An Empirical Investigation of the Stanford's "1.2 Rule" for Nitrogen Fertilizer Recommendation

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ABSTRACT

We evaluate an old and widely accepted rule of thumb for fertilizer management in corn production: *apply 1.2 pounds of nitrogen fertilizer per bushel of corn expected.* This "1.2 Rule" has dominated fertilizer management recommendations for almost fifty years, and similar algorithms have been used all over the world to make fertilizer recommendations for other crops. Here we show that the 1.2 Rule only makes economic sense if the production satisfies two restrictions: (1) to be of the von Liebig functional form, i.e. the function has a "kink" and a "plateau," and (2) the kinks of the von Liebig response curves for different growing conditions lie on a ray out of the origin with slope 1.2. Non-linear estimation techniques and non-nested hypothesis framework are used to test if the 1.2 Rule satisfies these restrictions. We conclude that there exists little scientific justification of the 1.2 Rule, and that its long-term and widespread use basically resulted from its long-term and widespread use.

Keywords: Stanford's 1.2 Rule, fertilizer recommendations, nitrogen, von Liebig, corn

JEL codes: Q10, Q12, Q16

Introduction

For almost fifty years, university extension and private consultants have widely used a yield goal-based algorithm, known as the "1.2 Rule," to recommend nitrogen (N) fertilizer rates to corn farmers (Lory and Scharf, 2003; Halbeisen, 2006). Similar rules have been established throughout the world for other crops. The 1.2 Rule is that the producer should apply 1.2 lb of N fertilizer per acre for every per-acre bushel of a type of yield, referred to usually as "potential yield," "target yield," or "yield goal," with adjustments for previous crops grown and other factors:

$$N_f \text{ (lb acre}^{-1}\text{)} = 1.2\, YG \text{ (bu acre}^{-1}\text{)} - N_s - N_a \quad (1)$$

where N_f is the recommended N rate , N_s is the quantity of N supplied by the soil before any fertilizer is applied, N_a is an adjustment for a previously grown crop, and *YG* is the yield goal or the target yield. The 1.2 Rule originates from research conducted by George Stanford (1966, 1973) and that the intellectual origins of Stanford's thinking are the mass balance theory of yield response and the von Liebig law of the minimum (Parr 1973; Karlen et al. 1985; Osmond et al. 2010; Chen et al. 2011). Stanford desired to develop a *less empirical* methodology for various crops, including corn, that would provide a basis for predicting the additional quantity of N required from fertilizer.

Since 1990s several studies have cast doubt on the appropriateness of yield goal-based approaches. The recommended N fertilizer rates, determined by the yield goal approach to implementing Stanford's 1.2 Rule, result to over fertilization (Lory and Scharf 2003). There exist poor relationships between 1.2 Rule-based recommendations and the economically optimal N rate (Blackmer et al., 1991; Vanotti and Bundy 1994a; Vanotti and Bundy 1994b; Fox and Piekielek, 1995; Kachanoski, et al., 1996; Andraski and Bundy, 2002; Lory and Scharf, 2003). There are also uncertainty about how yield goals should be determined and how adjustments for nonfertilizer N sources in yield goal approaches should be done (Sawyer et al., 2006). Despite all of these, the 1.2-Rule-type algorithms continue to be used to make fertilizer recommendations for many crops in many parts of the world including winter wheat in United States (Kansas State University 2015), cereals in Canada (Ontario Ministry of Agriculture, Food, and Rural Affairs 2011) and rice in Asia (International Rice Research Institute 2015).

Given the huge impact that the 1.2 Rule has had on fertilizer management throughout the world, it makes sense to reexamine its validity in the N rate recommendation system. There are no studies that have thoroughly investigated and verified Stanford's empirical contributions. Most of the studies are based on the assumption that what Stanford concluded from his data was justified. In this article, using non-linear estimation techniques and non-nested hypothesis framework, we rigorously test Stanford's conclusion: *the critical N concentration of the plant's dry matter is constant at 1.2 percent*. The analyses rely on the original datasets Stanford (1973) used in his research and the long-term corn experimental data from Illinois, Nebraska, and Iowa. To our knowledge, this is the first and only paper that provides empirical evidence of whether Stanford's 1.2 Rule is indeed faulty, and whether the yield goal-based approaches to N fertilizer recommendation should be completely abandoned. Although this study focuses on corn fertilizer recommendation, findings from this study may have broader implications for other crops, including rice and soybeans, whose N fertilizer requirements are also based on yield-goal based algorithm.

Our conclusion is that the 1.2 Rule, in itself, was a "ballpark" recommendation algorithm that may have done as much harm as good. The empirical results suggest that Stanford's 1.2 Rule can either result to either under- or over-application of fertilizer and the economic analyses indicate that the consequences of using the 1.2 Rule can be large.

Critiques of the 1.2 Rule

In Stanford's approach, at least strictly speaking, the use of yield goal to maximize profits makes little economic sense. After all, if a farmer's goal is to maximize profits, he cannot determine how to maximize profits by first assuming which yield will maximize profits. Conceptually more tenable may be the claim that if a farmer has insights to the maximum yield he can achieve (the "yield potential"), and this knowledge might somehow offer information about the optimal N rate. A farmer would need to have some idea about the maximum yield attainable on his field to have an idea of the yield potential, as suggested by Viets (1965). We argue that the Stanford's 1.2 Rule or yield potential approach for fertilizer recommendation only makes economic sense if the production satisfies two restrictions: (1) to be of the von Liebig functional form[1], i.e. the function has a "kink" and a "plateau," (Figure 1) so that input and output prices do not affect the (interior) solution to the profit maximization problem; and (2) the kinks of the von Liebig response curves for different growing conditions lie on a ray out of the origin with slope 1.2 (Figure 2). Under these two conditions, essentially input and output prices do not influence the input demands. If indeed the production function is von Liebig, the farmer will either choose 0 or $\overline{N}$ amount of input to maximize profit (Figure 1). Note that we are not claiming here that the von Liebig is the correct production function. The von Liebig function is only used as a starting point to test the validity of the 1.2 Rule.

