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**Testing for the Existence of Price Points in Retail Milk Scanner Data Using
Microeconomic Theory**

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*Selected Paper prepared for presentation at the American Agricultural Economics
Association Annual Meeting, Denver, Colorado, July 1-4, 2004*

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Testing for the Existence of Price Points in Retail Milk Scanner Data Using Microeconomic Theory

Abstract

Many in the private sector believe that price points have important implications for pricing strategy. Methods to identify and incorporate price points in demand systems were developed. Empirical tests identified price points in milk scanner data, but failed to find strategically meaningful differences among splines defined by the price points.

Introduction

Price perception by the consumer is a widely researched topic for over two decades. Reference prices, loss aversion (prospect theory), price discounting by the consumer (belief that consumers disregard marginal discounts), and retailers' pricing behavior (odd number pricing, price competition, and product discounts) are some of the concepts used to explain the demand discrepancy believed to occur at certain price thresholds. Demand discrepancy in general is defined as discontinuous price responses at specific points on the demand curve. In a theoretical framework, demand discrepancies can be explained using the concept of price points and price thresholds. The concept of price thresholds can be related to the psychological process of non-response to stimuli unless there is a perceptible difference in stimuli (Luce & Edwards, 1958). Price points are specific price levels at which there is a kink in the price response function (Kalyanam, 1997). Hunt and Levy (2000), described price points as unique or magical points that strongly affect customers' perceptions of price, causing dramatic changes in volume. Merrin (1990) classified price points based on the size of the thresholds into three

categories; primary, secondary, and tertiary price points, and also noted that presence of price points leads to demand curves that are downward sloping step functions. Here price points are defined as specific points on the demand curve where the consumer response causes kinks and flat ranges or discontinuities in demand. Identifying these price points and price thresholds can inform pricing strategy (everyday low prices, frequency and magnitude of discounts) and potentially affect profitability of wholesalers and retailers.

Rigorous empirical analysis of price points and thresholds is limited and the available literature used restricted functional forms and nonparametric techniques. Kalyanam and Shively (1998) used stochastic spline models to explore the effect of own price and cross price terms on the presence of price points and thresholds.

Milk demand elasticities are a subject of controversy and are widely debated. Elasticity estimates together with price points and price thresholds are of interest to milk producers, milk cooperatives, retailers, public and private sector policy analysts as it directly impacts their decisions. The primary objective of this study is to identify and estimate the kinks, flat ranges and discontinuities in retail milk demand caused by price points. The own price, cross price, and expenditure elasticities of retail consumer demand for whole milk, 2% milk, 1% milk, and skim milk were estimated using a conditional Almost Ideal Demand System (Deaton and Muellbauer, 1980) with additional spline terms. Spline terms were formulated using the knot selection scheme proposed by Zhou and Shen (2001). Weekly retail milk scanner data for the metropolitan area of Buffalo and Rochester, New York were used in the analysis.

Methods

The revealed preference testing program NORPOR (developed by Varian, 1995) was used to simultaneously test for the Generalized Axiom of Revealed Preference (GARP) and Strong Axiom of Revealed Preference (SARP). If consumer preferences have special rationality properties (SARP) then corresponding demand curves are strictly linear and downward sloping. Consumer preferences should be at least rational in order to derive demand functions from a maximizing utility function, and the corresponding demand curves for rational preferences should be continuous downward sloping step functions. The results showed no evidence of violation of GARP but there were two marginal violations of SARP. Violations of SARP suggest that the consumer preferences are not strictly utility maximizing (not special rationality preferences). Axiomatic results are consistent with utility maximizing behavior of consumers with rational preferences, implying that the demand functions to be estimated are rational demand systems. Hence, continuous downward sloping step functions can be estimated using a parametric functional form.

