Estimating US Crop Supply Model Elasticities Using PMP and Bayesian Analysis

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Abstract

This paper examines an innovative and practical way to model the supply of agricultural crops. This will be done by extending the technique developed by Howitt (1995), Positive Mathematical Programming (PMP), using Bayesian estimation. A key problem in the use of the PMP model is the relative difficulty of finding calibrating parameters such that the first and second order conditions are satisfied; with the added difficulty that many of the conditions needed to be satisfied are not exactly known. Thus the use of Bayesian analysis is a useful tool to try and determine these parameters. By employing a Markov chain Monte Carlo (MCMC) algorithm, specifically a Metropolis-Hasting Algorithm, a posterior distribution for the calibrating parameters can be found such that the resulting supply model will not only reproduce an optimum close to observed acreages, but also produce reasonable elasticities due to the prior information. The value of this style of estimation for a crop supply model lies in the limited amount of data needed to estimate the model.

Keywords: agricultural supply analysis, mathematical programming models, Bayesian econometrics, US agricultural

JEL classification: C11, C60, Q10

I. Introduction

The purpose of this work is to provide a new way to estimate an agricultural supply model. There are many reasons that crop supply models are useful. First, they can be used to test hypotheses regarding the impacts of policy changes on the supply of various crops. Second, these models can be utilized to forecast the supply of various crops. There are two methodologies that are useful in achieving a quantitative model for the supply of various crops. The method used in much of the literature surrounding the estimation of supply is the dual equation system which can specify the model with well established econometric techniques. The core idea of dual equations is to utilize the profit function of the various crops along side the various input demand equations which normally create a variety of resource restrictions. The disadvantage of this approach is that it often times oversimplifies the model and normally requires a lot of input data to be estimated. An example of this is given by Chambers and Just (1989) who employ a dual supply model that has explicit allocations for fixed factors. The second methodology is to use a programming model which can model complex problems. However, the programming approach is usually lacking statistical properties. The programming methodology has provided practical models for crop supply. One such form of programming model which has found a great deal of success was developed by Howitt (1995) who was the first to present the idea of Positive Mathematical Programing (PMP).

The basic idea of PMP is to form a non-linear programming problem for land allocation in a crop supply model by adding a quadratic portion to a known linear programming problem for crop supply. The key is that the quadratic portion added will calibrate the model such that it can not only reproduce an optimal solution that will be consistent with observations of land allocations, but will also calibrate the model so that it is consistent with economic theory. Thus, this model is consistent with the fact that the cost curve is convex. In addition to this, the model is calibrated to a correct belief of what the shadow price of land should be, and it properly reflects the correct price elasticity for the various crops. It is key that the model that is developed is consistent with other observations in the literature with respect to price elasticities, as the relevancy of this modeling is how well it can predict changes due to shifts in the supply curve. The benefits to PMP are that the policy analyst can easily form a crop supply model that not only is calibrated to the observed data, but also does not need a lot of data to carry out this calibration. The present research goes forward in exploring the PMP method and applies Bayesian estimation in order to not only estimate a crop supply model, but also to derive a crop supply model that is testable, forecastable, and can simulate a wide range of policy changes.

The work builds upon previous work in the estimation of a European Union (EU) model but with two key differences. First, the present application is made using US data, which adds some additional challenges in modeling. Secondly, the Metropolis Hasting Algorithm is used to estimate a model that provides for the creation of credible intervals and other forms of Bayesian inference not present in previous research in this area. With these estimations, forecasting supply as well as the effects of policy is now feasible.

II. Literature Review of PMP

The evolution of estimating agricultural supply models in a practical way is best seen though Heckelei and Britz's (2005) examination of PMP. The standard model to be considered is the simple linear problem:

$$
\begin{aligned}
\max Z &= p'x - c'x \\
\text{s.t.} & \\
Rx &\leq v, x \geq 0
\end{aligned}
\tag{1}
$$

Here p is a vector of revenues for various crops, x is a vector of acreages for the various crops, c is linear cost vector for the various crops, R is a matrix of coefficients in the resource constraints, and v is a vector of the various available resource quantities. The drawback of this is that it normally does not predict the observed value for x and normally predicts over specialization in crops because the model does not have decreasing marginal profits for any of the crops. This is a critical flaw in this model. The overall solution to this standard model would be specialization into a few crops since usually the number of constraints are below the observed activities. PMP was in part developed to create a model whose optimal solution could reproduce what is observed and would fit the standard properties of a profit function. The contribution of PMP by Howitt (1995) was to consider an additional non-linear portion to the objective function. This allows for the model to have decreasing marginal profits in all the crops. The non-linear model can be represented as:

$$\max Z = p'x - c'x - h'x - \frac{1}{2}x'Qx \tag{2}$$

$$\text{s.t.}$$

$$Rx \leq v, x \geq 0$$

The main issue in this non-linear function is that the parameters in Q and h are unknown. The only real restriction on the parameters is that Q has to be a positive semi-definite matrix in order to be consistent with standard economic theory of profit maximization. There are a number of ad-hoc methods that have been used to try to model this non-linear portion. In most cases these methods involve adding some element, such as risk, into the modeling. For example, Holt (1999) formulated the problem as a portfolio choice problem. However, a more practical solution usually used with the PMP method is to assume these parameters are unknown and use observed data to calibrate these parameters.

