Simulating the Impacts of Contract Supplies in a Spot Market-Contract Market Equilibrium Setting

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Abstract: This paper embeds a principal-agent model of producer-processor equilibrium within a market equilibrium model of contract and cash markets to analyze the impact of contracting on the spot market for hogs. The principal-agent model incorporates both quality differentiation in the contract market and an endogenously determined cash market price to account for processor-producer relationships in equilibrium. For five types of contracting scenarios, market equilibrium conditions are derived, and results are presented for a numerical example. Contrary to previous results, the paper finds that the increased supply of hogs under typical formula-price contracts can increase the cash market price and reduce its variance.

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Livestock contracting and other forms of vertical coordination can provide positive benefits by offering a mechanism to smooth production, share risk, and provide proper incentives for attainment of difficult-to-observe quality attributes (see, for example, Lawrence, Schroeder, and Hayenga; Hayenga et al.; Martin; Hueth and Ligon; Goodhue; Tsoulouhas and Vukina). However, increased acquisitions under contract or agreement, a phenomenon sometimes called captive supplies by beef industry observers, can also cause legitimate concern if adverse impacts arise from a thinning of the cash market (Hayenga et al.). For hogs, Hayenga et al. report that the cash market volume is dropping sharply, and price reporting will become more problematic. The same report also suggests that for the beef sector increased captive supplies and packer influence may lead to market price declines or outright price manipulation.

Empirical market analyses and other theoretical studies devoted to this issue show mixed results, with several concluding that the increasing use of contracting in meat packing reduces spot market prices and makes cash prices more volatile. Some empirical analyses that investigate beef prices, for example, find that increased captive supplies or forward contracting can reduce the cash price (Elam; Schroeder et al.; Ward, Koontz, and Schroeder, 1996) or increase price variability (Barkley and Schroeder). Alternatively, others (e.g., Ward, Koontz, and Schroeder, 1998) find ambiguous price effects due to shifts in both market supply and demand. Some theoretical studies (e.g., Xia and Sexton) show that contracts with special features such as best-price, top-of-the market clauses reduce cash market prices, while others (e.g., Azzam) find ambiguous results because captive-supply-induced shifts in market demand and supply are not explicitly modeled.

Most if not all of these studies concern the cattle rather than the hog industry. Moreover, these studies, which employ different empirical techniques, data, and model specifications, fall short of providing a definitive description of impacts of contracting on the cash prices. Despite evidence to the contrary, none of them incorporates asymmetric information into their models, especially imperfectly observed quality differences in the spot market and contract market. For example, several studies summarized by Hayenga et al. report significant quality differences in hog quality sourced from contracts and cash market transactions. In addition, reflecting quality differentials, average contract prices are consistently higher than spot market prices (Hayenga et al., Buhr and Kunkel).

Different from existing studies, this paper uses a structural model to analyze the impact of contracting on the spot market for hogs. To account for quality differentiation in the contract market, a principal-agent framework that incorporates asymmetric information on hog quality is used to model individual processor-producer relationships. For each type of contract analyzed, the market equilibrium is derived via a general equilibrium model by aggregating individual demand and supply. Further, in order to analyze the impact that contracting has on the hog spot market, a sensitivity analysis is performed by modifying the model parameters indicating the extent of contracting. Contributions to the existing literature come from (i) embedding the principal-agent model of processor-producer equilibrium behavior within a general equilibrium model of the hog market and (ii) endogenizing the producers' participation constraint by linking the producers' contracting decision to the general-equilibrium determined spot price of hogs.

Existing Marketing Contracts in the Hog Sector

Buhr and Kunkel and Hayenga et al. summarize the types of marketing contracts available in the hog sector. While more than a half dozen types of contracts exist, this paper focuses on four major types that offer substantial differences: fixed-price contracts, market price contracts, formula-price contracts with quality premiums, and cost-plus contracts with quality premiums.[1]

Formula-price contracts (which account for 47.2% of all contract types) are based on spot market prices plus a price premium or discount. Some observers have argued that formula price contracts do not provide price protections, as they fluctuate along with the market price on which they are based.

Cost-plus contracts specify a price based on feed costs, which comprise the greatest single cost of production. By implicitly setting a minimum price level, these contracts provide risk protection in addition to quantity assurance and market access. These contracts may also have a balancing clause where payments are made to contractors/processors when market prices are below the contract prices and vice versa.

Fixed-price and market-price contracts are relatively self-explanatory. The market-price contracts modeled below are a simplified synthesis of the basic features of price-floor contracts and price-window contracts, which are used in the hog sector to set a minimum and perhaps a maximum price. When the market price falls above the ceiling or below the floor price, the packer and the producer generally split the difference between the two prices.

The Model

The model developed below is compartmentalized in three stages: In stage I, processors compete for producers to whom they offer contracts, and each participating producer signs a contract with a processor. In stage II, each producer determines how many hogs to produce and deliver to the cash market. In stage III, when the cash market settles, each processor decides the

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[1] Since packer-fed supplies account for only a small portion of pork packers' procurement of hogs, they are excluded from this study. In addition, marketing contracts related to the futures market are also excluded because they are beyond the interest of this paper.

quantity to purchase in the cash market and both the contract and cash markets clear. There are N homogenous producers and M homogenous processors in this model of the pork sector, with $M<<N$. In the first stage, each producer decides whether to sign a contract or not. Suppose n_1^j producers sign a contract with processor j, where the subscript 1 denotes the contract market. For simplicity, we assume n_1^j is same for every processor j. Without loss of generality, we employ Xia and Sexton's (2004) assumption that each producer has a short-run supply function, $q = f(\varpi) = \varpi$, where q is the quantity of hogs produced and ϖ is the expected price the producer receives[2]. Each contract producer i independently produces a quantity of hogs q_0 based on the short-run supply function and sells a fixed proportion $\beta \in (0,1)$ of his hogs to a processor[3]. Thus, each processor j obtains $Q_1^j = n_1^j \beta q_0$ hogs from the contract market. On the other hand, those producers who do not participate in the contract independently decide to produce a quantity q_s, again based on the short-run supply function. Each processor converts procured hogs into a finished product according to a production function $g = g(Q \mid z)$, where z denotes the quality of hogs procured and is observable only to producers before delivery. The production function is assumed to be concave in Q and z with $g_Q(Q \mid z) > 0$, $g_{QQ}(Q \mid z) \leq 0$ and $g_z(Q \mid z) > 0$, $g_{zz}(Q \mid z) \leq 0$, $g_{zQ}(Q \mid z) > 0$. Further, each processor incurs costs $h = h(Q \mid z)$ depending on the quality of hogs procured, with $h(.)$ being convex in Q and with $h_Q(Q \mid z) > 0$, $h_{QQ}(Q \mid z) \geq 0$, and $h_z(Q \mid z) < 0$.

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2 Xia and Sexton (2004) studied market-price clause and captive supplies in the beef, not hog sector.
3 The specification of β allows one to investigate the effect of contract supplies on the market equilibrium while significantly simplifying the analysis. An economic interpretation of this specification is that β can be viewed as the exogenous hedge ratio of individual processors. Finally, treating β as an exogenous parameter can guarantee the existence of the cash market even in cases when contracts are widely preferred in a general equilibrium setting.

Since the true hog quality is unobservable to processors before delivery, we assume that processors observe the market price of the finished products, such as meat cuts, as an imperfect signal of the true quality of hogs delivered. More specifically, we assume that the market price of the finished product is random based on a PDF $f(P|\, z)$ and a corresponding CDF $F(P|\, z)$. It is assumed that the CDF $F(P|\, z)$ satisfies first-order stochastic dominance.

