Asset Fixity under State-Contingent Production Uncertainty

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Abstract

Asset fixity of inputs is tested under state-contingent production uncertainty. We construct a general dynamic dual model for U.S. agriculture that allows tests for full variability and strict fixity to be performed for each input as well as tests for functional form. We estimate the model using a generalized Box-Cox functional form. Most test results are robust to functional form, but test results of fixity are sensitive for two of four inputs. The generalized Leontief is found to be significantly preferred to the translog and normalized quadratic functional forms for the dynamic model. With this functional form, family labor exhibits strict fixity, while land, capital, and hired labor exhibit quasi-fixity. Production uncertainty has limited impacts on investment decisions for quasi-fixed inputs.

Keywords: dynamic duality, asset fixity, production uncertainty

JEL classification: D24, Q11

Introduction

Production is subject to uncertainty. In the agricultural production sector, risks and uncertainties are directly related to the nature of weather and climatic factors which cause fluctuations in yields and farm income. With production risk, producers need to make tradeoffs between adjustment costs and the potential benefits of investment on productive inputs to protect against unforeseen events. An important question arises: how fast do agricultural inputs adjust to their optimal levels when production uncertainty is present? This study advances the research frontier by providing insights on the interrelationship of risk and input adjustment rates at the industry level.

Input factors can be costly to adjust. Changing capital level may generate disruption costs during installation of the new or replacement capital, learning costs if structure of production changes, and delivery lags and value of time to install and/or build new equipment (Cooper and Haltiwanger 2006). Changing the level of demand for labor may involve severance pay or job advertisement and other labor search costs (Stefanou 2009). While firms can alter investment strategies to manage production uncertainty, adjustment costs may penalize rapid changes in input levels, leading to a potential discrepancy between actual and desired input levels. For instance, when a drought is expected to occur, a farmer can construct irrigation facilities to maintain or increase output level, but the associated adjustment costs may prevent the producer from making the response instantly. In the case that adjustment costs exceed the expected benefits, the producer may refrain from making the investment.

We construct a general dynamic dual model that allows asset fixity of inputs to be tested under production uncertainty, which is represented using a state-contingent approach that does not depend on agents' risk preferences. The literature on decision making under uncertainty generally uses the expected utility model (e.g., Saha, Shumway and Talpaz 1994; Sckokai and Moro 2006; Tveteras, Flaten and Lien 2011). However, an increasing number of studies provide accumulated evidence that the expected utility model fails to accurately represent agents' risk attitudes (Rabin 2000; Just and Peterson 2003; Friedman et al. 2014), and adopting a state-contingent approach offers a preferable framework for presenting uncertainty (Crean et al. 2013). When applied to a cost minimization problem, Chambers and Quiggin (1998, 2000) show that this approach accounts for producers' management of uncertainty through input allocation under different states of nature, and that standard duality theory applies the same as for nonstochastic technologies.

Through a state-contingent treatment of uncertainty, we are able to investigate the level of output substitution across states of nature. We represent production uncertainty by a set of mutually exclusive states of nature (e.g., a wet year and a dry year) and assume that producers can prepare differentially for different states of nature. Take an example from Chambers and Quiggin (2000) in which a producer makes decisions to construct irrigation infrastructure and/or flood control facilities using a fixed amount of resource. If irrigation facilities are developed instead of flood control, output will be relatively high in the event of a drought and low in the event of a flood. On the contrary, if efforts are devoted to flood control, output will be relatively low if a drought occurs and high with a flood. This assumption allows for greater flexibility and adaptability and implies a possibility of substitution between state-contingent outputs, i.e., output in a wet year and output in a dry year. In this study, we examine the substitution possibility between state-contingent outputs, as the zero substitution between outputs across states, referred to as an output-cubical technology, is a crucial assumption for studies based on the realized state of nature (e.g. productivity measurement and efficiency analysis).

The contribution of this study is twofold. First, our model broadens the factor adjustment literature to allow for testing of asset fixity under production uncertainty using a state-contingent approach. In this study, uncertainty is represented by mutually exclusive states of nature, and output is state-contingent for each production period. The firm chooses input allocations before uncertainty is resolved. Our model imposes no constraints on asset fixity. With no loss of generality, we initially allow all inputs to be quasi-fixed. After deriving the conditional optimal equations for input demand in a multivariate flexible form, both full variability and strict fixity are tested.

Second, to date the influence of functional form on the validation of asset fixity tests has not been examined. We specify a generalized Box-Cox functional form which nests the three functional forms (i.e., normalized quadratic, generalized Leontief, and translog) that are commonly used in dynamic duality analysis. The preferred functional form among this set is identified using Wald test statistics.

While most hypothesis test conclusions are robust to functional form, we find that the conclusions regarding fixity of capital and family labor are sensitive to functional form. Our functional form tests fail to reject the generalized Leontief but reject both the translog and normalized quadratic relative to the generalized Box-Cox. With the generalized Leontief chosen over the other two nested functional forms, we cannot reject the hypothesis of strict fixity for family labor. Land, capital, and hired labor exhibit quasi-fixity and they adjust 15 percent, 27 percent, and 47 percent, respectively of the difference between current and equilibrium levels each year. Impacts of uncertainty on investments in quasi-fixed inputs are insignificant. Empirical evidence is found to support an output-cubical technology, which suggests that analysis based on realized states of nature is appropriate.