By simply using one year of data involving observed acreage, prices, and crops, the model can be calibrated. Thus the optimal solution for x is the acreage observed which is ensured by the fact that the first order conditions of the model at the observed acreage are met. For this to occur, various artifices are employed. If the resource constraint only pertains to the land constraint, the simplest idea is to assume the shadow price of land is equal to the average profit of an additional unit of land across all the activities of the model. From this the parameters of h and Q could be defined in such a way that the optimal solution replicates what is observed. The only problem with this approach is that the first order condition create an ill-posed problem; meaning that multiple combinations of h and Q will solve the equation. Howitt's initial modeling used a simple specification for h and Q which could be used to calibrate the model to be optimal with respect to the observed acreages. However, the model could not accurately predict the optimal outcome if prices shift. What is needed is applying an additional restriction to the model using some known piece of information.

Heckelei and Britz (2005) show from the first order condition, that the optimal acreage x^* can be found. In addition to that, by taking the derivative of x^* with respect to price, $\frac{\partial x^*}{\partial p}$ is found and through that the parameters Q can be equated to the various crop elasticities which are (in some cases) assumed known. This additional information allows for the parameters of h and Q to have a unique set of outcomes for which the crop supply model will not only reproduce the observed acreage, but also reproduce an observed crop elasticity.

There are, however, two major problems associated with using exogenous elasticities. The first is that it is rare to exactly know what a crop elasticity is in a given year. The other complication lies in the fact that solving for what Q should be in order to reproduce exactly the exogenous supply elasticity is difficult and in some cases, as shown by Merel and Bucaram (2010), infeasible. Merel and Bucaram (2010) also raise the question of whether or not the calibration to observed acreage data is unique, which would ultimately need to be addressed in a PMP model. Because of the difficulties of exact calibration, a popular idea has emerged to use multiple observations to help estimate the parameters of the model by using a Maximum Entropy (ME) approach .

The first use of ME techniques to solve this problem was done by Paris and Howitt (1998). However, their use of it was rather limited in that they simply used one observation in their study. Heckelei and Wolff (2003) examined the performance of a an ME estimator though Monte Carlo simulations with small samples and found a general improvement in using a maximum entropy approach. The key idea of ME is that instead of making a specific parametric assumption, one assumes a uniform distribution for the parameters. In addition to that, the parameter must satisfy the given restrictions implied by the first-order conditions while being as close to the uniform distribution as possible. Next, multiple data observations could be included. There is a great deal of flexibility with the ME approach and this method opens up the possibility to calibrate the model uniquely without the need for knowledge of the elasticities.

There have been a number of articles which employ the PMP approach together with a number of ways of estimating the unknown parameters. On such approach was Kanellopous et al. (2010) with the Farm System SIMulator. They endeavored to assess the forecasting performance of a PMP model. They needed to do several modifications with respect to how the shadow price is estimated, apart from the standard method given by Howitt, in order for it to work with time series data.

There has also been work done by Buysse et al. (2007) in the estimation of the cost function of Belgian sugar beet through the use of a Generalized Maximum Entropy method on a farm programing model and with the additional problem of sugar beet quota rents. Other work has been done by Arfini et al. (2008) using the PMP estimation in a cross sectional problem in the Emilia-Romangna region of Italy. The calibration was done not only for the production of crops but also for the demand function of the crops which allowed the model to have endogenous prices. Another use of PMP modeling has been done by Gocht (2009), using PMP to estimate input allocations with the claim that this style of estimation can lead to more information on the input coefficients then previous estimation approaches.

There is, however, an alternate way to define the parameters of the model while still using a limited amount of information. Instead of the ME approach, a Bayesian approach could be used, as explained by Heckelei (2008). A Bayesian approach can recreate a ME approach, but is more flexible in incorporating prior information in the model, while also being simpler to compute.

This paper uses a model similar to that used by Jansson and Heckelei (2011) who utilized a Bayesian estimation method on time series data to estimate the supply model for the EU. The model used by Jansson and Heckelei is an errors in variables model. The benefits to using a measurement error model is explained by Carroll et al.(1995). They demonstrate a general improvement to the estimations done through Gibbs sampling when taking into account measurement error. In addition, this type of modeling is useful for filling in missing data by treating unobserved X's as an unobserved random parameter with a distribution based on some prior distribution. The ability to deal with incomplete data is a major reason to consider using an errors in variable model. While the general model this paper poses could allow for errors in variables, it will be restricted to assumed they are precisely measured. Thus the empirical portion will assume there is no measurement error in measuring the profit per acres of crops. This is done mainly to limit the number of parameters that are needed to be determined. Jansson and Heckelei's (2011) basic modeling and approach could be adapted to the US, but noticeable changes are necessitated when modeling a single country's agricultural supply model with information existing in subregions. There is also the problem that while Jansson and Heckelei (2011) could estimate a model using Bayesian estimation, they could not overcome the problem of determining the significance of the estimation since they used a form of optimization rather than determine the posterior distribution. Because of this the method used in the paper employed several alterations to allow for a posterior distribution to by sampled using an MCMC algorithm, a Metropolis-Hasting Algorithm. By using an alternative estimation method, one will not only be able to estimate the unknown parameters, but also will be able to give credible intervals and thus some significance to the estimation. First a review of the generalized model posed by Jansson and Heckelei is needed.