Since each processor purchases hogs from the cash market on live-weight or carcass weight basis, different qualities are not distinguished as precisely as in the contract market. To simplify the analysis, we assume that only average quality is observed in the cash market. Therefore, Akerlof's lemons argument applies and cash market prices would not provide sufficient incentives for hog producers to produce high-quality hogs. Hence, following this argument, we assume that independent producers not participating in contractual relationships will produce only low-quality hogs $\{\underline{z}\}$, while contract producers (who also sell some hogs in the cash market) will produce either high- or low-quality hogs depending on the individual contract. For simplicity, the quality of hogs available in the cash market is specified as the arithmetic mean of hog qualities sold by contract and independent producers to the spot market.[4]

The unobservability of quality also plays an important role in how payoffs are structured. Since quality is observable only to producers, it cannot be explicitly contracted. In addition, in order to procure high-quality hogs from the contract-participating producers, processors must provide enough incentive to encourage high quality from producers. Therefore, the contract price paid to producers by a processor, $w(P)$, must depend on the market price of the finished product, which can be regarded as the imperfect quality signal.

[4] An alternative assumption is that the quality of hogs available in the cash market is the weighted average of hog quality sold. However, this treatment would significantly complicate the derivation of equilibrium conditions without altering the nature of the results.

To simplify the analysis further, we assume that the output function of each processor is a linear function $g(Q_t \mid z) = \alpha_z Q_t$ with $\alpha_{\underline{z}} < \alpha_{\bar{z}}$ indicating the fact that high-quality hogs yield more finished product than low-quality hogs. The processing cost function for each packer takes a quadratic form $h(Q_t \mid z) = \frac{1}{2}\gamma_z (Q_t + \mu_t)^2$ where μ_t is a serially uncorrelated normally-distributed random variable with mean zero and variance σ_μ^2 affecting the processing cost function at time *t*. Additionally, it is assumed that $\gamma_{\bar{z}} < \gamma_{\underline{z}}$, reflecting the fact that low-quality hogs incur higher processing costs than high-quality hogs.

Contract producers have a time-invariant utility function $u(W) - v(z, q_0)$, where $W = \beta w(P) q_0 + (1-\beta) p_t^s q_0$ represents the total revenue of each contract-participating producer from both the contract market and the cash market, and p_t^s is the spot market price at time *t*. However, for independent producers, total revenue comes only from the spot market; that is, $W = p_t^s q_s$. Additionally, it is assumed that *u* is concave in *W* with $u'(W) > 0$ and $u''(W) \le 0$. In most cases, producers' utility functions are assumed to have the property of constant absolute risk aversion (CARA), $u(W) = 1 - \exp(-rW)$, where *r* is the Arrow-Pratt coefficient of absolute risk aversion. Then the expected utility $E[u(W)]$ is tantamount to $EW - \frac{1}{2} r \operatorname{var}(W)$. Finally, each producer incurs disutility according to the function $v(z, q_0) = c_z q_0^2$ with $c_{\underline{z}} < c_{\bar{z}}$.

Given the above assumptions, each processor maximizes its net profit:

$$(1) \quad \max_{w, q_2^j, n_1^j} \Pi = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid z) - h(Q_{1t}^j \mid z) - w(P) Q_{1t}^j] dF(P \mid z) + \int_{P\in\Omega} [Pg(q_{2t}^j \mid \tilde{z}) - h(q_{2t}^j \mid \tilde{z}) - p_t^s q_{2t}^j] dF(P \mid \tilde{z})$$

subject to:

$$(2) \quad \int_{P\in\Omega} E_{t-1}[u(\beta w(P) q_0 + (1-\beta) p_t^s q_0)] dF(P \mid z) - v(z, q_0) \ge E_{t-1}[u(p_t^s q_0)] - v(\underline{z}, q_0), \quad \forall z \in \{\underline{z}, \bar{z}\}$$

(3) $z \in \arg\max_{\hat{z}} \int_{P \in \Omega} E_{t-1}[u(\beta w(P) q_0 + (1-\beta) p_t^s q_0)] dF(P \mid \hat{z}) - v(\hat{z}, q_0), \qquad \forall z \in \{\underline{z}, \overline{z}\}$

where

E_{t-1} = Mathematical expectation operator of spot market price conditional on information available at time t-1,

$Q_{1t}^j = n_1^j \beta q_0$ hogs to be procured by processor j from the contract market,

q_{2t}^j = Hogs to be procured by processor j from the spot market,

$\widetilde{z}$ = Average quality of hogs sold in the cash market, and

p_t^s = Market price of hogs sold in the cash market at time t.

The individual rationality constraint (2) requires that the expected payoff to each participating contract producer should be no less than that when he sells all his hogs to the cash market. The incentive compatibility constraint (3), which also contains the endogenous cash price, ensures that under the compensation schedule $w(P)$ the producer's optimal quality choice is z.

The market equilibrium then requires that aggregated supply equals aggregated demand in both the contract market and the cash market. Further, we assume that the contract market supply is perfectly elastic; therefore, we only need to solve the equilibrium spot market price. Specifically, market-clearing in the spot market requires the following condition:

(4) $Q_{2s} = Q_{2d} \Rightarrow (N - Mn_1^j) q_s + M(1-\beta) q_0 n_1^j (E_{t-1} p_t^s \mid \overline{z}) = Mq_{2t}^j (p_t^s \mid \widetilde{z})$,

From this general "dual equilibrium" setting, five separate cases are analyzed: (i) In case one, the processor optimally offers the producer a fixed price $w(P) = \underline{w}$ independent of P, and the producer is risk neutral, i.e., $u(W) = W$. (ii) In case two, the contract price is again fixed, but now the producer is risk averse, i.e., $u(W) = 1 - \exp(-rW)$. (iii) In case three, the contract price is set equal to the spot market price, i.e., $w(P) = p_t^s$. (iv) In case four, which examines a formula-price contract with a quality premium, we assume the contract takes a linear form in terms of the market price of the finished product, *P*, i.e., $w(P) = p_t^s + a + bP$. (v) In case five, a cost-plus contract with a quality premium, we also assume that the contract takes a linear form, $w(P) = c_{\tilde{z}} + a + bP$ (where it is assumed that $c_{\tilde{z}} = (c_{\underline{z}} + c_{\bar{z}})/2$).

These five cases are essentially solved the same way: The overall market equilibrium is solved for simultaneously with the principal-agent equilibrium. In terms of the set up and solution methods, the only differences among the cases are the form of the payment *w(P)* and the form of the utility function *u(W)*. Because of these similarities, the solution for only one case, case (iv), is described below.[5] In terms of outcomes, however, we find that a fixed-contract price or a market-price contract can induce producers to produce only low-quality hogs, while formula-price and cost-plus contracts with quality premium can induce high-quality hogs from contract-participating producers.

Market conditions for formula-price contracts with quality premium

In this case, we assume the average quality of hogs in the cash market will be an arithmetic average of high quality and low quality: specifically, $\tilde{z} = (\bar{z} + \underline{z})/2$. Additionally, we assume the marginal product of finished hogs acquired from the spot market is $\alpha_{\tilde{z}} = (\alpha_{\bar{z}} + \alpha_{\underline{z}})/2$. However, producers who sign a formula-price contract with price premium will produce high-quality hogs only. The processing cost still takes the form $h(Q_t \mid z) = \gamma_z (Q_t + \mu_t)^2 / 2$ with $\gamma_{\bar{z}} < \gamma_{\tilde{z}} \le \gamma_{\underline{z}}$, where $\gamma_{\tilde{z}}$ is defined by $\gamma_{\tilde{z}} = (\gamma_{\bar{z}} + \gamma_{\underline{z}})/2$.