Even if agronomic theory makes von Liebig technology a plausible representation of true response functions (and it is not clear that it does), it remains unclear why the kinks should all lie on a common ray from the origin in an *(N,y)* diagram. If kinks do not line up, the critical N concentration of plant's dry matter will vary (Figure 2). In this case Stanford's 1.2 Rule's basis for fertilizer recommendations misleads.

[1] Many authors have interpreted this as an assumption of Leontief technology (Grimm, Paris, and Williams, 1987).

Data Description

The data used in this study come from two sources: (1) the published data in Pearson et al. (1961) and (2) the long-term corn experiments in Illinois, Nebraska, and Iowa (Table 1). Pearson et al. (1961) report results from corn field experiments in 1955 at three locations in Alabama, one in Georgia, and two locations in Mississippi, and in 1957 at one location in Georgia. This is the original data set that Stanford (1966, 1973) used in his analysis. Complete details of the data can be found in Rodriguez and Bullock (2015). The data set from long-term experiments in Illinois, Nebraska, and Iowa contains information on corn grain yields, dry matter yield, N fertilizer application rates, and N uptake. The experimental data from Illinois and Iowa, however, only contain information on corn grain yield and N fertilizer application rates. The Illinois data come from experimental plots in Monmouth and Perry conducted from 1980 to 2012 (n=720). Nitrogen fertilizer rates range from 0 to 320 pounds per acre in 20-60 pound increments, with three repetitions of each application rate performed annually at both locations.

There are two sets of data from corn experiments in Nebraska. The first experimental data set, conducted from 1969 to 1983, is from the Nebraska Agricultural Experiment Station Field Laboratory near Mead, NE. This data set is used in the previous work of Olson et al., (1986).[2] Nitrogen fertilizers are applied at 90, 180, and 270 pounds per acre and two check plots are included in each replication. The second experimental data set, which contains 1383 observations, comes from 17 experimental locations representing the main corn production areas of Nebraska including Mead from 2002 to 2004 (Dobermann et al., 2011). The N rates applied ranged from 0 to 300 pounds per acre. Individual plots are arranged in a randomized complete block design with four replications at each site.

[2] Note that this is one of the experimental sites in Stanford's 1966 and 1973 paper. Olson's data that were used by Stanford (1973) for his analysis were no longer available.

The Iowa data come from the earlier works of Binford, Blackmer, and Cerrato (1992) and Blackmer et al. (1989) in 15 experimental locations across the state between 1985 and 1990 (n=1998). Nitrogen fertilizer rates range from 0 to 300 pounds per acre in 25-50 pound increments, with three repetitions of each application rate performed annually at each experiment station site.

Estimation Procedures and Empirical Specification

The von Liebig Model Estimation

Using Paris's (1992) approach, the von Liebig formulation with linear potential yield function can be expressed as:

$$y_{it} = \min\{\theta_0 + \theta_N N_{it}, P\} + u_{it} \quad (2)$$

where y_{it} is the dry matter yield (pounds per acre) in the *i*th plot at time *t*, N_{it} is the N uptake level (pounds per acre) as the limiting input, $u_{it} \sim N(0, \sigma_e^2)$ is the disturbance term, P is the maximum or plateau yield, and θ_0 and θ_N are the parameters of the model.[3] The error, u_{it}, associated with the dependent variable is assumed to be unique and therefore, not subject to the minimum operator. Stanford simplifies the problem by assuming that year effect and temporal variability (i.e. rainfall, temperature, relative humidity, among others) can be ignored completely. The P is also assumed to be nonrandom, in spite of its determinants being stochastic (Ackello-Ogutu et al., 1985; Paris and Knapp 1989; LLewelyn and Featherstone, 1997). This assumption suggests that all factors that define P are fixed and completely controllable.

Equation (2) is an example of a non-linear regression function, $m(\boldsymbol{x}, \theta), \theta \epsilon \Re^P$ where m is a known function of $\boldsymbol{x}$, a *K*-vector, and θ, a $P \times 1$ parameter vector. The standard non-linear regression model can be defined by

[3] Since there are no available record on the dry matter yield and N uptake of plant's dry matter from the experimental field plots in Illinois and Iowa, we use the grain yield and the amount of N fertilizer applied in these states instead.

$$y = m(\boldsymbol{x}, \boldsymbol{\theta_0}) + u \quad (3),$$

where u are scalar i.i.d. random variables with $E(u|\boldsymbol{x}) = 0$ and σ_0^2. Unlike the linear model where $f(x, \beta_0) = (x'\beta_0)$, the dimensions of the vectors $\boldsymbol{x}$ and β_0 are not necessarily the same (Amemiya, 1983).

