# A Dynamic Economic Analysis of Nitrogen Induced Soil Acidification in China

Ziyan Yang
Department of Agricultural and Resource Economics
University of Maryland, College Park
Zyang33@umd.edu

Selected Paper prepared for presentation at the 2015 Agricultural & Applied Economics Association and Western Agricultural Economics Association Annual Meeting, San Francisco, CA, July 26-28

# 1 Background, Motivation, and Literature Review

Acidification of soil is a disruptive phenomenon in Europe and North America (Alcamo et al., 1987, Kauppi et al., 1990, Tarkalson et al., 2006). The observation of soil acidification in China was quite recent (Guo et al., 2010, Ju et al., 2004, Norse and Zhang, 2010, Huang et al., 2010). In previous studies of soil acidification, acid rain was regarded as the most important contributor. Kaitala et al. (1992) established a dynamic game between Finland and the nearby areas of the Soviet Union based on Holmberg's(1990) acidification model for investigating the optimal strategies of pollution control under forest value maximization. However, the significant decline of soil pH in China was not caused mainly by acid rain; instead, soil acidification was induced by nitrogen leaching. Guo et al. claimed that in China, topsoil pH declined significantly in all six arable soil types from 0.13 to 0.80 between 1980 to 1997 by tracking National Long-term Monitoring Fields at nationwide for 20 years. Along with declined soil pH, nitrogen application increased aggressively by 191% during the past 30 years (Guo et al., 2010), and nitrogen efficiency dropped simultaneously (Norse and Zhang, 2010). Guo et al., Ju et al. and Zhao et al. also pointed out that the power of nitrogen leaching on acidifying soil is about ten times that of acid rain. Environmental cost of nitrogen leaching is a classical question in economics, and the impacts of nitrogen leaching on water quality and climate changes have been studied for years (Yadav and Easter, 1994, Yadav, 1997, S. S. Vickner and Ascough, 1998, Palma, 2002) under scientific support (Kohl et al., 1971, Bremner and Blackmer, 1978). However, the impacts of nitrogen leaching on soil acidification have not been addressed due to the fact that much scientific evidence was not published until very recent. Hence, the first contribution of this paper to the literature of soil acidification is that it analyzes soil acidification itself as an environmental cost induced by human behaviors rather than an indirect consequence caused by air pollution. Instead of constructing a transboundary acid rain game, this paper constructs a dynamic model to solve for the optimal solution of nitrogen allocation for optimal soil acidity management.

In previous studies, lime application was the only approach to reverse soil pH. Land tenure was studied to understand the incentives of farmers' investments in lime application. Myyra et al. (2004) constructed a dynamic model to exam the impact of the length of contract on investment in lime. Literature of land improvement also showed the relationship between contract and land degradation (Johnson, 1950, Lichtenberg, 2007). Li et al. specifically derived a study to understand how land tenure changed farmer's investment incentives in China. They empirically showed that farmers were more likely to invest in agriculture if land tenure was secured. A special phenomenon in China is that, lime application was eliminated after the *Reform and Opening Up* in 1978. The Household Contract Responsibility System (*jia ting lian chang cheng bao ahi*) derived from institutional changes transformed the decision right of agricultural production from village collective communities to individual households. The policies of investment in lime application, which used to be compulsory, were not enforceable any more. In China, technology of lime application has not been updated, which is still labor and capital consuming and not affordable for each small holder. On the other hand, the agricultural extension offices cannot afford the transaction costs of soil tests of small areas of land, as well. So the first consequence of the Household Contract Responsibility System is that it makes cooperation among village members less available. As non-agricultural sectors developed faster than agricultural sectors after the Opening Up policies, farmland was taken away for commercial or residential purposes. So the second consequence of the big institutional change is that land tenure becomes less secured (Li et al., 1998, Ding, 2003, Lichtenberg and Ding, 2008). Since lime application is not a feasible approach to reverse soil pH in rural China, this paper attempts to solve soil acidification problems from the perspective of nitrogen control rather than lime application, which is the second contribution to the literature of soil acidification.

The main part of the paper is devoted to the identification of the structure and the dynamics of the nitrogen-induced soil acidification problem. The discrete dynamic programming model is introduced in Section 2. The dynamics are described by considering soil pH as a stock variable. The theoretic framework aims to reconcile the conflicting objectives of maximizing profit in the short run versus declining soil pH over the long run. The model is structured as a double-crop rotation cropping system. The earliest record of a rotation cropping system in China was in the Han Dynasty (202 BC-220 AD). In ancient times, farmers had realized the benefits of cover crops, namely, improving nitrogen efficiency, maintaining soil fertility, increase yield, and more fully utilizing farmland (Morgan et al., 1942, Rittera et al., 1998, Hartwig and Ammon, 2002). The triple-crop system is only feasible in a small area in the deep South, whereas the double-crop rotation system is available in most of the commercial agricultural production areas. Thus, a double-crop rotation model is able to derive potential policy implications for a large geographical area. Moreover, a double-crop rotation is enough to capture the essential features of a multi-season oration model. The literature of nitrogen leaching and water quality showed that the effects of nitrogen carryover across seasons made big changes in optimal water management compared with a single season model (Chiao and Gillingham, 1989, S. L. Johnson and Perry, 1991, S. S. Vickner and Ascough, 1998) Thus, double-crop rotation addresses the importance of interseasonal allocation of nitrogen in the optimal soil acidity management problem.

Section 3 is devoted to introducing the empirical framework. Two rotation systems are chosen to measure the distortion and environmental cost of nitrogen misallocation quantitatively. First, this section is devoted to the estimation of crop production functions and a soil pH transition equation. Second, I solve the dynamic programming model numerically to obtain the steady state solution of nitrogen and soil pH. Last but not least, by calculating the shadow prices of nitrogen of each season in each type of rotation combined with empirical analysis of the results, policy implications are discussed to address the problem of soil acidification. My results indicate that although rotation makes regular taxation incentive system not enforceable, rotation itself plays a positive role in maintaining soil pH levels.

## 2 Theoretical Modeling and Analysis

### 2.1 The Model

I use a discrete dynamic programming model to investigate optimal interseasonal allocation of nitrogen fertilizer in soil acidity management. I restrict my attention to the effects of nitrogen on soil pH value in a double-crop rotation system. This dynamic problem is fully described by considering the soil pH value ($S$) as a stock variable and nitrogen application in season 1 ($X^1$) and season 2 ($X^2$) as control variables.

The crop production function of season $i$ at time $t$, $f^i(E_t^i, S_t)$ ($i = \{1, 2\}$), is a function of absorbed nitrogen ($E_t^i$) and annually-tested soil pH value ($S_t$). The outcome of production of each season is $y_t^i$. Although this production function excludes the possibility of lime application, it still characterizes the essential features of the nitrogen-induced-soil-acidification problem in China, because lime application is rare in rural China after 1980s due to big institutional changes. The absorbed nitrogen $E$ is the product of applied nitrogen and an absorption coefficient $\alpha_i$, which varies in crops between 0 and 1. Thus, absorbed nitrogen in season 1 is $\alpha_1 X^1$, and absorbed nitrogen in season 2 is $\alpha_2[X^2 + (1 - \alpha_1)X^1]$. The reason to apply the concept of absorbed nitrogen instead of applied nitrogen is to consider nitrogen carryover and runoff in this dynamic system based on the *nitrogen mass balance method* derived from the principle of nitrogen cycle (Huang and Uri, 1993, Gilliam and Hoyt, 1987). I assume that crop yield $y^i$ increases at a decreasing rate as absorbed nitrogen $E$ increases.

I assume that variation of soil pH within a year does not influence crop production. Crop yield of every season is a function of the same soil pH value, which was recorded at the initial period of the year. Crop yield increases in $S$ if $S < \bar{S^i}$ and decreases if $S > \bar{S^i}$. This assumption is made based on the scientific facts that each crop has its desired soil pH range which is determined by its biological instinct, denoted by $\bar{S^i}$. Soil productivity reaches its peak around $\bar{S^i}$ and then drops when it moves away (Brady and Weil, 1996, Zhao et al., 2010). I also assume that $y^i$ is concave in $S$. The soil pH value increases the marginal productivity of nitrogen (letting subscripts denote derivatives, $f_{ES}^i > 0$). I focus on the interesting case in which increasing soil acidity impairs the marginal productivity of nitrogen. I thus assume that crop production is super modular in absorbed nitrogen and soil pH value.