The remainder of the paper is organized as follows. The next section presents the theoretical model. The empirical model follows a discussion of the data. In the results section, we estimate the state-contingent outputs, present the asset fixity results and their implications. The final section concludes the paper.

Theoretical Model

Consider a price-taking, single output firm which makes decisions under production uncertainty to minimize the discounted present value of costs over an infinite planning horizon. Production uncertainty is represented by S mutually exclusive states of nature, with each state representing a combination of rainfall, temperature, and other stochastic factors determined by nature that may affect production. Outputs are conditional on states, $Y_t = [y_{1t}, \dots, y_{st}]'$, where y_{st} is the output quantity realized under the state of nature s in time period t. Input decisions are made before uncertainty is resolved.

At any period of time, $t = 0$, conditional on the state-contingent outputs, the firm chooses variable input quantities and investment in quasi-fixed inputs consistent with the dynamic cost minimization problem:

(1) $$J(P, W, Z, Y, Q, H) = \min_{X,I} \int_0^\infty e^{-rt}[W'X + P'Z]dt$$

subject to

(1a) $$\dot{Z} = I - \delta Z,$$

(1b) $$Y = F(X, Z, I, Q, H),$$

(1c) $P(0) = P_0,\ W(0) = W_0,\ Z(0) = Z_0,\ Y(0) = Y_0\ ,\ Q(0) = Q_0, H(0) = H_0,$

where J is the optimal discounted present value of costs in the long run, which depends on the quasi-fixed inputs Z, their rental prices P, price vector W for variable inputs X, state-contingent outputs Y, fixed inputs Q, and technical change H; $\dot{Z}$ is the net change in Z that equals to the gross investment disposed of depreciation; r is the real discount rate; δ is a diagonal matrix of depreciation rates. The transformation function F provides a general *ex ante* representation of production technology under uncertainty. It is augmented with gross investment to account for internal adjustment costs in the form of foregone output (Lucas 1967) or increased use of inputs. Assume that all the input prices are observed at the time when the firm makes input quantity decisions and the price vectors W and P are normalized by a numeraire input price. All variables are implicit functions of time, so the subscripts t are dropped temporarily to minimize notational clutter. $P_0,\ W_0, Z_0, Y_0, Q_0$ and H_0 are the initial values of $P, W, Z, Y, Q,$ and H, respectively.

The Hamilton-Jacobi-Bellman equation of the problem takes the form:

(2) $$rJ(P, W, Z, Y, Q, H) = min\left[W'X + P'Z + J_Z\dot{Z} + \lambda\big(Y - F(X, Z, I, Q, H)\big)\right],$$

where λ is the Lagranger multiplier. After applying the envelope theorem, the conditional optimal demand equations for variable and quasi-fixed inputs are obtained as:

(3) $$X = rJ_W - J_{ZW}\dot{Z},$$

(4) $$\dot{Z} = J_{ZP}^{-1}(rJ_P - Z).$$

If the value function has a form such that $J_{ZP} = (rU - M)^{-1}$ where U is an identity matrix, equation (4) becomes a multivariate flexible accelerator model,

(5) $$\dot{Z} = M(Z - Z^*(P, W, Y, Q, H)),$$

where M is a constant adjustment matrix and $Z^*(P, W, Y, Q, H)$ is the optimal level of quasi-fixed inputs in long-run equilibrium.

To examine asset fixity, we impose no constraints on adjustment of inputs prior to the specification of the dynamic system. That is, none are constrained to be completely fixed or fully variable. Instead, all inputs are initially allowed to be quasi-fixed, and tests are conducted to determine whether any do not respond to price changes or production uncertainty or whether any fully respond in a single production period. The demand equation for variable inputs is nested within the equation for quasi-fixed inputs since equation (4) can be rewritten as $Z = rJ_P - J_{ZP}\dot{Z}$. The degree of factor fixity is investigated by conducting nested hypothesis tests with restrictions placed on the quasi-fixed inputs. Restrictions for full variability imply instantaneous adjustment to price changes and production uncertainty with no costs of adjustment. For strictly fixed inputs, restrictions imply no response to changes in market or uncertainty due to very large adjustment costs.

Empirical Model

Using the state-contingent approach to model uncertainty, one challenge is to recover the *ex ante* technology using the incomplete *ex post* observations, since for each time period t, $t = 1, \dots, T$, only one of the s possible output realizations is observed. In this study, we use the method proposed by Chavas (2008) that can generate all possible outputs Y under different states of nature.[1]

[1] The method of Chavas (2008) allows an empirically tractable analysis of all states of nature. It was chosen for convenience of estimation within a dynamic framework.

Assume that observations in each time period are associated with a different state of nature, so t represents both time and state of nature. Let the *ex ante* expected output quantities be represented as $y_{st} = \mu_t e_s^{\sigma_t}$, $s = 1, \dots, T$, where μ_t and σ_t are positive numbers, μ_t captures production technology at period t, σ_t is the spread parameter of the output distribution across states of nature, and e_s is a random variable that measures the relative changes in outputs across states of nature. This specification allows an arbitrary distribution of outputs across states of nature but assumes that their expected relative changes e_s are invariant across time.

