Incorporating Epidemiological Projections of Morbidity and Mortality into an Open Economy Growth Model: AIDS in South Africa

By

Terry L. Roe and Rodney B.W. Smith

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Abstract

HIV prevalence dynamics are introduced into a three sector, neoclassical growth model. The model is calibrated to South African national accounts data and used to examine the potential impact of HIV/AIDS on economic growth. Projections portend if left unchecked, the long run impact of HIV and AIDS could drive South African GDP to levels that are over 60% less than no-HIV levels, with AIDS death rates decreasing the long run stock of labor by over 60%.

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Incorporating Epidemiological Projections of Morbidity and Mortality

Into An Open Economy Growth Model:

HIV and AIDS in South Africa

1 Introduction

Of the 36.1 million people living with HIV and AIDS, 95 percent live in developing countries, with 29.4 million people infected with HIV and AIDS in sub-Saharan Africa. In 2002, approximately 3.5 million new cases of HIV occurred in Sub-Saharan Africa, and during the same year AIDS claimed the lives of an estimated 2.4 million Africans (FAO, 2002). Adult HIV prevalence rates in sub-Saharan Africa were estimated to range between 1% (Somolia) to 38% (Botswana) in 20002, while the South African rate was estimated to be between 15% to 25% (UNAIDS, 2002).[1]

Several studies have examined the likely impact of HIV and AIDS on economic growth. For example, Arndt and Lewis (2000, 2001) use a multi-sector computable general equilibrium (CGE) model to examine the impact of HIV and AIDS on South African economic growth. They predict, relative to a no-AIDS scenario, annual aggregate gross domestic product (GDP) in South Africa would be 0.8% to 1.0% lower in the presence of AIDS. Over (1992) performs a cross country regression study across 30 sub-Saharan African countries, and estimates AIDS could lead to a 0.56% to 1.08% drop in the level of annual aggregate GDP growth between 1990 and 2025. During the same period, Over estimates AIDS could lead to a 0.35% drop in per capita GDP. Sackey and Rarpala (2000) project Lesotho's aggregate GDP growth in 2010 would drop from 4.0% without AIDS to 2.4% with the disease, and in 2015 drop from 4.0% to 1.3%. Using cross country regressions, Bonnel (2000) estimates that relative to a no-AIDS case, over a twenty year period a typical sub-Saharan country with a prevalence rate of 20% would realize a 67% drop in aggregate GDP levels.

Using an overlapping generation model calibrated to South Africa data, Bell, et al. (2003)

explicitly model the impact of AIDS on human capital. In their baseline, no-AIDS, model the expected annual growth in household income between 1990 and 2050 (three generations) is 1.46% per year. Expected annual growth between 1990 and 2050 is -1.2% given AIDS and little or no intervention, and 1.22% with AIDS and government intervention. Although a very engaging study, the Bell, et al. (2003) results are not easy to interpret in terms of the impact of AIDS on aggregate GDP growth rates or levels because the only productive resource in the model is labor augmented by human capital: there is no physical capital.

The above studies take different approaches to ascertaining the impact of HIV/AIDS on economic growth. The studies by Bonnel, Over (1992), and Sackey and Raparla (2000) combine demographic modeling with cross country econometric analysis to ascertain the impact of the disease and aggregate GDP. These studies focus on aggregate GDP levels and are not designed to investigate the impact of the disease on sub-sectors of the economy. The studies by Arndt and Lewis (2000, 2001) are based on a multi-sector computable general equilibrium (CGE) model, and look at the short- and intermediate-run impact of the disease on GDP growth. The Arndt and Lewis (2000, 2001) model is quite involved, and to simplify the analysis they assume the wage bill and the rate at which capital accumulates are exogenous. Also, the Arndt and Lewis model focuses on the short and intermediate run (up to ten years) impact of the disease. Although these studies use different approaches to understanding the impact of HIV/AIDS on economic growth and development, they each lead to the same conclusion: HIV/AIDS is likely to affect, negatively, economic growth in sub-Saharan Africa.

This paper introduces HIV prevalence dynamics into a three sector Ramsey-type growth model of a small open economy. The model is then calibrated to South Africa national accounts data and used to examine the short, intermediate, and long run impact of HIV and AIDS on economic growth. Our results suggest that HIV and AIDS will decrease both per capita and aggregate GDP, and the potential impact of the disease on intermediate and long run aggregate GDP is staggering. South African per capita GDP in the presence of HIV and AIDS is estimated to range from 0.35% to 1.13% smaller than per capital GDP in a no HIV world. At the end of three generations, aggregate GDP in the presence of HIV and AIDS is

projected to be more than 60% smaller than aggregate GDP in a no HIV world.

The paper is organized as follows. The next section discusses the basic model. In addition to describing the economic environment, the discussion provides a brief introduction to the susceptible-infected epidemiological model of HIV prevalence dynamics. The disease dynamics are then linked to population growth and labor productivity. Section 3 presents a detailed characterization of the competitive equilibrium at the steady state and along the transition path. Section 4 provides a brief overview of the calibration proceedures, an informal report on the model's in sample performance (1993 – 2001), and a discussion of the economics underlying the model's behavior. Also presented in this section are the short, intermediate, and long run forecasts of GDP and sectoral output – with and without HIV and AIDS. The last section concludes and outlines a few additional features to be included in future work.

2 Basic model

We proceed by describing the environoment of the basic model, the essential primatives including the main features of disease prevelance, production and household behavior. Equilibrium is characterized, and features of long-run and transition equilibrium discussed.

2.1 The environment

The model depicts a small, open and perfectly competitive economy in which agents produce and consume three final goods: an agricultural, manufacturing, and service good, indexed at each instant in time by $j = a, m, s$, and traded at price p_j. The services of labor, L, and capital, K, are employed in the production of all three goods, while land, T, is a factor specific to production of the agricultural good, indexed $j = a$. The agricultural good is a pure consumption good that is internationally traded. The manufactured good, indexed $j = m$, is both a consumption and a capital good that is also internationally traded. The service good, indexed $j = s$, is a non-traded, pure consumption good. Labor services are not traded

internationally and domestic residents own the entire stock of domestic assets. At each instant in time, households earn income from providing labor services L in exchange for wages w, earn interest income at rate r on capital assets K, and receive rents from agriculture's sector specific resource, land T which is constant over time. The initial endowment of labor is normalized to unity, and initial capital stock, $K(0)$, is given.