[5] The solutions for the other cases can be obtained by contacting the authors.

Given these assumptions, each processor maximizes its net profit subject to each producer's participation constraint and incentive compatibility constraint. That is,

$$\max_{a,b,q_2^j,n_1^j} \Pi = \int_{P\in\Omega} [Pg(Q_{1t}^j \mid \bar{z}) - h(Q_{1t}^j \mid \bar{z}) - [p_t^s + a + bP]Q_{1t}^j] dF(P \mid \bar{z}) + \int_{P\in\Omega} [Pg(q_{2t}^j \mid \tilde{z}) - h(q_{2t}^j \mid \tilde{z}) - p_t^s q_{2t}^j] dF(P \mid \tilde{z}) \tag{5}$$

subject to

$$\int_{P\in\Omega} E_{t-1}[u(\beta q_0(p_t^s + a + bP) + (1-\beta)p_t^s q_0)] dF(P \mid \bar{z}) - v(\bar{z}, q_0) \geq E_{t-1}[u(p_t^s q_0)] - v(\underline{z}, q_0) \tag{6}$$

$$\bar{z} \in \arg\max_{\hat{z}} \int_{P\in\Omega} E_{t-1}[u(\beta q_0(p_t^s + a + bP) + (1-\beta)p_t^s q_0)] dF(P \mid \hat{z}) - v(\hat{z}, q_0) \quad . \tag{7}$$

Before deriving the first-order conditions, the parameters {a, b} in the contract price can be derived as follows. Given the specification of the contract price, conditions (6) and (7) must be binding because, otherwise, the processor can always reduce the contract price until both of the constraints become equalities. Given each producer's gross revenue,

$W = \beta q_0(p_t^s + a + bP) + (1-\beta)p_t^s q_0 = p_t^s q_0 + \beta q_0(a + bP)$, for any P we have

$EW = E_{t-1} p_t^s q_0 + \beta q_0(a + bP)$, and $\mathrm{var}(W) = q_0^2 \ \mathrm{var}(p_t^s)$.

Thus, the condition (6) is equivalent to

$$a\beta q_0 + b\beta q_0 E[P \mid \bar{z}] = v(\bar{z}, q_0) - v(\underline{z}, q_0) . \tag{8}$$

Similarly, the condition (7) becomes

$$\beta q_0 b E[P \mid \bar{z}] - \beta q_0 b E[P \mid \underline{z}] = v(\bar{z}, q_0) - v(\underline{z}, q_0). \tag{9}$$

Thus, the parameters {a, b} in the contract price can be computed by the conditions (8) and (9). Precisely,

$$a = -\frac{[v(\bar{z}, q_0) - v(\underline{z}, q_0)]E[P \mid \underline{z}]}{\beta q_0(E[P \mid \bar{z}] - E[P \mid \underline{z}])} = -\frac{[v(\bar{z}, q_0) - v(\underline{z}, q_0)]P^{\underline{z}}}{\beta q_0(P^{\bar{z}} - P^{\underline{z}})}, \text{ and} \tag{10}$$

(11) $$b = \frac{v(\bar{z},\, q_0) - v(\underline{z},\, q_0)}{\beta q_0 (E[P \mid \bar{z}] - E[P \mid \underline{z}])} = \frac{v(\bar{z},\, q_0) - v(\underline{z},\, q_0)}{\beta q_0 (P^{\bar{z}} - P^{\underline{z}})}.$$

Substituting (10) and (11) into the price specification, $w(P) = p_t^s + a + bP$, yields the contract price $w(P) = p_t^s + \frac{[v(\bar{z},\, q_0) - v(\underline{z},\, q_0)]}{\beta q_0 (P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}})$. Note that the optimal contract price consists of the spot market price and a quality premium, which is positively related to the difference between the observed price of finished products and the expected price of finished products of low quality.

Furthermore, given $v(z,\, q_0) = c_z q_0^2$, the contract price can be written as

(12) $$w(P) = p_t^s + \frac{c_{\bar{z}} q_0^2 - c_{\underline{z}} q_0^2}{\beta q_0 (P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}}) = p_t^s + \frac{(c_{\bar{z}} - c_{\underline{z}}) q_0}{\beta (P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}}).$$

Given the contract price (12) and producers' short-run supply function, each contract producer produces the following quantity:

$$\begin{aligned} q_0 &= \beta E[w(P) \mid \bar{z}] + (1-\beta) E_{t-1} p_t^s = \beta E[p_t^s + \frac{(c_{\bar{z}} - c_{\underline{z}}) q_0}{\beta (P^{\bar{z}} - P^{\underline{z}})}(P - P^{\underline{z}}) \mid \bar{z}] + (1-\beta) E_{t-1} p_t^s \\ &= E_{t-1} p_t^s + (c_{\bar{z}} - c_{\underline{z}}) q_0. \end{aligned}$$

Hence,

(13) $$q_0 = \frac{E_{t-1} p_t^s}{1 - (c_{\bar{z}} - c_{\underline{z}})}.$$

Again, independent producers choose to produce

(14) $$q_s = E_{t-1} p_t^s.$$

Thus, the first-order optimality conditions to this problem are ready to be derived. First, the optimal quantity of hogs demanded from the spot market, q_{2t}^j, must satisfy

$$\frac{\partial \Pi}{\partial q_{2t}^j} = \int_{P \in \Omega} [P \alpha_{\tilde{z}} - \gamma_{\tilde{z}} (q_{2t}^j + \mu_t) - p_t^s] dF(P \mid \tilde{z}) = 0,$$

from which

(15) $$q_{2t}^{j} = \frac{\alpha_{\tilde{z}} P^{\tilde{z}} - p_t^s}{\gamma_{\tilde{z}}} - \mu_t \,.$$

Second, the number of producers that each processor contracts with, n_1^j, must satisfy

$$\frac{\partial \Pi}{\partial n_1^j} = \int_{P \in \Omega} [P\alpha_{\bar{z}}\beta q_0 - \gamma_{\bar{z}}\beta q_0 (Q_{1t}^j + \mu_t) - \beta q_0 (p_t^s + a + bP)] dF(P \mid \bar{z}) = 0,$$

from which we can obtain

(16) $$n_1^j = \frac{\alpha_{\bar{z}} P^{\bar{z}} - p_t^s - a - bE[P \mid \bar{z}] - \gamma_{\bar{z}}\mu_t}{\gamma_{\bar{z}}\beta q_0} \,.$$

The spot market price can be obtained by setting market demand equal to market supply in the spot market. That is, $Q_{2s} = Q_{2d}$,

(17) $$(N - Mn_1^j) q_s + M(1-\beta) q_0 n_1^j (E_{t-1} p_t^s \mid \bar{z}) = Nq_s - Mn_1^j (E_{t-1} p_t^s \mid \bar{z})[q_s - (1-\beta) q_0] = Mq_{2t}^j (p_t^s \mid \tilde{z}).$$

Substituting the conditions (13), (14), (15), and (16) and taking the expectation operator E_{t-1} on both sides (applying the assumption $E_{t-1}\mu_t = 0$), we can derive the expected spot market price:

(18) $$E_{t-1} p_t^s = \frac{\dfrac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \dfrac{\alpha_{\bar{z}} P^{\bar{z}} [\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\dfrac{N}{M} + \dfrac{1}{\gamma_{\tilde{z}}} + [\dfrac{1}{\gamma_{\bar{z}}} - \dfrac{(1-\beta)(c_{\bar{z}} - c_{\underline{z}})^2}{\gamma_{\bar{z}}\beta^2 [1 - (c_{\bar{z}} - c_{\underline{z}})]}]} \,.$$

Substituting (18) back into (17) solves the spot market price:

(19) $$p_t^s = \frac{\dfrac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \dfrac{\alpha_{\bar{z}} P^{\bar{z}} [\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\gamma_{\bar{z}}\beta}}{\dfrac{N}{M} + \dfrac{1}{\gamma_{\tilde{z}}} + [\dfrac{1}{\gamma_{\bar{z}}} - \dfrac{(1-\beta)(c_{\bar{z}} - c_{\underline{z}})^2}{\gamma_{\bar{z}}\beta^2 [1 - (c_{\bar{z}} - c_{\underline{z}})]}]} - \frac{\gamma_{\bar{z}}\gamma_{\tilde{z}} [2\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}} (c_{\bar{z}} - c_{\underline{z}})} \mu_t \,.$$

Hence, the variance of the spot market price can be computed as

(20) $$\mathrm{var}(p_t^s) = \{\frac{\gamma_{\bar{z}}\gamma_{\tilde{z}} [2\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}} (c_{\bar{z}} - c_{\underline{z}})}\}^2 \sigma_\mu^2 \,.$$

Substituting (19) into (15) yields the quantity of hogs demanded from the spot market by each processor,

(21) $$q_{2t}^{j} = \frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} - \frac{1}{\gamma_{\tilde{z}}} \frac{\frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} + \frac{\alpha_{\bar{z}} P^{\bar{z}} [\beta - (c_{\bar{z}} - c_{\underline{z}})]}{\gamma_{\bar{z}} \beta}}{\frac{N}{M} + \frac{1}{\gamma_{\tilde{z}}} + [\frac{1}{\gamma_{\bar{z}}} - \frac{(1-\beta)(c_{\bar{z}} - c_{\underline{z}})^2}{\gamma_{\bar{z}} \beta^2 [1 - (c_{\bar{z}} - c_{\underline{z}})]}]} + \frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})(c_{\bar{z}} - c_{\underline{z}} - \beta)}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})} \mu_t .$$

Similarly, substituting (10), (11), (13), and (19) into (16) yields the number of producers with which each processor signs a contract:

(22) $$n_1^j = \frac{\beta[1 - (c_{\bar{z}} - c_{\underline{z}})] \alpha_{\bar{z}} P^{\bar{z}}}{\gamma_{\bar{z}} \beta^2 E_{t-1} p_t^s} - \frac{\beta + (1-\beta)(c_{\bar{z}} - c_{\underline{z}})}{\gamma_{\bar{z}} \beta^2} + \frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})[1 - (c_{\bar{z}} - c_{\underline{z}})]}{[\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})] E_{t-1} p_t^s} \mu_t .$$

We can now compute each processor's profit under the formula-price contract,

$$\begin{aligned}
\Pi_4^* = & \int_{P \in \Omega} [Pg(Q_{1t}^j \mid \bar{z}) - h(Q_{1t}^j \mid \bar{z}) - (p_t^s + a + bP) Q_{1t}^j] dF(P \mid \bar{z}) + \int_{P \in \Omega} [Pg(q_{2t}^j \mid \tilde{z}) - h(q_{2t}^j \mid \tilde{z}) - p_t^s q_{2t}^j] dF(P \mid \tilde{z}) \\
= & \frac{1}{2} \gamma_{\bar{z}} (\beta q_0)^2 \{ [\frac{\beta[1 - (c_{\bar{z}} - c_{\underline{z}})] \alpha_{\bar{z}} P^{\bar{z}}}{\gamma_{\bar{z}} \beta^2 E_{t-1} p_t^s} - \frac{\beta + (1-\beta)(c_{\bar{z}} - c_{\underline{z}})}{\gamma_{\bar{z}} \beta^2}]^2 + [\frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})[1 - (c_{\bar{z}} - c_{\underline{z}})] \mu_t}{[\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})] E_{t-1} p_t^s}]^2 \\
& + 2[\frac{\beta[1 - (c_{\bar{z}} - c_{\underline{z}})] \alpha_{\bar{z}} P^{\bar{z}}}{\gamma_{\bar{z}} \beta^2 E_{t-1} p_t^s} - \frac{\beta + (1-\beta)(c_{\bar{z}} - c_{\underline{z}})}{\gamma_{\bar{z}} \beta^2}][\frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})[1 - (c_{\bar{z}} - c_{\underline{z}})] \mu_t}{[\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})] E_{t-1} p_t^s}] \} \\
& - \frac{1}{2} \gamma_{\bar{z}} \mu_t^2 + \frac{1}{2} \gamma_{\tilde{z}} \{ [\frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} - \frac{1}{\gamma_{\tilde{z}}} E_{t-1} p_t^s]^2 + [\frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})(c_{\bar{z}} - c_{\underline{z}} - \beta)}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})} \mu_t]^2 \\
& + 2[\frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} - \frac{1}{\gamma_{\tilde{z}}} E_{t-1} p_t^s][\frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})(c_{\bar{z}} - c_{\underline{z}} - \beta)}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})} \mu_t] \} - \frac{1}{2} \gamma_{\tilde{z}} \mu_t^2 .
\end{aligned}$$

Hence, the expected profit under the formula price contract is

$$\begin{aligned}
E(\Pi_4^*) = & \frac{1}{2} \gamma_{\bar{z}} (\beta q_0)^2 \{ [\frac{\beta[1 - (c_{\bar{z}} - c_{\underline{z}})] \alpha_{\bar{z}} P^{\bar{z}}}{\gamma_{\bar{z}} \beta^2 E_{t-1} p_t^s} - \frac{\beta + (1-\beta)(c_{\bar{z}} - c_{\underline{z}})}{\gamma_{\bar{z}} \beta^2}]^2 + [\frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})[1 - (c_{\bar{z}} - c_{\underline{z}})]}{[\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})] E_{t-1} p_t^s}]^2 \sigma_\mu^2 \} \\
& - \frac{1}{2} \gamma_{\bar{z}} \sigma_\mu^2 + \frac{1}{2} \gamma_{\tilde{z}} \{ [\frac{\alpha_{\tilde{z}} P^{\tilde{z}}}{\gamma_{\tilde{z}}} - \frac{1}{\gamma_{\tilde{z}}} E_{t-1} p_t^s]^2 + [\frac{(\gamma_{\tilde{z}} - \gamma_{\bar{z}})(c_{\bar{z}} - c_{\underline{z}} - \beta)}{\beta(\gamma_{\tilde{z}} + \gamma_{\bar{z}}) - \gamma_{\tilde{z}}(c_{\bar{z}} - c_{\underline{z}})}]^2 \sigma_\mu^2 \} - \frac{1}{2} \gamma_{\tilde{z}} \sigma_\mu^2 ,
\end{aligned}$$

and variance of each processor's profit is

$$Var(\Pi_4^*) = \{\frac{\beta\alpha_{\bar{z}} P^{\bar{z}}(\gamma_{\bar{z}} - \gamma_{\bar{z}})}{\beta(\gamma_{\bar{z}} + \gamma_{\bar{z}}) - \gamma_{\bar{z}}(c_{\bar{z}} - c_{\underline{z}})} + \alpha_{\bar{z}} P^{\bar{z}}[\frac{(\gamma_{\bar{z}} - \gamma_{\bar{z}})(c_{\bar{z}} - c_{\underline{z}} - \beta)}{\beta(\gamma_{\bar{z}} + \gamma_{\bar{z}}) - \gamma_{\bar{z}}(c_{\bar{z}} - c_{\underline{z}})}]$$
$$- \frac{E_{t-1}p_t^s}{1-(c_{\bar{z}} - c_{\underline{z}})} \frac{(\gamma_{\bar{z}} - \gamma_{\bar{z}})[2(c_{\bar{z}} - c_{\underline{z}}) - (c_{\bar{z}} - c_{\underline{z}})^2]}{[\beta(\gamma_{\bar{z}} + \gamma_{\bar{z}}) - \gamma_{\bar{z}}(c_{\bar{z}} - c_{\underline{z}})]}\}^2 \sigma_\mu^2.$$

In a similar fashion, one can derive the spot price, the contract supply, the cash-market supply, processor profits, and producer welfare under the other four cases mentioned above.