The soil pH value at the end of the growing season is influenced by current soil pH value $S_t$ and nitrogen residual ($R_t$), which is caused by excessive nitrogen that cannot be absorbed or removed by crops. Nitrogen, if it cannot be taken up by crops, are left as residuals, which can flow into surface water through agricultural runoff or leach into deeper soil layers and underground water (Kohl et al., 1971, Kim and Hostetler, 1991, Huang and Uri, 1993). Employing the *nitrogen mass balance method* again, the nitrogen residual after an entire growing year is thus computed as

$$R_t = (1 - \alpha_2)[X_t^2 + (1 - \alpha_1)X_t^1]. \tag{1}$$

Hence, I strictly follow the principle of *materials-balance* (Pethig, 2006). Then, the soil pH value for the next period is given by the following state equation:

$$S_{t+1} = g(R_t, S_t). \tag{2}$$

Note that this definition of nitrogen residual and previous definition of absorbed nitrogen assume that the amount of nitrogen available for the growth of plants from atmosphere and nitrogen losses due to other factors, such as denitrification, volatilization, and water runoff are negligible. These two assumptions above can be removed if data of other nitrogen influx and efflux is available. Then, nitrogen residual can be recalculated by subtracting the amount of nitrogen lost due to other factors from Eq. (1); absorbed nitrogen can be recalculated by adding natural nitrification into the calculation of absorbed nitrogen. In the theoretical model, excluding other nitrogen influx and efflux does not influence the framework of the dynamic optimal soil acidity management model, because these nitrogen flows are exogenous. However, in the empirical framework, other sources of nitrogen influx and efflux are not negligible. If data of other nitrogen influx and efflux is not available, then total nitrogen could be presented by a proportion of applied nitrogen. Thus, under this empirical circumstances, $\alpha^i$ may be greater than 1 or less than 0.

The residuals ($R$) react with oxygen under normal temperature and pressure (Hinsinger et al., 2003). This reaction creates nitrates ($NO_3^-$) and hydrogen ions ($H^+$), both of which can acidify soil (Hinsinger et al., 2003, Guo et al., 2010, Huang et al., 2010). Thus, $g(.)$ decreases in $R$. Note that although soil pH decreases in nitrogen residual, the sign of $g_{X^1}$ and $g_{X^2}$ could be different. If both absorption coefficients are greater than 1, then as an additional unit of nitrogen in season 1 decreases soil pH, an additional unit of nitrogen in season 2 increases soil pH. This is consistent with previous literature that planting cover crops is the a feasible way to reduce nitrogen residual and maintain soil pH (Morgan et al., 1942, Muthoni and Kabira, 2010). I also assume that $g(.)$ is increasing and concave in $S$. This assumption is based on the feature of the natural acidity process and the buffering capacity of soil. Soils tend to become more acid naturally as time passes (decades to centuries) if there are no inputs of alkaline materials. Hence, in a short period of time $g_S$ should be close but less than 1. Buffering capacity means that soil pH has its resistance to acidification. This buffering capacity increases as soil pH increases. (Brady and Weil, 1996). Hence, as soil pH increases, decrease in soil pH reduces in its magnitude. This buffering capacity also determines that $g_{RS} > 0$. Since, as soil becomes more alkaline, marginal reduction of soil pH due to an additional unit of nitrogen residual decreases in absolute value. At last, I assume that $g(.)$ is a general concave function in both arguments and that it satisfies the second order condition such that the Hessian matrix

$$H = \begin{vmatrix} \dfrac{d^2 g(R_t, S_t)}{dS_t^2} & \dfrac{d^2 g(R_t, S_t)}{dR_t dS_t} \\ \dfrac{d^2 g(R_t, S_t)}{dS_t dR_t} & \dfrac{d^2 g(R_t, S_t)}{dR_t^2} \end{vmatrix}$$

is negative semi-definite.

## 2.2 Optimal Soil Acidity Management

Let $p_t^i$ denote the price of output produced in season $i$ at year $t$, $w_t$ denote the price of nitrogen at year $t$, and $\delta$ denote the discount factor. Optimal decision of nitrogen allocation in season 1 and season 2 maximizes the present value of return from crop production, which is revenue less the cost of production, and the discounted land value in all future periods, subject to the state equation (Eq. (2)) with given initial soil pH value $S_0$. The optimal management values the production profit of the current period and the present land value in all future periods, when land value is measured by its pH value. Unlike myopic agents, who only maximize the production profit but ignore the land value in all future periods. Thus, the recursion equation for the stationary problem is:

$$V(S_t) = p_t^1 f^1(\alpha_1 X_t^1, S_t) + p_t^2 f^2(\alpha_2(X_t^2 + (1-\alpha_1)X_t^1), S_t) - w_t(X_t^1 + X_t^2) + \delta V(S_{t+1}), \quad (3)$$

where

$$S_{t+1} = g([(1-\alpha_2)(X_t^2 + (1-\alpha_1)X_t^1)], S_t).$$

The first order conditions for interior maximum are:

$$\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha_1)p_t^2 f_{E_t^2}^2 - w_t + \delta(1-\alpha_1)(1-\alpha_2)g_{R_t} V_{S_{t+1}} = 0 \quad (4)$$

$$\alpha_2 p_t^2 f_{E_t^2}^2 - w_t + \delta(1-\alpha_2) g_{R_t} V_{S_{t+1}} = 0. \tag{5}$$

As is standard, optimal nitrogen application $X_t^1$ and $X_t^2$ are set to equate the current marginal net value of crop production with the present value of the marginal change in stock value in the future. I differentiate the value function (Eq. (3)) using the *Envelope Theorem* to rewrite the F.O.C:

$$\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha_1) p_t^2 f_{E_t^2}^2 - w_t + \delta(1-\alpha_1)(1-\alpha_2) g_{R_t} [p_{t+1}^1 f_{S_{t+1}}^1 + p_{t+1}^2 f_{S_{t+1}}^2 + \delta g_{S_{t+1}} A_{t+1}] = 0 \tag{6}$$

$$\alpha_2 p_t^2 f_{E_t^2}^2 - w_t + \delta(1-\alpha_2) g_{R_t} [p_{t+1}^1 f_{S_{t+1}}^1 + p_{t+1}^2 f_{S_{t+1}}^2 + \delta g_{S_{t+1}} B_{t+1}] = 0, \tag{7}$$

where

$$A_t = V_{S_{t+1}} = -\frac{\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha_1) p_t^2 f_{E_t^2}^2 - w_t}{\delta(1-\alpha_1)(1-\alpha_2) g_{R_t}}$$

$$B_t = V_{S_{t+1}} = -\frac{\alpha_2 p_t^2 f_{E_t^2}^2 - w_t}{\delta(1-\alpha_2) g_{R_t}}.$$

I substitute repeatedly for $\delta V_{S_{t+j}}$ $(j \geq 1)$ and derive the following necessary conditions for interior maximum:

$$\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha_1) p_t^2 f_{E_t^2}^2 - w_t + \sum_{\tau=1}^{\infty} \delta^\tau (1-\alpha_1)(1-\alpha_2) g_{R_t} [p_{t+\tau}^1 f_{S_{t+\tau}}^1 + p_{t+\tau}^2 f_{S_{t+\tau}}^2] = 0 \tag{8}$$

$$\alpha_2 p_t^2 f_{E_t^2}^2 - w_t + \sum_{\tau=1}^{\infty} \delta^\tau (1-\alpha_2) g_{R_t} [p_{t+\tau}^1 f_{S_{t+\tau}}^1 + p_{t+\tau}^2 f_{S_{t+\tau}}^2] = 0. \tag{9}$$

I take Eq. (8) as an example to show the intuitive interpretation of the necessary conditions. In Eq. (8), $\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha_1) p_t^2 f_{E_t^2}^2 - w_t$ is the value of the marginal product of an additional unit of nitrogen applied in season 1 less the marginal cost of nitrogen $w_t$. Hence, it is the marginal net profit of applying an additional unit of nitrogen in season 1 in the current growing year. Note that the nitrogen applied in season 1 influences production in both season 1 and season 2. That additional unit of nitrogen applied in season 1 has an impact on soil pH in future periods via the state equation. In all future periods, crop production in both seasons are influenced by the same soil pH value that has been changed by that additional unit of nitrogen applied in the current period. So the value of the marginal product of soil pH is the sum of its impacts in both seasons, $p_{t+\tau}^1 f_{S_{t+\tau}}^1 + p_{t+\tau}^2 f_{S_{t+\tau}}^2$. Suppose that that additional unit of nitrogen is not applied in the current period, there would be an increment of the soil pH in future period which is equal to the absolute value of $(1-\alpha_1)(1-\alpha_2)g_{R_t}$. Thus, the present value of the increases in profitability in all future period due to that small increment of soil pH value is presented by $\sum_{\tau=1}^{\infty}\delta^{\tau}(1-\alpha_2)g_{R_t}[p^1_{t+\tau}f^1_{S_{t+\tau}}+p^2_{t+\tau}f^2_{S_{t+\tau}}]$. So as is standard, the necessary conditions shows that the marginal benefit of applying an additional unit of nitrogen in season 1 of this period is equal to the marginal benefit of not applying this additional unit of nitrogen in this period.