A key new feature of the following environment is the presence of a disease that affects death rates and introduces population morbidity. Differential death rates influence population growth trajectories, while morbidity effects impact the supply of effective labor. In contrast to Roe and Saracoğlu (2004), this feature results in a system of differential equations governing the models state and control variables over time that is nonautonomous.

2.2 Prevalence and population dynamics

In this section, we describe a simple "structural" prevalence model. Note, in the empirical exercise that follows, to capture the richness of models developed by epidemiologists for the case of South Africa, we utilize a reduced form of their model.

2.2.1 Prevalence dynamics

Let $h(t)$ represent the share of the population who are HIV positive at time t. For the sake of illustration, consider a variant of the susceptible-infected epidemiological model of HIV prevalence dynamics invoked by Kremer (1996), where HIV prevalence rates evolve according to the following relationship:

$$\dot{h} = \beta(1-h) - \delta h. \tag{1}$$

Here $\dot{h}$ is the instantaneous rate of change in the prevalence rate, β is the rate at which new cases of the disease emerge, and δ is the (Poisson hazard rate) at which individuals with the disease die. With h representing the share of the population infected with the disease, $1-h$ percent of the population is HIV free. Then $\beta(1-h)$ is the rate at which the share of infected individuals increases, while δh is the rate at which the share of infected individuals

falls (typically due to AIDS deaths). For a more complex model of HIV prevalence dynamics see Anderson and May, 1991.

The solution to (1) is

$$h\left(t\right) = \frac{\beta}{\beta + \delta} - \frac{\beta - h_0\left(\beta + \delta\right)}{\beta + \delta}e^{-(\beta+\delta)t}, \tag{2}$$

where h_0 is the share of individuals with the disease at time $t = 0$. The steady state prevalence rate, denoted h_{ss}, is given by

$$h_{ss} = \lim_{t \to \infty} h\left(t\right) = \frac{\beta}{\beta + \delta}. \tag{3}$$

Note, expression (2) has an important property/limitation: if $h_0 < h_{ss}$, then h is an increasing, concave function that asymptotically approaches h_{ss} from below. On the other hand, if $h_0 > h_{ss}$, then h is an decreasing, convex function that asymptotically approaches h_{ss} from above.

Although this structure is sufficient for discussing the basics of prevalence dynamics in the analytical model presented below, the dynamics of prevalence modeled by epidemiologists tend to be more complex. For example, the "monotonic" trajectory in Figure 1
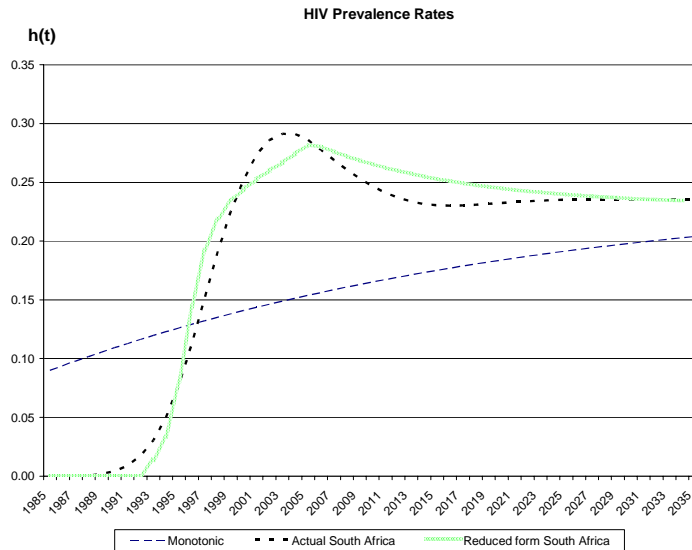

Figure 1. Monotonic, actual and reduced form prevalence rates

is typical of a trajectory from the class of prevalence dynamics consistent with (1). The actual and projected trajectory of HIV prevelence developed by South African epidemiologists over

the period 1985 though 2020 (with extensions through 2035) is shown in Figure 1. As this trajectory suggests, monotonic dynamics are likely to be encountered in one of two cases: (i) when the prevalence rate has reached its maximum and is beginning to fall, $h_0 > h_{ss}$, or (ii) seven to ten years after the disease has entered a population and the rate at which the disease is being transmitted is increasing exponentially, $h_0 < h_{ss}$. Thus, in the empirical model we encorporate the prevalence values of the South African epidemiologists using a reduced form structure, instead of equation (2). A comparison of the reduced form and actual trajectory is presented in Figure 1.

2.2.2 Population dynamics

Assume the population grows n percent each year without HIV and AIDS, where n is the difference between the crude birth rate and the crude death rate. Then at time t, the stock of labor cum HIV and AIDS is equal to $L\left(t\right) = L_0 e^{nt} = e^{nt}$. With HIV and AIDS, given (1), population growth at time t is given by $n - \delta h\left(t\right)$. It follows that with AIDS, the stock of labor at $t > 0$ is given by

$$L\left(t\right) = e^{nt - \delta \int_0^t h(\tau)d\tau}. \tag{4}$$

By (4), the impact of AIDS is felt directly via its influence on the rate at which the labor stock grows, i.e., via the term $-\delta h\left(\cdot\right)$.