Equation (2) can be estimated directly by maximizing its corresponding likelihood function (Paris and Knapp, 1989; Paris, 1992). However, the maximum likelihood method requires one critical assumption, i.e. the true Data Generating Process is known to lie within a specified probability distribution. This is to say that the model of the given data is correctly specified. If the correct distribution is something other than what is assumed, then the likelihood function is misspecified and the desirable properties of the maximum likelihood estimator (MLE) might not hold.[4] The most common probability distribution assumed when doing the maximum likelihood estimation is the normal distribution. The normal MLE is quasi-maximum likelihood and produces consistent estimates if the mean is correctly specified.[5]

Given this, we estimate the linear von Liebig model using nonlinear least squares. The idea behind this method is that it finds the non-linear least squares (NLLS) estimator, denoted by $\hat{\theta}$, which is defined as the value of θ that minimizes the sum of squared residuals between y and $m(\boldsymbol{x}, \theta)$. That is, $\widehat{\theta}$ solves

$$\min_{\theta \epsilon \Theta} N^{-1} \textstyle\sum_{i=1}^{N} [y - m(\boldsymbol{x_i}, \theta)]^2 \quad (4).$$

The θ appearing in (4) is an argument of the function $m(\boldsymbol{x}, \cdot)$ and θ_0 in (10) is a fixed true value. One only needs to supply the function $m(\boldsymbol{x}, \theta)$, in this case, (2). Initial values for the parameters are provided to begin the process. To find starting values for a nonlinear procedure can be

[4] The MLE is most attractive because of its large sample properties.
[5] The idea of quasi-maximum likelihood is that there is a family of densities whose first order condition (the score) with respect to the parameters in the mean is exactly the same. Such a family of distribution is called the exponential family or exponential models (Wooldridge, 2010).

difficult. We use the parameter estimates from the quadratic production function to determine the maximum possible yield, *P*, and nutrient uptake and then these values serve as starting points for the nonlinear procedure.

Using the parameter estimates from the linear von Liebig functional form, the critical N concentration of plant's dry matter, denoted by θ, can be derived by dividing the height of the plateau, *P*, by the minimum N required to achieve *P*, (N^k). That is,

$$\theta = \frac{P}{N^k} = \frac{P\theta_N}{P-\theta_0} \quad (5),$$

where

$$N^k = \frac{P-\theta_0}{\theta_N} \quad (6).$$

If the hypothesis given by

$$H_0{:}\, \theta = 1.2 \quad (7)$$

is rejected, then Stanford's 1.2 Rule misleads.

The Nonlinear Seemingly Unrelated Regression

Now, suppose there are:

$$\begin{aligned} y_1 &= m(x_1, \theta_1) + u_1 \\ y_2 &= m(x_2, \theta_2) + u_2 \\ &\vdots \\ y_G &= m(x_G, \theta_G) + u_G \end{aligned} \quad (8)$$

where *G* stands for locations or states and $E[u_g|\boldsymbol{x}] = 0, g = 1,2, \dots, G$. We test the null hypothesis:

$$H_0{:}\, \theta_1 = \theta_2 = \ldots = \theta_G \quad (9),$$

where θ_1 is the calculated critical N concentration of plant's dry matter for state 1, θ_2 is the critical N concentration for state 2, and θ_G is the critical N concentration for state *G*. We jointly estimate the equations in (8) described by a nonlinear equation system. The parameters are estimated by applying the nonlinear seemingly unrelated regression to the system of equations. From the fitted model, the null hypothesis about the estimated parameters is then tested. The rejection of the null hypothesis suggests that the critical N concentration of plant's dry matter is not constant at 1.2 and the Stanford's 1.2 Rule as basis for fertilizer recommendation is inappropriate.

The Non-nested hypothesis framework

Using a non-nested hypothesis framework as proposed by Davidson and MacKinnon (1982), we test

$$H_0 : y_i = m(x_i, \theta) + u_{0i} \tag{10},$$

where u_{0i} is assumed to be $N(0, \sigma_0^2)$. Suppose an alternative hypothesis is plausible:

$$H_1 : y_i = g(z_i, \gamma) + u_{1i} \tag{11},$$

where z_i is a vector of observations on exogenous variables, γ is the vector of parameters to be estimated and u_{1i} is $N(0, \sigma_1^2)$ if H_1 is true. For the purposes of this study, three alternative hypotheses are tested: quadratic, square-root and Mitscherlich-Baule specifications. The quadratic model is defined by

$$y_i = \gamma_0 + \gamma_1 N + \gamma_n N^2 + u \tag{12},$$

where $y_i$is the grain yield (bushels/acre) or the total dry matter weight (pounds/acre) and N is the rate of N application (pounds/acre) or N uptake (pounds/acre) and γ's are the parameters to be estimated. The square root model is defined by

$$y_i = \gamma_0 + \gamma_1 N + \gamma_n N^{1/2} + u \tag{13},$$

while the Mitscherlich-Baule model is defined by

$$y_i = P(1 - ke^{-\gamma N_j}) + u \tag{14}.$$

Following Davidson and MacKinnon (1982), the form of the compound model can be expressed as:

$$y_i = (1-\alpha)m(x_i,\theta) + \alpha g(z_i,\gamma) + u_i \qquad (15).$$

Simplifying (15),

$$y_i = m(x_i,\theta) + \alpha[g(z_i,\gamma) - m(x_i,\theta)] + u_i \qquad (16).$$

If $\alpha = 0$, then H_0 is the correct model and if $\alpha = 1$, then it implies H_1. In principle, H_0 could be tested by testing$\alpha = 0$. It is impossible, however, to estimate α, θ, and γ jointly. Davidson and MacKinnon (1983) suggest that a simple solution will be to replace γ by its predicted value, $\hat{\gamma}$, under H_1. The composite model becomes

$$y_i = (1-\alpha)m(x_i,\theta) + \alpha\hat{g} + u_i \qquad (17).$$

A test of $\alpha = 0$ is known as *J*-test and is a routine t-test.