III. US Crop Supply Model

The basic model used by Jansson and Heckelei (2011) at its core is Howitt's (1995) PMP model using time series data:

$$\max x_t' \left[Y_t p_t - A_t w_t \right] - q_t x_t' \left[h - \frac{1}{2} l_t Q x_t \right] \tag{3}$$

s.t.

$$R_t x_t = v_t$$

where x_t is a vector of acreages for J crops, Y_t is a $J \times J$ matrix of yields, p_t is a J vector of prices, A_t is a $J \times N$ vector of input coefficients for N inputs, w_t is an N vector of input prices, q_t is a J vector of price index, h is a J vector of the linear behavior parameters that are to be estimated, l_t is the land availability index is to be used to take into account any sifts in the total available land between years. Q is a $J \times J$ diagonal matrix of own crop effects to be estimated. The cross crops effects in Q were not considered for this model for several reasons. First the accurate information relating to direct interaction between crops in terms of the marginal costs does not exist. The only information that is known for the supposed synergy between crops for corn, soy and wheat, is that if the cross effect does exist, it would be a magnitude smaller than the crops own impact on the marginal cost. With that we can assume the form for Q should be approximately diagonal without losing much in its predictive power. Because of this, the model will assume Q to be diagonal so that the computation and determining the posterior distribution for the elements in Q will be easier. For the constraint, only the land constraint will be considered. Thus R_t is a J vector of ones and v_t is total land available. This model is a very general PMP model. $[Y_t p_t + s_t - A_t w_t]$ represents the observed marginal profits per acreage of crop j. $q_t x_t' \left[h - \frac{1}{2} l_t Q x_t\right]$ is just the PMP portion added in to be estimated. The use of q_t is a general price index employed to deflate opportunity costs, and the scalar l_t is used to prevent the possibility of land migration from influencing land rent. The parameter h is still an unknown linear portion in the problem and Q is the unknown quadratic portion. The first order and second-order conditions are then:

$$Y_t p_t - A_t w_t - q_t h - q_t l_t Q x_t - R_t \lambda_t = 0 \tag{4}$$

$$R_t x_t = v_t \tag{5}$$

$$Q = U'U \tag{6}$$

where $D = U'U$ ensures that the quadratic portion of the behavioral matrix is positive semidefinite. Thus the additional constraints to be considered when using time series data are:

Let $E_t = q_t l_t Q$, Solving for (4) it can be found that

$$x_t^* = E_t^{-1}\left[Y_t p_t - A_t w_t - q_t h - R_t \lambda_t^*\right] \tag{7}$$

from here substituting into (5) and solving for λ_t^* gives:

$$\lambda_t^* = \left[R_t E_t^{-1} R_t'\right]^{-1}\left[R_t E_t^{-1}\left(Y_t p_t - A_t w_t - q_t h\right) - v_t\right] \tag{8}$$

Substituting (7) into (9) then gives

$$x_t^* = E_t^{-1}\left[Y_t p_t - A_t w_t - q_t h\right] - E_t^{-1} R_t \left(\left[R_t E_t^{-1} R_t'\right]^{-1}\left[R_t E_t^{-1}\left(Y_t p_t - A_t w_t - q_t h\right) - v_t\right]\right) \tag{9}$$

Taking $\frac{\partial x_t^*}{\partial p_t}$ gives

$$\frac{\partial x_t^*}{\partial p_t} = E_t^{-1} Y_t - E_t^{-1} R_t \left[R_t E_t^{-1} R_t'\right]^{-1} R_t E_t^{-1} Y_t \tag{10}$$

From here η_t could be found

$$\eta_t = X_t^{-1}\left(E_t^{-1} Y_t - E_t^{-1} R_t \left[R_t E_t^{-1} R_t'\right]^{-1} R_t E_t^{-1} Y_t\right) P_t \tag{11}$$

X_t, Y_t, and P_t all represent diagonal matrices of x_t, y_t, and p_t respectively. This elasticity is consistent with the previous literature when considering the fact that $\frac{\partial \lambda_t^*}{\partial p_t} = \left[R_t E_t^{-1} R_t'\right]^{-1} R_t E_t^{-1} Y_t$. In some cases it may be simpler to just consider $\frac{\partial \lambda_t^*}{\partial p_t} = 0$ and solve for the behavioral estimates.

While this model is similar to that posed by Jansson and Heckelei's (2011), the Bayesian estimation of this model is what makes this model unique. Jansson and Heckelei's model and problems applying a Bayesian approach into their model stems from the priors not truly being a pre-data view of the problem. As a result they were only able to produce the highest point for the posterior density, which gives no information as to the significance of this point estimator. In no way is it proven that this highest point is in fact unique and there is no way of knowing whether or not the distribution has a single peak. It is with that caveat in mind that one major goal of this research is to extend the Bayesian setting to US data in such a way that some credibility for the estimated parameters will be obtained. A major reason why Jansson and Heckelei did not apply an MCMC algorithm is due to the size of their problem impacting the speed of the algorithm. The model this paper will use has the advantage of focusing on US crop supply rather than all of the EU's crop supply model. This model, however, will be scaled down slightly to include fewer range of parameters and excluding measurement errors in variables. Utilizing a MCMC algorithm is a major benefit in a Bayesian setting. Since the only information obtained by Jansson and Heckelei's approach is this highest point they have no knowledge of its overall significance, or the posterior distribution. It is difficult to use such results in any type of policy analysis. With the use of a Metropolis-Hasting algorithm, not only is a posterior distribution for the parameters obtained, but also a sample of the posterior is found making policy analysis feasible. This type of modeling also has a significant benefit compared to linearly approximating the supply curve. As with linear approximation, only one optimum is normally considered for the problem, and a single peak is assumed for its distribution, which does not need to be the case. In addition to that, without including any prior information on the elasticity, trying to linearly approximate the supply curve using a limited scope of this data would necessitate a huge range as to what the linear approximation could be. By imposing a prior, the range that the supply curve could lie is reduced.

