Rating Exotic Price Coverage in Crop Revenue Insurance

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Crop revenue coverage has continued to expand since its introduction in the 1990s and now accounts for roughly 85 percent of the $110 billion total insured value in the federal crop insurance program. The vast majority of this type of insurance is sold with a harvest–price replacement feature that pays indemnities on lost yields at the higher of the projected or the realized harvest price. Because of the public private nature of the heavily subsidized program, private companies that market and service the insurance policies cannot compete on insurance offerings covered under the federal program. Terms of coverage and premium rates are identical across these companies or approved insurance providers.

One dimension of the federal program that offers potential for more flexible private products is the price dimension: the manner in which prices that determine insurance guarantees are set. In the federal program, price discovery for most major crops in important growing areas is determined using a planting–time futures price (typically the February average) of a harvest–time futures contract (typically the October average of a November or December contract). However, a one size fits all approach to establishing price guarantees may not align with the needs of individual producers. A form of insurance that provides flexibility around this point involves establishing coverage on the basis of a maximum of prices observed over a fixed interval. For example, one might envision coverage that establishes a projected price guarantee using the highest observed value of a futures contract between January and May. Coverage could also be based on other functions of prices like geometric or arithmetic averages.

There are a number of conceptual approaches to measuring the risks associated with averages and order statistics. Options on extrema are often termed exotic options. The pricing of such exotics is an important area of financial research that requires the analyst to grapple with a number of dependencies. Zhang [8] provides a clear overview of the pricing of exotic options. It is possible to approach this problem in the crop insurance context by considering individual months. This leads to a multivariate distribution with important dependencies among the individual monthly average prices. Alternatively, one may approach the problem in terms of the joint distribution of the maximum over an interval and the harvest time price.

In this paper, we model the joint distributions using copula functions that capture tail dependence. Though we have made some assumptions on the form of the copula for convenience, it would be relatively easy to draw on a wide variety of copulas. Higher ordered but less flexible multivariate copulas that incorporate dependencies among a range of individual quotes spread over time are considered. Initial results indicate that dependence structures for policies with exotic price coverage are complex. However, it is possible to price these policies in a simple way using available financial and statistical tools. These policies provide an appealing alternative to standard revenue insurance offered under the federal crop insurance program.

1 Pricing Revenue Insurance

As the federal crop insurance program has grown in size, farmers have migrated toward the purchase of revenue insurance policies. Traditional insurance against crop yields does not necessarily protect the farmer from low prices. Though prices and yields typically have an inverse relationship, often termed the "natural hedge", it is possible for simultaneous declines to occur. Revenue insurance allows the farmer to protect himself from falling yields and falling prices. The most recent Farm Bill expanded federal crop insurance by adding a "Supplemental Coverage Option" on top of existing revenue insurance policies and calling for the development of additional insurance programs.

One of the most widespread revenue insurance products is revenue protection (RP) insurance. Coverage is available for both enterprise and whole farm units. The premium on these policies is calculated using the planting–time futures price but includes an adjustment to account for the possibility of a higher harvest–time price. For crop insurance to be actuarially sound, policies must be priced accurately. Ideally, pricing would occur at the actuarially fair rate where the premium on the policy is equal to expected loss. The true expected loss is rarely known and must be estimated. This estimation process depends on probability distributions of, in the case of revenue insurance, both yields and prices.

We abstract from consideration of the relationship between yields and prices to focus explicitly on dependence in prices alone. As noted, this is one dimension where private insurance companies can offer policies that differ from those stipulated under Federal crop insurance. There are several reasons why such policies may be more attractive from the farmer's perspective. From a behavioral standpoint, the purchase of insurance policies based on maxima are "no regret." The farmer is paid at the best possible price over the interval. In the case of coverage that depends on an average of prices, the distributions of such averages may have favorable properties. The average of many independent and identically distributed random variables will have a smaller variance that the individual random variables themselves. Policies built around these averages could offer a cheaper way for farmer's to insure against price risk. There may also be complex interactions between these types of policies and the type of expectations specifications described by Just and Rausser [5].

When estimating probability distributions in the context of price risk, it is possible to take either financial or purely statistical approaches. Financial pricing (market based) typically relies on financial theory for the distribution of prices. The Risk Management Agency (RMA) has employed this technique in their ratemaking. For example, volatility may be estimated from observed futures prices and then used in simulations under the assumption that prices are distributed lognormally. The assumption of lognormality follows the use of the Black–Scholes model. In contrast to financial pricing, we make use of what might be termed a purely statistical approach. No underlying distribution of prices is assumed. Distributions are chosen based on fit criteria and the statistical properties of prices. While we primarily consider statistical approaches in this paper – aside from our copula simulations – we are engaged in a broader effort to rate exotic price coverage under both paradigms.