To recover the *ex ante* outputs, assume an auxiliary variable v_t that satisfies the condition that the states of nature have the same relative effects on the output as they have on v_t, $v_t = k_t e_t^{\sigma_t}$. In this study, we use the total factor productivity (TFP) as the ancillary variable, since TFP accounts for all the effects in total output not caused by changes in quality-constant inputs, and in U.S. agriculture, TFP has nearly synonymous growth with output as the total resource base has barely increased (Heisey, Wang and Fuglie 2011). The equation $v_t = k_t e_t^{\sigma_t}$ becomes a linear model when taking logarithms of both sides, $\ln(v_t) = \ln(k_t) + \sigma_t ln(e_t)$, where $\ln(v_t)$ is the dependent variable, $\ln(k_t)$ is the regression line, and $\sigma_t ln(e_t)$ is the error term. σ_t captures possible heteroscedasticity and $ln(e_t)$ has mean zero and variance one. After obtaining consistent estimates of k_t and σ_t, we can generate a vector of T realized values of the random variable $e = [\left(\frac{v_1}{k_1}\right)^{1/\sigma_1}, \dots, \left(\frac{v_T}{k_T}\right)^{1/\sigma_T}]$. These values in turn can be used to obtain the simulated state-contingent outputs Y_t given by

(6) $$Y_t = \left\{ y_{rt} : y_{rt} = y_t \frac{\left({}^{v_r}/_{k_r} \right)^{\frac{\sigma_t}{\sigma_r}}}{\left({}^{v_t}/_{k_t} \right)}, r = 1, \dots, T \right\},$$

where y_t is the realized output at time t.

The state-contingent outputs incorporate T variables for each time period and their elements tend to be correlated in the sample. To avoid generating collinearity problems, Chavas (2008) proposed a parsimonious specification by classifying the T states into L intervals with $V_{1t} = (-\infty, m_{1t}]$, $V_{2t} = (m_{2t}, m_{3t}]$…, and $V_{Lt} = [m_{Lt}, \infty)$, $m_{1t} < m_{2t} < \cdots < m_{Lt}$.[2] The output means for each interval are used to define the reduced state space.

Data

We apply the model to the U.S. agriculture production sector over the period 1948-2011. U.S. agriculture is modeled as a representative firm with one aggregated output produced using five input categories: land, non-land capital, hired labor, family labor, and materials.

Price and quantity data for the aggregate output and input categories come from the U.S. Department of Agriculture (USDA, 2014). These data comprehensively cover inputs used by the agricultural production sector and outputs produced by the sector. The aggregate output includes production of livestock, crops, and output of goods and services from secondary activities. For purposes of this study, physical capital is classified into land and non-land capital. The non-land capital category is an aggregate of the quantity of durable equipment, service buildings, and inventories. Labor input is

[2] The m_{it}'s are chosen such that there is at least one observation in each interval (Chavas 2008).

classified into two categories: hired labor, and self-employed family workers. Materials include all intermediate goods used in production such as energy, fertilizer, pesticides, and purchased services. Because each input category is an aggregate of many individual inputs, aggregate price for each category is formed as a Tornqvist index, and quantity is computed as total expenditures on the category divided by the respective price index. Data for TFP over 1948-2011 are also obtained from USDA (2014). There is little change in aggregate agricultural input levels over the data period and the TFP closely represents changes in aggregate output levels.

Public and private agricultural research and public extension expenditures are used to create a proxy for technical change. Huffman and Evenson (2008, pp.105-107) provide total annual real public and private research expenditures for the period 1948-2000. Public agricultural research funding for the period 2001-2010 were updated by Huffman (2014). Private research expenditures for 2001-2010 come from Fuglie et al. (2011). We converted the research expenditures to constant dollar values using the price index for agricultural research from Jin and Huffman (2013). Public and private research stocks were created using the trapezoid-shaped timing weights with a total lag of 35 years for public expenditures and 19 years lag for private expenditures from Wang et al. (2013). State-level agricultural extension data for 1951-1976 are from Huffman, Ahearn, and Yee (2005). Data for the period 1977-2010 were updated by Huffman (2013). They were aggregated to the national level and linearly extrapolated back from 1950 through 1944 and forward for 2011.[3] Extension stocks were computed using the exponentially declining weights over 5 years from Jin and Huffman (2013).

[3] We used the linear extrapolation technique provided by STATA (command package "ipolate").

Measuring State-Contingent Outputs

The equation $\ln(v_t) = \ln(k_t) + \sigma_t ln(e_t)$ is fitted as an autoregressive conditional heteroscedastic (ARCH) model with the mean equation $\ln(v_t) = \ln(k_t) + a_t$ and the volatility equation $a_t = \sigma_t ln(e_t),\ \sigma_t^2 = \alpha_0 + \alpha_2 a_{t-2}^2$.[4] Total factor productivity (TFP) is treated as the dependent variable v. The explanatory variables k are specified to control for the effects of market conditions and technology development. They include all input prices normalized by lagged output price and the log of public and private research and extension stock.

Results of the ARCH model estimation are presented in Table 1. Six of the eight explanatory variables are significant at the 0.05 level. We find that more expensive family labor or materials relative to output price lead to a significant decrease in productivity while an increase in the relative price of land or hired labor stimulates a significant increase in productivity. Both the public and private agricultural research stocks have significant and positive effects on TFP. Changes in the relative price of capital or in the extension stock do not significantly impact TFP.