IV. Bayesian estimation and Metropolis-Hasting Algorithm

The first goal in the estimation is to properly set up a model with reasonable priors and likelihood densities. To simplify the model, let θ be the set of parameters that define the unobservable quadratic portion of the problem, or $\theta = (h, U)$ where $D = U'U$. Another simplification is to condense the observations from $(y_t p_t)$ and $(A_t w_t)$ into simply $revenue_t$ and $cost_t$, which are J vectors of the average revenue or cost per acre. As for the data, we are able to observe x, $revenue$, and $cost$, and from that we also infer the shadow price λ which is an average of the observed gross margins of all the crops. Thus the observed data is $z = \left(x^{obs}, revenue^{obs}, cost^{obs}, \lambda^{obs}\right)$. Let us also define $x^* = x(\theta)$, $\lambda^* = \lambda(\theta)$, and $\eta^* = \eta(\theta)$. For times series these would be:

$$x_t(\theta) = E_t^{-1}(revenue_t - cost_t - q_t h) - E_t^{-1} R' [R E_t^{-1} R']^{-1} \left(R E_t^{-1}(revenue_t - cost_t - q_t h) - v_t\right), \tag{12}$$

$$\lambda_t(\theta) = [R E_t^{-1} R']^{-1} \left(R E_t^{-1}(revenue_t - cost_t - q_t h) - v_t\right), \tag{13}$$

and

$$\eta_t(\theta) = (diag(x(\theta)))^{-1} \left(E_t^{-1} - E_t^{-1} R' [R E_t^{-1} R']^{-1} R E_t^{-1}\right) diag(revenue) \tag{14}$$

The goal is to obtain a posterior density, $g(\theta|z)$,which is based on observations $z = \left(x^{obs}, revenue^{obs}, cost^{obs}, \lambda^{obs}\right)$. The basic Bayes rule should now be:

$$g(\theta|z) \propto f(z|\theta)\, g(\theta) \tag{15}$$

Here the biggest question is how to form a prior for $g(\theta)$ when the information that can be drawn from to form the prior is from other literature which can basically be seen as a prior on the elasticity that is a function of θ or $g_\eta(\eta(\theta))$. The basic information for that distribution $g_\eta(\eta(\theta))$ is that mode of the distribution should be around 0.25 for corn and soybeans and 0.15 for wheat and that most of the distribution should fall between 0 and 1. From this information, given that average data for x^{obs} and expected price for the crop this prior belief can be transformed into a prior belief on $\frac{\partial x_i^*}{\partial p_i}$ so then the prior belief we have is $g_{slope}\left(\frac{\partial x_i^*}{\partial p_i}(U)\right)$. In trying to form a distribution for $g_{slope}\left(\frac{\partial x_i^*}{\partial p_i}(U)\right)$, the basic idea is to examine the slope at the mean of the data over the five years and examine $\frac{\partial x_i^*}{\partial p_i}(U)$ at various values of U to determine a general distribution for U such that its mode would be at 4.5 million for corn, 1.5 million for soy, and 1 million for wheat and its standard deviation would allow a .1 to .2 shift in the elasticity. From here a $g(\theta)$ is found such that draws from that distribution would simulate $g_{slope}\left(\frac{\partial x_i^*}{\partial p_i}(U)\right)$. One complication was that that Q ultimately gets inverted, so small values of θ would account for large elements of Q^{-1}, which in turn would significantly impact the optimal amount of acreage chosen by the farmer. Thus the parameters for θ where examined over its inverted parameters for the quadratic parameters i.e. $U_{ii}^* = \frac{1}{U_{ii}}$so that the jumping distribution used in the algorithm would not cause large shifts in the acreage with each new point. This was done and ultimately the distribution used was $g(U_{11})\,\tilde{}\,N(3100, 1400)$, $g(U_{22})\,\tilde{}\,N(3200, 1400)$, and $g(U_{33})\,\tilde{}\,N(3000, 1100)$.A prior still needs to be specified for $g(h)$. This distribution was formed based on the distribution proposed for $g(U)$ and based on mean data that formed $x(\theta)$, such that a general form of the distribution would be found; $g(h_1)\,\tilde{}\,N(-9, 5), g(h_2)\,\tilde{}\,N(-7, 4)$, and $g(h_3)\,\tilde{}\,N(-7, 3)$.With that draws can be done from $g(\theta)$ such that the distribution is close to what is know about the elasticity, $g_\eta(\eta(\theta))$.

Next the likelihood function is $f(z|\theta) = f\left(x^{obs}, revenue^{obs}, cost^{obs}, \lambda^{obs}|\theta\right)$. Note, there is an assumption that there is no measurement errors in *revenue* and *cost*. In order to construct this likelihood function there needs to be an assumption that the measurement errors are independent, thus the likelihood function could be redefined as:

$$f(z|\theta) = f_x\left(x^{obs}|\theta\right) f_\lambda\left(\lambda^{obs}|\theta\right) f_{rev}\left(revenue^{obs}|\theta\right) f_{cost}\left(cost^{obs}|\theta\right)$$

Since, there is an assumption that there is no measurement errors in *revenue* and *cost*, and they are not determined by θ, it can be assumed that $f_{rev}\left(revenue^{obs}|\theta\right) = f_{cost}\left(cost^{obs}|\theta\right) = 1$.