A Numerical Example and Market Performance Results

For each of these scenarios, a numerical example shows how the various contracts affect contract supply and the spot market price, and producers' and processors' welfare. Finally, the example also shows the impact of market power, in terms of N/M, on the performance of each contract.

To start, we assume that the randomness associated with the market price of the finished product, *P*, is governed by an exponential distribution function: $f(P \mid z) = (e^{-P/z})/z$, $0 \le P < \infty$, and $z > 0$. Thus $E(P|z) = z$. For the numerical example, the values of parameters $\{\beta, \alpha_{\bar{z}}, \alpha_{\underline{z}}, \gamma_{\bar{z}}, \gamma_{\underline{z}}, c_{\bar{z}}, c_{\underline{z}}, \bar{z}, \underline{z}, r, M, N, \sigma_\mu^2\}$ are listed in Table 1. Given these parameter values, Table 2 shows the equilibrium expected prices and quantities from the numerical example and also forms the basis for Figures 1 to 9.[6]

Table 1: Parameters used in the numerical example

$\bar{z}$	4	$\underline{z}$	3
β	60%-95%	c_z	0.1
$\alpha_{\bar{z}}$	0.5	M	10
α_z	0.4	N	50
$\gamma_{\bar{z}}$	0.2	σ_u^2	0.5
γ_z	0.3	r	0.1-2
$c_{\bar{z}}$	0.3		

[6] Results were also calculated but are not presented for cases where $N = 20$ and 100. In addition, although the numerical example is conducted with the risk aversion parameter in the range 0.1 to 2, only the results for $r = 0.5$ are presented. The results for other values of r and N are very similar to those in Table 2.

Case (i), fixed-price contracts under risk neutrality: With risk neutrality, the contract price takes the form $\underline{w} = E_{t-1} p_t^s$. Under this contract, the expected spot market price is the lowest among all types of contracts except the market price contract. In this case, contract supplies do not have any causal effect on the expected spot market price or its variance, and both contract supplies and the expected spot market price stay constant. Thus, by producing low-quality hogs, a risk-neutral producer is always indifferent between signing a contract and selling to the spot market regardless of values of β.

Case (ii), fixed- price contracts under risk aversion: Under this contract, the contract price takes the form $\underline{w} = E_{t-1} p_t^s - \frac{1}{2} r q_0 (2-\beta) \operatorname{var}(p_t^s)$. Figure 1 demonstrates that contract supplies have a positive relationship with the expected spot market prices and the variance of the spot market prices. This result suggests that as β increases, processors have the incentive to raise the contract price to make risk-averse producers indifferent between signing a contract and selling to the spot market. Increases in the contract price reduce the quantity demanded by each processor from the contract market and, hence, raise the quantity supplied to the spot market. Consequently, the quantity supplied to the spot market exceeds the quantity demanded from the spot market and the expected spot market price decreases.

Case (iii), market-price contracts with risk averse producers: Similar to the fixed-price contract with risk neutrality, contract supplies through the market-price contract do not affect the expected spot market price or its variance. Under the market-price contract, a contract producer is indifferent both *ex ante* and *ex post* between signing a contract and selling to the spot market, and strictly prefers to produce low-quality hogs regardless of the parameter β. Hence, given any value of β, a processor optimally purchases half of his hogs from the contract market and half from the spot market under expectation. While equilibrium market conditions are similar to the scenario with fixed-price contracts and risk neutrality, market-price contracts cause a smaller variance of spot market price.

Case (iv), formula-price contracts with premiums: Under this scenario, Figure 2 shows that contract supplies are positively related to the expected spot market price and negatively related to its variance under the formula-price contract. These effects highlight the link between market equilibrium and the participation and incentive-compatibility constraints: As the parameter β increases, processors reduce the contract price to make contract producers indifferent between signing a contract and selling to the spot market. Decreases in the contract price raise the quantity demanded by each processor from the contract market and, hence, reduce the quantity supplied to the spot market. As a result, the quantity demanded from the spot market exceeds the quantity supplied to the spot market and the expected spot market price increases.

Note also that the expected market price under formula-price contracts is greater than those under the fixed-price or market-price contracts due to quality differences between the contract market and the cash market. Moreover, the formula-price contract causes the smallest variability of spot market prices among all types of contracts. Another important property of this contract is that it makes the spot market thinner than the fixed-price contract and the market-price contract. Given the example shown in Table 2, spot market supply accounts for about 40.5%, on average, of total supply. Therefore, this effect of the formula-price contract is consistent with what has been observed in the hog and beef markets.

Case (v), cost-plus contract with premium: Similar to the formula-price contract, here both contract supply and the expected spot market price increase as β increases. Further, Figure 3 shows that the increase in contract supply due to the cost-plus contract raises the expected spot market price and its variance (unlike results under the formula-price contract scenario). Results also show that the contract price decreases as β increases. Thus, each processor purchases more hogs from the contract market and, hence, the quantity supplied to the spot market decreases. Consequently, the excess demand in the spot market drives up the equilibrium spot market price, which in turn raises the output of independent producers as well as contract producers.

Note that for each value of β, the expected spot market price under a cost-plus contract is the greatest among all types of contracts. However, the variance of the spot market price is also greater than that under the formula-price contract and the market-price contract. Similar to the formula-price contract, the spot market becomes thinner under the cost plus contract and is, in fact, the thinnest of all the contract scenarios considered.

Welfare Effects and the Impact of Market Power

Welfare effects also tend to highlight the tradeoff between risk and returns. Under the fixed-price contract – case (i), processors' expected profit stays constant as β increases. However, processors obtain a relatively greater profit than producers. In addition, processors can eliminate all risk in their profit by adjusting the quantities demanded from the spot market and the contract market. On the other hand, changes in β do not affect producers' expected utility, and contract producers earn the same expected utility as independent producers. However, as β increases, contract producers face a smaller variance of their income relative to independent producers.

For case (ii), Figure 4 shows that an increase in contract supplies raises both processors' expected profit and the variance of processors' profit. On the other hand, as each contract producer signs a greater proportion of his hogs with a processor, total contract supply decreases and both contract producers and independent producers obtain a smaller expected utility.

Further, since processors can depress the contract price as producers' degree of risk aversion increases, processors capture more surplus and, hence, contract producers earn a lower utility relative to independent producers under this contract. In addition, increases in contract supply raise the variance of producers' income. However, since the contract price is fixed given each r and β, contract producers face a relatively smaller variance of their income than independent producers. Figure 5 shows these impacts of contract supplies on both contract producers' and independent producers' profit.

Under the market-price contract – case (iii), just as in the fixed-price contract with risk neutrality, changes in β do not affect the amount of contract supplies, processors' profit, and producers' profit. As β increases, the variance of both contract producers' and independent producers' income stays constant. Compared to the fixed-price contract with risk neutrality, however, contract producers face a larger variance of income, while independent producers face a smaller variance of income under the market-price contract. Further, under the market-price contract, both processors and producers obtain smaller profit or utility relative to those under the fixed-price contract with risk neutrality; and processors earn the smallest profit among all types of contracts.