2 Statistical Approaches

To measure price risk under the types of exotic price coverage we propose, it is necessary to be able to accurately estimate joint distributions of several statistics that are functions of prices. Actuarial soundness of the insurance program depends on this assessment. Because our present application uses a statistical approach, we do not rely on economic theory that would imply distributional assumptions for prices. The accuracy of the statistical approach depends on its flexibility. We would like to be able to capture idiosyncrasies in the marginal distributions and dependence structures of these functions of prices.

In our first pricing exercise, we estimate eight different models using a variety of univariate distributions. The goodness of fit of these models is captured by statistics that are functions of the likelihood or the empirical cumulative distribution function. Given a vector of data x of size n, and a distribution function with k parameters, the formulas in Table 1 are used to calculate the fit statistics. The first three statistics are based on the likelihood function L, while the latter three are based on comparison of the empirical distribution function (EDF) and the cumulative distribution function (CDF).

Fit Criteria		
Statistic	*Acronym*	*Formula*
Akaike Information Criterion	AIC	$-2\log(L)+2p$
Corrected Akaike Information Criterion	AICC	$-2\log(L)+\frac{2np}{n-p-1}$
Scwarz Bayesian Information Criterion	BIC	$-2\log(L)+p\log(n)$
Kolmogorov–Smirnov	KS	$sup_x\lvert EDF-CDF\rvert$
Anderson–Darling	AD	$n\int_{-\infty}^{\infty}\frac{(EDF-CDF)^2}{CDF(1-CDF)}dCDF$
Cramér–von Mises	CvM	$n\int_{-\infty}^{\infty}(EDF-CDF)^2 dCDF$

Table 1: Fit Statistics and Formulas

The first eight models do not capture joint dependence between monthly average prices. We also estimate four models based on copulas, allowing us to account for this structure. Copulas have recently seen increased application in crop insurance. Goodwin and Hungerford [3] applied copula models to investigate dependence between yields and prices. A copula is a function that joins two or more marginal distributions to form a single joint distribution. Comprehensive treatments of the theory of copulas can be found in Cherubini, Luciano, and Vecchiato [2] and Joe [4] while one of the earliest works on copulas was by Sklar [7]. In what follows, we make use of the Student's t or t copula, which is a member of the elliptical copula family. This copula is a generalization of a multivariate t distribution and is able to capture dependence in extreme values of prices. The t copula is

$$C(u)=\int_{-\infty}^{F_v^{-1}(u_1)}\cdots\int_{-\infty}^{F_v^{-1}(u_n}\frac{\Gamma(\frac{v+n}{2})}{\Gamma\frac{v}{2}\sqrt{(\pi v)^n|P|}}\left(1+\frac{x'P^{-1}x}{v}\right)^{-\frac{v+n}{2}}dx \tag{1}$$

where v is the degrees of freedom, P is the correlation matrix, d is the number of dimensions of the copula, $\Gamma(\cdot)$ is the gamma function, x is a vector of data, and F_v^{-1} are the marginal quantile functions.

It is well–known that the Gaussian copula is tail independent. The t copula is tail dependent and is generally more flexible than the Gaussian in terms of the dependence structures it can capture. However, it does impose symmetry in the tails of the distribution. A brief overview of the features of the t copula is given by McNeil and

Demarta [6]. While this paper concentrates on a single copula, there are other copulas that could be applied to this problem and we plan to investigate this issue.

3 Maximum Over an Interval

The data consists of monthly averages for corn and soybean futures contracts from 1960 to 2014 from the Chicago Board of Trade. Two types of price instruments are considered. Under the first, the insurer pays the higher of the February or October average futures price. Under the second price instrument, the insurer pays the higher of the average contract price in October or the maximum of the average prices in January, February, March, or April. We will refer to the first instrument as the Feb–Oct instrument and the second as the Maximum Over Interval (MOI) instrument. Prices are normalized about one so that the price charged for coverage is

$$\text{Coverage Price} = (PF - 1) \times \text{Commodity Price} \tag{2}$$

The normalizations used to construct the price factors (PF) are

$$PF = 1 + \log \frac{P_F}{P_O} \tag{3}$$

$$PF = 1 + \log \frac{\max(P_J, P_F, P_M, P_A)}{P_O} \tag{4}$$

$$\tag{5}$$

where PF is the price factor for a given year and $P_{(\cdot)}$ is the monthly average price in January, February, March, April, or October. The construction in equation 3 is used for the Feb–Oct instrument and equation 4 for the MOI instrument.

Because we do not rely on an assumed distribution of prices, we first fit various probability distributions to these price factors. There are two types of price instruments and two commodities, giving four models. We also varied the observable history of prices. Each of the four previously mentioned models was applied to both the full history of prices and a truncated history of prices. In the latter case, the history of price factors is left–truncated at one. This accounts for the possibility that the price factors may not be observable to certain parties when the price factor is less than one. After varying the data history, there are a total of eight models estimated in this initial exercise.