The most general form of an aggregable and flexible demand system that is consistent with utility maximization behavior can have a maximum rank of three. Following Banks et al. (1997), a rank three demand system can be written as

$$w^i = a^i(p) + b^i(p) \log x + c^i(p) g(x),$$

for goods $i = 1, \dots, J$, w^i is the vector of budget shares for good i , p is the I -vector of prices, a^i , b^i and c^i are differentiable functions of price, and x and $g(x)$ are total expenditures and a smoothing function of expenditures. A rational rank four demand system proposed by Lewbel (2003) is not considered as this study limits the demand

system to have exactly aggregable properties. The general rank three form includes the homothetic, quasi-homothetic, quadratic expenditure, PIGL, PIGLOG, and LINLOG demand systems as special cases (Lewbel, 1989). Lewbel (1991) defined the rank of any demand system to be the maximum dimension of the function space spanned by the Engel curves of the demand system. Cragg and Donald (1996) formulated an asymptotic Wald-type chi-square test of whether the elements of the sub-matrix of random matrix A are zero. Random matrix A will have a rank r iff a sub-matrix of A of dimension $r \times r$ is non-zero.

Following Cragg and Donald (1996) the rank of a coefficient matrix of the demand system is found by estimating the following model

$$w^i = PX^i * A + u^i$$

for goods $i=1, \dots, J$, PX^i is a vector of higher order (m) polynomial in $x \{1, x, x^2, \dots, x^m\}$, u^i is the vector of OLS residuals, and A is the coefficient matrix of dimension $j \times q$, the rank of which is tested below. The rank of the demand system is identified by simultaneously testing the null hypothesis $\rho(A) = r$ against the alternative hypothesis that $\rho(A) > r$ when $j \geq q$ otherwise A^* is used instead of A to identify the rank. The test statistic is distributed asymptotically $\chi^2(j-r)(q-r)$ under the null hypothesis.

$$\hat{\xi} = \min_A \left\{ n \left(\text{vec}[\hat{A} - A] \right)' \hat{V}^{-1} \left(\text{vec}[\hat{A} - A] \right) : \rho(A) = r \right\},$$

where n is the number of observations, A is the matrix of coefficients, \hat{A} is the consistent estimate of A , and $\hat{V} = I \otimes (PX^* PX / n)^{-1} (1/n) \sum_{i=1}^n \hat{u}_i \hat{u}_i' \otimes px_i px_i' (I \otimes (PX^* PX / n)^{-1})$ is the White's heteroscedasticity consistent covariance matrix. Results for the rank tests

given in Table 1 suggests a rank two demand system is appropriate for the retail milk scanner data used in this study.

In order to account for the kinks and flat ranges (step functions) in the demand curve, spline terms were included in the demand system. Identifying appropriate knots is vital for formulating the spline functions. An accurate knot selection scheme proposed by Zhou and Shen (2001) was implemented for selecting the knots. This procedure overcomes knot compounding problem observed in stepwise, forward and backward selection schemes. The basic idea behind the selection scheme lies in the fact that the knot requirement is directly tied to smoothness of the function. Although not required, initial set of knots for use in Zhou and Shen's knot selection scheme were obtained by identifying statistically significant knot locations using the stepwise selection method (Smith, 1979; Marsh, 1986) as it speeds up the knot selection process. Stein's unbiased risk estimate (1981) was used as model selector to select the optimal set of knots through knot addition, knot deletion, and knot relocation. Stein's unbiased risk estimate is given

$$\text{as } R\left(\hat{f}\right) = \sum_{i=1}^n \left(y_i - \hat{f}(x_i : t) \right)^2 / n + C(k + m)\sigma^2 / n, \text{ where } n \text{ is the sample size, } k \text{ is the}$$

number of knots, C is the smoothing parameter, m is the order of the spline, σ^2 is the robust median estimator given by $\left\{ |y_{2i} - y_{2i-1}| / (0.9539) : i = 1, \dots, n/2 \right\}$. The order of the spline (m) depends on the number of derivatives required to estimate a smooth spline function. The function $\hat{f}(x_i : t)$ is approximated by cubic b-splines ($m = 4$) as they provide numerically more stable estimates than other power functions (de Boor, 1978). Cubic b-splines were formulated using three sets of minimum and maximum values as

external knots on either side, and for the first loop the internal knots were the sets of knots obtained from the stepwise selection scheme. Let $R(f)^*$ be the model selector value with the addition of new knot t and let $R(f)$ be the model selector value without the new knot. The new knot t is added as an internal knot if $R(f)^*$ is less than $R(f)$. The knot selection process gives higher priority to subintervals where a knot was added in the previous loop and lower priority to subintervals where no knot was added. A similar procedure is used for knot deletion and knot relocation. Zhou and Shen suggested using $C=2$ for getting an optimal set of knots.