## 2.3 Steady State Analysis

The analysis of steady state is of interest to derive optimal regulation and allocation of nitrogen in the long run. At the steady state, optimal nitrogen application $X^1$ and $X^2$ are constant across time. Soil pH value is maintained at a constant level. Plugging $X^i_t = X^i_{t+1}$ and $S_t = S_{t+1}$ in Eqs. (6) and (7) and combining them with the state equation (Eq. (2)), I derive the following three conditions at the steady state:

$$\alpha_1 p_1 f^1_{E_1} + \alpha_2(1-\alpha_1)p^2_t f^2_{E_2} - w = \frac{\delta(1-\alpha_1)(1-\alpha_2)g_R(p_1 f^1_S + p_2 f^2_S)}{\delta g_S - 1} \tag{10}$$

$$\alpha_2 p_2 f^2_{E_2} - w = \frac{\delta(1-\alpha_2)g_R(p_1 f^1_S + p_2 f^2_S)}{\delta g_S - 1} \tag{11}$$

$$g([(1-\alpha_2)(X_2 + (1-\alpha_1)X_1)], S) - S = 0. \tag{12}$$

The denominator of the right-hand side of Eqs. (10) and (11) is negative, because $g_S$ is less than 1 for all $S$. The sign of the numerator of the RHS of Eqs. (10) and (11) is jointly determined by $g_R$( which is negative) and $p^1 f^1_S + p^2 f^2_S$. Since the desired soil pH range for each crop is unique, the sign of $f^1_S$ and $f^1_S$ can have different. In general, there are three cases:

(i): Alkaline Rotation, $p^1 f^1_S + p^2 f^2_S < 0$

(ii): Acid Rotation, $p^1 f^1_S + p^2 f^2_S > 0$

(iii): Ideal pH Rotation, $p^1 f^1_S + p^2 f^2_S = 0$

Rather than analyzing crops separately into each season, it makes more practical sense to analyze them jointly as a rotation. It is not important at all whether $f^i_S$ is negative or positive or zero. What does matter is how steady state soil pH influences production of the entire rotation. An alkaline rotation indicates that at the steady state soil pH level, the total value of the marginal product of soil pH within a growing year is negative. An acid rotation, on the opposite, has a positive value of the marginal product of soil pH at the steady state. An ideal pH rotation occurs when both $f_S^1$ and $f_S^2$ are both zero or the positive marginal value of soil pH in one season is offset completely by the other season with a negative marginal value of soil pH.

The shadow price of soil pH value at the steady state is measured by $\delta V_{S_*}$. From Eqs. (4), (5), (10) and (11), steady state shadow price of soil pH $\lambda^* = \dfrac{\delta(p_1 f_S^1 + p_2 f_S^2)}{1 - \delta g_S}$. The steady state shadow price of soil pH is negative in an alkaline rotation. Consistently, the shadow price of nitrogen as measured by the RHS of (10) and (11) is negative as well. This indicates that additional nitrogen increases the marginal product of soil pH by acidifying soil so that soil pH moves back to the increasing part of the production function with respect to soil pH. A amount of subsidy that is equivalent to the RHS of Eqs. (10) and (11) should be given to encourage the use of nitrogen. In an acid rotation, the shadow prices of soil pH and nitrogen are positive. Thus, reducing additional unit of nitrogen increases soil pH. Therefore, the value of the marginal product of soil pH increases. A amount of tax that is equivalent to the RHS of Eqs. (10) and (11) should be added to the current price of nitrogen to reduce nitrogen application. In case (iii), the shadow prices of soil pH and nitrogen are zero in an ideal pH rotation. In this case, there is no environmental cost or benefit of additional unit of nitrogen.

For myopic agents, the value of marginal product of nitrogen in each season equals to the unit price of nitrogen. The impact of nitrogen on the productivity of soil pH, which is the shadow price of nitrogen is ignored. Thus, steady state conditions for myopic management are:

$$\alpha_1 p^1 f_{E_1}^1 + \alpha_2(1 - \alpha_1) p^2 f_{E_2}^2 - w = 0 \tag{13}$$

$$\alpha_2 p_2 f_{E_2}^2 - w = 0 \tag{14}$$

and Eq. (12).

The comparison of optimal and myopic soil acidity management is discussed under three cases listed above.

(i) Alkaline Rotation

Comparing Eq. (10) with (13) and Eq. (11) with (14), by the concavity of the profit function, optimal nitrogen level is greater than myopic nitrogen level. The benefits of declined soil pH is ignored by myopic agents. The social marginal value of nitrogen is higher than the private marginal value of nitrogen. For myopic agents, price paid for an additional unit of nitrogen is thought to be higher than the value of the marginal product of nitrogen. They have no incentives to increase their use of nitrogen. Thus. myopic agents cannot drag soil pH back to the desired soil pH range in order to gain nonnegative soil pH productivity. The soil pH under myopic management stays at the downward-sloping part of the production function.

(ii) Acid Rotation

In acid rotation, since the shadow price of nitrogen is positive. By the concavity of the profit function, optimal nitrogen level is less than myopic nitrogen level. In this case, the cost of declined soil pH is ignored by myopic agents, who overvalue the marginal value of nitrogen. They keep high levels of nitrogen to pursue increment of profit from marginal productivity of nitrogen because the price paid for nitrogen is lower than the private marginal value of nitrogen. However, keeping high levels of nitrogen further decreases soil pH because $f_{ES} > 0$, marginal product of nitrogen decreases as soil pH decreases. The net profit of production decreases further in future periods. The social marginal value of nitrogen is lower than the private one. Thus, optimal management, on the contrary, gain higher yield in future periods by having a higher soil pH with less cost because of its low nitrogen application level.

(iii) Ideal pH Rotation

In this case, optimal and myopic soil acidity management are identical. Both optimal and myopic agents have no incentives to change the level of soil pH because the marginal cost of nitrogen is equal to the value of the marginal product of nitrogen. There is no externality induced by nitrogen on the productivity of soil pH in this case, Thus, there is no difference between optimal and myopic management.

Hence I summarize the series of results as:

PROPOSITION 1: *At the steady state, nitrogen application under optimal management is greater (less) than the myopic management if the total value of the marginal product of soil pH at the steady state for an entire rotation is negative (positive). If there is no externality generated by nitrogen at the steady state soil pH level, there is no difference between optimal and myopic management.*

## 2.4 Transition Dynamic Analysis

As the previous section concentrates on the steady state analysis, this section develops transition dynamic analysis. Since both nitrogen and soil acidity contribute to crop production and nitrogen might offset the productivity of soil, optimal nitrogen application decisions are tradeoffs. The analysis of transitional dynamics is a preparation to derive the policy function. What matters here is how soil pH ($S$) and nitrogen application in season 1 ($X^1$) and season 2 ($X^2$) interact simultaneous.

LEMMA 1:*As optimal soil pH increases, optimal nitrogen application in both seasons does not decrease at the same time.*

LEMMA 1 is proved by contradiction in three steps as shown below.

*Step 1*: prove that $\alpha_2 p_t^2 f^2_{E_t^2}$ is greater in $X^1(S^0)$, $X^2(S^0)$, and $S^0$.

Suppose there exists $S^* < S^0$, such that $X^2(S^*) > X^2(S^0)$ and $X^1(S^*) > X^1(S^0)$, by having $f_{ES} > 0$ and the concavity of $f(.)$ in $E$ and $S$, I derive:

$$\alpha_2 p_t^2 f^2_{E_t^2}(X^1(S^0), X^2(S^0), S^0) > \alpha_2 p_t^2 f^2_{E_t^2}(X^1(S^0), X^2(S^0), S^*) > \alpha_2 p_t^2 f^2_{E_t^2}(X^1(S^*), X^2(S^*), S^*). \tag{15}$$

*Step 2*: prove that $\delta(1-\alpha_2) g_{R_t} V_{S_{t+1}}$ is greater in $X^1(S^0)$, $X^2(S^0)$, and $S^0$.

Since $X^2(S^*) > X^2(S^0)$ and $X^1(S^*) > X^1(S^0)$, I get $R_t^0 < R_t^*$. Because $g(.)$ is concave in $R$, I have $g_{R_t^0} > g_{R_t^*}$. Since $g(.)$ is super modular in $R$ and $S$, $g(R_t^0, S_0) > g(R_t^*, S_0) > g(R_t^*, S_*)$.

In other words, $S^0_{t+1} > S^*_{t+1}$. Again, the concavity of value function makes $V_{S^0_{t+1}} < V_{S^*_{t+1}}$. Thus,

$$\delta(1-\alpha_2)g_{R^0_t}V_{S^0_{t+1}} > \delta(1-\alpha_2)g_{R^*_t}V_{S^*_{t+1}}. \tag{16}$$

*Step 3*: prove that Eq. (5) is greater in $X^1(S^0)$, $X^2(S^0)$, and $S^0$.

Adding Eqs. (15) to (16)

$$\alpha_2 p^2_t f^2_{E^2_t}(X^1(S^0), X^2(S^0), S^0) + \delta(1-\alpha_2)g_{R^0_t}V_{S^0_{t+1}} > \\ \alpha_2 p^2_t f^2_{E^2_t}(X^1(S^*), X^2(S^*), S^*) + \delta(1-\alpha_2)g_{R^*_t}V_{S^*_{t+1}} = 0. \tag{17}$$

Here, in Eq. (17), there is a contradiction because as $X^1(S^0)$ and $X^2(S^0)$ maximize the value function when soil pH is $S^0$, $X^1(S^*)$ and $X^2(S^*)$ also maximize the value function when soil pH is $S^*$. The contradiction is derived by comparing Eq. (17) with Eq. (5). Thus, I arrive at LEMMA 1. As soil pH goes up, the optimal nitrogen application of season 1 and season 2 do not decrease at the same time. Intuitively, since nitrogen induces soil acidification, the higher the soil pH value, the more tolerate the soil is to acidification. Thus, absolute decrease of nitrogen in both seasons is not reasonable.