Since H_0 involves a nonlinear model, (24) is also a nonlinear regression, and one which may be computationally difficult if H_0 and H_1 are very similar. To overcome this problem, (17) can be linearized around the point $\alpha = 0$ and $\theta = \hat{\theta}$, so as to obtain the linear regression

$$y - \hat{m} = \widehat{\boldsymbol{M}}\theta + \alpha(\hat{g} - \hat{m}) + u \qquad (18)$$

where $\hat{m} = m(x_i,\hat{\theta}), \hat{g}_j = g_j(z_j,\hat{\gamma})$ and $\widehat{\boldsymbol{M}}$ is the matrix of derivatives of m with respect to θ, evaluated at the non-linear square estimates $\hat{\theta}$. This procedure is called a *P* test. If the null hypothesis that $\alpha = 0$ is not rejected, then von Liebig model is the correct model specification.

The *P* test can be easily extended to handle several alternative hypotheses. Let the null hypothesis still be H_0, given by (3), and the alternative hypotheses be

$$H_j: y = g_j(z_j,\gamma_j) + u_j, j = 1,2,\dots,J \qquad (19).$$

The compound model becomes

$$H_c: \left(1 - \textstyle\sum_{j=1}^{J}\alpha_j\right)m(x,\theta) + \textstyle\sum_{j=1}^{J}\alpha_j g(z_j,\gamma_j) \qquad (20),$$

and the corresponding P test regression is

$$y_i - \hat{m} = \sum_{j=1}^{J} \alpha_j (\hat{g}_j - \hat{m}) + \widehat{\boldsymbol{M}}\theta + u_j \quad (21).$$

The appropriate test statistic is then an asymptotic F test of the hypothesis that $\alpha_1 = \alpha_2 = \cdots = \alpha_j$.

Results

The summary statistics for all the explanatory variables in the non-linear estimation that are used throughout the study are presented in Tables 2. In this section, we present formal statistical and empirical evidence about whether the two restrictions mentioned above are satisfied.

Is the critical N concentration of the plant's dry matter is constant at 1.2 percent?

Table 3 presents the estimation results using the von Liebig model by U.S. state. All the parameters are found to be significant at the one percent level, indicating a clear response for corn to applied N, except for the intercept, θ_0, in Georgia in section A. The values of θ, which represent the critical N concentration of corn yield in each U.S. state, range from 0.62-0.86. The hypothesis, $H_0{:}\theta = 1.2$, is rejected in F tests for each state (in Alabama, $F(1,77) = 140.19$, *p-value* = 0.00; Georgia, $F(1,45) = 54.42$, *p-value* = 0.00; Mississippi, $F(1,58) = 359.41$, *p-value* = 0.00; Nebraska, $F(1,1630) = 3158.06$, *p-value* = 0.00). This implies that the maximum attainable yield is not associated with 1.2 percent N concentration in total dry matter. Using the parameter estimates in section A, fertilizer recommendations based on the 1.2 Rule overestimate the minimum N requirement of corn, (N^k), necessary to achieve maximum yield potential. Fertilizer recommendations given to farmers can result to over-fertilization. In Alabama for example, the estimated θ was 0.62 of plant's dry matter. Since Stanford assumes that corn typically has a harvest index of 50 percent and a bushel of shell corn contains 49.3 pounds dry matter, making total above ground dry matter 98.6 pounds (grain plus stover), a corn plant only needs to absorb 0.61 pounds of N to achieve one bushel of corn (98.6 x 0.62%) with adjustments on other factors, and not 1.2 pounds of N.

Using the estimates in Section B, where grain yield and N rate applied are used in the estimation, a farmer in Alabama needs to apply about 0.63 pounds N per bushel of corn instead of 1.2 pounds.[6] If the yield goal is set at estimated P=80.39 bushels per acre (which is assumed to have 12% water), then its equivalent dry matter is equal to 70.74 bushels/acre grain and there is about 70.74 bushels/acre of stover. The total dry matter is then equal to 141.48 bushels/acre. A fertilizer recommendation using a factor of 1.2 pounds N per bushel of expected yield will have predicted a fertilizer need of about 170 pounds of N per acre. This is equivalent to about 81 pounds N per acre in excess of the predicted N based on this analysis with adjustments for fertilizer efficiency and existing nutrients in the soil. The excessive N use due to Stanford's 1.2 Rule is not acceptable from either an economic or environmental viewpoint. The farmer will decrease his profit by $34 per acre if the 1.2 Rule is followed. This varies with N and corn prices. The cost presented here is based on N costing 42 cents per pound and corn price at $5 per bushel. The excessive N use is also a potential pollution hazard as fertilizer N application in excess of crop need dramatically increases residual N in the soil, which is likely to move into the ground or surface waters (Olsen et al., 1970; Lory, et al., 1995).

There are cases however when the Stanford's 1.2 Rule may be correct and/or can also result to under-fertilization. We fail to reject the hypothesis, $H_0: \theta = 1.2$, in Georgia (F(1,45) = 0.96, *p-value* = 0.3322), Iowa (F(1,1995), *p-value* = 0.5522) and Illinois (F(1,717), *p-value* = 0.7795). If indeed the correct functional form is linear von Liebig, then Stanford's 1.2 Rule does not mislead in these states. On the other hand, the hypothesis in Nebraska (F(1,1630) = 80.83, *p-*

[6] A bushel of corn is assumed to be 56 pounds. The total above ground yield on a per bushel basis is 112 pounds.

value = 0.00) is rejected suggesting that the 1.2 factor in Stanford's rule needs to be adjusted given the correct functional form is indeed linear von Liebig.

