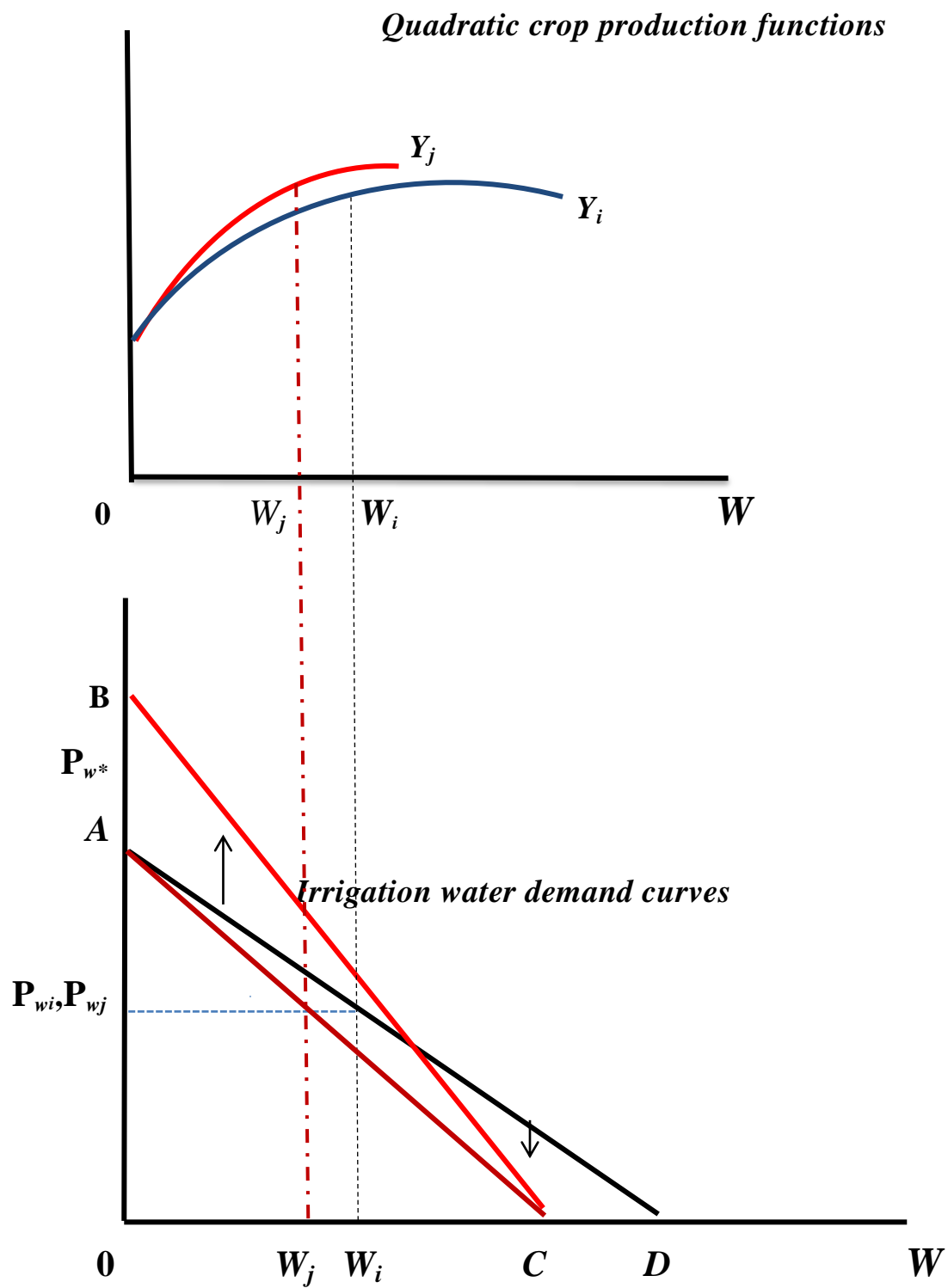


Figure 1. Effects of improving the irrigation efficiency on groundwater use for irrigation.