After knowing how nitrogen in season 1 and season 2 influenced by soil pH, it is the right time to know exactly how nitrogen in season1 and season 2 interact. A similar contradiction is constructed by using Eq. (4):

Suppose there exists $X^*_{1t} > X^0_{1t}$, such that $X^*_{2t} > X^0_{2t}$; in this case, I arrive at the similar contradictory condition that (compared with Eq. (4)):

$$\alpha_1 p^1_t f^1_{E_{1t}}(E^0_{1t}, S_t) + \alpha_2(1-\alpha_1)p^2_t f^2_{E^0_{2t}}(E^0_{2t}, S_t) + \delta(1-\alpha_2)g_{R^0_t}V_{S^0_{t+1}} > \\ \alpha_1 p^1_t f^1_{E^*_{1t}}(E^*_{1t}, S_t) + \alpha_2(1-\alpha_1)p^2_t f^2_{E^*_{2t}}(E^*_{2t}, S_t) + \delta(1-\alpha_2)g_{R^*_t}V_{S^*_{t+1}} = 0. \tag{18}$$

Thus, I prove that when $X^1_t$ increases at the optimal, $X^2_t$ does not increase.When optimal nitrogen application in season 1 increases, keeping the absorption coefficient constant, the nitrogen residual from season 1 increases, as well. The residual is involved in the crop production in season 2. Thus, without increasing nitrogen application in season 2, the availability of nitrogen in season 2 improves. For any level of soil pH, if increase in $X_t^1$ increases $X_t^2$, then the residual after the entire rotation goes up. Soil pH cannot stay at the same level if residual after season 2 goes up via the state equation. Thus, for optimal management, at any level of soil pH, carryover effect of season 2 reduces the application of nitrogen. I summarize this results as:

LEMMA 2: *For any levels of soil pH value, any increment of optimal nitrogen in season 1 decreases optimal nitrogen in season 2.*

The contradiction in Eqs. (17) and (18) occurs whenever $(X_{2t}^* - X_{2t}^0) > -(1-\alpha_1)(X_{1t}^* - X_{1t}^0)$ and $X_{1t}^* > X_{1t}^0$. Hence, with LEMMA 1 and LEMMA 2, I conclude that for any increment in optimal soil pH value, optimal nitrogen in season 1 increases and optimal nitrogen in season 2 decreases, and the sum of nitrogen of a rotation increases. Intuitively, LEMMA1 shows that any increment in soil pH increases the resistance of soil to acidity. Thus, increment in soil pH leads to increase of total nitrogen in an entire rotation. LEMMA 2 shows that optimal nitrogen level in season 2 declines because of its carryover effects. However, the room left for nitrogen by a higher soil pH is higher than the nitrogen saved from a cover crop in season 2. This conclusion is consistent with previous literature that states that crop rotation plays an important role in reducing excess nitrogen fertilizer use (Huang and Uri, 1993). I summarize this result as:
PROPOSITION 2: *For an increment in optimal soil pH value, the marginal decrease of optimal* $X_t^2$ *is no greater than the marginal increase in* $X_t^1$ *adjusted by* $1-\alpha_1$*. Thus, any increment in optimal soil pH value, the sum of nitrogen of a rotation increases.*

## The Policy Function of Soil pH Value

After deriving the analysis of the steady state and transition dynamics, it is time to analyze the policy function of the soil pH value. The policy function of soil pH value explains how $S_t$ maps into $S_{t+1}$. By having the policy function, the transition path can be derived to see how soil pH moves dynamically towards the steady state and whether the steady state soil pH is stable or not.

Since optimal total nitrogen increases in soil pH value, and nitrogen application across seasons offsets each other, I write $X_t^i$ as a function of $S_t$ ($\Psi^i(S_t)$) and substitute it into the state equation:

$$S_{t+1} = g((1-\alpha_2)(\Psi^1(S_t) + (1-\alpha_1)\Psi^2(S_t)), S_t). \tag{19}$$

The sign of $\dfrac{\partial S_{t+1}}{\partial S_t}$ determines the slope of the policy function. Differentiating Eq. (19) with respect to $S_t$, I have:

$$\frac{\partial S_{t+1}}{\partial S_t} = g_{R_t}\{(1-\alpha_2)[\frac{\partial X_t^2}{\partial S_t} + (1-\alpha_1)\frac{\partial X_t^1}{\partial S_t}]\} + g_S. \tag{20}$$

The sign of the brace ($\{.\}$) is positive as I elaborated in Prop. 2. Since the sign of $g_R$ is negative and that of $g_S$ is positive, the sign of Eq. (20) is undetermined. Without having an explicit relationship among $g_{RR}$, $g_{SS}$. and $\dfrac{\partial X_t^i}{\partial S}/\partial S$, I can still make some quantitative analysis of the policy function. As $S_t$ increases, on the one hand, $g_S$ is positive but decreases because $g(.)$ is concave in $S$. On the other hand, $g_R$ decreases in absolute value because $g_{RS} > 0$. Without rigorous proof, I expect that the brace ($\{.\}$) increases at an increasing rate in $S$. Intuitively, as soil becomes more tolerant to acidity, optimal total nitrogen increases greater. Thus, the shape of the policy function has three cases:

(i) $|g_{R_t}\{(1-\alpha_2)[\frac{\partial X_t^2}{\partial S_t} + (1-\alpha_1)\frac{\partial X_t^1}{\partial S_t}]\}| < g_S$ for all $S$;

(ii) $|g_{R_t}\{(1-\alpha_2)[\frac{\partial X_t^2}{\partial S_t} + (1-\alpha_1)\frac{\partial X_t^1}{\partial S_t}]\}| < g_S$ for some $S$; and

(iii) $|g_{R_t}\{(1-\alpha_2)[\frac{\partial X_t^2}{\partial S_t} + (1-\alpha_1)\frac{\partial X_t^1}{\partial S_t}]\}| > g_S$ for all $S$.

*Case (i): The policy function is upward sloping. The steady state is locally stable.*

The steady state exists because $g_S < 1$ for all $S$. Adding a negative factor to $g_S$ ensures that slope of the policy function is flatter than the $45^o$ line around the steady state. As shown in Figure 1(a), that steady state is locally stable. This is a case when the direct impact of $S_t$ is greater than the indirect impact of $S_t$ on future soil pH through its impact on nitrogen. Intuitively, the magnitude of soil buffering capacity towards acidity is stronger than the marginal decrease of soil pH caused by increasing nitrogen. Policy function goes up because the buffering capacity offset the decline in soil pH induced by nitrogen. Since the steady state soil pH is stable, soil pH moves towards the steady state. In this case, the damage of over-application of nitrogen is the least severe case among the three. This might be the situation in places where soil is not very sensitive to acidity or has a very strong buffering capacity. Soil acidification does not happen at any soil pH value.

*Case (ii): The policy function reaches its peak at its steady state and then goes down. The steady state is locally stable.*

Since $g_S$ decreases in $S$, the possibility of a "V"-shape policy function is ruled out. The policy function reaches its peak at the steady state because at the $45^o$ line $\frac{\partial S_{t+1}}{\partial S_t} = 0$. The indirect impact of nitrogen residual on soil acidification becomes stronger when soil pH increases gradually. The increasing negative impact of nitrogen residual mainly comes from the impact of increasing $S$ on $X_t^i$. Thus, the soil is very sensitive to acidity. Since $g_{RS} > 0$, as $S$ increases, $|g_R|$ decreases. Thus, $\frac{\partial X_{it}}{\partial S}/\partial S > 0$ is the only explanation of the downward policy function after the steady state. The steady state is stable, so soil pH moves towards the steady state pH from either side. This is the case when soil acidity goes up, buffering capacity of soil becomes weaker, extra acidity induced by increasing nitrogen offsets the buffering capacity gradually and forces the policy function go down. Since the soil is very sensitive to acidity, the buffering capacity of soil can only offset the decline of soil pH for some values of soil pH. The damage of over-application of nitrogen in case (ii) is more severe than in Case (i).

*Case (iii): The policy function is downward sloping. The stability of the steady state depends on the slope of the policy function.*

In this case, for all $S$, soil pH decreased by the nitrogen residual is more influential than the buffering capacity of soil for all possible $S$. This situation is true in some areas when soil is very sensitive to acidity or buffering capacity is pretty weak. The stability of the steady state depends on the slope. A flatter slope, as shown in Figure. 1(c) makes the steady state stable.

A stable steady state ensures that soil pH moves closer and closer towards the steady state soil pH spirally. Unstable steady state under a steeper slope is an even worse situation because soil pH moved away from the steady state. A steeper slope ($|\frac{\partial S_{t+1}}{\partial S_t}| > 1$) indicates that the absolute value of $g_R\{.\}$ is even greater than that of a flatter-slope-function. In other words, the decrease of soil pH caused by nitrogen is much greater than the buffering capacity of the soil.

# 3 Empirical Results and Policy Implications

In this section, I establish an empirical model, which is consistent with the double-crop rotation dynamic model to estimate the magnitude of the shadow price of nitrogen in each season and then calculate optimal nitrogen application levels and the steady state of the soil pH value in the objective area, Anhui Province in China. These empirical results and analysis are derived from estimations of production functions of crops and the state equation. Potential policy implications are discussed subsequently.