To account for knots in the demand system, the expenditure function for a rank two consistent PIGL demands (Muellbauer, 1975) can be written as

$$\ln e(p, knot, z, u) = (1 - u) \ln a(p, knot, z) + u \ln b(p, knot, z) \quad \text{----- 1}$$

where $a(p, k, z)$ and $b(p, k, z)$ are linearly positive homogeneous functions of prices, knots, and other exogenous variables (z), and $e(p, u)$ is the minimum expenditure necessary to attain a given level of utility u . The homogeneous functions of prices, $\ln a(p, k, z)$ and $\ln b(p, k, z)$ may be specified in a similar manner as in the AIDS model (Deaton and Muellbauer, 1980).

$$\ln a(p, k, z) = \alpha_0 + \sum_i \alpha_i \ln p_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \frac{1}{2} \sum_{j=i, k=1}^K \lambda_{jk} d_{jk} \ln(p_j - knot_{jk}) \ln p_j$$

$$\ln b(p, k, z) = \ln a(p, k, z) + \beta_0 \prod_h p_h^{\beta_h}$$

Substituting $\ln a(p, k, z)$ and $\ln b(p, k, z)$ in equation 1 and differentiating with respect to $\ln p_j$ we get a modified version of a rank two AIDS model. The following rank two spline AIDS model is used for further analysis:

$$w_i = \alpha_i + \sum_{j=1}^J \gamma_{ij} \ln p_j + \sum_{j=i} \sum_{k=1}^K \lambda_{jk} d_{jk} \ln(p_j - knot_{jk}) + c_i s_i + \phi_i p r_i + \beta_i \ln(X / P)$$

where w_i is the budget share for good i , p_j is the price of the j th good, X is the total expenditure, $knot_{jk}$ is the knot k for p_j , s_i captures seasonality, pr_i captures product promotion effect, d_{jk} is a dummy variable defined as follows:

$$d_{jk} = \left\{ \begin{array}{l} 1 \text{ if } knot_k \geq p_j \geq \min(p_j) : k = 1 \\ 1 \text{ if } knot_{k+1} \geq p_j > knot_{k-1} : k = 1, \dots, K; knot_{K+1} = \max(p_j) \\ 0 \text{ otherwise} \end{array} \right\},$$

and $\ln(P)$ is given as follows:

$$\ln(P) = \alpha_0 + \sum_{i=1}^J \alpha_i \ln p_i + \frac{1}{2} \sum_{j=1}^J \sum_{k=1}^K \lambda_{jk} d_{jk} \ln(p_j - knot_{jk}) \ln p_j + \frac{1}{2} \sum_{j=1}^J \sum_{i=1}^J \gamma_{ij} \ln p_j \ln p_i.$$

Spline terms $\ln(p_j - knot_{jk})$ in the demand systems were derived using the optimal set of knots obtained earlier. In this formulation, the budget share is assumed to be the function of own-price spline terms alone.

Elasticity estimates at the expenditure means can be obtained using the following derivations obtained from the demand model. The expenditure elasticity is

$$\eta_{ij} = 1 + \frac{\beta_i}{w_i}.$$

The uncompensated price elasticities of demand are

$$\eta_{ij} = \delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i} \frac{d \ln P}{d \ln p_j} \text{ where } \delta_{ij} = 1 \text{ if } i=j, \text{ and } \delta_{ij} = 0 \text{ otherwise.}$$

Compensated price elasticities of demand are

$$\eta_{ij} = \delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i} \frac{d \ln P}{d \ln p_j} + w_j + w_j \frac{\beta_i}{w_i}$$

$$\text{where } \frac{d \ln P}{d \ln p_j} = \alpha_j + \frac{1}{2} \sum_{i=1}^J \gamma_{ij} \ln p_i + \frac{1}{2} \sum_{k=1}^K \lambda_{jk} d_{jk} \ln(p_j - knot_{jk})$$

Theoretical demand restrictions imposed on the parameters in the model based on statistical significance are as follows:

$$\text{Adding-up: } \sum_{i=1}^J \alpha_i = 1; \sum_{i=1}^J \gamma_{ij} = 0, \text{ for all } j; \sum_{i=1}^J \beta_i = 0; \sum_{i=1}^J \lambda_{ik} = 0, \text{ for all } k$$

$$\text{Homogeneity: } \sum_j \gamma_{ij} = 0 \text{ for all } i$$

$$\text{Symmetry: } \gamma_{ij} = \gamma_{ji} \text{ for all } i, j$$

where subscript i represent the equations and subscript j represent price terms within each equation i and subscript k represent the knots for each price term j . F-test was used to test the significance of the restrictions.