Burr, inverse Gaussian, lognormal, gamma, and Weibull distributions were fit to each set of price factors. These distributions were chosen because they can capture various types of tail behavior. Estimates of the probability density functions for each model are given in Figure 1. A comparison of the estimated cumulative distribution functions with the empirical CDF is shown in Figure 2. The effect of truncation depends largely on the choice of underlying distribution. Lognormal and inverse Gaussian distributions for truncated data tend to have over–accentuated modes. The Weibull distribution is fairly consistent across truncated and full data situations.

The best distribution for each model was selected according to fit statistics including the Akaike information criterion (AIC), corrected Akaike information criterion (AICC), Schwarz Bayesian information criterion (BIC), Kolmogorov-Smirnov statistic (KS), Anderson-Darling statistic (AD), and Cramér-von Mises statistic (CvM). Fit statistics for each model with a Feb–Oct price instrument are shown in Table 3. Table 4 contains fit statistics for the models with MOI price instruments.

In most cases the Weibull distribution has best fit. The primary exception is the estimate using the full history for the MOI contract for soybeans. In this case the gamma distribution fits best. For some models the Burr distribution is selected for several criteria. In the cases where it is not clear which distribution is best, we default to the Weibull distribution. Table 2 gives the parameter estimates for the best distribution for each model.

Parameter Estimates					
Price Factor	*Parameter*	*Estimate*	*Standard Error*	*t Value*	*Approx Pr > |t|*
Corn Feb-Oct Truncated	*Theta*	1.18384	0.02823	41.93	<.0001
	Tau	11.04376	2.63746	4.19	0.0002
Corn Feb-Oct	*Theta*	1.11067	0.02386	46.54	<.0001
	Tau	6.70928	0.74893	8.96	<.0001
Soybeans Feb-Oct Truncated	*Theta*	1.04613	0.11235	9.31	<.0001
	Tau	6.04127	2.59018	2.33	0.0280

Parameter Estimates					
Price Factor	Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|
Soybeans Feb-Oct	Theta	1.06000	0.02334	45.42	<.0001
	Tau	6.59190	0.69484	9.49	<.0001
Corn MOI Truncated	Theta	1.16484	0.05139	22.66	<.0001
	Tau	7.32752	1.91734	3.82	0.0005
Corn MOI	Theta	1.14486	0.02482	46.12	<.0001
	Tau	6.66723	0.72312	9.22	<.0001
Soybeans MOI Truncated	Theta	1.04261	0.11825	8.82	<.0001
	Tau	5.49807	2.24218	2.45	0.0202
Soybeans MOI	Theta	0.02827	0.00550	5.14	<.0001
	Alpha	36.20392	7.00072	5.17	<.0001

Table 2: Parameter Estimates for Selected Distributions

In addition to the eight models given above, we also generate four models based on copulas. We consider both price instruments and both crops. The exercise for pricing the copula based models is slightly different from previous methods. Instead of fitting distributions to observed price factors, we model the joint distribution of the average prices from each month. Price factors are generated through simulation following estimation of the copulas. This modeling approach is able to capture additional information and incorporate it into the estimation process. Accounting for dependence among monthly average prices suggests increased accuracy in ratemaking.

We fit a t copula to both corn and soybean average monthly futures prices for January, February, March, April and October. The empirical CDF was used to transform the data prior to estimation. Parameter estimates and correlation matrices for each crop are shown in Table5. Correlation between October and the other months is generally stronger for corn than soybeans. Correlation among the first four months of the year is fairly similar across crops. Scatter diagrams are shown in Figures 3 and 4. Note that in these diagrams $p0$ denotes the October price, $p9$ the January price, $p8$ the February price, and so on.

The degrees of freedom parameter v that is estimated for each copula is also a general measure of dependence structures. As the degrees of freedom increase, the t copula converges to the Gaussian copula. Less degrees of freedom implies an increase in the probability that an extreme event will occur. The degrees of freedom estimates of 4.4389 for corn and 1.3852 for soybeans suggest that the probability of a tail event is greater for soybeans.

For the first eight models, pricing is accomplished via simulation with 10,000 draws taken from the quantile function of the best fitting distribution. Table 6 gives the mean of all price factors and the mean of price factors greater than one. Standard deviations for each group are also shown. For the copula models, 10,000 draws are taken from the copulas and then raw prices are constructed assuming that prices follow a lognormal distribution with a mean of 400 and variance of 0.2. To make the price factors from the copula models comparable with those from the initial eight models, add one to each factor.

Initial results show that the way coverage is constructed, and the assumptions embedded in the parameters, can have a significant effect on the pricing of insurance contracts. In many cases, the mean price factors for the copula based models and the initial eight models are considerably different. This difference is economically significant when viewed with respect to the amount of money in crop insurance programs. These types of insurance contracts may also be appealing to private insurers. However, the viability of exotic price coverage in crop insurance will ultimately depend on proper assessment of risk.