## 3.1 Data Description

The data used to estimate the production functions and the state equation in the dynamic programming model (e.g $y_t^i$ and $S_{t+1}$) were obtained using the experimental data from the Anhui Agricultural Extension Office (Qian, 2005-2010). This experiment data is derived from the Anhui's Annual Report of Soil Quality from 2005 to 2010. These reports cover all National Long-term Monitoring Fields in Anhui Province, which were chosen as representatives of soil types within the province. Evidences of nationwide soil acidification observed by Guo et al. (2010) were derived from the National Long-term Monitoring Fields in 31 provinces. In Anhui Province, double-crop rotation is very common. The first season started from mid-October to late May. The second season started from late May to early October in the next year. The order of a rotation is determined by climate factors. Rice that cannot be grown in harsh weather is planted in late spring. The dataset provided 11 types of double-crop rotation. In the following estimation and analysis, I only concentrate on wheat-rice and rapeseed-rice rotation to maximize the number of observation I can use. The unbalanced panel data includes 23 observations of wheat in season 1, 28 observations of rapeseed in season 1, and 31 observations of rice in season 2. Among these observations, there are 10 wheat-rice rotation observations, and 9 rapeseed-rice rotations. In Anhui Province, rapeseed-rice and wheat-rice rotation occupy about 70% of the farmland (Wang and Luo, 2006). Anhui is located in the eastern part of China across the basins of the Yangtze River and the Huai River (Wang and Luo, 2006). Climate and agriculture differs from the north to the south within the province. Thus, cropping system and crop species are different. Rapeseed-rice rotation is more commonly seen in the south of Anhui province, while wheat-rice is more commonly seen in the north.

Crop yield and nitrogen application levels were recorded separately for each season. N application in wheat production was between 3.20 kg/mu and 20.60 kg/mu with a mean of 13.04 kg/mu. Wheat yield varied from 350 kg/mu to 1134 kg/mu with a mean of 740.31 kg/mu. N application in rapeseed production was between 5.8 kg/mu and 28.37 kg/mu with a mean of 12.49 kg/mu. The yield of rapeseed varied from 431.67 kg/mu to 269.09kg/mu. In rice production, N application varied from 6.40 kg/mu to 18.35 kg/mu with a mean of 12.99 kg/mu. The yield of rice was between 450 kg/mu and 1392 kg/mu. Soil property variables, such as soil pH, were tested annually rather than seasonally. There are 48 observations of soil pH value in the dataset to support the estimation of the state equation. The range of the soil pH value in the dataset is between 4.60 to 8.40, which covers weak acid soil to weak alkaline soil. The soil pH range associated with observations of rice in season 2 was between 4.69 to 8.40 with a mean of 6.09. The soil pH range associated with observations of wheat in season 1 was between 5.40 to 8.40 with a mean of 6.67. The soil pH range associated with observations of rapeseed in season 1 was between 4.60 to 8.20.

### 3.2 Parameter Estimation

Both crop production functions and the state equation are estimated in reduced form with arguments $X_t^i$ and $S_t$, because absorbed nitrogen ($E_t^i$) and residual ($R_t$) were not observed in the dataset. To be consistent with the assumptions I made in the theoretical model, I impose the following quadratic functional forms for each crop production and the state equation:

$$f^1(X_t^1, S_t) = a_0^1 + a_1^1 X_t^1 + a_2^1 X_t^{12} + a_3^1 S_t + a_4^1 S_t^2 + a_5^1 S_t X_t^1 + e_t^1 \tag{21}$$

$$f^2(X_t^2, S_t) = a_0^2 + a_1^2 X_t^2 + a_2^2 X_t^{22} + a_3^2 S_t + a_4^2 S_t^2 + a_5^2 S_t X_t^2 + a_6^2 X_t^1 + e_t^2 \tag{22}$$

where $a_1^1$ to $a_5^1$ and $a_1^2$ to $a_6^2$ are fixed parameters for production in season 1 and season 2, $e_t^1$ and $e_t^2$ are error terms, and $a_6^2$ captures the nitrogen carryover from season 1 to season 2. Besides $X_t$, $S_t$, and their quadratic terms, interaction between $S_t$ and $X_t$ is included in the production functions.

Based on properties of the state transition equation (Eq. 2), the state equation is estimated in a quadric form in $S_t$ and a linear form in $X_t$.

$$S_{t+1} = k_0 + k_1 S_t + k_2 S_t^2 + k_3 X_t^1 + k_4 X_t^2 + k_5 S_t X_t^1 + k_6 S_t X_t^2 + \epsilon_t \tag{23}$$

where $k_0$ to $k_6$ are fixed parameters, and $\epsilon_t$ is an error term. Since soil pH was not tested after each season, the soil pH in the next period is determined by N application in both seasons and the interaction between N application and $S_t$ in each season.

I assume that all error terms are not correlated and each satisfies a normal distribution with a mean of zero. Observations of wheat, rapeseed, and rice were from 29 plots in 5 years. Among all the observations, only 23% of the observations were rotated in the same plots in the same year. Within the 19 observations of rotation (10 for wheat-rice, 9 for rapeseed-rice), there were only one three-year-sequential rotation of wheat-rice, and a four-year-sequential rotation of rapeseed-rice. All other 12 observations of rotation were not repeated in the same plot. Thus, it is safe to assume that the error terms of the 23 observations of wheat or the 28 observations of rapeseed are not correlated with the error term of the 31 observations of rice, because most of them come from different plots in different years. The 48 observations of soil pH cover 7 crops. There were 45% of the soil pH observations also in the observations used in production function estimations. Similarly, observations of soil pH value of the same plot across years were only a small proportion of the sample. Thus, it is reasonable to believe that $\epsilon_t$ is not correlated with any $e_t^i$. Thus, I apply the *Ordinary Least Square* estimation to Eqs. (21)-(23). Results are shown in Table 1.

Due to the fact that observations are limited for both crop production and state equation estimation, I apply two simulation methods, *Monte Carlo* and Bootstrapping to test whether the OLS results are unbiased. In the *Monte Carlo* simulation, nitrogen application, crop yield, and soil pH are randomly created based on uniform distributions within the range of the dataset. Error terms $e_t^1$ and $e_t^2$ are randomly generated from normal distributions with a mean of zero and variance that equals to the variance calculated from the OLS regression. *Monte Carlo* simulation is applied to compare competing statistics for small samples under realistic data conditions. The OLS results are unbiased if they are close to *Monte Carlo* coefficients, because *Monte Carlo* coefficients are derived from a random process that runs thousands of time. I setup a sample of 100 observations of nitrogen application, soil pH, and yield and then run the simulation for 1000 times. The OLS coefficients are very close to the mean of *Monte Carlo* coefficients. Thus, OLS results are unbiased and reliable for further empirical analysis. OLS results are shown in Table. 1 and results from *Monte Carlo* and bootstrapping are shown in Table 2.

In wheat production, parameters are jointly significant at a 5% confident level. Coefficients of $X_t^1$, $X_t^{12}$, and $X_t^1 S_t$ are jointly significant at a 1% confident level, while coefficients of $S_t$ and $S_t^2$ are not jointly significant. Wheat production is not sensitive to soil pH in the range of 5.40 to 8.40. The production of wheat is a concave function in nitrogen, since the coefficient of the quadratic term is negative. It is an increasing function in nitrogen if nitrogen is less than 32 kg/mu.

In rapeseed production, parameters are jointly significant at a 1% confident level. Coefficients of $X_t^1$, $X_t^{12}$, and $X_t^1 S_t$ are jointly significant at a 5% confident level. Coefficients of $S_t$ and $S_t^2$ are both significant at a 1% confident level. The desired range of soil pH for rapeseed production is between 8.03 to 8.79 by plugging in the range of nitrogen application of all rapeseed observations. Rapeseed yield increases in soil pH as soil pH less than $S^{ra\bar{p}eseed}$. The rapeseed production is concave in soil pH and nitrogen because the quadratic term of each arguments has a negative coefficient. The yield of rapeseed increases in N application when N application is less than 12.76 kg/mu.

In rice production, parameters are jointly significant at a 1% confident level. Coefficients of $X_t^2$, $X_t^{22}$, and $X_t^2 S_t$ are jointly significant at a 5% confident level. Coefficients of $X_1^t$, $X_t^2$, and $X_t^{22}$ are jointly significant at a 1% confident level. This shows that carryover effect from previous season is not negligible. Coefficients of soil pH and its quadratic term are jointly significant at a 5% confident level. The rice production function increases if soil pH is less than $S^{\bar{rice}}$. The estimated $S^{\bar{rice}}$ is between 4.23 to 7.41. The function increases in nitrogen if nitrogen is less than 12.4 kg/mu. The rice production function is also concave in nitrogen and soil pH.