While Stanford (1973) claimed that the critical N concentration of the plant's dry matter is constant, our results show that θ across U.S. states take on different values and are statistically different from each other (Table 4). The tests are performed by temporarily holding each θ in each state as null and testing in a pair-wise fashion with θ from a different state and against all the other θs. Results imply that the kinks of the linear von Liebig response curves do not line up on a common ray, which suggests that the critical concentration for corn (on N) varies in every state. For example, we reject the hypothesis that $\theta_{AL} = \theta_{GA} = \theta_{MS} = \theta_{NE}$ (*chi-square*(3) = 99.98, *p-value* = 0.00). This is also evident when the predicted values of dry matter yield are plotted against the N uptake (Figure 3). Although the kinks seem to be quite close to each other especially those from Alabama, Georgia, and Mississippi, they do not line up on a common ray,[7] hence not corroborating with Stanford's findings. The estimated optimum N rate needed in each specific state differs. On the other hand, using data on grain yield and N applied, the critical concentration of N is similar in Georgia and Iowa, Georgia and Nebraska, and Iowa and Illinois (Table 4). We also test Stanford's 1.2 Rule at the experimental station level and similar results are found.

What is the correct functional form?

The non-nested hypothesis results based on a *P* test are reported in Table 5. The tests are performed by temporarily holding each hypothesis as null and testing in a pair-wise fashion with each alternative and against all alternatives. The quadratic functional form outperforms all the rival specifications in Illinois while the Mitscherlich-Baule model is more appropriate than any

[7] Nebraska appears to be much different than the other states in terms of yield plateau because the state has the highest irrigated land per country. The water supply comes from the Ogallala Aquifer and reservoirs that capture water from snow-melt and rains.

other alternatives in Nebraska, both on a pairwise comparison as well as in a collective test against all alternatives. As for the other states, the results are inconclusive. In Alabama the null hypothesis that $\alpha = 0$ rejected at 10 percent level suggests that the von Liebig model is not the correct specification when it is the null hypothesis. Information is insufficient however to choose the correct model specification from among the alternative models. Neither the polynomial functions nor the Mitscherlich-Baule function is rejected over any other model when they are the null hypothesis. In Iowa, the non-nested hypothesis test rejects square root and linear von Liebig functions but fails to reject quadratic and Mitscherlich-Baule functions. Failure to reject these alternatives in Alabama and Iowa suggest that the corn response on yield and N fertilizer tends to be smooth and allows diminishing marginal productivity. If this is the case, then the marginal product schedule and the input and output prices matter in the determination of the economically optimum N rate. Given a non-zero price ratio, there is a difference between the yield maximizing and profit maximizing input levels. An optimum fertilization level is attained when the marginal product of fertilizer is equal to the fertilizer price and output price ratio. In Georgia and Mississippi, none of the four specifications is rejected. The data do not allow us to say much about which functional form best represents corn response to N fertilizer.

Out of the 42 experimental locations in the study, only in two locations that the linear von Liebig model outperforms all the rival specifications, both on a pairwise comparison as well as in a collective test against all alternatives. The non-nested hypothesis tests reject the linear von Liebig but fail to reject quadratic, square-root, and Mitscherlich-Baule in other 18 locations. In all the remaining experimental locations, the non-nested hypothesis tests favor none of the four rival specifications. The results are inconclusive on what the best specification is to interpret the data set.

Conclusion

The empirical results indicate no strong empirical evidence to support Stanford's 1.2 Rule. The linear von Liebig production function is rejected in various locations and the kinks of the von Liebig response curves for different growing conditions do not lie on a ray out of the origin with slope 1.2. The production function and the critical concentration of N can vary widely both among states and within states, and therefore the level of optimal N can also vary. Site-specificity matters in making fertilizer recommendations.

We conclude that there exists little scientific justification of the 1.2 Rule, and that its long-term and widespread use basically resulted from its long-term and widespread use. Given these, it is noteworthy to revisit the fertilizer recommendation algorithms that rely on the 1.2 Rule or yield goal-based approaches and test if they satisfy the two restrictions mentioned earlier. Unlike before, data from high-quality agronomic experiments and the necessary statistical and empirical procedures for such an empirical test are now available.

Several states in the Corn Belt have in recent years abandoned the 1.2 Rule and moved toward more data-driven recommendations. One example is the use of the MRTN ("Maximum Return to N") algorithm to make N make N fertilizer recommendations by many U.S. land grant universities (Sawyer et al. 2006; Sawyer, Laboski, and Nafzier 2012; Camberato and Nielsen 2015; Iowa State University Agronomy Extension 2015). The underlying premise of MRTN is to provide rate guidelines based directly from the results of many N response trials and flexibility for producers in addressing risk and rice fluctuation. Note however that this approach cannot be used to predict site-specific N requirements (Sawyer, Laboski, and Nafziger 2012). The N fertilizer rate guidelines need to be more site-specific to account for farmer's specific crop growing conditions, crop and soil management, and climate which can vary greatly among fields, seasons, and years. This is an area of research in great need of interdisciplinary research among agronomists and agricultural economists.

There is one caveat to keep in mind when interpreting our results. So far we only assume that the farmer is risk-neutral and the response function is non-stochastic. Farmer's input decisions, including fertilizer use, are typically influenced by risks (e.g. risks from pests and other unmanageable inputs) and stochastic factors (e.g. soil variability, weather). That is the recognition that the nutrient choice does not determine mean response alone. And given farmer objectives other moments of the distribution might be important. How risk-aversion affects nutrient management depends on whether fertilizer is seen as a risk-reducing or risk-enhancing input. An interesting extension of this study is the inclusion of risks and stochastic factors in the analysis.