Table 3: Fit Statistics for Feb-Oct Price Factors

All Fit Statistics: Corn Feb-Oct Truncated														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Igauss	−63.77660		−59.77660		−59.36281		−56.84513		0.71222		0.59913		0.08064	
Logn	−64.09163		−60.09163		−59.67783		−57.16015		0.61406	*	0.58536		0.08554	
Burr	−66.97035		−60.97035		−60.11321		−56.57315		0.68459		0.49699	*	0.07296	*
Weibull	−66.98313	*	−62.98313	*	−62.56933	*	−60.05166	*	0.68191		0.49883		0.07336	
Note: The asterisk (*) marks the best model according to each column's criterion.														

All Fit Statistics: Corn Feb-Oct														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Gamma	−19.46301		−15.46301		−15.23224		−11.44834		0.71935		0.89199		0.12097	
Igauss	−13.65028		−9.65028		−9.41951		−5.63562		0.95094		1.03133		0.10255	
Logn	−14.49095		−10.49095		−10.26018		−6.47628		0.84037		1.16830		0.16659	
Burr	−30.89409		−24.89409		−24.42350		−18.87209		0.58022	*	0.38009	*	0.04451	*
Weibull	−30.91775	*	−26.91775	*	−26.68698	*	−22.90308	*	0.58368		0.38221		0.04490	
Note: The asterisk (*) marks the best model according to each column's criterion.														

All Fit Statistics: Soybeans Feb-Oct Truncated														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Igauss	−59.04031		−55.04031		−54.54031		−52.44864		0.56073		0.39393		0.05593	
Logn	−60.14000		−56.14000		−55.64000		−53.54833		0.36450	*	0.25433		0.02702	
Burr	−60.49474		−54.49474		−53.45126		−50.60723		0.38939		0.23810		0.02427	
Weibull	−60.50001	*	−56.50001	*	−56.00001	*	−53.90833	*	0.38844		0.23785	*	0.02418	*
Note: The asterisk (*) marks the best model according to each column's criterion.														

All Fit Statistics: Soybeans Feb-Oct														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Gamma	−38.74367		−34.74367		−34.51290		−30.72901		0.57951		0.25061		0.04044	
Igauss	−37.47523		−33.47523		−33.24446		−29.46056		0.48120		0.35837		0.03523	
Logn	−37.46822		−33.46822		−33.23745		−29.45355		0.66105		0.34970		0.05710	
Burr	−39.44273	*	−33.44273		−32.97215		−27.42074		0.41179	*	0.14959	*	0.01888	*
Weibull	−39.17579		−35.17579	*	−34.94502	*	−31.16112	*	0.41193		0.17629		0.02218	
Note: The asterisk (*) marks the best model according to each column's criterion.														

Table 4: Fit Statistics for MOI Price Factors

All Fit Statistics: Corn MOI Truncated														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Gamma	−62.39521		−58.39521		−58.03157		−55.22817		0.41291		0.25081		0.03062	
Igauss	−61.42247		−57.42247		−57.05883		−54.25543		0.50956		0.33340		0.03317	
Logn	−62.15130		−58.15130		−57.78767		−54.98427		0.42140		0.26935		0.03354	
Burr	−63.35048		−57.35048		−56.60048		−52.59992		0.36743		0.18029		0.01877	*
Weibull	−63.35784	*	−59.35784	*	−58.99420	*	−56.19080	*	0.36499	*	0.18018	*	0.01889	
Note: The asterisk (*) marks the best model according to each column's criterion.														

All Fit Statistics: Corn MOI														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Gamma	−22.49188		−18.49188		−18.26111		−14.47721		0.59057		0.64551		0.09568	
Igauss	−19.17333		−15.17333		−14.94256		−11.15866		0.70008		0.75660		0.07195	
Logn	−19.42225		−15.42225		−15.19148		−11.40758		0.68810		0.86940		0.13244	
Burr	−29.15893		−23.15893		−22.68834		−17.13693		0.36999	*	0.13479		0.01690	*
Weibull	−29.17045	*	−25.17045	*	−24.93968	*	−21.15578	*	0.37357		0.13475	*	0.01698	
Note: The asterisk (*) marks the best model according to each column's criterion.														

All Fit Statistics: Soybeans MOI Truncated														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Gamma	−66.11206		−62.11206		−61.69827		−59.18059		0.38663		0.23805		0.02536	
Igauss	−64.43540		−60.43540		−60.02160		−57.50392		0.52509		0.44271		0.05564	
Logn	−65.97741		−61.97741		−61.56362		−59.04594		0.38896		0.24504		0.02626	
Burr	−66.59649		−60.59649		−59.73935		−56.19929		0.38169		0.22463		0.02346	
Weibull	−66.60454	*	−62.60454	*	−62.19075	*	−59.67307	*	0.38142	*	0.22458	*	0.02344	*
Note: The asterisk (*) marks the best model according to each column's criterion.														