Based on the estimation results, we obtain a total of $T = 64$ simulated state-contingent output levels for each year using equation (6). Since multicollinearity problems quickly arise when the partition of state space becomes more accurate, we classify the simulated output space into two states for each period – an unfavorable state

[4] Both the Ljung-Box and the Lagrange multiplier test on the squared residuals reject the hypothesis of homoscedasticity with p-values smaller than 0.05 up to 12 lag windows. The ARCH (0, 2) model is chosen from a set of competing models based on the Akaike Information Criterion (AIC).

Y_1 and a favorable state Y_2.[5] Figure 1 shows the evolution of the ratio Y_1/Y_2 over the time period 1948-2011.

Functional Form

The behavior of the industry is modeled as a representative firm using the aggregate data. The necessary and sufficient condition for consistent aggregation across firms is that the value function has a form such that $J_{ZZ} = 0$. A value function in general Box-Cox (BC) form that satisfies the consistent aggregation condition takes the form:

$$\begin{aligned} J(P,W,Z,Y,Q,H) &= AR + .5R'BR + P'C^{-1}Z + [P'\ W']D(\log Y) \\ &+ [P'\ W']Evech(\log Y)\,(\log Y)' + [P'\ W']GH + [P'\ W']LQ, \end{aligned} \tag{7}$$

where A, B, C, D, E, G and L are parameter vectors or matrices of appropriate dimension; B is symmetric; The vector $R = [\frac{P^{\lambda}-1}{\lambda} \frac{W^{\lambda}-1}{\lambda}]$ is a generalized Box-Cox transformation of prices that nests the commonly used three functional forms as special cases: $\lambda \to 0$ for the translog (TL), $\lambda = 0.5$ for the generalized Leontief (GL), and $\lambda = 1$ for the normalized quadratic (NQ); *vech* is a half-vectorization operator; and $\log Y$ is the natural logarithm of Y.

The functional form only takes Box-Cox transformation in prices. J_{PZ} is specified as a matrix of constants to maintain the multivariate flexible accelerator hypothesis of input adjustment. We take natural logarithms of the positively skewed state-contingent outputs Y to improve the fit of model. The technical change H and fixed inputs Q are in

[5] The 64 states for each year are ordered by output levels and classified into two intervals by the mean value. There are 29 states in the lower interval and 35 states in the upper interval. The outputs Y_1 and Y_2 are the means for each interval. Multicollinearity problems seriously impact standard errors when the number of states is increased above two.

their original values by which we assume that they have a linear relationship with input demand.

Prices and cost are normalized by one of the factor prices to maintain the theoretically implied property of linear homogeneity. The price of a variable factor is generally used as the numeraire, but it is not applicable in this study since all inputs are initially treated as quasi-fixed. For the initial estimation, we use the price of intermediate goods as the numeraire.[6]

Treating all inputs as potentially quasi-fixed, we derive the optimal demand equations for quasi-fixed inputs based on equation (4). Since the demand equations of variable inputs and fixed inputs are special cases of equation (4) in which some inputs in Z are treated as fully variable or strictly fixed, terms involving W and Q (i.e., prices of variable inputs and quantities of fixed inputs) in the value function are initially removed, which generates the following system of demand equations conditional on the state-contingent outputs:

$$\dot{Z} = (rU - C)Z + rC(AR_P + R_P'BR + D(\log Y) + Evech(\log Y)(\log Y)' + GH). \tag{8}$$

Replacing $\dot{Z}$ by a discrete approximation of $Z_t - Z_{t-1}$, the above equation can be written as:

$$Z_t = (U + M)Z_{t-1} + rC(AR_P + R_P'BR) + \widehat{D}(\log Y) + \hat{E}vech(\log Y)(\log Y)' + \hat{G}H, \tag{9}$$

where $\widehat{D} = rCD$, $\hat{E} = rCE$, $\hat{G} = rCG$, and the adjustment matrix $M = rU - C$.

[6] By using the price of intermediate goods as the numeraire, we implicitly treat it as a variable input. If fully variable factors are found to be present based on the fixity tests, they become candidates for the numeraire.

The demand equations for the four possibly quasi-fixed inputs (land, capital, hired labor and family labor), i.e., equation (9), constitute the initial estimation system, which is estimated using nonlinear iterative seemingly unrelated regression (ITSUR).[7] In all equations, r is the real discount rate (4 percent).[8] If the tests of asset fixity reveal that some inputs are variable, demand equations derived from equation (3) will be included in the estimation system in place of their quasi-fixed input demand equations. If evidence of fixed inputs is found, they will become independent variables in both variable and quasi-fixed input demand equations, and their quasi-fixed input demand equations will be removed from the system.

Results

In this section, we test for asset fixity and functional form conditional on the simulated state-contingent outputs. The adjustment process and input demand elasticities are investigated based on the preferred functional form.

Asset Fixity Tests

The asset fixity hypotheses and their implied restrictions on the adjustment matrix, $M = rU - C$, are reported in Table 2. The ith row of M represents the adjustment process for the ith input. The diagonal parameter M_{ii} is the adjustment rate of input i. The off-diagonal elements capture the dynamic interaction between pairs of inputs. If the change in input instantaneously fills the gap between its actual level and optimal level in one production period and its adjustment is independent of the adjustment path of other

[7] The demand equation for the numeraire factor (i.e., materials) is not estimated as part of the system due to its complexity and because most of the relevant information about materials demand can be obtained from other estimated parameters of this system of equations.
[8] The real discount rate is calculated as the average annual nominal yield on Moody's Baa corporate bonds over all maturities less the rate of inflation over the data period, 1948-2011.

inputs, then there are no adjustment costs and the input is fully variable. If the adjustment of input does not respond to relative changes in prices or production uncertainty and if it does not depend on the adjustment path of other inputs, then the input is strictly fixed. If it is between these two cases, i.e., it adjusts partially in one production period, then it is quasi-fixed.