McGuirk et al. (1995) pointed out that single equation tests for econometric violations in a system setting can lead to erroneous inferences and may fail to account for cross equation influences. The system misspecification testing procedure developed by McGuirk et al. was implemented to test for econometric violations in the demand model. Specifically, a joint conditional mean test simultaneously tests for parameter instability, appropriateness of functional form, and serial independence, and a joint conditional variance test simultaneously tests for static heteroskedasticity, dynamic heteroskedasticity, and error variance instability. The tests were implemented by regressing systems of auxiliary equations. The variables trend and trend squared, lagged estimate of residuals, and squared predicted values were proxies for parameter instability, independence, and appropriateness of functional form, respectively. The squared predicted values from the original equation, lagged squared estimated residuals, trend, and trend squared terms were proxies for static heteroskedasticity, dynamic heteroskedasticity, and error variance instability, respectively. A Rao test statistic for

parametric restrictions is distributed $F(pq, rt-g)$ and it is given as follows:

$$F = \left[\frac{1 - \Lambda^{1/t}}{\Lambda^{1/t}} \right] \frac{rt - g}{pq}; t = \left[\frac{p^2 q^2 - 4}{p^2 + q^2 - 5} \right]^{1/2}; r = v - (p - q + 1)/2; g = \left(\frac{pq - 2}{2} \right)$$

where Λ is the determinant of the unrestricted error covariance matrix over the determinant of the restricted error covariance matrix, v is the error degrees of freedom from the unrestricted system of equations, p is the number of additional independent variables in the unrestricted model, q is the number of equation in the model. If the null is rejected, causes for the rejection can be identified using Rao test statistics for individual econometric violations and equation-by-equation F-tests.

After correcting for econometric violations, a system of auxiliary regression equations with the potentially endogenous variables (lnp_j) on the left-hand-side, predetermined instruments (exogenous variables, lagged exogenous variables, and lagged endogenous variables) and residuals from the original equations on the right-hand-side was implemented to test for simultaneity bias (McGuirk et al., 1995; Davidson and Mackinnon, 1993 p. 239). The Rao F-test given above was used for testing the restrictions on the residuals. An IV estimator like 3SLS can be used instead of SUR if simultaneity bias is present.

Data

Milk retail scanner data corresponding to retail stores with over \$2 million in annual sales was provided by A.C. Nielsen for the weeks ending March 2, 1996 through June 13, 1998 ($n = 120$) for Buffalo and Rochester, New York. Products consist of whole milk, reduced fat milk (2 %), low fat milk (1 %), and skim milk. The descriptive statistics are provided in Table 2. A conditional demand system was estimated as expenditure on milk accounts for a small share of overall consumer spending.

Results

The primary objective is addressed by testing the data for consumer preferences, constructing spline terms using price knots, identifying and testing the validity of the demand structure based on axiomatic tests, rank tests and system misspecification tests, and finally testing the null hypotheses that (i) all the spline coefficient terms are zero, (ii) the spline coefficient terms are equal for different milk categories.

The optimal set of knots for $p1$ (2.38, 2.56), $p2$ (2.23, 2.51), and $p4$ (2.07) obtained using the knot selection scheme were used to construct the spline terms. No specific set of knots were available for $p3$.

The system joint conditional variance tests were not rejected but the system joint conditional mean tests were rejected. Autocorrelation was identified as the cause of violation by the strong significance of the proxies used, significance of the individual system Rao test for autocorrelation, and also by equation-by-equation tests for autocorrelation. A second-order autoregressive process for $w1$ and $w2$ and a first order autoregressive process for $w3$ and $w4$ were used to correct the violation. Theoretical restrictions significant at the 5 % level were imposed in the model. Results of the system DWH test of exogeneity of prices was rejected at the 5 % level therefore the spline AIDS model was estimated using IT3SLS.