All Fit Statistics: Soybeans MOI														
Distribution	*-2 Log Likelihood*		*AIC*		*AICC*		*BIC*		*KS*		*AD*		*CvM*	
Gamma	−39.78159		−35.78159	*	−35.55082	*	−31.76692	*	0.48616		0.22220		0.02983	
Igauss	−38.67394		−34.67394		−34.44317		−30.65928		0.42070		0.33070		0.02630	
Logn	−38.65718		−34.65718		−34.42641		−30.64251		0.56961		0.31852		0.04438	
Burr	−40.12202	*	−34.12202		−33.65143		−28.10002		0.38128	*	0.13548	*	0.01612	*
Weibull	−39.73975		−35.73975		−35.50898		−31.72508		0.39146		0.19206		0.02407	
Note: The asterisk (*) marks the best model according to each column's criterion.														

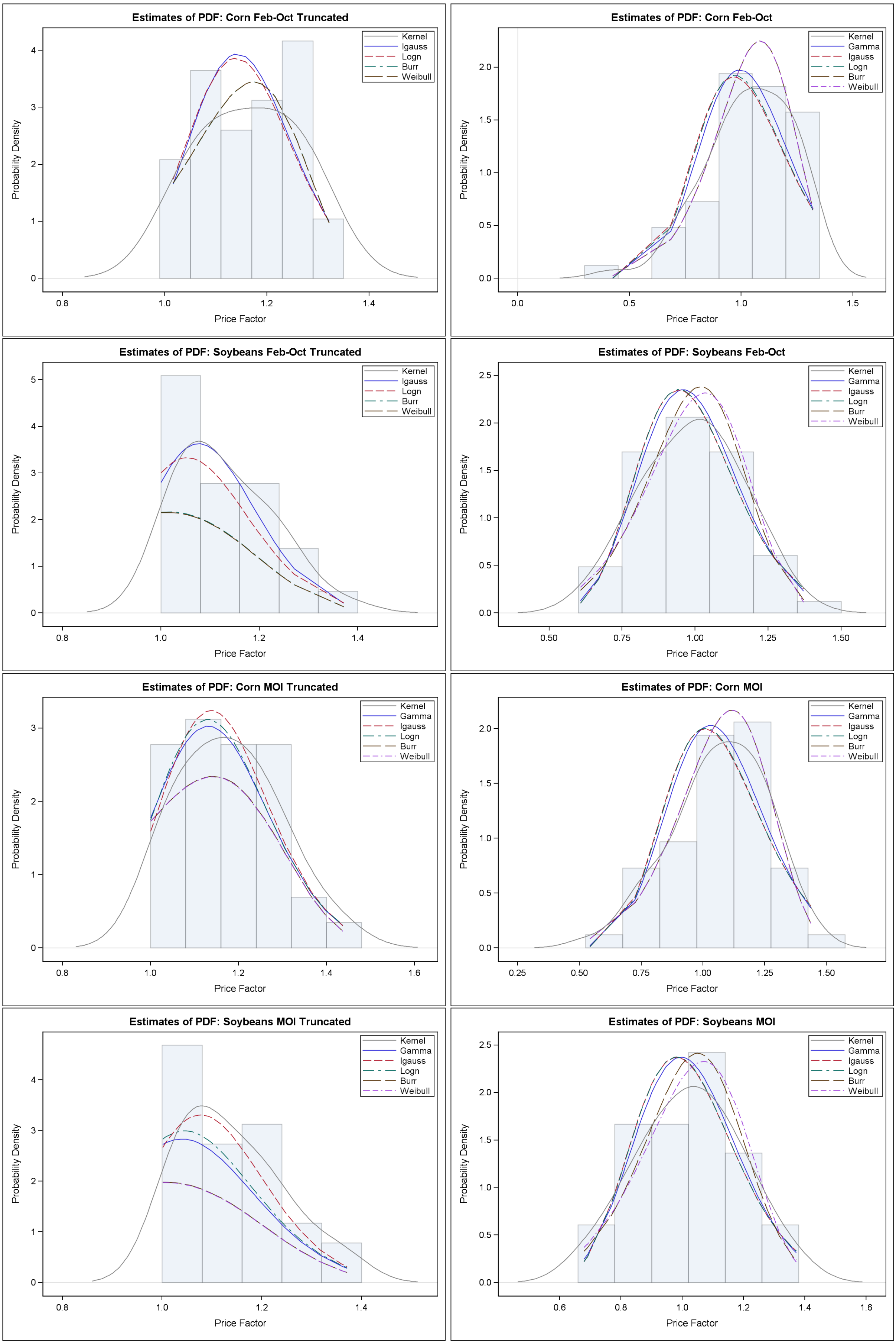

Figure 1: PDF Plots

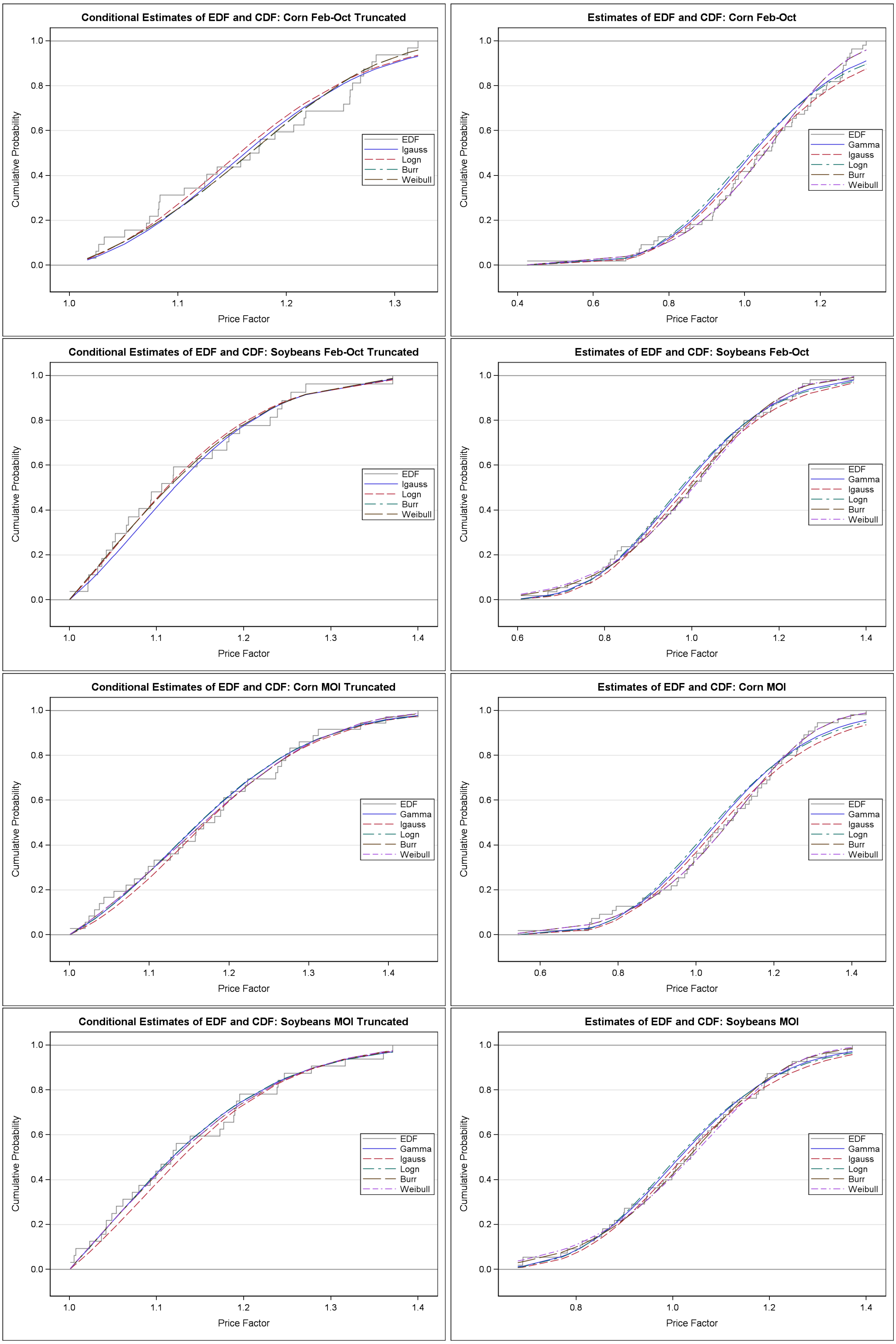

Figure 2: CDF Plots

Parameter Estimates: Corn				