Another important economic impact of HIV occurs when the disease affects the productive efficiency of labor, i.e., when HIV introduces morbidity effects into the labor force. Assume each individual potentially provides one efficiency unit of labor, and assume labor productivity grows according to the labor productivity function $A\left(t\right) = e^{xt}$, where $x > 0$ is the coefficient of labor productivity growth. Then, in the absence of any morbidity effects, the total number of time t labor efficiency units is equal to

$$A\left(t\right)L\left(t\right) = e^{xt + nt - \delta \int_0^t h(\tau)d\tau}.$$

If an individual is HIV free, then that person provides a full efficiency unit of labor. If the individual is HIV positive, then assume, on average, he or she provides γ efficiency units of

labor, where $\gamma \in (0, 1]$. The parameter γ is meant to represent a notion of the "average" or modal impact of HIV on labor productivity. One simple interpretation of γ is if, on average, an HIV positive individual does not report to work one day out of five, then set $\gamma = 0.8$.[2] With this assumption, if share $h(\cdot)$ of the population is HIV positive, then the adjusted number of labor efficiency units is denoted $\hat{L}(t)$ and is defined as

$$\hat{L}(t) = A(t) H(t) L(t), \tag{5}$$

where $H(t) = 1 - (1 - \gamma) h(t)$ is the average efficiency of a unit of labor. It follows from (3) that this element of labor efficiency approaches the long run value

$$H_{ss} = \lim_{t \to \infty} H(t) = \frac{\delta + \gamma \beta}{\beta + \delta}. \tag{6}$$

2.3 Production

Suppressing the time argument t, and assuming production is nonjoint in inputs, production at each instant in time is represented by the constant returns to scale (CRS) technology $\mathbf{Y}(t) : \mathbb{R}_+^3 \times \mathbb{R}_+^3 \times \mathbb{R}_+ \to \mathbb{R}_+^3$, where $\mathbf{Y}$ is defined as

$$
\begin{aligned}
\mathbf{Y}\left(\tilde{L}, K, T, \Gamma, t\right) \;=\; & \left\{ (Y_a, Y_m, Y_s) : Y_a \leq F^a\left(\gamma_a A(t) L_a, K_a, B(t) T\right), Y_m \leq F^m\left(\gamma_m A(t) L_m, K_m\right), \right. \\
& \left. Y_s \leq F^s\left(\gamma_s A(t) L_s, K_s\right); \; \tilde{L} \geq L_a + L_m + L_s, \; K \geq K_a + K_m + K_s \right\}.
\end{aligned}
$$

and L_j, and K_j denote the level of labor and capital in the $j - th$ sector, and $F^j(\cdot)$ are production functions that are increasing and strictly concave in each argument. The parameters $\Gamma = (\gamma_a, \gamma_m, \gamma_s)$ are factors that further modify the impact of HIV and AIDS on labor productivity for the respective sectors. For instance, γ_s and γ_a are the labor productivity modifiers for the service and agricultural sectors, with $\gamma_j \in (0, 1]$. $B(t)$ represents the growth in land productivity.

Given the technologies are CRS, the minimum cost per-unit of the manufacturing and service output are respectively given by:

$$C^j\left(\hat{w}, r, \gamma_j\right) \equiv \min_{\{\hat{k}_j\}} \left\{\hat{w} + r\hat{k}_j : 1 \leq f^j\left(\gamma_j, \hat{k}_j\right)\right\}, \; j = m, s$$

and maximum agricultural rent per-unit of output in effective labor units is given by

$$G^a\left(p_a,\hat{w},r,\gamma_a\right)b\left(t\right)T\equiv\max_{\{\hat{k}_a\}}\left\{\left[p_af^a\left(\gamma_a,\hat{k}_a\right)b\left(t\right)T-\left(\hat{w}+r\hat{k}_a\right)\right]l_a:1\leq f^a\left(\gamma_a,\hat{k}_a\right)b\left(t\right)T\right\}.$$

where $\hat{w}=w/A\left(t\right)H\left(t\right)$ is the labor productivity adjusted wage rate,

$$b\left(t\right)=\frac{B\left(t\right)}{A\left(t\right)H\left(t\right)L\left(t\right)}\tag{7}$$

and $\hat{k}_j=K_j/A\left(t\right)H\left(t\right)L_j$ is capital per labor efficiency unit.

2.4 Households

The representative household receives utility from the sequence $\{\hat{q}_a,\hat{q}_m,\hat{q}_s\}_{t=0}^{t=\infty}$ expressed as a weighted sum of all future flows of utility

$$U=\int_0^\infty\frac{u\left(\hat{q}_a,\hat{q}_m,\hat{q}_s\right)^{1-\theta}}{1-\theta}e^{(n-\rho)t-\delta\int_0^t h(\tau)d\tau}dt.$$

where $\hat{q}_j=Q_j/A\left(t\right)H\left(t\right)L\left(t\right)$, are expressed in the per-effective-unit levels of agricultural, manufacturing, and service consumption, $\hat{q}_j\in\mathbb{R}_+$, $j=a,m,s$, and $u\left(\hat{q}_a,\hat{q}_m,\hat{q}_s\right)$ is the felicity function. Given normalized prices $\left(p_a,p_s\right)$, the minimum cost of achieving $u\left(\hat{q}_a,\hat{q}_m,\hat{q}_s\right)$ is given by the expenditure function

$$\hat{E}=\mu\left(p_a,p_s\right)\hat{c}\equiv\min_{(\hat{q})}\left\{\left(\hat{q}_m+p_a\hat{q}_a+p_s\hat{q}_s\right)\mid\hat{c}\leq u\left(\hat{q}_a,\hat{q}_m,\hat{q}_s\right)\right\}.$$

Then, suppressing t, the flow budget constraint is

$$\dot{\hat{k}}=\hat{w}+\left[r-\frac{\dot{A}}{A}-\frac{\dot{H}}{H}-\frac{\dot{L}}{L}\right]\hat{k}+G^a\left(p_a,\hat{w},r,\gamma_a\right)b\left(t\right)T-\mu\left(p_a,p_s\right)\hat{c},\tag{8}$$

where $\hat{k}=K\left(t\right)/\tilde{L}\left(t\right)$, and $\dot{A}/A$, $\dot{H}/H$, and $\dot{L}/L$ are obtained from (5).