Now the only part of the distribution that has relevance to the likelihood function is $f_x\left(x^{obs}|\theta\right) f_\lambda\left(\lambda^{obs}|\theta\right)$. $f_x\left(x^{obs}|\theta\right)$ is a joint truncated normal distribution such that $f_x\left(x^{obs}|\theta\right) = \prod f_{x_j}\left(x_j^{obs}|\theta\right)$ where j represents the three crops. Each of the distributions $f_{x_j}\left(x_j^{obs}|\theta\right)$ are truncated normal with mean at $x_j(\theta)$, truncated at 0 and with standard deviation being $\sigma^2 = (0.5 * \mu_{xi})^2$. $f\left(\lambda^{obs}|\theta\right)$ is assumed to be normally distributed, truncated at 0 with a standard deviation equal to 25% of λ, that is $f_\lambda\left(\lambda^{obs}|\theta\right) = \frac{\phi\left(\frac{363-\lambda(\theta)}{.25*\lambda(\theta)}\right)}{.25*\lambda(\theta)\left(1-\Phi\left(\frac{-\lambda(\theta)}{.25*\lambda(\theta)}\right)\right)}$.

All that is needed is to find $g(\theta|z)$ based on these distributions. The method used here to find $g(\theta|z)$ is an MCMC algorithm, namely the Metropolis-Hasting algorithm. There are several instances in the literature of using an MCMC algorithm to obtain posterior distributions. An MCMC algorithm has been used by Ehlers (2011) to determine how efficient a production function is in describing real life data. MCMC algorithms have been used by O'Donnel, Shumway, and Ball (1999) for determining input demand functions. Thus it seems natural to apply an MCMC algorithm to this problem, specifically Metropolis-Hasting. With regards to the MH algorithm for this problem, it should start with θ^0 then for $j = 1, 2......$

1.Generate$\left(\theta^{j*}\right)$ ~$J_j\left(|\theta^{j-1}\right)$ where$J_j\left(|\theta^{j-1}\right)$ are draws from normal distributions for the quadratic values ,U, being means based by θ^{j-1}, and the linear portion , h, is drawn conditional quadratic values. This is because of U and h needing to be drawn as a pair to ensure the draw has a non-zero posterior density.

2. Compute

$$r_j = \frac{f_x\left(x^{obs}|\theta^j\right) f_\lambda\left(\lambda^{obs}|\theta^j\right) g_1(\theta^j) g_\eta\left(\theta^j\right)}{f_x\left(x^{obs}|\theta^{j-1}\right) f_\lambda\left(\lambda^{obs}|\theta^{j-1}\right) g_1(\theta^{j-1}) g_\eta\left(\theta^{j-1}\right)}$$

and generate

$$W_j \;\tilde{}\; \text{Bernoulli}\left(\min\left(1, r_j\right)\right)$$

3. Take

$$\left(\theta^j\right) = W_j \;\left(\theta^j,\right) + \left(1 - W_j\;\right)\left(\theta^{j-1}\right)$$

The important point is that this algorithm generates a point to jump to, then either accepts the point and jumps to it, or rejects it and stays at its current point. In most cases since the number of parameters to consider are large, the algorithm tends to reject most points unless the point proposed is relatively close to the current point. It is important that the prior is correct since the data set is limited.

V. Preliminary view of the Data

The data that is used is total national acreage planted obtained from the NASS from 2009-2013 for the crops of corn, soybeans, and wheat. The total acreage includes the prevented acreage (acreage that would have been planted, but were prevented most likely due to natural disaster). That data is shown in the table below:

Table 1: Observed and Prevent Acres in Millions

Years	Corn Acres Planted	Corn Acres Prevented	Soy Acres Planted	Soy Acres Prevented	Wheat Acres Planted	Wheat Acres Prevented
2013	95.365	3.617	76.533	1.704	56.156	2.013
2012	97.155	0.262	77.198	0.160	55.666	0.587
2011	91.936	3.013	75.046	1.447	54.409	4.117
2010	88.192	2.102	77.404	1.347	53.593	3.258
2009	86.382	1.879	77.451	0.933	59.168	0.917

Looking at the data there are a few interesting things to note, mostly looking from 2011-2013, where commodity prices go in an interesting direction. One key point is the fact that prevented acres seem to rise from year to year, except a drop in 2012 in which there was a drought. This makes sense since that is the time when commodity prices for corn and soybeans shot up by a significant margin. Data surrounding cost and revenue were obtained from USDA-ERS over the same time span. The crops cost were measured in per acre total operating costs, not including the total allocated overhead that the ERS reports and shown below:

Table 2: Average Cost in $ per acre

Year	Corn	Soy	Wheat
2013	355.98	180.36	128.08
2012	349.59	172.29	126.72
2011	332.33	136.87	121.89
2010	286.41	131.89	102.78
2009	295.01	130.49	112.92

Here it is seen, with the exception of wheat, that those per acre costs of a crop are increasing over time. Revenue was measured by the product of expected price and yield. Expected yield is given by a linearization of the yields, $Y_t = \alpha + \beta t$. This was done due to the fact that unexpected disasters like droughts could impact yields in a way farmers could not plan for. The data used in the linearization were actual average yields reported by the ERS from 1980-2013. The form of the linearization estimated in bu/acres is:

$$
\begin{aligned}
CornExpectYield_t &= 97.57 + 1.73T \\
SoyExpectYield_t &= 27.35 + 0.59T \\
WheatExpectYield_t &= 28.99 + 0.34T
\end{aligned}
$$