The state equation is concave in soil pH with a negative coefficient of $S_t^2$. The marginal product of soil pH is less than 1 within the range of nitrogen in the dataset. Nitrogen applications in season 1 and season 2 are jointly significant at a 1% confident level. Interaction terms of nitrogen and soil pH are jointly significant at a 1% confident level. Within the range of soil pH of the dataset, per unit of nitrogen in season 1 decreases soil pH by at least 0.02. Per unit of nitrogen in season 2 increases soil pH by no more than 0.03. This state equation shows that rotation help maintaining soil pH. Huang and Uri (1993) stated that rotation can reduce nitrogen residual. Since nitrogen residual is the main contributor to soil acidification this result that rotation is good for maintaining soil pH is consistent with Huang's work. However, this state equation indicates that $g_{X_t^1}$ and $g_{X_t^2}$ have different signs. The reason that empirically nitrogen in each season functions differently in the state equation is that nitrogen influx and efflux beside nitrogen application is not negligible. Therefore, missing information about these nitrogen flows is presented as a proportion of nitrogen application measured in $\alpha_1$ and $\alpha_2$. In this case, both $\alpha_1$ and $\alpha_2$ is greater than 1, which indicates that nitrogen influx is greater than nitrogen efflux in total.

### 3.3 Numerical Solutions at the Steady State

The steady state is characterized by Eqs. (10), (11) and (12). Specific function forms and estimated parameters are applied to calculate the steady state of the optimal soil management in wheat-rice rotation and rapeseed-rice rotation. The parameters that are not significant in estimations of crop production and state equation are not eliminated because all non-constant terms are jointly significant. The prices of the N fertilizer and crops are farm-gate-price in 2011. In 2011, the average transaction price of the N fertilizer was 2.2 RMB per kilogram of pure nitrogen (*Annural report of nitrogen fertilizer's production*, n.d.). This price is equivalent to market price less the subsidies. Subsidies of the N fertilizer are given from both local and central government, which is hard to be measure because it has a high degree of local variability. The average farm-gate price of wheat was 2.04 RMB/kg (NRDC, 2011*c*); average farm-gate price of rapeseed was 4.5 RMB/kg (NRDC, 2011*a*); and average farm-game price of rice was 2.4 RMB/kg (NRDC, 2011*b*). In the following analysis of steady state, $\delta$ is set to be 0.9. Numerical solutions are reported in Table 3.

*Wheat-Rice Rotation*

Optimal nitrogen for wheat is 30.62 kg/mu, and optimal nitrogen for rice is 10.88 kg/mu. Both of the optimal levels are in the range of the increasing part of production functions. At the steady state, the soil pH is 5.89, which is a weak acid soil environment. The shadow price of N in season 2, which is equal to the RHS of Eq. (11), is 2.70, which is about 1.1 times of its market price. Applying the same calculation, the shadow price of N in season 1, which is equal to the RHS of Eq. (10), is negative 1.94, which is about 94% of the market price of wheat. The reason that the sign of the N shadow price is different is that the sign of $g_{X_t^1}$ and $g_{X_t^2}$ is different at the steady state. Thus, the taxation policies towards season 1 and season 2 are different.

In wheat production, the social marginal value of nitrogen is undervalued by myopic agents. Price paid by agents is greater than the value of the marginal product of N. The myopic application level of N in season 1 is 30.40 kg/mu, which is less than optimal N application 30.62 kg/mu. In rice production, the social marginal value of nitrogen is overvalued because the price paid by agents is less than the value of the marginal product of N. Myopic application level of N in season 2 is 11.01 kg/mu, which is greater than the optimal N level 10.88 kg/mu. Thus a tax of 2.70 RMB should be taken for every kilogram of N applied in season 2 and a subsidy of 1.94 RMB should be given for each kilogram of N applied in season 1. However, this taxation system has no enforceability if farmers invest in N fertilizer jointly for both seasons at the beginning of the year. The steady state of myopic behavior is 5.05, which is less then the steady state soil pH. The sum of the value of the marginal product of soil pH at the steady state is negative 1.92 RMB, which indicates that wheat-rice rotation is an alkaline rotation. Thus, optimal management applies more nitrogen in total than the myopic agents to drag soil pH backward. As shown in Table. 3, the sum of nitrogen in myopic management is less than optimal management. This result is consistent with Prop. 1. Moreover, N-induced soil acidification is not only influenced by the sum of nitrogen but also the interseasonal allocation of nitrogen, because the sum of nitrogen in optimal and myopic management are very close but end with different steady state soil pH levels. Thus, the agricultural extension offices should pay attention to the allocation of nitrogen across seasons beside regulating the sum of N fertilizer.

*Rapeseed-Rice Rotation*

Optimal N application for rapeseed is 13.29 kg/mu, and optimal N application of rice is 12.06 kg/mu. The steady state soil pH is 7.86, which is a weak alkaline environment. The shadow price of N in rapeseed production is positive 26.27 RMB, which is 5.8 times of its market price. The shadow price of N in rice production is negative 4.92 RMB, which is 2 times of its market price. Myopic N application in season 1 is 14.90 kg/mu, and myopic N application in season 2 is 11.50 kg/mu. The steady state of soil pH under myopic management is 7.09. Thus, unlike the wheat-rice rotation, the sum of N application in myopic management is 26.4 kg/mu, which is greater than the sum of N application under optimal management at 25.6 kg/mu. The reason that myopic management uses more nitrogen than the optimal management is that rapeseed-rice rotation is an acid rotation with a positive value of the sum of the marginal product of soil pH of 2.72 RMB. Thus, myopic agents who set the shadow price of nitrogen to zero have no incentive to reduce nitrogen and prevent acidification. Although reallocation of nitrogen cannot completely solve the soil acidification problem, it could offset a great proportion of the problem.

### 3.4 Policy Implications

From the empirical results of wheat-rice and rapeseed-rice rotation, there are three essential points that associated with three coherent policies. First, education programs that focus on introducing the shadow price of nitrogen should be organized for farmers. The steady state soil pH under myopic management is less than that of optimal management. Thus, ignoring the shadow price of nitrogen not only misallocate nitrogen but also accelerate soil acidification. The shadow price of nitrogen could be ignored in two ways: ignoring soil productivity or ignoring soil acidity induced by nitrogen. The productivity of soil is less likely to be ignored by farmers. The real situation is that after the boom of domestic nitrogen production in the late 1970s, nitrogen has been heavily subsidized and strongly encouraged to use by the agricultural extension offices. The understanding of the mechanism of soil acidification induced by nitrogen is still at the stage of textbook. The new challenge of nitrogen in China is no longer the shortage in supply but the excessive demand. Thus, the agricultural extension offices should adjust their education programs from solely encouragement of fertilization to help improving nitrogen efficiency and introduce environment cost associated with nitrogen including the soil acidification problem to farmers. Second, reduction in nitrogen application is important, but what is more important is the interseasonal allocation of nitrogen. As is shown in Table. 3, the difference between the sum of nitrogen in optimal and myopic manage is small, but the difference between the steady state soil pH is not trivial. Thus, the agricultural extension offices should work on the nitrogen recommendation guidance under the framework of the optimal management by taking N-induced soil acidification into consideration. Third, rotation makes regular taxation policy unenforceable, but itself plays a positive role in preventing the decline of soil pH. The steady state of a rapeseed-rice rotation is higher than the steady state of a wheat-rice rotation. Thus, if climate and other production factors are feasible and regardless of the functional difference between wheat, a stamp crop, and rapeseed, an oil crop, converting wheat-rice rotation into rapeseed-rice rotation increases the steady state soil pH.

## 4 Conclusion

The problem of soil acidification has concerned economists for ages. However, the soil acidification problem has not been structured under the framework of nitrogen control without lime application. Although lime application is feasible in developed countries, regulation of nitrogen is still vital in improving nitrogen efficiency and avoiding unnecessary investment in lime application. Thus, regulation on nitrogen should be considered as an option to deal with soil acidification even in places where lime application is available.

I formally set up the theoretical framework of optimal soil acidity management under a double-crop rotation system. From this discrete dynamic programming model, I first show that the optimal soil acidity management is able to adjust nitrogen application according to its impacts on soil productivity and soil acidity. When soil pH is too high for a rotation system, optimal management applies more nitrogen than myopic agents to offset the negative value of the marginal product of soil pH induced by nitrogen. When soil pH is too low for a rotation, optimal management prevents further decline of soil pH by reducing nitrogen. Myopic agents, on the contrary, ignored the shadow price of nitrogen and results with a lower steady state soil pH value. Second, I show the interaction of nitrogen application and soil pH at the optimal. A higher steady state soil pH leads to a higher sum of nitrogen application in two seasons, because with a higher soil pH, the soil has a higher tolerance to the acidity produced by nitrogen. Last but not least, the policy function of soil pH determined by two factors: the sensitivity of soil to acidity and soil buffering capacity. I show that soil acidification is most severe when the buffering capacity cannot cover the decline of soil pH induced by nitrogen. Then, consistently and empirically, these results are also shown by an empirical model using data from Anhui Province. I numerically solve the optimal solution of two types of rotation. The results show that the shadow prices of nitrogen are different in each season in both rotation systems. The magnitude of the shadow prices varies from 0.94 to 5.8 times of the farm-gate price of crops. The empirical results further show that myopic management with misallocation of nitrogen ends with a lower soil pH value at the steady state than optimal management. Thus, to understand the soil acidification problem, both the sum of nitrogen and the interseasonal allocation of nitrogen should be focused. The difference of soil pH between the two representative rotations implies that rotation can be used as a feasible tool to reverse soil pH. From the theoretical and empirical findings, I suggest three potential policies to address the soil acidification problems in China. Taxation policies are not enforceable in a rotation system. However, rotation itself plays a positive role in relieving soil acidification by reallocating nitrogen among seasons. Without lime application, China could still reserve the soil pH by adjusting agricultural education programming, making a new interseasonal nitrogen recommendation handbook, and taking advantage of the effects of rotation on soil pH.