Figure 6 shows that, for formula-price contracts with quality premiums – case (iv), both processors' expected profit and variance of processors' profit increase as contract supplies increase. On the other hand, Figure 7 shows contract supply is positively related to producers' expected utility and variance of producers' income. Compared with independent producers, contract producers obtain a greater expected utility, but also face a greater variance of their income. Because processors can acquire high-quality hogs from the contract market, they earn a greater profit than that under the fixed-price contract and the market price contract due to greater profitability of high-quality hogs. Similarly, although producers incur high production costs by providing high-quality hogs to the market, both contract producers and independent producers can obtain a greater utility from high spot market prices and high contract prices. Risk-averse producers also benefit from low variance of spot market prices.

The performance of cost-plus contracts with quality premium – case (v) – is very similar to formula price contracts. Figure 8 shows that increased contract supplies raise processors' profit and variance of processors' profit, and the variance of processors' profit rises relatively slower than expected profit as contract supplies increase. The cost-plus contract offers the greatest profit to processors among all types of contracts. Compared to the formula price contract, however, processors incur a greater variance of profit. Figure 9 shows that both contract producers' and independent producers' expected utilities increase as contract supplies increase. However, increased contract supplies raise the variance of independent producers' income, while they reduce the variance of contract producers' income. In addition, contract producers obtain a greater expected utility and a greater variance of income relative to independent producers for each level of contract supply. Compared to the formula-price contract, contract producers earn a lower expected utility but face a smaller variance of income, while independent producers obtain a greater expected utility but face a greater variance of their income.

To demonstrate the effects of market power on the performance of the five types of contract scenarios, one can vary, N/M, the ratio between the number of producers and the number of processors, given the same set of parameters. As N/M increases, processors gain more market power in the sense that they can manipulate the market equilibrium more significantly. Without loss of generality, we analyze the impact of market power by fixing the number of processors M = 10 and setting the number of producers N = 20, 50, 100.

For each value of β under each type of contract, the expected spot market price is pushed down as N increases. However, the variance of spot market price stays unchanged. As a result, processors purchase more hogs from both the contract and spot markets due to the lower prices and, hence, both the contract and spot markets expand. As N increases, processors gain market power as buyers; hence, more surplus is captured by processors through both the contract market and the spot market. Thus, under each type of contract, processors obtain a greater profit as N increases. However, each processor incurs a greater variance of profit under each contract as N increases. On the other hand, each producer earns a smaller expected utility due to the reduced spot market price and the reduced amount of hogs produced by each producer. However, each producer faces a smaller variance of income as well.

Conclusion and Discussion

This paper investigates the relationship between the hog contract and spot markets, and provides a general methodology for analyzing this type of problem. Different from most studies, this paper embeds a principal-agent model of processor-producer behavior within a general equilibrium model of the hog market and accounts for the endogenous relationship between contract supplies and the spot market price. The paper also incorporates asymmetric information concerning hog qualities into the equilibrium model. Finally, the paper investigates the relationship between the contract and the spot markets under five different contract scenarios.

Major findings from the structural model and the numerical example are summarized in Table 3. At least two main results differ from those of previous studies: First, the paper finds that contract supplies raise the expected spot market price under a formula-price and cost-plus contracts while reducing (increasing) the variance of spot market price under formula-price (cost-plus) contracts. Second, the paper finds that the formula-price contract offers the second highest expected profit to processors, highest expected utility to contract producers, and the second highest expected utility to independent producers relative to other contracts. This second result is at odds with studies that report producers' complaints of formula-price contracts not providing price protection. Here, both processors and producers prefer the formula-price contract to the fixed-price or market-price contacts if asymmetric information about hog quality is taken into account. Compared to cost-plus contracts, formula-price contracts offer processors smaller expected profit and independent producers lower expected utility, but offer contract producers greater expected utility. In fact, performances of the cost-plus and formula-price contracts are both better than the fixed-price and market-price contracts. These results are consistent with current observations that formula-price contracts are dominant in the hog sector.

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Table 2: A numerical illustration of the model (*r* =0.5 except where noted, *N*=50)

β	$E_{t-1}p^s_t$	$Var(p^s)$	q_0	q_s	q_2^j	n_1	contract supply	spot supply (Mq_2^j)	processor profit $E[\Pi]$	$var(\Pi)$*	u_1*	$varInc_1$*	u_2*	$varInc_2$*
(i) Fixed-price contracts with risk neutrality (r = 0)														
0.6	0.6857	0.18	0.6857	0.6857	1.7143	4.1667	17.143	17.143	0.88163	0	0.42318	1.354E-02	0.4232	0.0846
0.7	0.6857	0.18	0.6857	0.6857	1.7143	3.5714	17.143	17.143	0.88163	0	0.42318	7.617E-03	0.4232	0.0846
0.8	0.6857	0.18	0.6857	0.6857	1.7143	3.1250	17.143	17.143	0.88163	0	0.42318	3.386E-03	0.4232	0.0846
0.9	0.6857	0.18	0.6857	0.6857	1.7143	2.7778	17.143	17.143	0.88163	0	0.42318	8.464E-04	0.4232	0.0846
(ii) Fixed-price contracts														
0.6	0.7092	0.19232	0.6816	0.7092	1.6362	4.3747	17.891	16.362	0.89214	8.24E-05	0.41457	1.430E-02	0.4284	0.0967
0.65	0.7082	0.19184	0.6796	0.7082	1.6393	4.0431	17.86	16.393	0.89162	7.39E-05	0.41295	1.085E-02	0.4273	0.0962
0.7	0.7073	0.19137	0.6778	0.7073	1.6425	3.7581	17.83	16.425	0.89111	6.62E-05	0.41144	7.912E-03	0.4263	0.0957
0.75	0.7063	0.1909	0.6761	0.7063	1.6455	3.5103	17.8	16.455	0.89063	5.92E-05	0.41002	5.454E-03	0.4252	0.0952
0.8	0.7054	0.19043	0.6746	0.7054	1.6486	3.2928	17.77	16.486	0.89016	5.29E-05	0.40871	3.466E-03	0.4242	0.0948
0.85	0.7045	0.18996	0.6733	0.7045	1.6516	3.1001	17.742	16.516	0.88972	4.71E-05	0.40748	1.938E-03	0.4232	0.0943
0.9	0.7036	0.1895	0.6721	0.7036	1.6546	2.9283	17.713	16.546	0.88928	4.19E-05	0.40635	8.560E-04	0.4221	0.0938
0.95	0.7028	0.18904	0.6711	0.7028	1.6575	2.7739	17.685	16.575	0.88886	3.72E-05	0.4053	2.129E-04	0.4211	0.0934
(iii) Market-price contracts														
0.6	0.6857	0.045	0.6857	0.6857	1.7143	4.1667	17.143	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.7	0.6857	0.045	0.6857	0.6857	1.7143	3.5714	17.143	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.8	0.6857	0.045	0.6857	0.6857	1.7143	3.125	17.143	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
0.9	0.6857	0.045	0.6857	0.6857	1.7143	2.7778	17.143	17.143	0.73163	0	0.41789	0.021159	0.4179	0.0212
(iv) Formula-price contracts														
0.6	0.9449	0.0258	1.1812	0.9449	2.5202	4.6658	33.067	25.202	1.7763	5.413E-04	0.96761	0.036032	0.7979	0.0231
0.65	0.9587	0.0257	1.1984	0.9587	2.4653	4.3174	33.629	24.653	1.7796	5.431E-04	0.99601	0.036936	0.8213	0.0236
0.7	0.9708	0.0256	1.2135	0.9708	2.4167	4.017	34.123	24.167	1.7834	5.491E-04	1.0214	0.037746	0.8422	0.0242
0.75	0.9816	0.0256	1.2270	0.9816	2.3736	3.7555	34.56	23.736	1.7876	5.577E-04	1.0443	0.038478	0.8610	0.0246
0.8	0.9913	0.0255	1.2391	0.9913	2.335	3.5258	34.949	23.35	1.7919	5.678E-04	1.0649	0.03914	0.8781	0.0251
0.85	0.9999	0.0254	1.2499	0.9999	2.3003	3.3225	35.299	23.003	1.7963	5.789E-04	1.0837	0.039743	0.8935	0.0254
0.9	1.0078	0.0254	1.2597	1.0078	2.2689	3.1413	35.614	22.689	1.8008	5.905E-04	1.1008	0.040294	0.9076	0.0258
0.95	1.0149	0.0254	1.2686	1.0149	2.2403	2.9787	35.9	22.403	1.8051	6.023E-04	1.1164	0.040799	0.9205	0.0261
(v) Cost-plus contracts														
0.6	0.9640	0.0901	1.1772	0.9640	2.4438	4.8186	34.035	24.438	1.9354	0.0323	0.96506	1.998E-02	0.8155	0.0838
0.65	0.9783	0.0928	1.1925	0.9783	2.3869	4.465	34.609	23.869	1.9427	0.0339	0.99136	1.617E-02	0.8391	0.0889
0.7	0.9907	0.0952	1.2057	0.9907	2.3373	4.1595	35.106	23.373	1.95	0.0356	1.0145	1.245E-02	0.8600	0.0934
0.75	1.0016	0.0972	1.2173	1.0016	2.2937	3.8927	35.539	22.937	1.9571	0.0372	1.035	9.003E-03	0.8785	0.0975
0.8	1.0112	0.0990	1.2275	1.0112	2.2553	3.6577	35.919	22.553	1.964	0.0389	1.0533	5.966E-03	0.8949	0.1012
0.85	1.0197	0.1005	1.2366	1.0197	2.2213	3.4491	36.254	22.213	1.9705	0.0404	1.0696	3.459E-03	0.9097	0.1045
0.9	1.0272	0.1019	1.2448	1.0272	2.191	3.2625	36.55	21.91	1.9766	0.0419	1.0843	1.579E-03	0.9228	0.1075
0.95	1.0340	0.1031	1.2522	1.0340	2.164	3.0945	36.813	21.64	1.9823	0.0434	1.0976	4.043E-04	0.9347	0.1103