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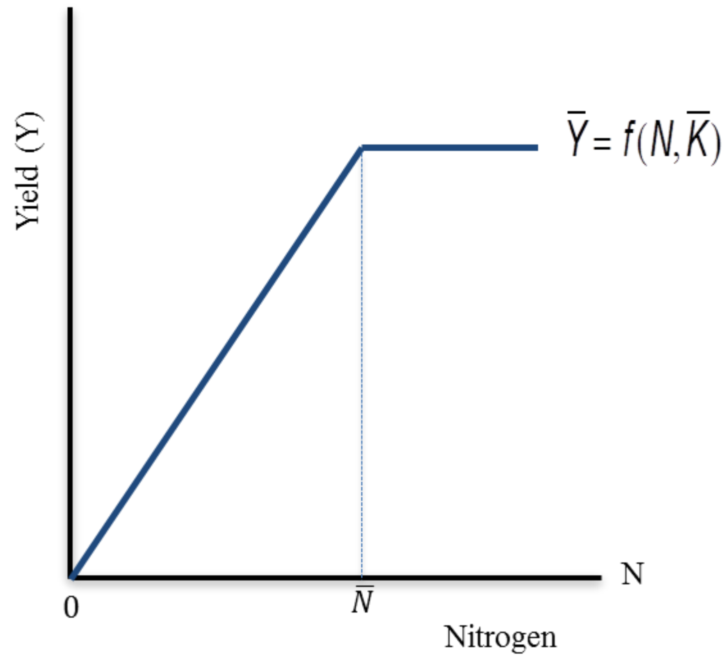

Figure 1. Yield response under Leontief technology

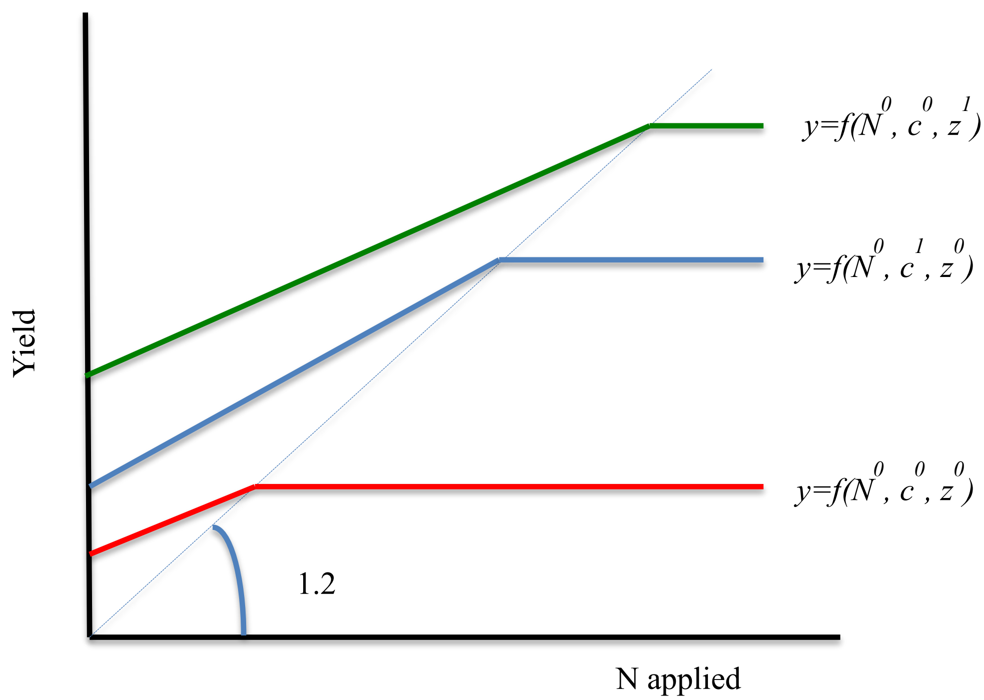

Figure 2. Kinks line up on a ray out of the origin with slope 1.2

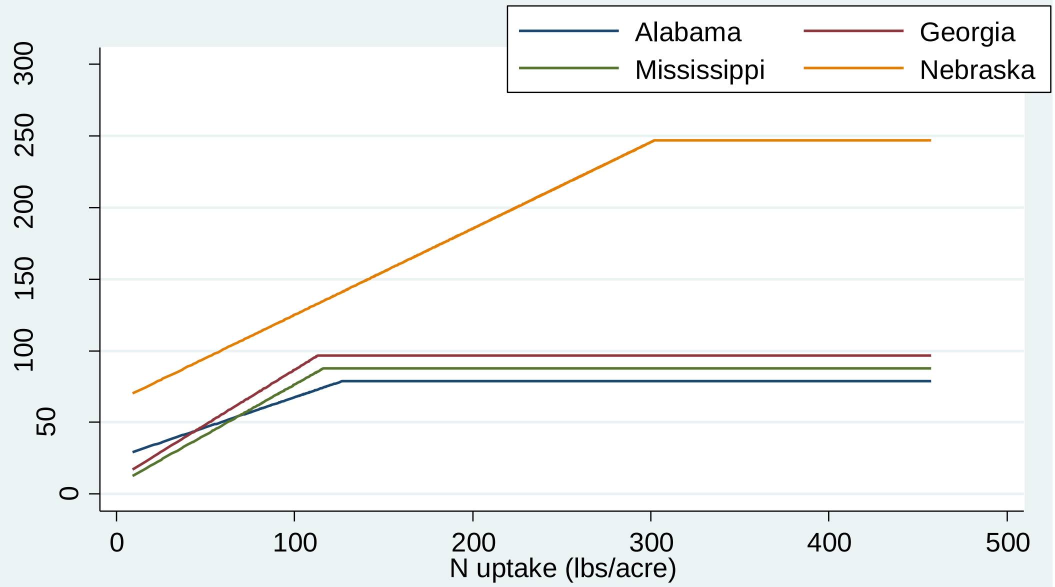

Figure 3. Relation of total dry matter yield to total N uptake for corn experiments