For the case where $\theta\searrow 1$, the first order conditions obtained from the corresponding present value Hamiltonian yields the following Euler equation

$$\frac{\dot{\hat{E}}}{\hat{E}}=r-x-\rho-\frac{\dot{H}}{H}\tag{9}$$

which, together with the transversality condition,

$$\lim_{t \to \infty} \left[v\dot{\hat{k}} \right] = 0. \tag{10}$$

and the equation of motion (8), characterize the household's optimization problem.

Condition (9) suggests the optimal choice of expenditure levels over time depends on both the Harrod rate of growth in effective labor x, and the rate of change in the average efficiency of a unit of labor due to morbidity, $\dot{H}/H$. For a steady state to exist, (1) must be zero.

3 The competitive equilibrium

3.1 Characterization

Given the endogenous sequence of values $\left\{ \hat{k}(t), \hat{E}(t) \right\}_{t \in [0,\infty)}$, at each t the five-tuple sequence of positive values $\{\hat{w}(t), r(t), \hat{y}_m(t), \hat{y}_s(t), p_s(t)\}_{t \in [0,\infty)}$ must satisfy the following intra-temporal conditions:

(i) zero profits in manufacturing and services

$$c^j(\widehat{w}, r) = p_j, \; j = m, s \tag{11}$$

(ii) labor market clearing

$$\sum_{j=m,s} \frac{\partial}{\partial \hat{w}} c^j(\widehat{w}, r) \hat{y}_j - \frac{\partial}{\partial \hat{w}} G^a(p_a, \hat{w}, r) b(t) T = 1 \tag{12}$$

(iii) capital market clearing

$$\sum_{j=m,s} \frac{\partial}{\partial r} c^j(\widehat{w}, r) \hat{y}_j - \frac{\partial}{\partial r} G^a(p_a, \hat{w}, r) b(t) T = \hat{k} \tag{13}$$

and (iv) clearing of the market for non-traded goods

$$\frac{\partial}{\partial p_s} \hat{E} = \hat{y}_s. \tag{14}$$

This system can, in principle, be solved to express each endogenous variable $\{\hat{w}, r, \hat{y}_m, \hat{y}_s, p_s\}$ as a function of the exogenous variables (p_m, p_a, T), and the remaining endogenous variables $\left(\hat{k}, \hat{E} \right)$.

In the next sections we derive the steady-state solution and two equations of motion that, combined with expressions (11) – (13), constitute a solution to the entire sequence of endogenous variables.

3.2 The long-run equilibrium

Our approach is to first derive the steady-state values for r, p_s, and $\hat{w}$, and then substitute these values into the budget constraint and solve for $\hat{k}$.

Using the zero profit condition (11) express $\hat{w}$, and r as a function of p_s.

$$\hat{w} = W(p_s) \tag{15}$$

$$r = R(p_s). \tag{16}$$

where we suppress the exogenous variables p_a, and p_m to minimize clutter. Substitute (15) and (16) into the factor market clearing conditions (12) and (13), and use these resulting equations to solve for $\hat{y}_m$ and $\hat{y}_s$ as a function of the endogenous variables $\left(p_s, \hat{k}\right)$[3]. Express the result for $\hat{y}_s$ as

$$\hat{y}_s = Y^s\left(p_s, \hat{k}, t\right) \tag{17}$$

where t appears as a separate argument due to the term $b(t)$ in expressions (12) and (13), and other exogenous variables are suppressed. The supply function (17) is linear in $\hat{k}$ for the same reasons as in the static Hecksher-Ohlin model. Next, substitute (17) into the non-traded good market clearing condition (14) to obtain

$$\hat{E} = \frac{p_s}{\lambda_s} Y^s\left(p_s, \hat{k}, t\right) \tag{18}$$

where λ_s is the non-traded good share of total expenditure on goods.

If a steady state exists, for the case where $\theta \to 1$, $\dot{H}/H = 0$, the Euler condition (9) implies the steady state capital rental rate

$$r_{ss} = \rho + x.$$

Using (16), the steady state price of the non-traded good $p_{s,ss}$ is recovered using[4]

$$p_{s,ss} = R^{-1}(\rho + x) \tag{19}$$

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and from (17), the wage rate is given by

$$\hat{w}_{ss} = W\left(p_{s,ss}\right).$$

Substituting these values into the budget constraint (8), and using (18), yields

$$0 = \dot{\hat{k}} = \hat{w}_{ss} + \hat{k}(r_{ss} - x - (n - \delta h_{ss})) + G^a\left(p_a, \hat{w}_{ss}, r_{ss}\right) b\left(t\right) T - \frac{p_{ss}}{\lambda_s} Y^s\left(p_{ss}, \hat{k}, t\right) \quad (20)$$

If the contribution to land's productivity in effective labor units is

$$b\left(t\right) = \left\{ \begin{array}{c} \text{constant, } \forall t \\ \text{constant} \geq 0, \ t \searrow \infty \end{array} \right\} \quad (21)$$

then (20) contains a single unknown $\hat{k}$[5]. In other words, either t does not appear in (20), or $\lim_{t \to \infty} \dot{\hat{k}} = 0$. The root $\hat{k}_{ss}$ satisfying (20) is the steady state level of capital stock. Knowing the values $\left(r_{ss}, w_{ss}, p_{s,ss}, \hat{k}_{ss}\right)$ permits calculation of the remaining endogenous variables.