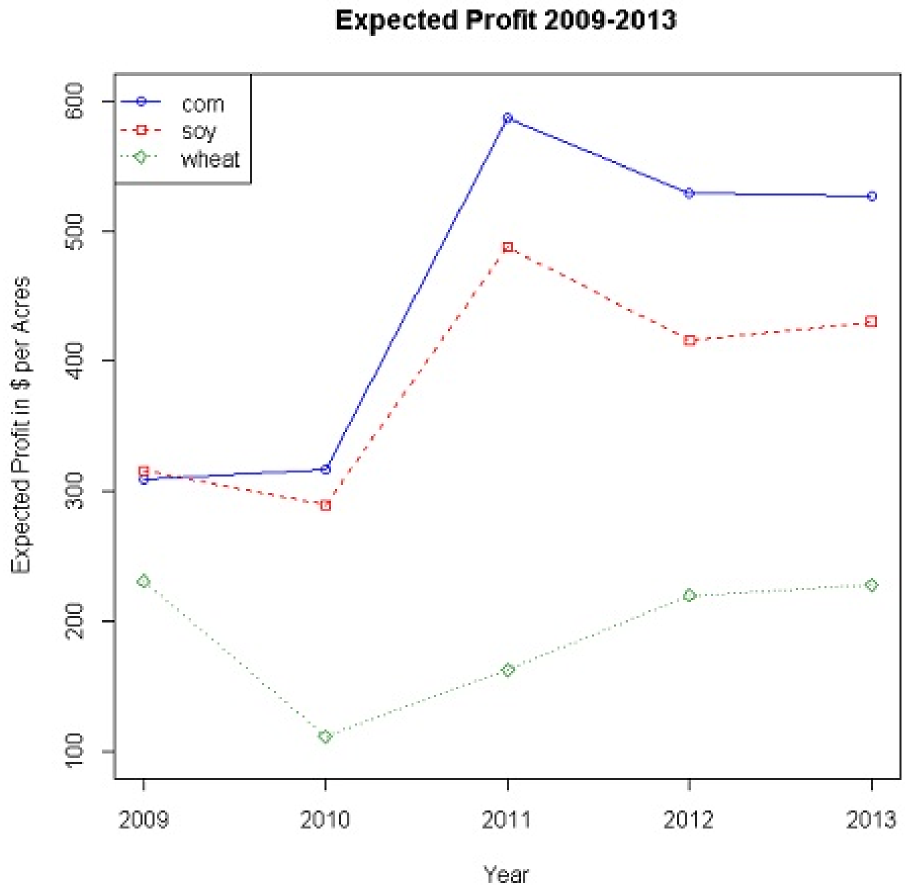
where T is each year goes from 1 to 34. As can be seen, by linearizing the yields to form expected yields has basically made the yields close to constant from 2009 to 2013,.

Expected price is given by the approved projected price reported by USDA Risk Management Agency as shown in Table 3.

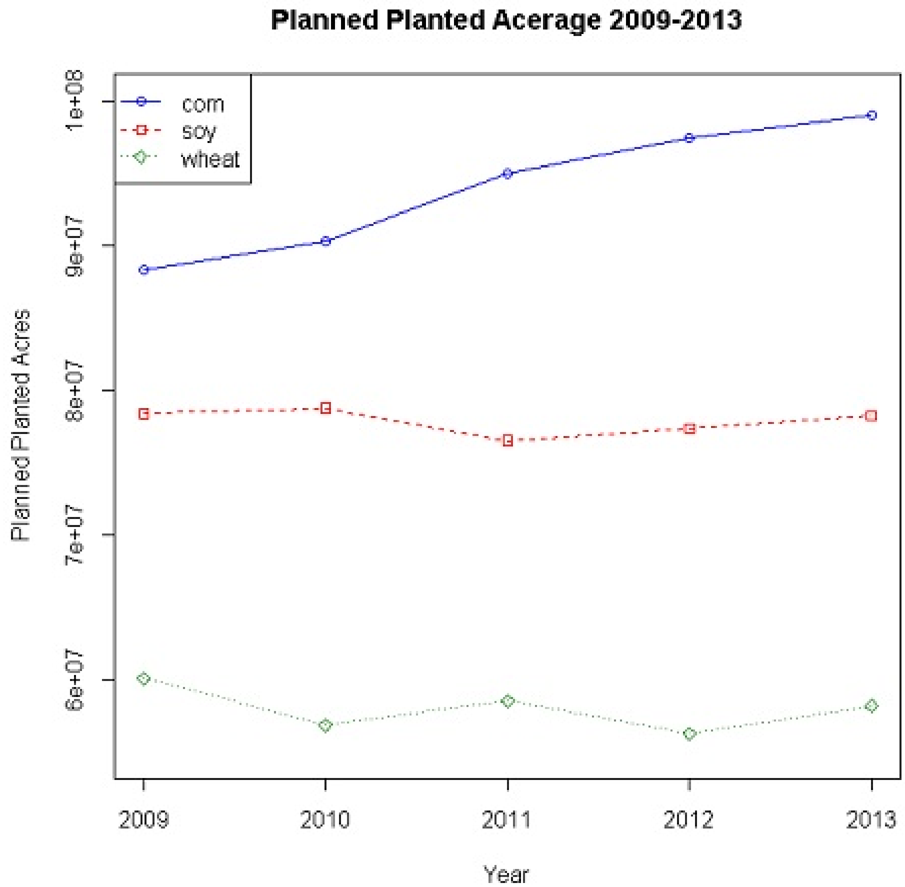
Table 3: Expected Price of Crop in $ per Bushel