Table 3 provides the asset fixity test results for the Box-Cox functional form as well as the three nested functional forms. The hypotheses of full variability, strict fixity, and independent adjustment of all inputs are rejected by all functional specifications at the 0.05 significance level, implying the quasi-fixity of the dynamic system. All functional forms also reject the hypothesis of full variability for each input at the 0.05 significance level, implying the existence of adjustment costs in input demand for each input. Rejection of the hypothesis of strict fixity for land and hired labor is also consistent across functional forms. However, the tests for strict fixity of capital and family labor inputs are sensitive to functional form. The NQ and BC fail to reject the hypothesis of fixed capital while the TL and GL reject it at the 0.05 level. The TL, GL and NQ fail to reject the hypothesis that family labor is strictly fixed while the BC rejects this hypothesis at the 0.05 level.

The conclusion that the U.S. agricultural production system adjusts to its optimal level sluggishly is robust to functional form. This result is in accord with the findings of Vasavada and Chambers (1986), Luh and Stefanou (1996), and Asche, Kumbhakar and Tveteras (2008), and Yang and Shumway (2015) although none considered production uncertainty. Strong support is found for the existence of adjustment costs in individual adjustment paths of inputs, as all the functional forms reject the hypothesis of full variability for each input. Vasavada and Chambers (1986) also found the adjustments of aggregated labor, capital and land are sluggish in U.S. agriculture. The hypothesis of full variability of capital (including land) and labor were also soundly rejected by Luh and Stefanou (1996) and Asche, Kumbhakar and Tveteras (2008). The rejection of fixity for land and hired labor is robust to functional form, suggesting they are both quasi-fixed inputs, but the hypothesis tests of fixity for capital and family labor are sensitive to functional form. Most support is found for strict fixity of family labor since it cannot be rejected by three of the four functional forms. Only a few previous studies have examined the fixity characteristics of inputs. The work of Asche, Kumbhakar and Tveteras (2008) suggest that capital (including land) and aggregated labor do not exhibit strict fixity when outputs are allowed to be fully variable. Consequently, our results based on cost minimization differ in this important respect from prior dynamic literature.

The preferred functional form among the TL, GL, and NQ is identified using the Wald test. Nested test results are reported in Table 4. With the estimated value of 0.45 for the parameter λ in the unrestricted BC estimation, the hypothesis that the GL is the correct functional form cannot be rejected against the BC. Both the TL and NQ are rejected at the 0.05 significance level. These results indicate that the GL is the preferred functional form for the dynamic model with the provided data. Unlike many prior functional form tests based on the Box-Cox (Giannakas, Tran and Tzouvelekas 2003; Koebel, Falk and Laisney 2003; Lafrance 2008), there is clear preference for one functional form in that it is not rejected and both of the alternative common nested forms are rejected.

Implications

Based on the test results for asset fixity and choice of functional form, we re-specify the model with the generalized Leontief functional form and treat family labor as strictly fixed. The final estimation system consists of demand equations for the three quasi-fixed inputs, i.e., land, capital and hired labor. The demand equation for family labor is removed from the system. Given the value function in (7), quantity of family labor is included as an explanatory variable in all equations in (9) (i.e., the matrix term $\hat{L}Q$ is added with $\hat{L} = rCL$).

Table 5 provides the nonlinear ITSUR estimates for the final model. The number of observations used for estimation is 63 and the degrees of freedom for each equation is 45. The GL model explained nearly all of the variation with R^2 values for land, capital and hired labor equal to .999, .97, and .99 respectively. Three of the six price parameters (B_{ij}) are significant at the 0.05 level and one at the 0.10 level. Six of the nine adjustment parameters (C_{ij}) are significant at the 0.05 level, including all own-adjustment parameters and three off-diagonal elements. One public research parameter and one fixed input coefficient are significant at the 0.05 level. One extension stock parameter is significant at the 0.10 level.

The adjustment matrix M is derived from the estimated parameters of the matrix C with $M = rU - C$. The diagonal parameters are own adjustment rates, which are -0.1504 for land, -0.2705 for capital and -0.4746 for hired labor. They imply that land, capital, and hired labor adjust in one year by 15 percent, 27 percent, and 47 percent, respectively, of the divergence between actual and equilibrium levels. These values suggest that the adjustment lags to equilibria are about 7 years for land, 4 years for capital, and less than 3 years for hired labor. The off-diagonal parameters in the adjustment matrix reflect how the disequilibrium in one input affects the adjustment rate of the other. The adjustment paths of land and hired labor are significantly interrelated to each other but have asymmetric effects. When land and hired labor are both below or above their own optimal levels, disequilibrium in hired labor increases the adjustment rate of land while disequilibrium in land slows the adjustment speed of hired labor. When land and hired labor are on the opposite sides of equilibrium, e.g., land is above but hired labor is below the equilibrium, the adjustment rate of land decreases while the adjustment rate for hired labor increases. The adjustment path of hired labor also depends on the disequilibrium in capital. If they are on the same side of equilibrium, adjustment speed of hired labor increases. If they are on the opposite sides, its adjustment speed decreases.