## Appendix (A)

Following Mann (1975), let the irrigation industry profit function be given by:

$$\pi(py(p, K)) = \int_0^{\infty} [py(p, K) - C(y(p, K)) - s(t | k, g)] e^{-\delta t} dt \quad (\text{A.1})$$

where,  $p$  is unit price of irrigation technology,  $y$  is quantity of irrigation technology,  $C$  is cost function,  $s$  is annual capital investment flow,  $k$  is a pumping cost per acre-foot of groundwater per foot of lift,  $g$  is the rate of government subsidy,  $\delta$  is the rate of discount, and  $K(t)$  is capital stock, such that:

$$K(t) = \omega \int_{-\infty}^t s(u | k, g) e^{-\omega(t-u)} du, \quad (\text{A.2})$$

where  $\omega e^{-\omega(t-u)}$  is the Nerlove-Arrow weighting function governing the rate at which capital investment flow is manifested into capital stock.

Using the Leibniz integral rule, the time derivative of equation (A.2) is represented by:

$$\begin{aligned} \frac{dK(t)}{dt} &= K'(t) = \omega [s(t | k, g) - \omega \int_{-\infty}^t s(u | k, g) e^{-\omega(t-u)} du] \\ &= \omega s(t | k, g) - \omega K(t), \end{aligned} \quad (\text{A.3})$$

which can be rewritten as:

$$s(t | k) = \frac{K'(t)}{\omega} + K(t) \quad (\text{A.4})$$

Integrating equation (A.4) by parts is given by:

$$\begin{aligned} \int_0^{\infty} s(t | k, g) e^{-\delta t} dt &= \int_0^{\infty} \left( \frac{K'(t)}{\omega} + K(t) \right) e^{-\delta t} dt = \left( \frac{K(t)}{\omega} e^{-\delta t} \right) \Big|_0^{\infty} + \int_0^{\infty} \left[ \delta \frac{K(t)}{\omega} e^{-\delta t} + K(t) e^{-\delta t} \right] dt \\ &= \int_0^{\infty} \left( 1 + \frac{\delta}{\omega} \right) K(t) e^{-\delta t} dt \end{aligned} \quad (\text{A.5})$$

Using equation (A.5), equation (1) can be rewritten as follow:

$$\int_0^{\infty} [py(p, K) - C(y(p, K)) - s(t | k, g)] e^{-\delta t} dt = \int_0^{\infty} [py(p, K(t)) - C(y(p, K(t)) - (1 + \frac{\delta}{\omega}) K(t | k, g) e^{-\delta t} dt] \quad (\text{A.6})$$

Equation (A.6) shows that corporate plan to develop an induced irrigation technology can be explained by net profits and the capital stock,  $K(t)$ , whose magnitude is influenced by rising energy costs of groundwater pumping and the rate of government subsidies.

## Appendix (B)

The necessary conditions for optimality are:

$$\frac{\partial H}{\partial W_0} = 0 \text{ implies} \quad (\text{B.1})$$

$$e^{-\delta t}(1 - M_i N_i)[\beta^2 P_y a_1 - k(SL - h(t)) - \beta^3 P_y a_0 W_0] = \lambda_1 \frac{\beta}{AS}$$

or,  $e^{-\delta t}(1 - M_i N_i)[\beta P_{w_0} - k(SL - h(t))] = \lambda_1 \frac{\beta}{AS}$

$$\frac{\partial H}{\partial W_i} = 0 \text{ implies} \quad (\text{B.2})$$

$$e^{-\delta t}(M_i N_i)[\beta^2 (1 + \theta_i)^2 P_y a_1 - k(SL - h(t)) - \beta^3 (1 + \theta_i)^3 P_y a_2 W_i] = \lambda_1 \frac{\beta(1 + \theta_i)}{AS}$$

or,  $e^{-\delta t}(M_i N_i)[\beta(1 + \theta_i)P_{w_i} - k(SL - h(t))] = \lambda_1 \frac{\beta(1 + \theta_i)}{AS}$

$$\frac{\partial H}{\partial g_i} = 0 \text{ implies} \quad (\text{B.3})$$

$$e^{-\delta t} \left\{ M_i(t) N_i(t) z_i + [(1 - M_i(t))(N_i(t) \alpha t (\frac{\partial m_i}{\partial g_i})) + M_i(t)(1 - N_i(t)) \sigma t (\frac{\partial n_i}{\partial g_i})] (L_i(t) - L_0(t)) \right\}$$