Parameter	*Estimate*	*Standard Error*	*t Value*	*Approx Pr > |t|*
DF	4.438858	1.868387	2.38	0.0175

Parameter Estimates: Soybeans				
Parameter	*Estimate*	*Standard Error*	*t Value*	*Approx Pr > |t|*
DF	1.385201	0.481244	2.88	0.0040

Correlation Matrix: Corn					
	Oct	*Apr*	*Mar*	*Feb*	*Jan*
Oct	1.0000	0.8488	0.8368	0.8476	0.8479
Apr	0.8488	1.0000	0.9853	0.9806	0.9693
Mar	0.8368	0.9853	1.0000	0.9903	0.9744
Feb	0.8476	0.9806	0.9903	1.0000	0.9876
Jan	0.8479	0.9693	0.9744	0.9876	1.0000

Correlation Matrix: Soybeans					
	Oct	*Apr*	*Mar*	*Feb*	*Jan*
Oct	1.0000	0.8104	0.7943	0.7946	0.7784
Apr	0.8104	1.0000	0.9883	0.9744	0.9665
Mar	0.7943	0.9883	1.0000	0.9879	0.9840
Feb	0.7946	0.9744	0.9879	1.0000	0.9959
Jan	0.7784	0.9665	0.9840	0.9959	1.0000