Matrix $\widehat{D}$ and $\widehat{E}$ represent the effects of alternative states of nature on input demands. None of the parameters are significantly different from zero, indicating that the production uncertainty has very limited impact on investment decisions in land, capital and hired labor. A possible explanation is that costs related to input adjustments may exceed the potential benefit from investments on inputs to protect against uncertainty. Substitution between outputs across states of nature (i.e., output in unfavorable state and output in favorable state) implies that resources are allocated in a way that improves output level in one state of nature, while leaving it unchanged or reduced in another state. For each demand equation i, given the input level, substitution between state-contingent outputs is captured by the elasticity of Y_1 with respect to Y_2, which is represented as $\epsilon_i = -\frac{\widehat{D}_{i2}+2\widehat{E}_{i2}\log(y_2)+\widehat{E}_{i2}\log(y_1)}{\widehat{D}_{i1}+2\widehat{E}_{i1}\log(y_1)+\widehat{E}_{i2}\log(y_2)}, i = 1, 2, 3$. For the demand equations of land, capital and hired labor, the p-values of the elasticities are 0.5453, 0.8099, and 0.2292 respectively.

Thus, none is significant, which provides evidence in favor of an output-cubical technology and suggests that an *ex post* analysis conditional on the realized states of nature is appropriate. This finding is consistent with that of Chavas (2008) and Serra, Stefanou and Lansink (2010), both of which found very limited output substitution between states of nature.

The parameters of matrix $\hat{G}$ capture the impacts of public and private agricultural research and public extension on input uses. An increase in agricultural extension significantly reduces the demand for land at the 0.10 level, and an increase in public agricultural research significantly reduces the demand for hired labor at the 0.05 level. The dependences of quasi-fixed inputs on the level of the fixed input (i.e., family labor) are provided by matrix $\hat{S}$. The demand for hired labor is significantly and positively related to the level of family labor, which implies that they are complements.

Price elasticities for input demands in both the short run (one production period) and long run are presented in Table 6. Since the long-run elasticities are highly nonlinear, their significance levels are determined using the bootstrap technique in which we obtain lower and upper bounds of confidence limits at the 0.05 and the 0.10 levels.[9] The hypothesis that the long-run elasticity equals to zero is rejected if zero lies outside the approximate confidence interval. In the short-run, the own-price elasticities of capital and hired labor are negative and significant at the 0.05 level. Two short-run cross-price elasticities are significant at the 0.05 level and two at the 0.10 level. Land is a substitute for hired labor and a complement to capital and materials. Hired labor is a substitute for

[9] We generate 1,000 bootstrap samples from the original sample with replacement to derive replicated estimates of long-run elasticities, from which we obtain two-sided equal-tailed confidence intervals at the 0.05 and 0.10 levels.

materials. In the long run, the own-price elasticity of hired labor and its cross-price elasticity with materials are significant at the 0.10 level.

All estimated own-price demand elasticities are negative except the very small and insignificant short-run demand for land. All long-run values are at least as large in absolute values as the corresponding short-run elasticities, which is consistent with the Le Chatelier theorem. Except for long-run demand elasticities of capital with respect to the prices of capital and materials (both of which are insignificant), all demands are inelastic. Consistent with its estimated rate of adjustment toward equilibrium being the slowest among all the quasi-fixed inputs, the inelasticity is particular pronounced for land in both the short run and the long run.

Conclusions

When production uncertainty is present, producers' investment decisions are outcomes from the tradeoff between costs related to input adjustments and the benefits from investment in inputs to manage risk. Examination of asset fixity under uncertainty is especially important for the agricultural production sector because production uncertainty due to weather fluctuations is an essential feature. This study employs a general dynamic dual model that allows asset fixity to be tested. A state-contingent approach is used to represent production uncertainty within a dynamic framework. We specify a Box-Cox functional form which nests the translog, generalized Leontief, and normalized quadratic that are commonly used to estimate dynamic adjustment costs. We apply the model to U.S. agricultural production over the period 1948-2011.

Hypotheses of instantaneous adjustment, independent adjustment, and fixity of all inputs are strongly rejected by all functional forms, indicating quasi-fixity of the overall production system. The test results against treating individual inputs as variable inputs are also robust to functional form, which documents the existence of adjustment costs for each input. Results indicate that the delineation of inputs between quasi-fixed and strictly fixed categories is sensitive to the choice of functional form for two of the inputs – capital and family labor.

Based on the tests for nested functional forms, the generalized Leontief is chosen over the translog and normalized quadratic. It is not rejected against the alternative of the Box-Cox functional form, and both the translog and normalized quadratic are rejected. Tests results with the generalized Leontief support strict fixity for family labor and quasi-fixity for land, capital, and hired labor. The estimated adjustment rates for the three quasi-fixed inputs are 15 percent, 27 percent, and 47 percent, respectively. Their adjustment lags to adjust all the way to equilibrium levels are 7, 4, and less than 3 years, respectively.

We find that production uncertainty has very limited impact on quasi-fixed input investment decisions. It appears that adjustment costs may exceed the benefits of investment that protects against uncertainty. The insignificant elasticity between state-contingent outputs support the hypothesis of an output-cubical technology. Therefore, we find no evidence that *ex post* analysis of stochastic technology conditional on realized states of nature in U.S. agricultural sector is inappropriate.