Year	Corn	Soy	Wheat
2013	$5.65	$12.87	$8.78
2012	$5.68	$12.55	$8.62
2011	$6.01	$13.49	$7.14
2010	$3.99	$9.23	$5.42
2009	$4.04	$9.90	$8.77

This table again shows the jump in commodity prices in 2011 and onwards. The above tables and linearized yields, as well as the general price index, are all that is needed to simulate this posterior distribution in order to estimate this supply model. Graphically the impacts of the observed expected profitability of the crop and the observed acres that were intended to be planted, can be seen in a general sense.



The general trend shows a dip initially and then a spike occurs in 2011. Wheat for the most part is not influenced as much in the 2011 shift in commodity prices. Looking at the planned acres to plant(observed acreage plus prevented acres), is the following:



From the graphs it appears that the total aggregate acres the farmers plan to plant match with the expected profitability at least with corn. However, there are certain trends that don't match up. The most obvious is the decrease in wheat as expected profit clearly has an upward trend, as well as the flatness of the line in the amount of soybeans planted. It is clear the competition between the crops for land usage is needed to better understand the shifts in the acreage allowed to specific crops. Thus the simulations are needed to find this interaction by estimating unknown parameters generated by the PMP method.

VI. Results of the Simulated National model

For the national crop supply model, the posterior distribution was sampled using three different starting points for θ^0. The starting point θ^0 was found by fitting Howitt's (1995) initial technique in calibrating to one of the five years between 2009 and 2014. A guess for own price elasticity was drawn between 0 and 1 for each crop and used to calibrate the unknown quadratic as a diagonal matrix for a given year randomly selected between 2007 and 2014. This was all done to produce varying θ^0s while at the same time making sure that point is in fact a feasible result.

Thus we have a starting point where for a randomly selected year $\lambda\left(\theta^0\right) = \lambda^{obs}$, $x\left(\theta^0\right) = x^{obs}$ and $\eta\left(\theta^0\right)$ is at some value in the ball park of its distribution, η_{guess}. At this starting point, the only major error that the MCMC algorithm is trying to correct for is what the believed prior is and the inaccuracy in its prediction of the other years. Therefore the algorithm is selecting θ such that $g\left(\theta\right)$, $f_x\left(x^{obs}|\theta\right)$ and $f_\lambda\left(\lambda^{obs}|\theta\right)$ in a hill climbing algorithm trying to find θ that best satisfies these distributions. Three chains of three hundred thousand iterations with a burn in of one hundred thousand was used.

The estimated median of the resulting posterior distribution formed from this MCMC algorithm for the unknown θ is given in Table 4.

Table 4: Posterior distribution of θ at the Median & 95% Credible Interval

Quadratic portion of θ	U_{11}	U_{22}	U_{33}
Estimated Median	3185	3357	2674
(95% Credible Interval)	$(2060, 5460)$	$(2113, 5593)$	$(1863, 3885)$

Linear portion of θ	h_1	h_2	h_3
Estimated Median	-8.92	-6.90	-8.82
(95% Credible Interval)	$(-21.68, -2.88)$	$(-17.41, -2.41)$	$(-17.61, -4.60)$

These results show a relatively tight range on all the parameters for θ^*. The fact that $Q = U'U$ implies by construction that the unknown quadratic cost term is such that the marginal cost of producing a specific crop is increasing with respect to its own crop.

Next one must examine how well the optimum acreage of the model matches the data it is based upon. The following Table 5 examines the distributions of the optimum acreage based on the posterior distribution of θ^*. The measurement is both at the median of the distribution for the simulation and a 95% credible interval in parentheses.

Table 5: Distribution of Predicted Crop Acreages 2009-2013

at the Median and 95% Credible Interval

Years	Corn Predicted	Corn Observed	Soy Predicted	Soy Observed	Wheat Predicted	Wheat Observed
2013	97.51 $(93.32, 101.66)$	98.98	78.76 $(74.67, 82.86)$	78.23	59.08 $(55.84, 62.39)$	58.16
2012	95.96 $(91.98, 99.93)$	97.41	77.12 $(73.54, 81.07)$	77.35	57.89 $(54.92, 60.88)$	56.25
2011	96.67 $(92.46, 100.86)$	94.94	78.69 $(75.06, 82.90)$	76.49	54.51 $(50.75, 57.88)$	58.52
2010	91.03 $(87.23, 94.66)$	90.29	77.10 $(73.65, 81.14)$	78.75	57.70 $(54.68, 60.78)$	56.85
2009	89.33 $(84.96, 93.17)$	88.26	76.84 $(73.04, 81.11)$	78.38	60.56 $(56.99, 64.52)$	60.08

Measured in millions of acreage, and includes prevented acres in both predicted and observed

The predicted values are from the 95% credible interval from a sample distribution of three chains. As observed from above, the resulting prediction based on the optimization modeling have intervals which for all years encompass the observed acreage and in many cases the observed is relatively close to the median of the distribution of note the only year where the model has trouble predicting is 2011, the year a major shift in expected profits occurred. Other than that instance these results reflect the benefit of employing a programming method creating a model optimization model that can reproduced observations.