Notes (*): u_1 and u_2 represent the expected utilities of contract and independent producers, and *$varInc_1$ and $varInc_2$* represent the variances of contract and independent producers' incomes.

Table 3: Summary of impacts of contract supplies under each contract scenario

	Expected Spot market price	Variance of spot price	Processor profit	Variance of processors' profit	Expected utility of contract producers	Variance of contract producers' income	Expected utility of indep. producers	Variance of indep. producers' income
Fixed-price with risk neutrality	*No change and lowest*	*No change 2nd highest*	*No change and 2nd lowest*	*No change and lowest*	*No change and 3rd lowest*	*Decrease with beta and lowest*	*No change*	*No change*
Fixed-price with risk aversion	*Positive and 2nd lowest*	*Positive and highest*	*Positive and 3rd lowest*	*Positive and 2nd lowest*	*Positive and lowest*	*Positive and 2nd lowest*	*Positive*	*Positive*
Market-price contract	*No change and lowest*	*No change 2nd lowest*	*No change and lowest*	*No change and lowest*	*No change and 2nd lowest*	*No change and 2nd highest*	*No change and lowest*	*No change and lowest*
Formula-price contract	*Positive 2nd highest*	*Negative and lowest*	*Positive 2nd highest*	*Positive 2nd highest*	*Positive and highest*	*Positive and highest*	*Positive and 2nd highest*	*Positive and 2nd lowest*
Cost-plus contract	*Positive and highest*	*Positive 3rd lowest*	*Positive and highest*	*Positive and highest*	*Positive and 2nd highest*	*Negative and 3rd highest*	*Positive and highest*	*Positive*

Notes:

1. "No change" indicates that contract supplies have no effect on the variable listed in the column heading. "Positive" indicates that contract supplies have a positive relationship with that variable; "negative" indicates a negative relationship.
2. The order (ranking) is based on the relative magnitude of variable listed in the column heading for all five contract scenarios. If no order is indicated, relative rankings are indeterminate. The shaded boxes reflect the two most preferred rankings.

Figure 1: Contract supplies v. expected spot market price and variance under fixed-price contracts with risk aversion

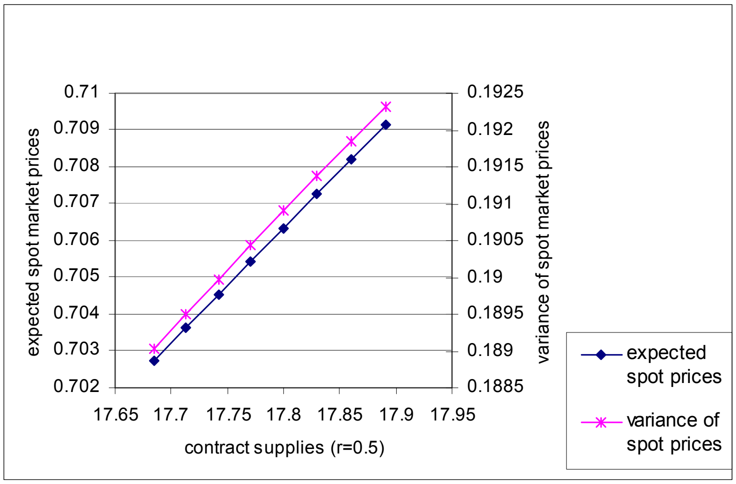

Figure 2: Contract supplies v. expected spot market price and variance under formula-price contracts

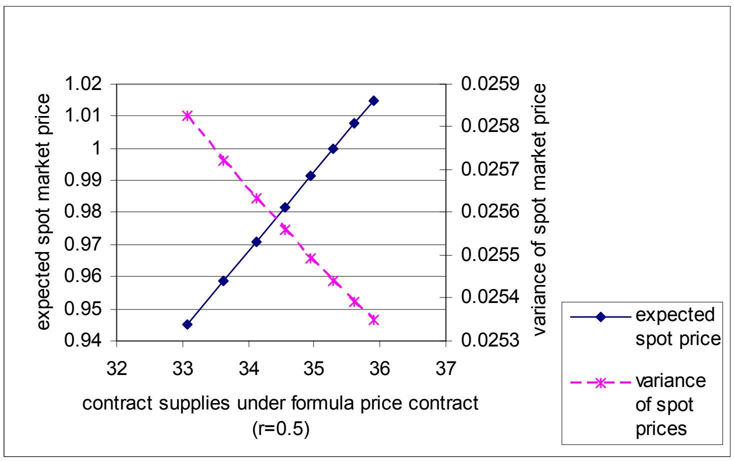

Figure 3: Contract supplies v. expected spot market price and variance under cost-plus contracts