The empirical results derived in this paper are limited by the dataset. A longer year panel of rotation observations in the same plots across years should be able to give more precise estimation of crop productions and state equation by estimating as a system using *Seemly Unrelated Regression.* Future empirical studies of the nitrogen-induced soil acidification problem should concentrate on the dynamic transitional analysis to investigate the path of dynamics. Future theoretical studies should address the importance of lime application as well as nitrogen control to see the tradeoff between precaution (reducing nitrogen) and prevention behavior (applying lime) in a dynamic structure. To address the elimination of lime application in China, institutional studies on how land property right changes farmers' incentive of investment and collective action should be studied both theoretically and empirically.

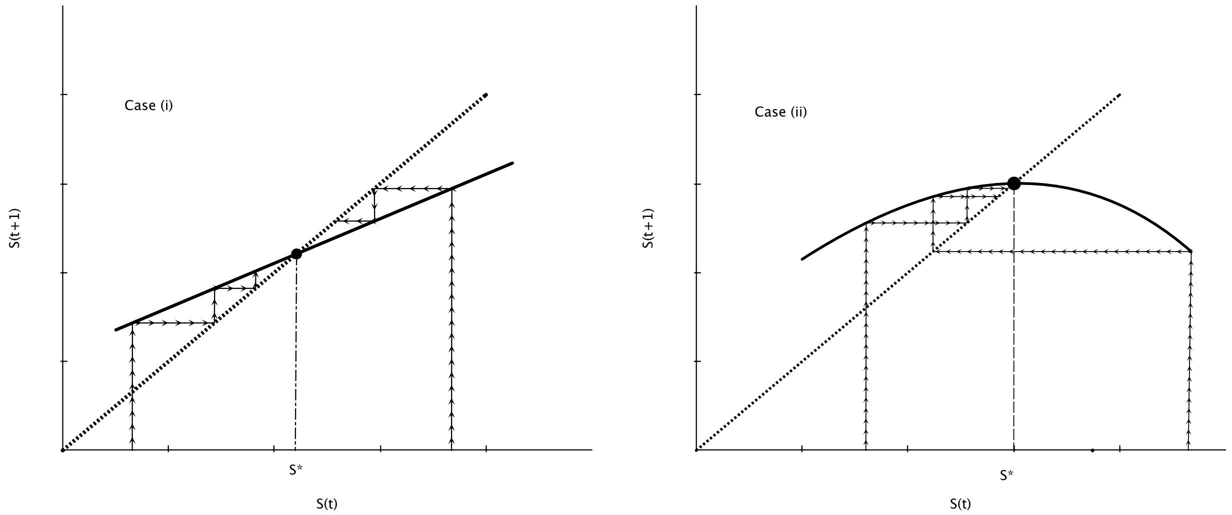

Figure. 1(a): Policy Function of Case (i)  Figure. 1(b): Policy Function of Case (ii)

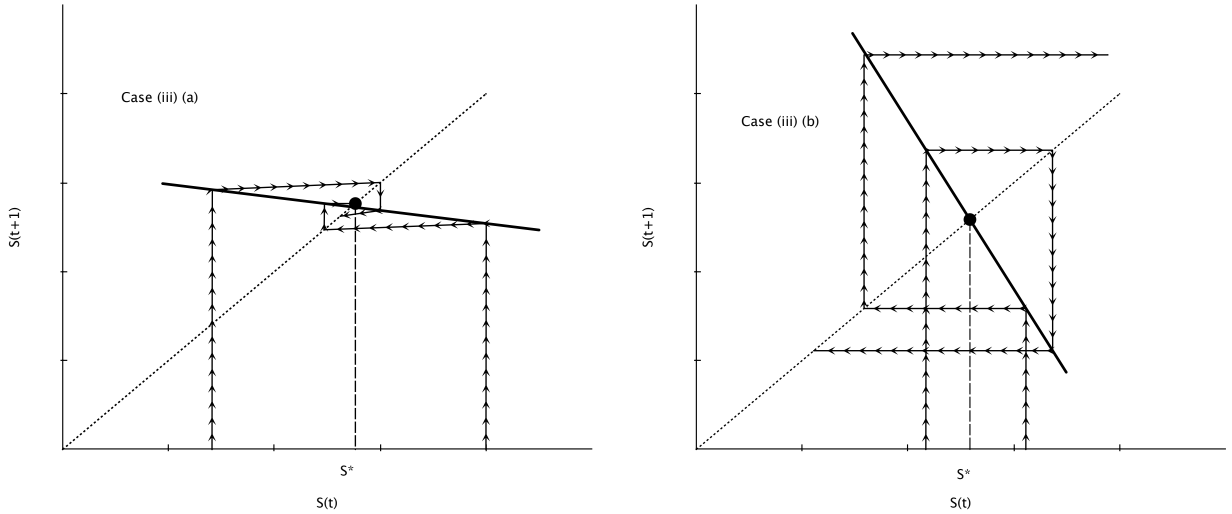

Figure. 1(c): Policy Function of Case (iii-a)  Figure. 1(d): Policy Function of Case (iii-b)

Figure 1: Policy Functions of Case (i) to (iii)

# References

Alcamo, J., Amann, M., Hettelingh, J. P., Holmberg., M. and Kamari, L. (1987), 'Acidification in europe: A simulation model for evaluation control strategies', *Ambio* **16**, 232–245.

*Annural report of nitrogen fertilizer's production* (n.d.).

Brady, N. C. and Weil, R. R. (1996), *The Nature and Properties of Soils*, A Simon and Schuster Company.

Bremner, J. M. and Blackmer, A. M. (1978), 'Nitrous oxide: Emission from soils during nitrication of fertilizer nitrogen', *Science* **199**, 295–296.

Chiao, Y. S. and Gillingham, A. (1989), 'The value of stabilizing fertilizer under cerry-over conditions', *American Journal of Agricultural Economics* **71**, 352–62.

Ding, C. (2003), 'Land policy reform in china: Assessment and prospects.', *Land Use Policy* **20**, 109–120.

Gilliam, J. W. and Hoyt, G. D. (1987), *Effect of conservation tillage on the fate and transport of nitrogen.*, Lewis Publishers, Chelsea, MI.

Guo, J. H., Liu, X. J., Zhang, Y., Shen, J. L., Han, W. X., Zhang, W. F., Chrisite., P. and Zhang, F. S. (2010), 'Significant acidification in major chinese croplands', *Science* **327**(1008-1010).

Hartwig, N. L. and Ammon, H. U. (2002), 'Cover crops and living mulches', *Weed Science* **50**(6), 688–699.

Hinsinger, P. H., Passard, C., Tang, C. and Jaillard, B. (2003), 'Origins of rootmediated ph changes in the rhizosphere and their responses to environmental constraints: A review', *Plant and Soil* **248**, 43–59.

Holmberg, M. (1990), *Model of ion dyanmics and acidification of soil: Simulating recovery of base saturation*, number 359-368, ElsevierScience Publishers, B. V. Amsterdam.

Huang, S., Zhang, W., Yu, X. and Huang, Q. (2010), 'Effects of long-term fertilization on corn productivity and its sustainability in an ultisol of southern china', *Agriculture, Ecosystems and Environment* **138**, 44–50.

Huang, W. Y. and Uri, N. D. (1993), 'The effect of crop rotation on reducing excess nitrogen fertilizer use.', *Applied Mathemetics Modelling* **17**.

Johnson, D. G. (1950), 'Resource allocation under share constracts', *Journal of Political Economy* **58**, 111–123.

Ju, X., Liu, X., Zhang, F. S. and Roelcke, M. (2004), 'Nitrogen fertilization, soil nitrate accumulation, and policy recommendations in several agricultural regions of china.', *Royal Swedish Academy of Sciences* **33**, 300–305.

Kaitala, V., Pohjola, M. and Tahvonen, O. (1992), 'Transboundary air pollution and soil acidification: a dynamic analysis of an acid rain game betwwen finland and the ussr', *Environmental and Resource Economics* **2**, 161–181.

Kauppi, P., Anttila, P. and Kenttamies, K. (1990), *Acidification in Finland. Finnish Acidification Research Programme HAPRO 1985-1990*, Springer-Verlag.

Kim, C. S. and Hostetler, J. E. (1991), 'Policy implications of a water quality-constrained dynamic model of nitrogen ferilizer use', *Canadian Journal of Agricultural Economics* (39), 781–91.

Kohl, D. H., Shearer, G. B. and Commoner, B. (1971), 'Fertilizer nitrogen:contribution to nitrate in surface water in a corn belt watershed.', *Science* **174**, 1331–1334.

Li, G., Rozello, S. and Brandt, L. (1998), 'Tenure, land rights, and farmer investment incentrives in china', *Agricultural Economics* **18**(1), 63–71.