The methods used in this paper are subject to important limitations. The limitations include our restricting the number of state-contingent outputs to two. Although the approach we used is tractable and theoretically consistent for any number of state-contingent outputs, a collinearity problem arises in empirical analysis when a third state is added. A finer partition of state space would render a more flexible representation of the underlying technology. It would be useful for future research to explore advanced econometric methods that can handle high collinearity more gracefully (e.g., the maximum entropy or cross entropy approach), but also are easily applicable to highly nonlinear models.

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Table 1. Parameter Estimates of the ARCH Model

Variable	Estimates	Standard Error
Intercept	-3.4167**	0.3865
Land	0.0560**	0.0162
Capital	-0.0048	0.0072
Hired Labor	2.5194**	0.7776
Family Labor	-2.2585**	0.7573
Materials	-0.1939**	0.0838
Public Research	0.3356**	0.0698
Private Research	0.1050**	0.0441
Extension	-0.0049	0.0775
ARCH0	0.0004**	0.0002
ARCH2	0.4264	0.3977

Level of significance: ** p<0.05, * p<0.1

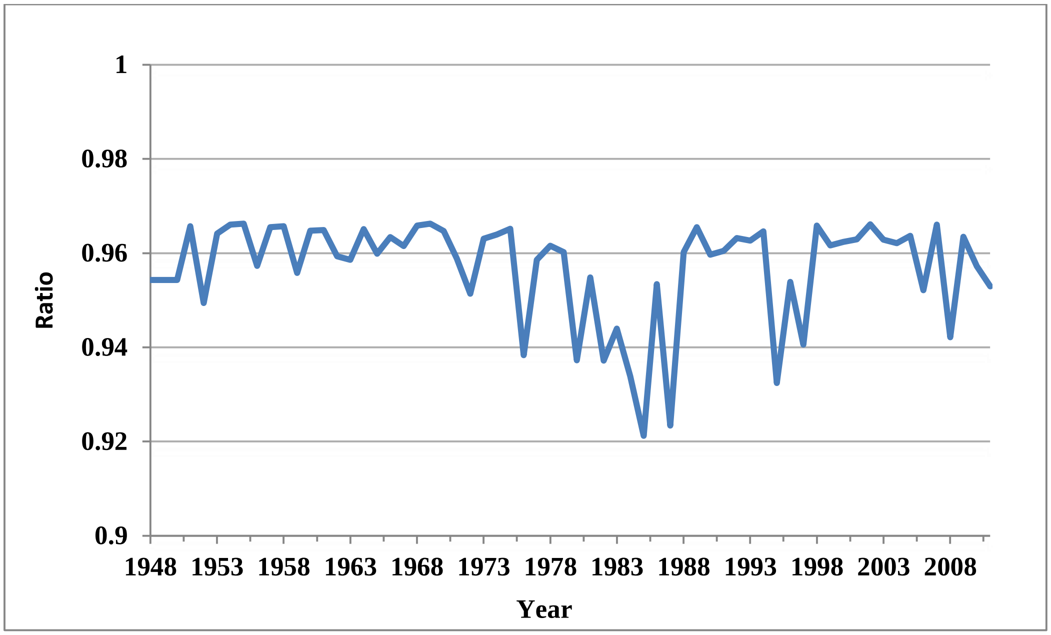

Figure 1: Output ratio under bad and good states of nature, 1948-2011

Table 2. Parameter Restrictions for of Asset Fixity Hypothesis Tests

Tested Hypotheses	Parameter Restrictions
All Inputs Variable	$C_{ii} = 1 + r, \forall\, i = 1, \ldots, 4; C_{ij} = 0, \forall\, i \neq j$
All Inputs Fixed	$C_{ii} = r\,, \forall\, i = 1, \ldots, 4; C_{ij} = 0, \forall\, i \neq j$
Independent Adjustment	$C_{ij} = 0, \forall\, i \neq j$
Land Variable	$C_{11} = 1 + r; C_{1j} = 0, \forall\, j \neq 1$
Capital Variable	$C_{22} = 1 + r; C_{2j} = 0, \forall\, j \neq 2$
Hired Labor Variable	$C_{33} = 1 + r; C_{3j} = 0, \forall\, j \neq 3$
Family Labor Variable	$C_{44} = 1 + r; C_{4j} = 0, \forall\, j \neq 4$
Land Fixed	$C_{11} = r; C_{1j} = 0, \forall\, j \neq 1$
Capital Fixed	$C_{22} = r; C_{2j} = 0, \forall\, j \neq 2$
Hired Labor Fixed	$C_{33} = r; C_{3j} = 0, \forall\, j \neq 3$
Family Labor Fixed	$C_{44} = r; C_{4j} = 0, \forall\, j \neq 4$