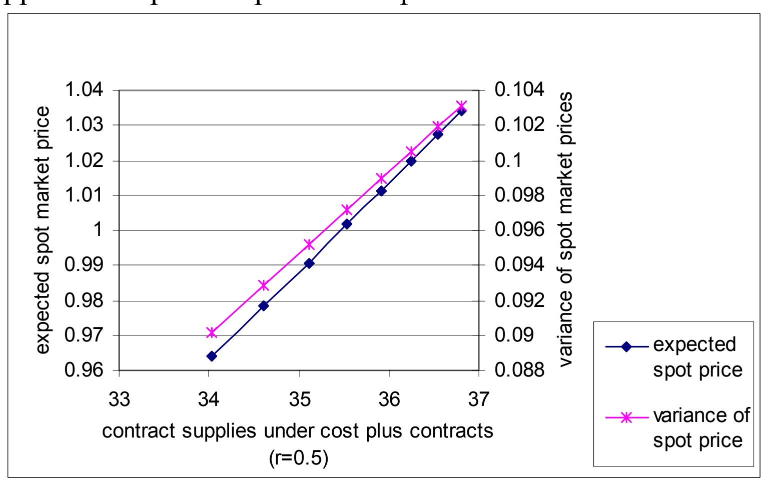

Figure 4: Contract supplies vs. processors' expected profit and variance of processors' profit under fixed-price contract with risk aversion

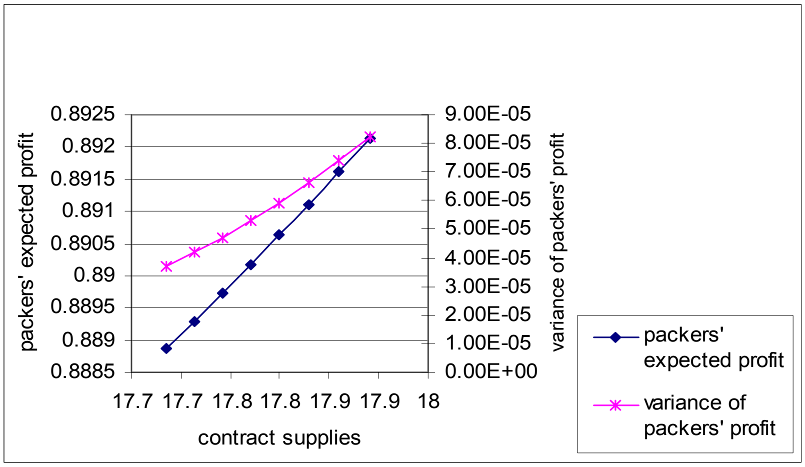

Figure 5: Contract supplies vs. producers' expected profit and variance under fixed-price contracts with risk aversion

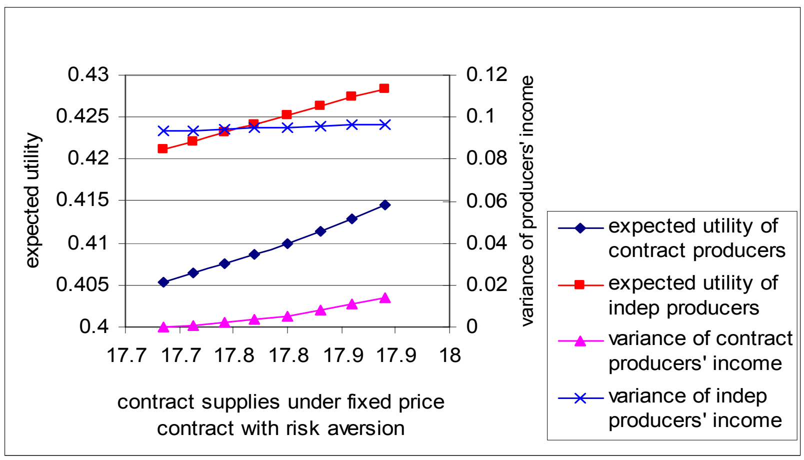

Figure 6: Contract supplies vs. processors' expected profit and variance of processors' profit under formula-price contract

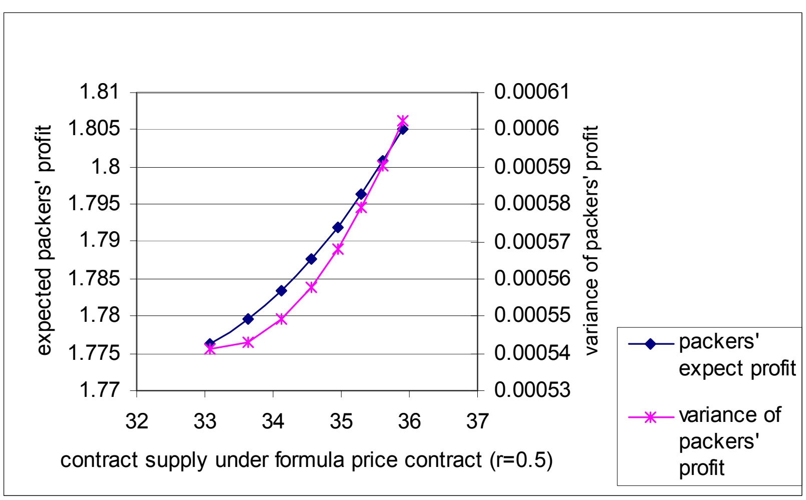

Figure 7: Contract supplies vs. producers' expected profit and variance under formula-price contract

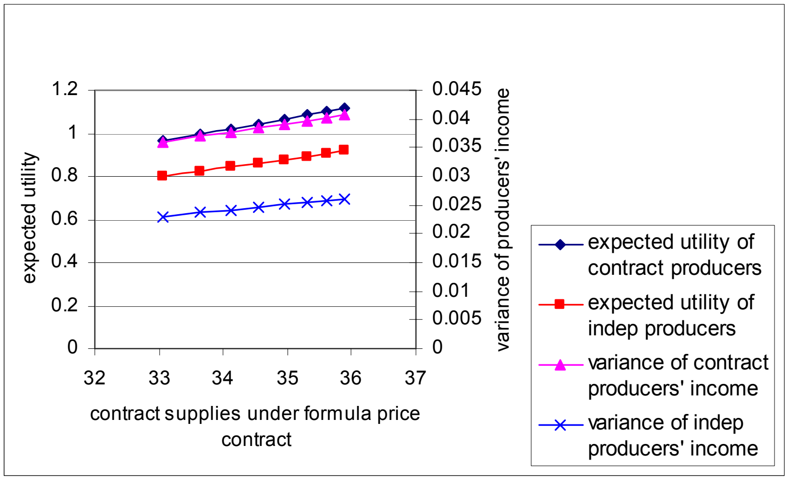

Figure 8: Contract supplies vs. processors' expected profit and variance of processors' profit under cost-plus contracts

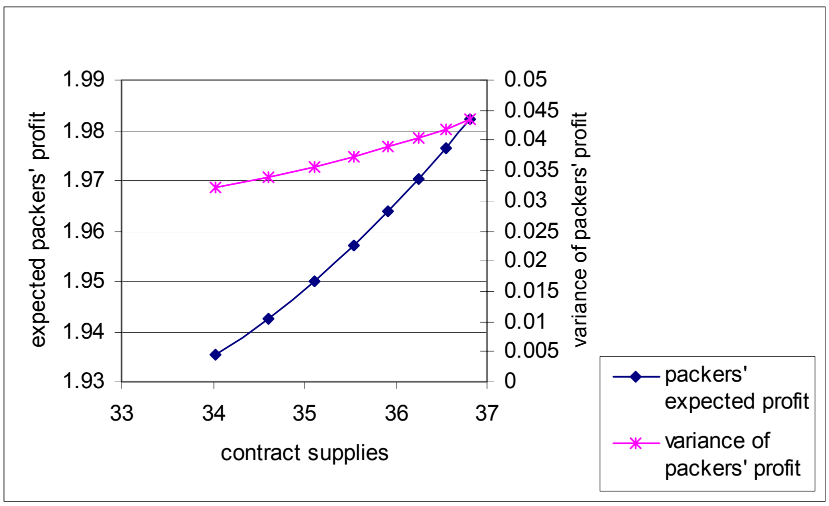

Figure 9. Contract supplies vs. producers' expected profit and variance under cost-plus contracts