Table 1. Description of experiment duration and N fertilizer rate, all sites

Location	Year of Experiment	Nitrogen Fertilizer Rate
Alabama		
Belle Mina	1955, 1957, 1959	0, 50, 100, 200
Pratville	1955, 1957, 1960	0, 50, 100, 201
Thorsby	1956, 1958, 1959	0, 50, 100, 202
Georgia		
Tifton	1958, 1959	0, 30, 60, 90, 120
Watskinville	1957, 1958, 1959	0, 30, 60, 90, 121
IA		
Site0	1987	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site1	1986-1988	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site3	1986-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site4	1988	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site5	1985-1987	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site6	1985-1987	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site8	1986-1988	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site9	1986-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site10	1987-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site11	1987-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site12	1987-1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site13	1987-1989	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site14	1989	0, 50, 100, 150, 200, 250, 300
Site15	1989	0, 50, 100, 150, 200, 250, 300
Site16	1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Site17	1990	0, 25, 50, 75, 100, 125, 150, 200, 250, 300
Illinois		
Monmouth	1983-2012	0, 60, 120, 180, 240
Perry	1980-1992	0, 60, 80, 120, 160, 180, 240, 320
Mississippi		
Brooksville	1956, 1957, 1959	0, 50, 75, 100, 150, 200
Poplarville	1956-1959	0, 50, 75, 100, 150, 200

Table 1. Continued…

Location	Year of Experiment	Nitrogen Fertilizer Rate
Bellwood	2002-2003	87, 105, 112, 145, 162, 185, 187, 212, 235, 287, 335
Box Butte	2002-2004	0, 75, 100, 125, 150, 175, 200, 223, 300
Brosius	2004	0, 100, 150, 171, 200, 300
Brunswick	2002-2004	0, 50, 75, 100, 125, 150, 170, 175, 250
Cairo	2002-2004	0, 92, 100, 125, 132, 150, 156, 175, 192, 200, 210, 225, 300
Clay Center	2002	0, 92, 125, 175, 225, 300
Concord	2002-2004	0, 50, 75, 100, 110, 125, 150, 175, 250
Funk	2004	0, 100, 150, 200, 207, 300
Mead	1969-2004	0, 50, 75, 90, 100, 119, 125, 131, 140, 150, 175, 180, 250, 270
N. Platte	2002-2003	0, 100, 125, 150, 175, 180, 195, 200, 225, 300
North Bend	2004	0, 50, 100, 110, 150, 250
Paxton	2002-2003	0, 100, 125, 150, 175, 200, 212, 214, 225, 300
Pickrell	2003-2004	0, 50, 100, 123, 131, 150, 250
Scal	2002-2004	0, 50, 75, 100, 115, 125, 150, 175, 250
Scottsbluff	2002-2003	0, 75, 100, 125, 150, 175, 200, 225, 300
Spurgin	2004	0, 100, 150, 193, 200, 300
Wymore	2002	0, 75, 112, 125, 175, 250

Table 2. Descriptive statistics

Variable	No.of observation	Mean	S.D.	Min	Max
<u>Alabama</u>					
Grain yield (bu/acre)	72	69.49	18.83	18.50	108.90
Dry matter yield (grain + stover, cwt/acre)	72	68.49	18.56	18.23	107.33
Nitrogen applied (lbs/acre)	72	94.44	50.04	0.00	200.00
Nitrogen uptake (grain + stover, lbs/acre)	72	106.44	29.04	46.00	202.00
<u>Georgia</u>					
Grain yield (bu/acre)	43	75.83	21.00	19.70	113.20
Dry matter yield (grain + stover, cwt/acre)	43	74.73	20.70	19.42	111.57
Nitrogen applied (lbs/acre)	43	82.33	34.70	0.00	120.00
Nitrogen uptake (grain + stover, lbs/acre)	43	87.02	26.83	23.00	149.00
<u>Iowa</u>					
Grain yield (bu/acre)	1998	127.66	45.32	4.12	218.08
Nitrogen applied (lbs/acre)	1998	127.93	93.45	0.00	300.00
<u>Illinois</u>					
Grain yield (bu/acre)	720	122.01	56.58	0.40	217.47
Nitrogen applied (lbs/acre)	720	136.67	99.68	0.00	320.00
<u>Mississippi</u>					
Grain yield (bu/acre)	58	49.94	22.53	6.70	90.30
Dry matter yield (grain + stover, cwt/acre)	58	49.22	22.21	6.60	89.00
Nitrogen applied (lbs/acre)	58	94.83	55.16	0.00	200.00
Nitrogen uptake (grain + stover, lbs/acre)	58	62.81	31.81	9.00	153.00
<u>Nebraska</u>					
Grain yield (bu/acre)	1633	212.84	43.45	41.31	302.80
Nitrogen applied (lbs/acre)	1633	164.99	90.01	0.00	335.00
Nitrogen uptake (grain + stover, lbs/acre)	1483	251.53	62.98	53.00	457.30