3.3   Transition path equilibrium

To characterize the transition path we follow an approach similar to that found in Elbasha and Roe (1996). Substitute (15), (16) and (18) into the budget constraint (8) to yield a differential equation in two unknowns, p_s and $\hat{k}$:

$$\dot{\hat{k}} = \mathbf{k}\left(\hat{k}, p_s, t\right) = W\left(p_s\right) + \hat{k}(R\left(p_s\right) - x - \frac{\dot{H}}{H} - \frac{\dot{L}}{L}) + \pi\left(p_a, W\left(p_s\right), R\left(p_s\right)\right) b\left(t\right) T - \frac{p_s}{\lambda_s} Y^s\left(p_s, \hat{k}, t\right) \quad (22)$$

To derive the differential equation for p_s, totally differentiate the non-traded good market clearing condition (18) with respect to time. Then, use the Euler condition (9) to solve for $\dot{p}_s$ giving

$$\dot{p}_s = \frac{p_s Y^s\left(p_s, \hat{k}, t\right)\left(r - \rho - x - \dot{H}/H\right) - p_s \frac{\partial}{\partial \hat{k}} Y^s\left(p_s, \hat{k}, t\right) \dot{\hat{k}}_s}{Y^s\left(p_s, \hat{k}, t\right) + p_s\left(\frac{\partial}{\partial p_s} Y^s\left(p_s, \hat{k}, t\right) + \frac{\partial}{\partial t} Y^s\left(p_s, \hat{k}, t\right)\right)}. \quad (23)$$

Finally, substitute (22) for $\dot{\hat{k}}$ in (23) to yield the differential equation

$$\dot{p}_s\left(t\right) = \mathbf{p}\left(\hat{k}\left(t\right), p_s\left(t\right), t\right). \quad (24)$$

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The roots, $p_{s,ss}$ and k_{ss} satisfying (19) and (20), must also satisfy (22) and (23), in which case it can be seen that the numerator of (23) is zero at the steady state. The steady state is thus a fixed point of the transition path equilibria

$$\left\{ \hat{k}\left(t\right), p_s\left(t\right) \right\}_{t \in [0,\infty)}. \tag{25}$$

If $b\left(t\right)$ is constant for all t, the system of differential equations given by (22) and (24) can be solved empirically using the Time-Elimination Method developed by Mulligan and Sala-i-Martin (1991). If $b\left(t\right)$ approaches a positive constant as t becomes large, then the system is nonautonomous and the method of Brunner and Strulik (2000) is invoked. Knowing (25) allows the calculation of $\left\{ \hat{E}^* \right\}_{t \in [0,\infty)}$ using (18) Together with the intra-temporal system, the remaining sequence of factor payments, firm and household allocations are easily recovered.

3.4 Some comparative statics

In this section we briefly sketch some of the key comparative statics associated with the evolution of prices and output. This discussion helps to interpret the empirical results.

3.4.1 The path of prices

We first observe that if $\hat{k}\left(0\right) < \hat{k}_{ss}$ and $\dot{\hat{k}} \geq 0 \; \forall t$, then the path of $\hat{w}$, and p_s depends upon the factor intensity of sector s relative to sector m. To see this note that the zero profit condition (11) includes only the technology parameters of sectors m, and s. If $\hat{k}\left(0\right) < \hat{k}_{ss}$ and $\dot{\hat{k}} \geq 0 \; \forall t$, then diminishing returns to $\hat{k}$ imply

$$\varepsilon^r \frac{\dot{p}_s}{p_s} = \frac{\dot{r}}{r} \leq 0, \tag{26}$$

where ε^r is the elasticity of (16) with respect to p_s. If sector s is labor intensive relative to sector m, the Stopler-Samuelson "like" condition implies $\varepsilon^r \leq 0$, and the price of the non-traded good rises, i.e.,

$$\frac{\dot{p}_s}{p_s} \geq 0.$$

If sector s is capital intensive relative to sector m, then $\varepsilon^r \geq 0$ in which case

$$\frac{\dot{p}_s}{p_s} \leq 0.$$

In either case, if follows from capital deepening that

$$\frac{\dot{\hat{w}}}{\hat{w}} = \varepsilon^w \frac{\dot{p}_s}{p_s} \geq 0 \qquad (27)$$

since the elasticity ε^w of (15) is positive if s is labor intensive and negative if s is capital intensive, relative to sector m.

3.4.2   The path of output supplies

Consider the case of agriculture. In non-intensive form, output supply is

$$Y_a = \frac{\partial}{\partial p_a} G^a (p_a, \hat{w}, r) B(t) T$$

and, since $\dot{p}_a / p_a = 0$, its evolution per worker is given by

$$\frac{\dot{Y}_a}{Y_a} - \frac{\dot{L}}{L} = \varepsilon_{\hat{w}}^a \frac{\dot{\hat{w}}}{\hat{w}} + \varepsilon_r^a \frac{\dot{r}}{r} + \frac{\dot{B}}{B} - \frac{\dot{L}}{L} \qquad (28)$$

where it follows from the envelope conditions that the elasticities $\varepsilon_{\hat{w}}^a = (\partial Y_a / \partial \hat{w})(\hat{w}/Y_a)$, and $\varepsilon_r^a = (\partial Y_a / \partial r)(r/Y_a)$ are negative. In the steady state, (28) grows at the rate $\dot{B}/B - \dot{L}/L$. Thus, given (26) and (27), the transition $\dot{Y}_a/Y_a$ depends on the intensity of labor in production relative to capital (i.e., the relative magnitude of the elasticities: $\varepsilon_{\hat{w}}^a$, ε_r^a given the rate of increase (decrease) in $\dot{\hat{w}}/\hat{w}$ and $\dot{r}/r$, respectively), and consequently the path of output per worker is not necessarily a monotonic convergence to its long-run growth rate.