Likelihood ratio test statistics (3.28, $p = 0.9158$) failed to reject the null hypothesis (a joint test of 7 restrictions) that the spline coefficient terms have no influence. The null hypothesis that the spline coefficient terms are equal for different milk categories was also not rejected. Results of likelihood ratio tests that spline terms are zero, spline terms

are equal, system joint conditional tests, and the system DWH test for exogeneity of prices are given in table 3.

The own price elasticity of milk is a highly debated issue. Previous articles suggested that demand for fluid milk products is inelastic (Huang, 1993; Suzuki and Kaiser, 1997), but wholesalers and retailers argue that milk demand is elastic. Compensated elasticities were estimated at the mean values for different spline sections. Own price elasticity estimates for whole milk, 2 % milk, 1 % milk, and skim milk were -0.45, -1.33, -1.55, and -0.81, respectively. Own price compensated elasticity estimates for all spline sections across different milk categories were consistent with the estimates given above. The elasticity estimates for 2 % milk and 1 % milk were elastic compared to previous studies dealing with aggregate data while the elasticity estimates for whole milk and skim milk more closely resembled those in the literature. The compensated price elasticity matrix estimated using the spline AIDS model is given in table 4.

Lack of storability of milk discourages stockpiling during product discounts. Lack of storability and higher purchase frequency could be the reason for the insignificant differential demand response for price movements away from the knots. Price increases above the knots, or price decreases below the knots, do not seem to have a statistically significant differential impact on the quantity of milk purchased. Although clear evidence of kinks and flat ranges would be valuable information for managers, the lack of significant price points found in this study could also be important from a managerial perspective. Demand curves predicted by the spline AIDS model for whole milk, 2 % milk, 1 % milk, and skim milk are given in figures 1-4, respectively. Even though the

spline terms are insignificant, the predicted demand curves do show evidence of kinks and flat ranges.

The primary contribution of this study is the development of feasible methods by which price points may be rigorously identified and incorporated into demand systems analysis. The data selected for application did not produce evidence of differential price sensitivity at price points, but application to other products may well produce different results. Future work might apply the framework developed in this paper to more disaggregated data than were used in this study, to data on storable products, and to products with wider price variation.

References

- Banks, J. W., R. W. Blundell, and A. Lewbel, "Quadratic Curves and Consumer Demand." *The Review of Economics and Statistics* 79,4(Nov., 1997):527-539.
- Cragg, J. G., and S. G. Donald, "On the Asymptotic Properties of LDU-Based Tests of the Rank of a Matrix." *Journal of the American Statistical Association* 91,435(Sep., 1996):1301-1309.
- Davidson, R., and J.G. MacKinnon. *Estimation and Inference in Econometrics*. New York: Oxford University Press, 1993.
- Deaton, Angus, and John Muellbauer, "An Almost Ideal Demand System." *The American Economic Review* 70,3(Jun., 1980): 312-326
- Huang, K. S. "A Complete System of U.S. Demand for Food." Technical Bulletin No. 1821, USDA, Economic Research Service, Washington, DC, September 1993.
- Hunt, P, and O. Levy, "Price Perception Research." *Imprints* (March, 2000):12-13.
- Kalyanam, K, and T. S. Shively, "Price Point Effects: Why Some Prices are Better Than Others for Price Competition." Fourth Workshop on Bayesian Statistics in Science and Technology: Case Studies, September 25 - 27, June 1997.
- Kalyanam, K, and T. S. Shively, "Estimating Irregular Pricing Effects: a Stochastic Spline Research Approach." *Journal of Marketing Research* 35(Feb., 1998):16-29.
- Lewbel, A., "Exact Aggregation and A Representative Consumer." *The Quarterly Journal of Economics* 104,3(Aug., 1989):621-633.
- Lewbel A., "The Rank of Demand Systems: Theory and Nonparametric Estimation." *Econometrica* 59,3(May, 1991):711-730.

Lewbel A., "A Rational Rank Four Demand System." *Journal of Applied Econometrics* 18,2(March/April, 2003): 127-135.

Maynard, L. J. "Empirical Tests of the Argument that Consumers Value Stable Retail Milk Prices." *Journal of Agribusiness* 18,2(Spring 2000):155-172

McGuirk, A., P. Driscoll, J. Alwang, and H. Huang. "System Misspecification Testing and Structural Change in the Demand for Meats." *Journal of Agricultural and Resource Economics* 20(1995):1-21.

Muellbauer, J. "Aggregation, Income Distribution and Consumer Demand." *Review of Economic Studies* 42(1975): 525-43.