Table 3. Asset Fixity Test Results

Tested Hypotheses	Translog	Generalized Leontief	Quadratic	Box-Cox	Critical Value	Test Degrees of Freedom
All Inputs Variable	3,346.40**	1,132.80**	1,159.80**	152,640.00**	26.30	16
All Inputs Fixed	153.01**	180.78**	173.18**	350.69**	26.30	16
Independent Adjustment	83.36**	78.72**	78.95**	43.42**	21.03	12
Land Variable	2,634.60**	451.75**	472.11**	12,945.00**	9.49	4
Capital Variable	70.73**	74.69**	80.96**	437.42**	9.49	4
Hired Labor Variable	37.01**	32.52**	36.84**	28.18**	9.49	4
Family Labor Variable	177.06**	165.55**	120.29**	34,203.00**	9.49	4
Land Fixed	42.36**	52.79**	54.30**	47.59**	9.49	4
Capital Fixed	12.13**	10.27**	6.73	4.75	9.49	4
Hired Labor Fixed	36.94**	28.75**	24.55**	24.82**	9.49	4
Family Labor Fixed	2.27	3.27	7.47	45.99**	9.49	4

Level of significance: ** p<0.05, * p<0.1

Table 4. Functional Form Test Results

Functional Form	Transformation	Wald Statistic
Translog	$\lambda = 0$	165.16**
Generalized Leontief	$\lambda = 0.5$	1.91
Normalized Quadratic	$\lambda = 1$	243.93**
Estimated by the Generalized Box-Cox	$\lambda = 0.45$	

Level of significance: ** p<0.05, * p<0.1

Table 5. Nonlinear ITSUR Estimates of the Generalized Leontief Cost Function (with Family Labor Fixed)

Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
A_1	78.4621**	31.4840	$\widehat{D}_{31}$	1.8064	7.0959
A_2	-318.1240	358.1000	$\widehat{D}_{32}$	-0.5860	7.0486
A_3	93.9190	89.4738	$\hat{E}_{11}$	0.7544	4.2022
B_{11}	39.0444**	15.6815	$\hat{E}_{12}$	-2.0298	8.3489
B_{12}	-0.5408*	0.3217	$\hat{E}_{13}$	1.1995	4.1470
B_{13}	-1.2633**	0.4946	$\hat{E}_{21}$	76.4199	91.6687
B_{22}	-188.7770	178.7000	$\hat{E}_{22}$	-150.8380	182.1000
B_{23}	-0.5147	0.4784	$\hat{E}_{23}$	74.0653	90.5058
B_{33}	38.9379	44.5619	$\hat{E}_{31}$	-2.6565	62.2901
C_{11}	0.1904**	0.0540	$\hat{E}_{32}$	5.9900	123.9000
C_{12}	-0.0002	0.0043	$\hat{E}_{33}$	-2.9253	61.5558
C_{13}	-0.0455**	0.0074	$\hat{G}_{11}$	-0.0169	0.0186
C_{21}	-1.4333	1.0987	$\hat{G}_{12}$	0.0137	0.0102
C_{22}	0.3105**	0.1052	$\hat{G}_{13}$	-0.0364*	0.0200
C_{23}	-0.0051	0.1593	$\hat{G}_{21}$	0.3568	0.4406
C_{31}	1.6307**	0.7350	$\hat{G}_{22}$	0.0417	0.2049
C_{32}	-0.1323**	0.0654	$\hat{G}_{23}$	0.5024	0.4422
C_{33}	0.5145**	0.1088	$\hat{G}_{31}$	-0.8319**	0.2857
$\widehat{D}_{11}$	-0.0735	0.4793	$\hat{G}_{32}$	-0.0475	0.1518
$\widehat{D}_{12}$	0.0218	0.4757	$\hat{G}_{33}$	-0.0388	0.2958
$\widehat{D}_{21}$	6.9621	10.3799	$\hat{L}_1$	0.0033	0.0049
$\widehat{D}_{22}$	-7.4110	10.3172	$\hat{L}_2$	0.1086	0.1116
			$\hat{L}_3$	0.1561**	0.0732

Level of significance: ** $p<0.05$, * $p<0.1$. Parameters codes refer to the parameter vectors and matrices in equation (9). For example, C_{ij} is the ijth entry of matrix C, i,j=1,2,3, 1 is land, 2 is capital, and 3 is hired labor; i has the same meaning for all matrices; in matrix $\widehat{D}$, j=1, 2, 1 is unfavorable state and 2 is favorable state; in $\hat{E}$, j=1,2,3, 1 is unfavorable state, 2 is interaction of the two states of nature, and 3 is favorable state; in $\hat{G}$, j=1, 2, 3, 1 is public research stock, 2 is private research stock, and 3 is public extension stock.

Table 6. Estimated Demand Elasticities, Short- and Long-Run

	Elasticity with Respect to the Price of			
Quantity	Land	Capital	Hired Labor	Material
Short-Run:				
Land	0.0016	-0.0047*	0.0111**	-0.0081*
	(0.0019)	(0.0027)	(0.0043)	(0.0048)
Capital	0.0095	-0.2545**	0.0910	0.1543
	(0.0342)	(0.0947)	(0.1148)	(0.1562)
Hired Labor	-0.0299	-0.0022	-0.1985**	0.2306**
	(0.0197)	(0.0284)	(0.0472)	(0.0612)
Long-Run:				
Land	-0.0033	-0.0713	-0.0154	0.0900
Capital	0.0168	-1.3211	0.2491	1.0559
Hired Labor	-0.0471	-0.1279	-0.2959*	0.4712*

Level of significance: ** p<0.05, * p<0.1. Standard errors are in parentheses. Elasticities are calculated at the means of the variables. The tests of statistical significance for long-run elasticities are conducted using confidence intervals from bootstrap percentiles.