The case of the other two sectors is most easily seen by appealing to the country's gross product function. The economy's supply functions in non-intensive form, per worker, are implied by envelope properties of this function. In elasticity terms, we have

$$\frac{\dot{Y}_m}{Y_m} - \frac{\dot{L}}{L} = \varepsilon_{p_s}^m \frac{\dot{p}_s}{p_s} + \varepsilon_{AL}^m \left(x + \frac{\dot{H}}{H} \right) + \varepsilon_K^m \frac{\dot{K}}{K} + \varepsilon_{AT}^m \frac{\dot{B}}{B} - \frac{\dot{L}}{L} (1 - \varepsilon_{AL}^m) \qquad (29)$$

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$$\frac{\dot{Y}_s}{Y_s} - \frac{\dot{L}}{L} = \varepsilon_{p_s}^s \frac{\dot{p}_s}{p_s} + \varepsilon_{AL}^s \left(x + \frac{\dot{H}}{H} \right) + \varepsilon_K^s \frac{\dot{K}}{K} + \varepsilon_{AT}^s \frac{\dot{B}}{B} - \frac{\dot{L}}{L} (1 - \varepsilon_{AL}^s) \qquad (30)$$

where homogeneity of degree one in factors of production implies that the factor elasticities sum to unity. If sector j is the most capital intensive, it can be shown that ε_K^j is positive and

$$\varepsilon_{AL}^j < \varepsilon_K^j > \varepsilon_{AT}^j$$

In transition to long-run growth, for an interval where $\hat{k}(t) < \hat{k}_{ss}$, where $\dot{\hat{k}} > 0$, it follow that, $\dot{K}/K > x + \dot{H}/H + \dot{L}/L$. That is, capital accumulation will tend to increase the per capita output of manufactures relative to services unless $\dot{p}_s/p_s > 0$. Since the demand for the non-traded good must evolve at the same rate as supply (30), using the expenditure funtion and the Euler condition, we obtain

$$r - \rho - \frac{\dot{p}_s}{p_s} = \frac{\dot{Y}_s}{Y_s} - \frac{\dot{L}}{L} \qquad (31)$$

4   Fitting the model to data

Three primary sources of data were used to calibrate the model to the South African economy for the year 1993. This year was chosen because it is consistent with the Social Accounting Matrix (SAM) avalable from the International Food Policy Research Institute, and the onset of HIV was relatively small. Further, calibrating the model to this point in time permits us compare the model's prediction with observations for the period 1994 to 2001. Rather than using the "structural" prevalence equation (2), for reasons mentioned, we instead calibrate to the prevalence forecasts of the South African epidemiological "model" (ASSA, 2000). To assess the model's peformance, we "adjust" the World Bank's World Development Indicators data base for South Africa to comply with our SAM's definitions of agriculture $(j = a)$, the rest of the internationally traded goods sector $(j = m)$, and the non-internationally traded goods sector $(j = s)$.

The essential parameters calibrated from the data are reported in Table 1.

Table 1.

	Production cost shares			Consumption shares
	Labor	Capital	Land	
Agriculture	0.371	0.500	0.129	0.0546
Manufacturing	0.522	0.478	–	0.3305
Services	0.595	0.405	–	0.6149

The model was also calibrated **without** the HIV prevalence structure. In this case, we simply replaced the prevalence structure in the model with the country's rate of labor force growth reported in the Development Indicators data for the years 1993-94. The results from this calibration provide some insight into how the economy might have evolved in the absence of HIV.

4.1 Evaluating model performance

Figures 2.a through 2.d show the *per capita* level of agriculture, industry, services and economy GDP in constant local currency units (LCU), normalized to unity in the base year, 1993, based on the World Development Indicators. Corresponding values based on model
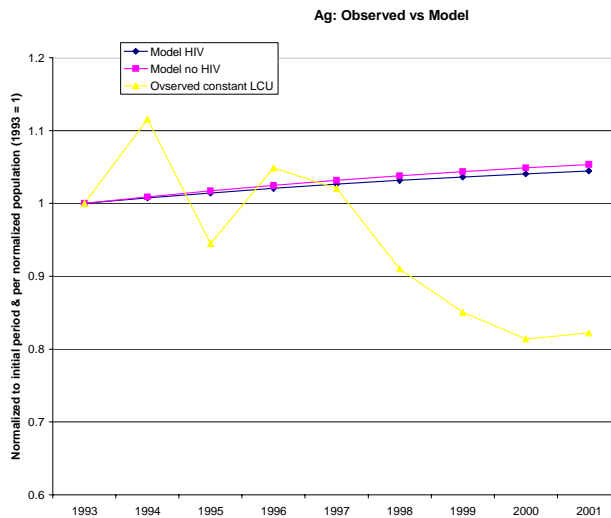

Figure 2.a

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forecasts, normalized to unity in the base year, 1993 are also shown for the two model results, one without the HIV structure, and one with the HIV structure.

Consider figure 2.a for the case of agriculture. Clearly, weather and other factors outside our model are affecting the performance of South African agriculture. Neither the HIV nor the no-HIV model results seem to approximate well agriculture's peformance over the period 1993 - 1997, after which the data suggest negative rates of growth. This could be caused by the extended drought during the latter 1990s, but post apartheid policies in agriculture may also play a role.

Industry: Observed vs Model
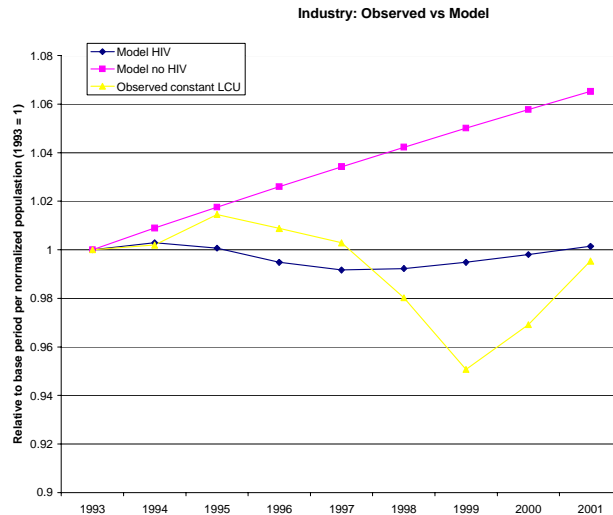

Figure 2.b

The HIV model appears to perform much better for the case of industry (figure 2.b). The model appears to capture the down turn in industrial output relative to the base year during the 1994-1997 period, and then, it misses the continued downturn of 1998-1999, but continues upward through 2001 as does the real economy. The no HIV model clearly over predicts industrial output.