Stein, C. "Estimation of the Mean of a Multivariate Normal." *The Annals of Statistics* 9(1981):1135-1151.

Suzuki, N., and H. M. Kaiser. "Imperfect Competition Models and Commodity Promotion Evaluation: The Case of U.S. Generic Milk Advertising." *Journal of Agricultural and Applied Economics* 29,2(Dec., 1997): 315-325

Zhou, Shanggan and Xiaotong Shen, "Spatially Adaptive Regression Splines and Accurate Knot Selection Schemes." *Journal of the American Statistical Association* 96, 453(Mar 2001): 247-259

Table 1. Rank Test for Retail Milk Scanner Data

| Test | χ^2 Statistics (df) | <i>p</i> -value |
|------|--------------------------|-----------------|
| r=1 | 2088607.4 (12) | < 0.0001 |
| r=2 | 0.01856 (6) | 0.99 |

Table 2. Descriptive Statistics for Weekly Milk Scanner Data

| | <i>Whole Milk (gallons)</i> | <i>2 % Milk (gallons)</i> | <i>1 % Milk (gallons)</i> | <i>Skim Milk (gallons)</i> | <i>Price of Whole Milk (\$/gallon)</i> | <i>Price of 2 % Milk (\$/gallon)</i> | <i>Price of 1 % Milk (\$/gallon)</i> | <i>Price of skim milk (\$/gallon)</i> |
|--------------------|-----------------------------|---------------------------|---------------------------|----------------------------|--|--------------------------------------|--------------------------------------|---------------------------------------|
| Mean | 138246 | 383315 | 137286 | 192412 | 2.57 | 2.31 | 2.36 | 2.40 |
| Standard Deviation | 10810 | 43438 | 15651 | 17850 | 0.13 | 0.17 | 0.18 | 0.16 |
| Minimum | 117019 | 281254 | 105793 | 151771 | 2.26 | 1.94 | 1.96 | 2.02 |
| Maximum | 170626 | 504215 | 176612 | 233736 | 2.87 | 2.72 | 2.78 | 2.75 |

Table 3. Endogeneity, System Misspecification, and Model Performance Test Statistics for the Spline AIDS model

| System Misspecification Tests | | |
|-------------------------------|-----------------|-------------------------------|
| Spline AIDS model | Rao F-statistic | Critical Value |
| Joint conditional mean | 0.0511 | $F_{0.05}^c(28,344) = 1.5091$ |
| Joint conditional variance | 1.3155 | $F_{0.05}^c(28,380) = 1.5061$ |

| System DWH Endogeneity Test | | |
|-----------------------------------|-----------------|------------------------------|
| Spline AIDS model | Rao F-statistic | Critical Value |
| $\text{Lnp}1, \dots, \text{lnp}4$ | 11.8855 | $F_{0.05}^c(4,107) = 2.4566$ |

| Likelihood Ratio Test that all spline terms are zero | |
|--|---------|
| L.R. Statistic | p-value |
| 2.74 | 0.9495 |

| Likelihood Ratio Test that all spline terms are equal for milk categories | | |
|---|----------------|---------|
| Milk category | L.R. Statistic | p-value |
| whole milk | 0.41 | 0.8152 |
| 2 % milk | 0.38 | 0.8282 |
| skim milk | 0.43 | 0.5122 |

Table 4. Estimated Compensated Price Elasticity Matrix from the Spline AIDS model for the Spline term $PI < 2.38$

| | Whole Milk | 2 % Milk | 1 % Milk | Skim Milk |
|------------|----------------------|-----------------------|-----------------------|-----------------------|
| Whole Milk | -0.4547 (1.0897) | 0.8650 * (0.2989) | 0.0601 (0.2703) | -0.0262 (0.2596) |
| 2 % Milk | 0.3423 * (0.1202) | -1.3269 * (0.3005) | 0.5980 * (0.2259) | 0.3189 (0.2465) |
| 1 % Milk | 0.1109 (0.3214) | 1.2264* (0.1881) | -1.5510 * (0.5680) | 0.6129 * (0.2522) |
| Skim Milk | -0.0182 (0.1959) | 0.5573 * (0.2571) | 0.4170 * (0.1774) | -0.8143 * (0.4057) |

Standard errors are in parentheses.