Lichtenberg, E. (2007), 'Tenants, landlords, and soil conservation', *American Journal of Agricultural Economics* **89**(2), 294–307.

Lichtenberg, E. and Ding, C. R. (2008), 'Assessing farmland protection policy in china', *Land Use Policy* **25**, 59–68.

Morgan, M. F., Jacobson, H. and LeCompte, J. S. (1942), 'Drainage water losses from a sandy soil as affected by cropping and cover crops', *Connecticut Agric. Exp. Station Bul.* pp. 731–759.

Muthoni, J. and Kabira, J. N. (2010), 'Effects of crop rotation on soil macronutrient content and ph in potato producing areas in kenya: Case study of kari tigoni station', *Journal of Soil Science and Environmental Management* **1**(9), 227–233.

Myyra, S., Ketoja, E., Yli-Halla, M. and Pietola, K. (2004), Land improvements under land tenure insecurity the case of ph and phosphate in finland. MTT Agrifood Research Finland. Discussion papers.

Norse, D. and Zhang, F. S. (2010), *Improved nutrient management in agriclture-A neglected* *opportunity for China's low carbon growth path*, UK-China Sustainable Agriculture Innovation Network.

NRDC (2011*a*), State minimum purchasing prices for rapeseed, Technical report, State Administration of Grain of China.

NRDC (2011*b*), State minimum purchasing prices for rice, Technical report, State Administration of Grain of China.

NRDC (2011*c*), State minimum purchasing prices for wheat, Technical report, State Administration of Grain of China.

Palma, C. R. (2002), 'Groundwater management when water quality is endogenous.', *Journal of Environmental Economics and Management* **44**(1), 93–105.

Pethig, R. (2006), 'Non-linear production, abatement, pollution, and materials balance reconsidered.', *Journal of Environmental Economics and Management* **51**, 185–204.

Qian, X. H. (2005-2010), Annural report of anhui's national long-term monitoring field, 2005-2010., Soil property report and annual aeo programs report, Anhui Agricultural Extension Office.

Rittera, W. F., Scarborough, R. W. and Chirnside, A. E. M. (1998), 'Winter cover crops as a best management practice for reducing nitrogen leaching', *Journal of Contaminant Hydrology* **34**(1-2), 1–15.

S. L. Johnson, R. M. A. and Perry, G. (1991), 'The on-farm costs of reducing groundwater pollution.', *American Journal of Agricultural Economics* (73), 1063–73.

S. S. Vickner, D. L. Hoag, W. M. F. and Ascough, J. C. (1998), 'A dynamic economic analysis of nitrate leaching in corn production under nonuniform irrigation conditions.', *American Journal of Agricultural Economics* (80), 397–408.

Tarkalson, D. D., Payero, J. O., Hergert, G. W. and Cassman, K. G. (2006), 'Acidification of soil in a dry land winter wheat-sorghum/corn-fallow rotation in the semiarid u.s. great plains.', *Plant and Soil* **283**, 367–379.

Wang, J. N. and Luo, F. (2006), *Yearbook of Anhui Province*.

Yadav, S. N. (1997), 'Dynamic optmization of nitrogen use when groundwater contamination is internalize at the standard in the long run', *American Journal of Agricultural Economics* **79**(3), 931–945.

Yadav, S. N. and Easter, K. W. (1994), Dynamic optimzation of nitrogen use under ground water contamination., AAEA Annual Meeting, San Diego CA.

Zhao, B., Li, X., Shi, X., Huang, S. and Powlson, D. (2010), 'Long-term fertilizer experiment network in china: Crop yields and soil nutrient trends', *Agronomy* **102**, 216–230.

Table 1: Estimation of Crop Production Functions and State Equation

|  | Wheat ($y_t^1$) | Rapeseed ($y_t^1$) | Rice ($y_t^2$) | State Equation ($S_{t+1}$) |
|---|---|---|---|---|
| (Intercept) | 204.90 | -1295.44*** | 471.80 | -5.12** |
|  | (633.74) | (277.12) | (307.22) | (2.00) |
| Nitrogen in Season 1($X_t^1$) | 53.90 | 21.07 | 9.17 | 0.39** |
|  | (66.25) | (39.90) | (8.92) | (0.16) |
| Square of N in season 1($X_t^{12}$) | -1.01 | -1.34** |  |  |
|  | (2.51) | (0.53) |  |  |
| N in season 2($X_t^2$) |  |  | 71.98 | 0.03 |
|  |  |  | (50.01) | (0.29) |
| Square of N in season 2($X_t^{22}$) |  |  | -4.11** |  |
|  |  |  | (1.32) |  |
| Soil pH ($S_t$) | 33.61 | 413.52*** | 31.09 | 2.09*** |
|  | (65.87) | (71.04) | (88.33) | (0.42) |
| Square of Soil pH ($S_t^2$) | -6.41 | -26.25*** | -6.19 | -0.03 |
|  | (4.82) | (4.99) | (9.95) | (0.07) |
| Interaction between Soil pH and N in season 1 ($S_tX_t^1$) | 0.53 | 2.04 |  | -0.07 |
|  | (2.33) | (5.42) |  | (0.03) |
| Interaction between Soil pH and N in season 2 ($S_tX_t^2$) |  |  | 3.31 | -0.00005 |
|  |  |  | (7.82) | (2.00) |
| No. Observation | 23 | 28 | 31 | 45 |
| R-Squared | 0.37 | 0.32 | 0.41 | 0.80 |
| *F*-statistics of non constant terms | 4.13** | 77.55*** | 14.27*** | 23.72*** |
| *Chi*-statistics of nitrogen related terms | 53.89*** | 11.58** | 2.79** | 4.59** |
| *Chi*-statistics of soil pH related terms | 2.46 | 161.68*** | 554.22*** | 23.73*** |

Note: Robust Standard errors are in parenthesis
* Significant at 10%
**Significant at 5%
***Significant at 1%

Table 2: Monte Carlo and Bootstrapping Simulation of Crop Productions

| | Wheat ($y_t^1$) | | Rapeseed ($y_t^1$) | | Rice ($y_t^2$) | |
|---|---|---|---|---|---|---|
| | Bootstrapping | Monte Carlo | Bootstrapping | Monte Carlo | Bootstrapping | Monte Carlo |
| (Intercept) | 286.12<br>(1923.43) | | -1295.44<br>(956.83) | | 471.80<br>(3046.39) | |
| Nitrogen in Season 1($X_t^1$) | 45.35<br>(50.66) | 54.20**<br>(23.78) | 21.07<br>(68.42) | 20.62<br>(16.92) | 9.17<br>(14.13) | 9.13***<br>(3.37) |
| Square of N in season 1($X_t^{12}$) | -0.69<br>(2.25) | -1.01<br>(1.20) | -1.34<br>(1.56) | -1.32**<br>(0.65) | | |
| N in season 2($X_t^2$) | | | | | 71.98<br>(163.81) | 74.26**<br>(34.30) |
| Square of N in season 2($X_t^{22}$) | | | | | -4.11<br>(3.14) | -4.21*<br>(2.40) |
| Soil pH ($S_t$) | 29.11<br>(560.67) | 33.99<br>(26.28) | 413.52<br>(274.38) | 411.47 ***<br>(103.50) | 31.09<br>(934.34) | 34.52<br>(99.47) |
| Square of Soil pH ($S_t^2$) | -6.41<br>(42.39) | -6.40<br>(1.59) | -26.25<br>(23.47) | -25.28<br>(23.88) | -6.19<br>(75.64) | -6.40<br>(24.97) |
| Interaction between Soil pH and N in season 1 ($S_t X_t^1$) | 0.54<br>(2.26) | 0.51<br>(1.32) | 2.04<br>(13.76) | 1.94<br>(6.74) | | |
| Interaction between Soil pH and N in season 2 ($S_t X_t^2$) | | | | | 3.31<br>(18.63) | 2.96<br>(6.70) |
| No. Observation | 23 | | 28 | | 31 | |
| R-Squared | 0.46 | | 0.32 | | 0.41 | |
| $F$-statistics of non constant terms | 13.07** | | 8.61 | | 17.29** | |

Note: Robust Standard errors are in parenthesis
* Significant at 10%
**Significant at 5%
***Significant at 1%

Table 3: Myopic and Optimal Management at Steady State

| | Optimal Management | | | | Myopic Management | | | |
|---|---|---|---|---|---|---|---|---|
| | $X^1$ | $X^2$ | $S$ | $X^1 + X^2$ | $X^1$ | $X^2$ | $S$ | $X^1 + X^2$ |
| Wheat-Rice Rotation | 30.62 kg/mu | 10.88 kg/mu | 5.89 | 41.5 kg/mu | 30.40 kg/mu | 11.01 kg/mu | 5.05 | 41.41 kg/mu |
| Shadow Price of Nitrogen | -1.94 RMB | 2.7 RMB | | | | | | |
| Rapeseed-Rice Rotation | 13.29 kg/mu | 12.06 kg/mu | 7.86 | 25.35 kg/mu | 14.90 kg/mu | 11.50 kg/mu | 7.09 | 26.4 |
| Shadow Price of Nitrogen | 26.26 RMB | -4.92RMB | | | | | | |