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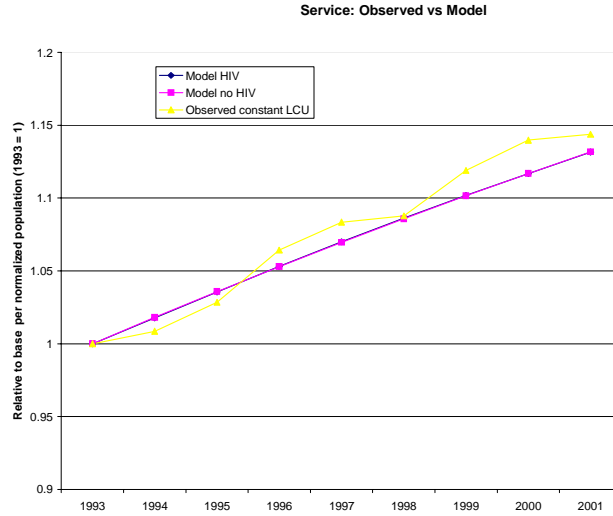
Figure 2.c

In the case of the service sector (figure 2.c), both the HIV and no HIV models track sector output surprisingly well throughout the 1993-2001 period. Combining these results for economywide GDP, figure 2.d shows that the no-HIV model tracks GDP during the early periods when the prevalence of HIV in the SA economy was relatively small, while the HIV model appears to track the data more closely during the last half of the 1990s.
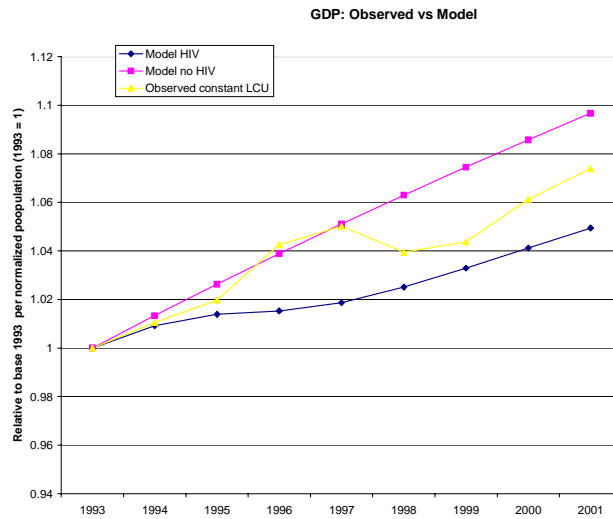

Figure 2.d

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We conclude from these comparisons that, at least in a qualitative manner, the HIV model should provide insights into the effects of the disease on the SA economy.

4.2 Basic economic forces causing model results

We focus on the basic forces driving model results for the case of the no HIV model, and then discuss how the presence of HIV modifies these basic forces.

4.2.1 The no HIV model

Model results for the specification with no HIV and with HIV are presented in Table 2 for five year intervals over the period 1993 - 2027. Within sample period results for production are presented in figure 2.a-2.d. Common to both models is a a rise in wages, a decline in capital rental rates, and a rise in the price of the non-traded good over the period. The rise in price of non-traded goods is predicted by equation (26) when the manufacturing sector is relatively more capital intensive than is the production of non-traded goods. Table 1 shows that capital cost in manufacturing is a slightly larger share in total costs than is capital cost in services, which leads to a negative elasticity $\varepsilon^r < 0$ in equation (26). A decline in returns to capital, $(\dot{r}/r < 0)$, is thus consistent with a rise in the price of non-traded goods $\dot{p}_s/p_s > 0$.

Growth in output of manufacturing and services per worker is positive but declining over time in the no HIV model. In services, the negative Rybczynski like effect of growth in capital stock per worker is compensated by the positive effects from growth in the labor force and growth in the price of the service good. These effects are reversed in manufacturing. The exception is agriculture. We use equation (28) to explain this case. The growth in agricultural output per worker $\left(\dot{Y}_a/Y_a - \dot{L}/L \right)$ is positive over the period during which the effect of a decline in the capital rental rate plus the rate of growth in land productivity dominates the growth in effective wages, i.e.,

$$\varepsilon_r^a \left(\dot{r}/r \right) + \left(\dot{B}/B \right) > \varepsilon_{\hat{w}}^a \left(\dot{\hat{w}}/\hat{w} \right).$$

Then, as $\dot{r}/r$ becomes a smaller absolute value, the change in wages per effective worker eventually dominates, and the growth in agricultural output per worker becomes negative, starting in about 2014.

Table 2 Output per worker relative to base period, results with and without HIV, five year mean

Period	Model with no HIV						Model with HIV					
	w	r	p_s	y_a	y_m	y_s	w	r	p_s	y_a	y_m	y_s
93-97	1	1	1	1	1	1	1	1	1	1	1	1
98-02	1.060	0.944	1.008	1.031	1.040	1.070	1.031	0.948	1.007	1.026	1.003	1.047
03-07	1.112	0.901	1.015	1.049	1.075	1.130	1.073	0.908	1.014	1.056	1.015	1.101
08-12	1.157	0.867	1.020	1.057	1.107	1.182	1.115	0.877	1.019	1.110	1.029	1.157
13-17	1.197	0.839	1.025	1.055	1.137	1.226	1.154	0.851	1.023	1.167	1.041	1.209
18-22	1.233	0.818	1.029	1.047	1.166	1.265	1.188	0.830	1.026	1.219	1.052	1.254
23-27	1.264	0.800	1.032	1.033	1.192	1.298	1.219	0.812	1.029	1.264	1.064	1.293

All variables are normalized to the base period. The y_j are in terms of output per worker.