The elasticity of the posterior can also be seen by looking at the median and 95% credible interval (parenthetic) from a sample distribution. By examining the elasticity matrix for a single year (2013)the median of the elasticity is shown in Table 6.

Table 6: Distribution of Supply Elasticity for 2013, Median & 95% Cred. Interval

2013 Elasticity	**Corn Price**	**Soy Price**	**Wheat Price**
Corn Acres	0.265	-0.193	-0.165
	$(0.137, 0.522)$	$(-0.487, -0.074)$	$(-0.372, -0.059)$
Soy Acres	-0.108	0.238	-0.124
	$(-0.268, -0.042)$	$(0.122, 0.468)$	$(-0.292, -0.047)$
Wheat Acres	-0.040	-0.054	0.144
	$(-0.090, -0.015)$	$(-0.128, -0.020)$	$(0.080, 0.254)$

These results are remarkably close to Hendricks, Smith, and Sumner (2014) measurement of long run own price and cross price elasticities between corn and soybeans, their estimates have an own price for corn at 0.29 and soybeans at 0.26. There is a slight difference in the cross price elasticity with Hendricks, Smith, and Sumner estimation and that may be due to the interaction of wheat. A regional model might better replicate the resulting elasticities shown by Hendricks, Smith, and Sumner, again only needing a limited amount of information. Looking at the elasticities generally, the own price elasticity is inelastic and generally soybeans and corn are similar in terms of own price elasticity, while wheat is the most inelastic. Its also interesting to note that the strongest cross price elasticity lies between corn and soybeans and the cross price elasticity for wheat tends to be fairly weak, if it does exist.

VII. Convergence Test

The robustness of the model can be checked through the Brooks-Gelman-Rubin (BGR) test. This was done for 3 sample models for over thirty-five million iterations. The general formula for this test for an individual element is as follows. Let m be the number of samples, and n the number of iterations in sample and let ψ be a sample element within θ so the simulated draws are $\psi_{i,j}$ for $i = 1,..,n$ and $j = 1,..,m$ Then the between and within sequence variances are calculated as:

$$B = \tfrac{n}{m-1}\sum_{j=1}^{m}\left(\overline{\psi}_{*j} - \overline{\psi}_{**}\right)^2$$

$$W = \tfrac{1}{m}\sum_{j=1}^{m}\left(\tfrac{1}{n-1}\sum_{i=1}^{n}\left(\psi_{i,j} - \overline{\psi}_{*j}\right)^2\right)$$

With that the variance can be derived as :

$$\widehat{var} = \tfrac{n-1}{n}W + \tfrac{1}{n}B$$

From here the convergence is then measured by

$$\widehat{R} = \sqrt{\tfrac{\widehat{var}}{W}}$$

This will converge to 1 as $n \to \infty$.This will allow us to determine how many iterations are suitable for the sampling to converge. However, as this is test for a multivariate model, the added complication to this test will be dealt with by using an R package called coda. This will allow for the measurement of this variability at a multivariate level. The results are shown in Table 7.

Table 7: Measurement of $\widehat{R}$ for all elements in θ^*

Point estimate	U_{11}	U_{12}	U_{13}	h_1	h_2	h_3	Multivariate
Measurement of $\widehat{R}$	1.00	1.01	1.00	1.00	1.00	1.00	1.01

The rule of thumb for this test is that the chains have converged if $\widehat{R}$ is at 1.1 or lower. it is clear that over these 200,000 iterations convergence occurred, and more than likely that a smaller sample size would show convergence.

VIII. Forecasting

For prediction I used the year 2014. It is easy enough to project the expected yields and an expected cost for 2014. The expected price is already given by USDA Risk Management Agency. From this the model, one can attempt to predict 2014 acreage, the results are:

Table 8: Prediction for Crop Acreage for 2014, Median & 95% Cred. Interval

2014	**Predicted**	**Observed**
Corn	94.19 $(88.70, 99.04)$	92.46
Soy	81.28 $(77.17, 85.54)$	85.01
Wheat	60.31 $(56.94, 63.66)$	58.20

As can be seen, this aggregate model has modes fairly close to observed for Corn and Wheat.. While the observation is slightly higher than the mode for soybeans it is still within the 95% credible interval of the simulations. Ultimately this Bayesian technique can do a decent job at forming a posterior distribution that can actually reflect the observations while at the same time forming reasonable estimates for the elasticities and gives reasonable forecasts.

IX. Conclusion

The work done in this paper has shown how to employ an MCMC algorithm to determine the set of parameters that are consistent with the beliefs of what a national crop supply model should be in the context of a PMP model. The general results of the model show promise in that they reflect what is observed and appear fairly consistent with the literature. Improvements to the model, specifically to a more informed prior, could be made that allow for credible intervals without a tremendous amount of sampling needed.

Future work will be to expand this national model into a regional model. This should not be too difficult with respect to the data, as regional data exist from USDA-ERS for the US. The only real hurdle to this problem is to develop the basic techniques used to form a prior at the national level to the regional level. Further analysis will also examine effects of implementing policy both at a national level and a regional level. The results of the model show how this basic programming model can be useful in estimating and forecasting supply at a national level for the US. Further work will extend these estimates and forecast to a regional level, and offer flexibility in policy analysis with added statistical analyses.

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