Table 5: Copula Parameter Estimates and Correlation Matrices

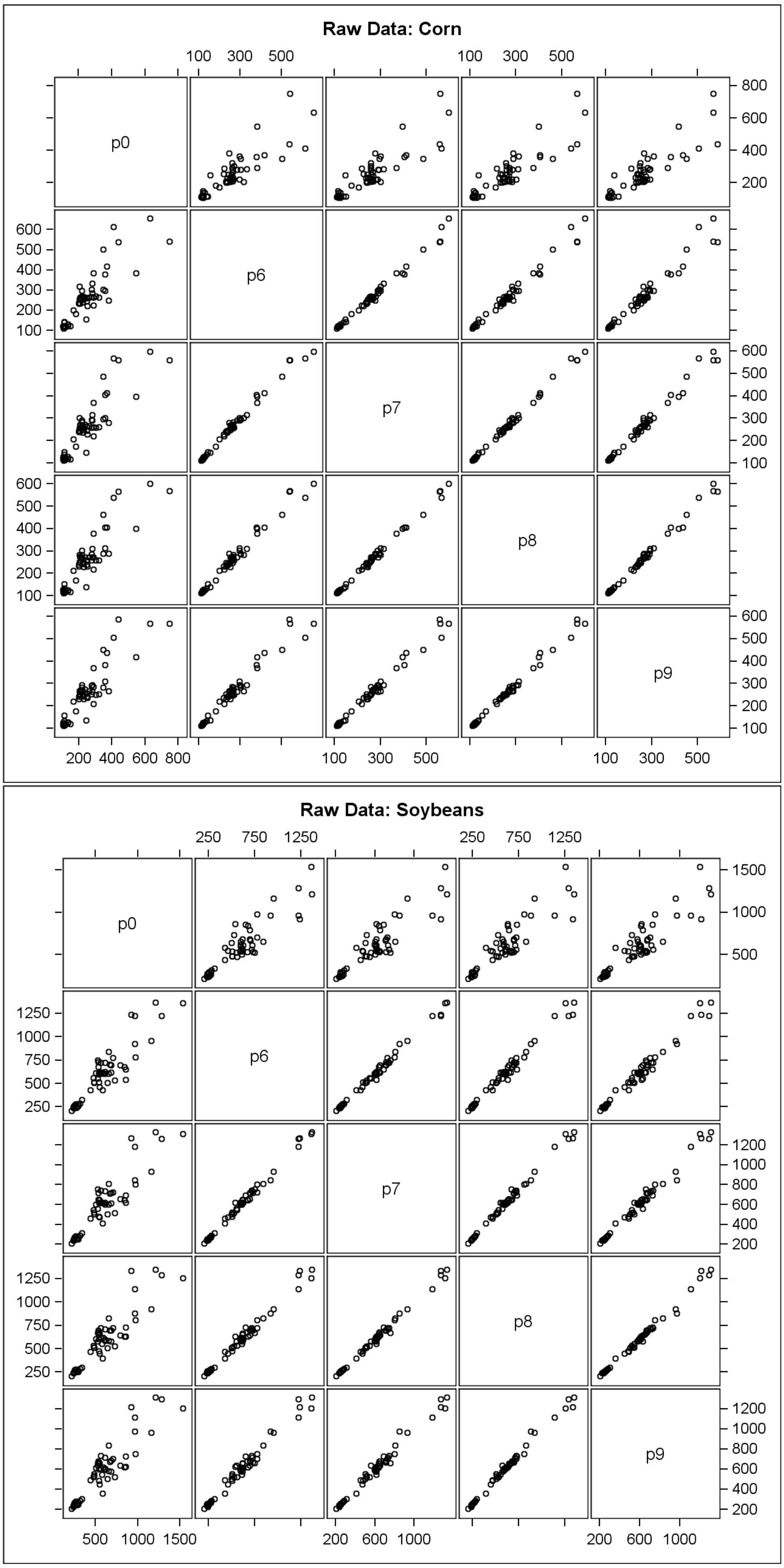

Figure 3: Copula Scatter Diagrams

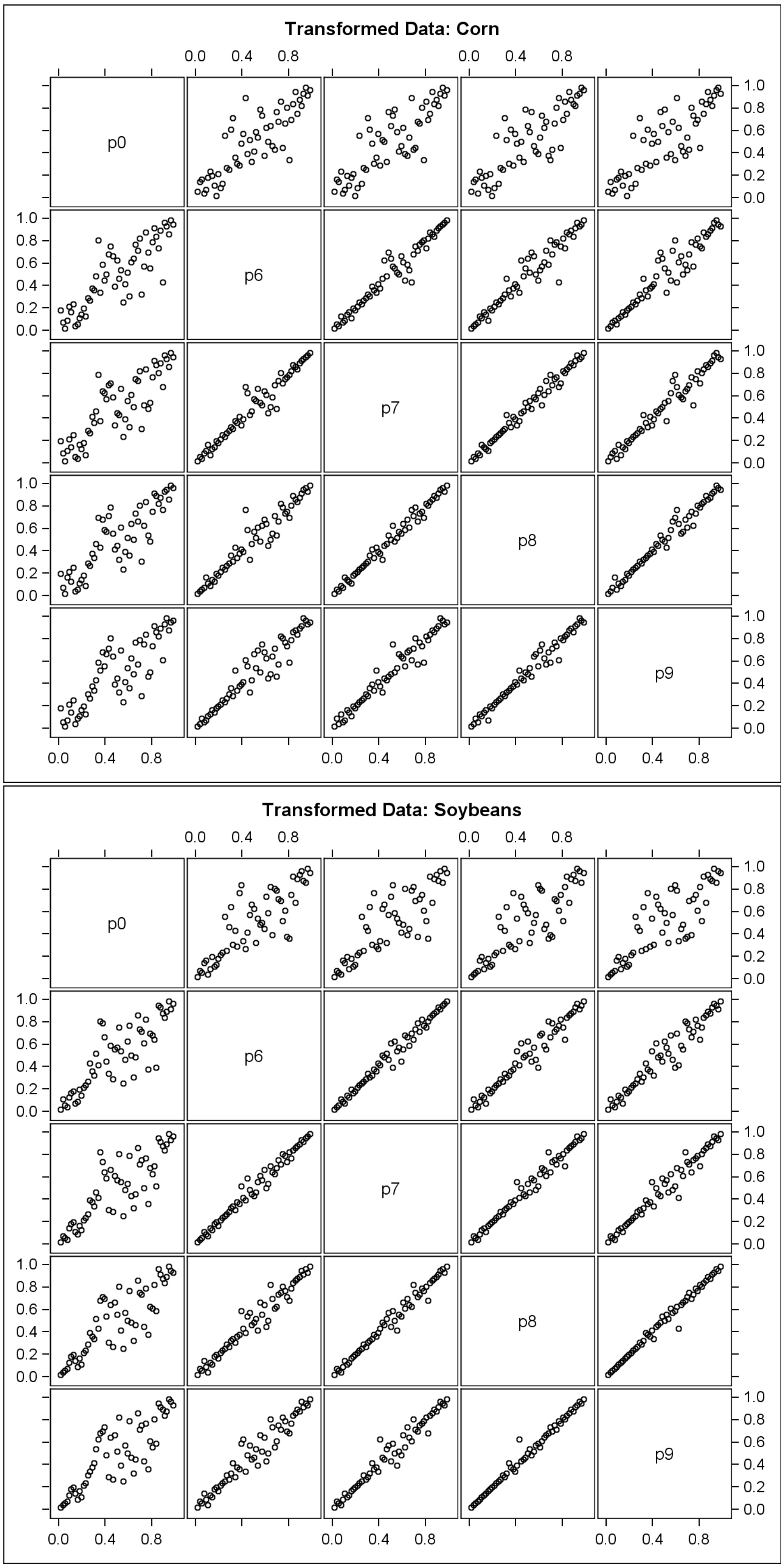

Figure 4: Copula Scatter Diagrams

Pricing Estimates from Initial Eight Models				
Model	*Mean PF*	*Mean PF ≥ 1*	*SD PF*	*SD PF ≥ 1*
Corn Feb-Oct Truncated	1.1320	1.1680	0.1241	0.0869
Corn Feb-Oct	1.0359	1.1501	0.1804	0.1009
Soybeans Feb-Oct Truncated	0.9685	1.1263	0.1887	0.0894
Soybeans Feb-Oct	0.9889	1.1275	0.1756	0.0886
Corn MOI Truncated	1.0939	1.1799	0.1776	0.1099
Corn MOI	1.0651	1.1745	0.1890	0.1120
Soybeans MOI Truncated	0.9648	1.1402	0.2021	0.0994
Soybeans MOI	1.0242	1.1493	0.1704	0.1194

Table 6

Pricing Estimates from Copula Based Models				
Model	*Mean Feb-Oct PF*	*Mean MOI PF*	*SD Feb-Oct PF*	*SD MOI PF*
Corn Copula	0.0406	0.0545	0.0671	0.0771
Soybeans Copula	0.0432	0.0543	0.0877	0.0981

Table 7

4 Conclusion

Though we have only examined price instruments based on a maximum of prices over an interval, similar application of these techniques will also allow us to price contracts based on averages of prices. The statistical approach to this problem shows that these contracts can be constructed fairly easily. In forthcoming work we will expand on these issues, consider different types of policies, and also compare statistical approaches with financial approaches based on asset pricing theory (e.g Black–Scholes).

In addition to developing these types of crop insurance policies, it would be useful to have a better understanding of the way that farmers choose insurance policies and manage risk. The expected utility model has received some criticism based on several studies showing observed behavior that does not conform to its predictions. As described by Buschena [1], choice patterns may violate transitivity and individuals views of risk may depend on reference points. In either case, challenges to the expected utility model will have implications for crop insurance programs. We would be interested in seeing how the demand for various types of policies could change under different modeling approaches and assumptions.

References

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