* denotes statistical significance at the 0.05 level

Figure 1. Predicted Demand Curve for Whole Milk

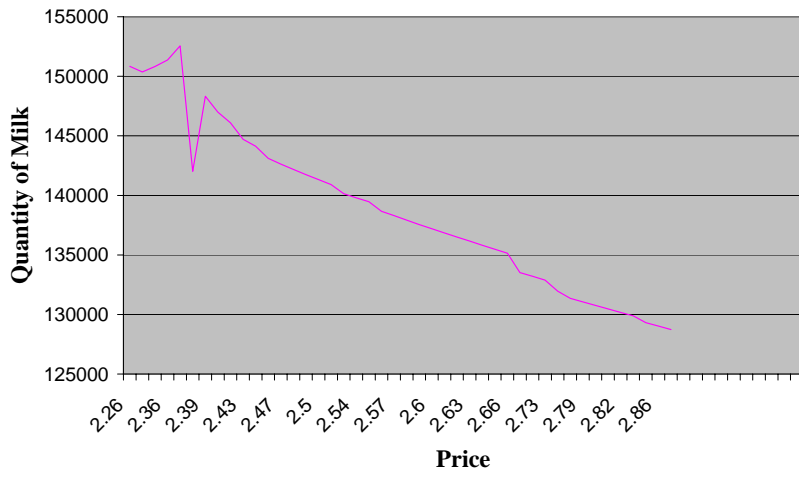


Figure 2. Predicted Demand Curve for 2 % Milk

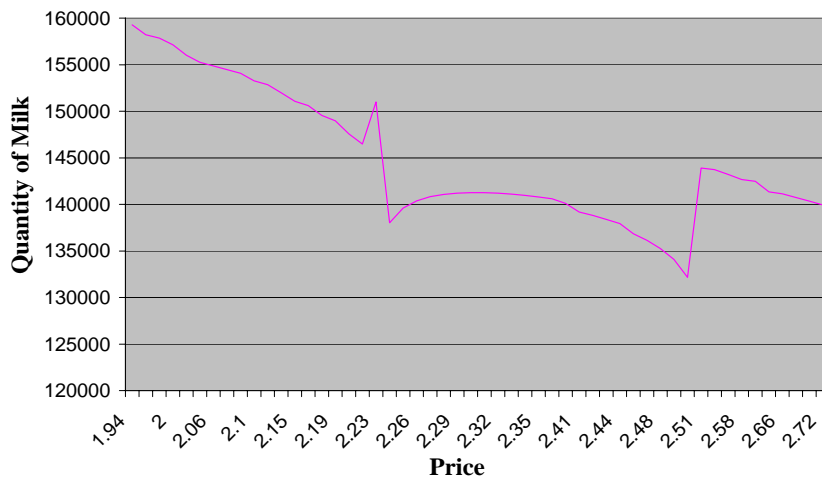


Figure 3. Predicted Demand Curve for 1 % Milk

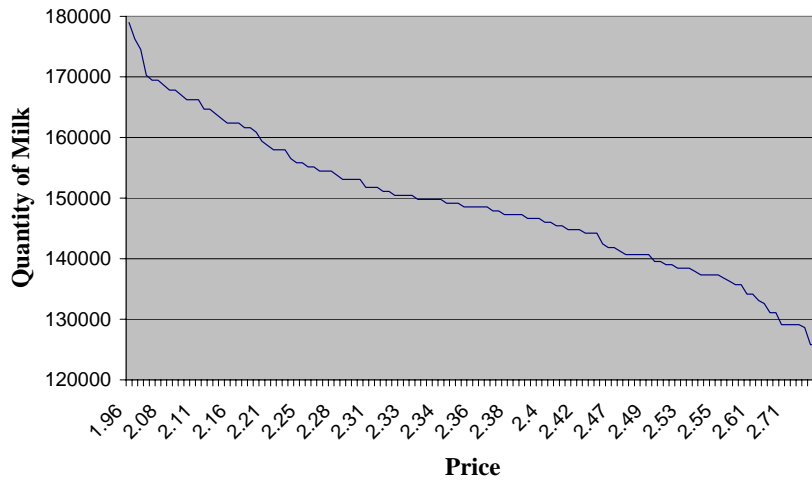


Figure 4. Predicted Demand Curve for Skim Milk