$$= \lambda_2(t)(1 - M_i(t))[1 - \alpha_i m_i t] (\frac{\partial m_i}{\partial g_i}) + \lambda_3(1 - N_i(t))[1 - \sigma_i n_i t] (\frac{\partial n_i}{\partial g_i}),$$

where  $L_i(t) = [P_y \beta^2 (1 + \theta_i)^2 a_1 - k(SL - h(t))] W_i(t) - 0.5 P_y \beta^3 (1 + \theta_i)^3 a_2 W_i^2 - (1 - g_i) z_i]$  and

$$L_0(t) = [(P_y \beta^2 a_1 - k(SL - h(t))) W_0(t) - 0.5 P_y \beta^3 a_2 W_0^2]$$

$$-\frac{\partial H}{\partial h} = \frac{\partial \lambda_1}{\partial t} \text{ implies} \quad (\text{B.4})$$

$$\frac{\partial \lambda_1}{\partial t} = -e^{-\delta t} k[\beta W_0 + \beta(1 + \theta_i) W_i]$$

$$-\frac{\partial H}{\partial M} = \frac{\partial \lambda_2}{\partial t} \text{ implies} \quad (\text{B.5})$$

$$\begin{aligned} \frac{\partial \lambda_2}{\partial t} = & -e^{-\delta t} N(t) \{ [((\beta^2(1 + \theta_i)^2 P_y a_1 - k(SL - h(t))) W_1(t) - 0.5\beta^3(1 + \theta_i)^3 P_y a_2 W_i^2) \\ & - (1 - g_i) z_i(t) - ((\beta^2 P_y a_1 - k(SL - h(t))) W_0(t) - 0.5\beta^3 P_y a_2 W_0^2))] \} \\ & + \lambda_2 m_i(k, z_i(t), g_i, K_i(t)) \end{aligned}$$

$$-\frac{\partial H}{\partial N} = \frac{\partial \lambda_3}{\partial t} \text{ implies} \quad (\text{B.6})$$

$$\begin{aligned} \frac{\partial \lambda_3}{\partial t} = & -e^{-\delta t} M(t) \{ [((\beta^2(1 + \theta_i)^2 P_y a_1 - k(SL - h(t))) W_1(t) - 0.5\beta^3(1 + \theta_i)^3 P_y a_2 W_i^2) \\ & - (1 - g_i) z_i(t) - ((\beta^2 P_y a_1 - k(SL - h(t))) W_0(t) - 0.5\beta^3 P_y a_2 W_0^2))] \} \\ & + \lambda_3 n_i[E(t), mc, (1 - g_i) z_i(t)] \end{aligned}$$

$$\frac{\partial H}{\partial \lambda_1} = \frac{\partial h(t)}{\partial t} \text{ implies } \frac{\partial h(t)}{\partial t} = \frac{\lambda_1}{AS} [R - (\beta W_0(t) + \beta(1 + \theta_i) W_i(t))] \quad (\text{B.7})$$

$$\frac{\partial H}{\partial \lambda_2} = \frac{\partial M_i(t)}{\partial t} \text{ implies } \frac{\partial M_i(t)}{\partial t} = m_i[z_i(t), K(t | k, g_i)](1 - M_i(t)) \quad (\text{B.8})$$

$$\frac{\partial H}{\partial \lambda_3} = \frac{\partial N_i(t)}{\partial t} \text{ implies } \frac{\partial N_i(t)}{\partial t} = n_i[E(t), mc(t), (1 - g_i) z_i(t)] [1 - N_i(t)] \quad (\text{B.9})$$

$$\lim_{t \rightarrow \infty} \lambda_1 = 0, \quad \lim_{t \rightarrow \infty} \lambda_2 = 0, \quad \lim_{t \rightarrow \infty} \lambda_3 = 0, \quad \lim_{t \rightarrow \infty} \lambda_1 h \geq 0, \quad \lim_{t \rightarrow \infty} \lambda_2 M_i \geq 0, \quad \lim_{t \rightarrow \infty} \lambda_3 N_i \geq 0. \quad (\text{B.10})$$

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