Table 3. Production function parameter estimates using von Liebig model by state

VARIABLE	STATE					
	Alabama	Georgia	Iowa	Illinois	Mississippi	Nebraska
A. Dry matter yield vs N uptake [a]						
θ_o	25.50**	10,35	-	-	6.410**	64.79***
	(9.20)	(7.92)	-	-	(2.06)	(3.86)
θ_N	0.421***	0.764***	-	-	0.700***	0.603***
	(0.10)	(0.11)	-	-	(0.03)	(0.02)
θ	0.62***	0.86***	-	-	0.75***	0.82***
	(0.05)	(0.05)	-	-	(0.02)	(0.01)
P	78.96***	96.50***	-	-	87.87***	247.0***
	(3.51)	(3.33)	-	-	(0.82)	(1.12)
N^k	127.06***	112.72***	-	-	116,38	302.17***
	(12.50)	(7.83)			(9.02)	(3.60)
No. of obs	80	48	-	-	61	1633
adj. R-sq	0.95	0.98	-	-	0.98	0.98
B. Grain Yield vs N rate applied						
θ_o	45.61***	41.14***	95.12***	74.90***	22.31***	165.5***
	(4.66)	(7.46)	(2.32)	(4.16)	5.31)	(2.63)
θ_N	0.278***	0.846*	0.401***	0.578***	0.303***	0.404***
	(0.05)	(0.35)	(0.04)	(0.09)	(0.06)	(0.03)
θ	0.63***	1.73***	1.25***	1.24***	0.44***	1.55***
	(0.07)	(0.54)	(0.08)	(0.14)	(0.05)	(0.08)
P	81.71***	80.39***	140.2***	140.3***	72.10***	224.0***
	(5.19)	(2.71)	(1.34)	(2.43)	(6.51)	(1.19)
N^k	129.64***	46.39***	112.37***	113.14***	164.46***	144.77***
	(21.84)	(14.81)	(7.56)	(13.41)	(27.91)	(7.98)
No. of obs	80	48	1998	720	61	1633
adj. R-sq	0.95	0.95	0.90	0.86	0.90	0.97

[a]No available data in Iowa and Illinois
Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 4. Do kinks line up on a common ray?

STATE	Alabama (θ_{AL})	Georgia (θ_{GA})	Iowa (θ_{IA})	Illinois (θ_{IL})	Mississippi (θ_{MS})	Nebraska (θ_{NE})	All other states
A. Dry matter yield vs *N* uptake							
Alabama (θ_{AL})	-	63.20***	-	-	42.07***	30.99***	99.88***
Georgia (θ_{GA})	63.20***	-	-	-	19.41***	281.63***	99.88***
Iowa (θ_{IA})	-	-	-	-	-	-	-
Illinois (θ_{IL})	-	-	-	-	-	-	-
Mississippi (θ_{MS})	42.07***	19.41***	-	-	-	21.90***	99.88***
Nebraska (θ_{NE})	30.99***	281.63***	-	-	21.90***	-	99.88***
All other states	99.88***	99.88***	-	-	99.88***	99.88***	-
B. Grain yield vs *N* rate applied							
Alabama (θ_{AL})	-	24.81***	40.65***	84.71***	45.20***	41.06***	807.29***
Georgia (θ_{GA})	24.81***	-	0.36	7.02***	98.78***	0.00	807.29***
Iowa (θ_{IA})	40.65***	0.36	-	1.37	9.63***	42.71***	807.29***
Illinois (θ_{IL})	84.71***	7.02***	1.37	-	194.54***	18.72***	807.29***
Mississippi (θ_{MS})	45.20***	98.78***	9.63***	194.54***	-	38.35***	807.29***
Nebraska (θ_{NE})	41.06***	0.00	42.71***	18.72***	38.35***	-	807.29***
All other states	807.29***	807.29***	807.29***	807.29***	807.29***	807.29***	-

The test statistic is distributed as a chi-square.
* p<0.10, ** p<0.05, *** p<0.01

Table 5. Nonnested Hypothesis Results Based on a *P* Test by state

State/ Alternative Hypothesis	Null Hypothesis			
	Linear von Liebig	Quadratic	Square-root	Mitscherlich-Baule
ALABAMA				
Linear von Liebig	-	0.14	1.13	0.11
Quadratic	1.48	-	2.45	0.79
Square-root	3.23*	2.47	-	0.36
Mitscherlich-Baule	3.87*	2.26	1.9	-
All alternatives	2.27*	1.54	1.02	1.57
GEORGIA				
Linear von Liebig	-	1.04	0.75	1.5
Quadratic	0.48	-	0.07	0.48
Square-root	0.16	0.07	-	0.11
Mitscherlich-Baule	0.23	0.13	0.01	-
All alternatives	0.1	0.58	0.76	0.82
IOWA				
Linear von Liebig	-	2.7	5.42**	1.28
Quadratic	74.95***	-	4.38**	1.39
Square-root	3.01*	2.62	-	1.83
Mitscherlich-Baule	3.92**	1.8	6.65**	-
All alternatives	1.51	1.81	2.27*	0.69
ILLINOIS				
Linear von Liebig	-	0.4	1.06	5.24**
Quadratic	1.09	-	8.73***	15.32***
Square-root	49.77**	2.59	-	17.55***
Mitscherlich-Baule	18.43***	5.89**	7.86***	-
All alternatives	17.02***	1.55	7.88***	8.09***
MISSISSIPPI				
Linear von Liebig	-	0.00	0.01	0.37
Quadratic	2.07	-	0.06	0.78
Square-root	1.81	0.06	-	0.80
Mitscherlich-Baule	1.93	0.04	0.07	-
All alternatives	0.73	0.03	1.09	0.28
NEBRASKA				
Linear von Liebig	-	2.49	3.15*	0.00
Quadratic	39.45***	-	4.13**	0.01
Square-root	39.20***	4.21**	-	0.01
Mitscherlich-Baule	39.29***	4.40**	2.44	-
All alternatives	6.57***	1.5	0.22	0.04

* p<0.10, ** p<0.05, *** p<0.01