A more intuitive explanation of this evolution is the following. As capital accumulates at a higher rate than the growth in labor, labor productivity rises to a larger extent in the capital intensive sector relative to the least capital intensive sector (services). Thus, at the period $t = 0$ wage rate, this accumulation gives rise to an excess demand for labor in the more capital intensive sectors. If the price of the non-traded good p_s were to remain constant, labor would be pulled from this sector. However, growth in income induces households to increase their demand for non-traded goods, which the service sector can only accomodate by raising the price of non-traded goods. The rise in the price of non-traded goods causes a rise in effective wages according to ε^w (27), thus dampening the demand for labor in agriculture through the parameter, $\varepsilon^a_{\hat{w}}$, (28), and manufacturing through the parameter $\varepsilon^m_{p_s}$, (29). Diminishing returns in agriculture, given land as a fixed sector specific factor, cause output per worker to fall as the rise in wages eventually dominate the decline in the rental rate of capital.

4.2.2 The HIV model

The fundamental forces discussed above prevail in both models, but they are conditioned in the HIV model by the prevalence function, i.e., the terms $H(t)L(t)$, equation (5). The effect of HIV on morbidity, and mortality (i.e. effect on the supply of labor), are shown in Table 3, column one and two. The effect of HIV on morbidity is particularly pronounced from 1993 to about 2010 (see also figure 1), and then relatively constant from about 2020 onward. The effect on mortality is pronounced, suggesting that relative to the average population in the base period, South Africa's population will only be about 58 percent of the population that would have existed during 2023-27 if HIV were totally absent from the population over the entire period. Morbidity, $H(t)$, affects the "augmentation" of effective labor. As figure 1 suggests, and as shown in Table 3, augmentation declines most rapidly in the earlier periods.

Since the morbidity term $H(t)$ appears in the Euler condition, (9), household savings are directly affected by HIV in the short-run, $\dot{H}/H < 0$ (Table 3). The stock of capital per worker without HIV is about 5 % larger than the stock of capital in the economy with HIV (Table 3). Effectively, the productivity of capital is less in the HIV model due to the smaller supply of effective labor, thus decreasing, at the margin, household incentives to save[6]. While the evolution of r and p_s are similar in both models, it can be seen from Table 2 that w evolves more slowly, and averages about 4.2% less than the wage in the no HIV model over the 1993-2027 period.

The effect on agricultural production $\left(\dot{Y}_a/Y_a - \dot{L}/L\right)$ is such that, in contrast to the no HIV case, the effective wage effect $\varepsilon_{\hat{w}}^a\left(\dot{\hat{w}}/\hat{w}\right)$ does not dominate the other terms in (28) so that output per worker grows throughout the transition to the steady state. Although in the initial periods, the level of agricultural output per worker is lower in the HIV case, during the 2003-07 interval it surpasses the corresponding level in the no HIV case (see Table 2).

Table 3. Evolution of morbidity, labor, and capital stock per worker

(relative to base period)

Period	Percent Less Labor	Morbidity	Cap. Stock/Worker Relative to base		
			no HIV	with HIV	Percent Less
93-97	1.74	0.981	1.000	1	0
98-02	3.87	0.968	1.119	1.083	3.30
03-07	6.12	0.961	1.227	1.173	4.65
08-12	10.93	0.958	1.325	1.261	5.12
13-17	16.91	0.958	1.413	1.340	5.40
18-22	23.17	0.959	1.491	1.412	5.61
23-27	29.50	0.962	1.562	1.478	5.67

The manufacturing sector output per worker is affected negatively in the HIV case over the period 1993-1997, (Figure 2.b), and then grows at low positive rates relative to the no HIV case, as can be seen by comparing the corresponding y_m columns in Table 2. The negative effect of HIV on manufacturing output per worker is caused by the small positive Rybczynski like effect from growth in capital stock, $\varepsilon_K^m(\dot{K}/K)$, compared to the no HIV model. This "loss" is greater than the effect of the decline in the growth of labor due to the mortality effect, which operates the term $(1 - \varepsilon_{AL}^s)(\dot{L}/L)$, equation (29). The effect of morbidity, $\varepsilon_{AL}^m \left(x + \dot{H}/H \right)$, equation (29), is positive but small during the 1990s, and negative, but small, during the rest of the period. Since the negative effect of the increase in the price of the non-traded good $\varepsilon_{p_s}^m (\dot{p}_s/p_s)$ is almost identical in the two models, this term causes little effect in the sector's growth rate between the two models.

As both Figure 2.b and Table 2 suggest, that the production of the non-traded good, on a per worker basis, is least affected by the presence of HIV. This result occurs because the slower rate of growth in the capital stock has a smaller negative Rybczynski like effect than in the no HIV model, and this smaller negative effect is almost balanced by the slower growth in labor, which amounts to a smaller positive Rybczynski like effect. The negative supply effect of morbidity, $\dot{H}/H$ operating through the term $\varepsilon_{AL}^s \left(x + \dot{H}/H \right)$, is larger in during the first ten years, and relatively small there after. The evolution of $\dot{p}_s/p_s$ does not

affect differences between the two models since the path is virtually the same in both.

4.3   Longer-term forecasts and contrasts

We begin by examing GDP growth rates. Figure 3.a presents the aggregate GDP growth rates for the HIV and no-HIV base scenario, assuming $\gamma = 0.85$. Although examining a different country, Sackey and Rarpala (2000) assume that with no HIV, GDP growth in Lesotho would be 4% between 2000 and 2015. With HIV, however, they estimate the growth rate would drop to 2.5% in 2010 and 1.3% in 2015. Arndt and Lewis (2002) estimate without HIV and AIDS, GDP growth over the period 1997 through 2010 would increase from about 2.3% in 1997 to 3.7% in 2010. Our estimates, however, indicate an opposite trend: GDP growth without HIV declines steadily from 3.59% in 1993 to a long run rate of growth of about 2.5%. With HIV, Arndt and Lewis estimate a GDP growth rate of about 2% in 1997 to about 1.3% in 2010. Hence, Arndt and Lewis predict a 0.3% difference in aggregate GDP growth in 1997 and a 2.4% difference in 2010. Our estimates predict a difference in aggregate GDP growth rates of 1.55% during 1998 – 2002, and a difference of about 1.65% in 2010. The difference in aggregate GDP growth rates reaches is projected to exceed 1.9% by 2050.
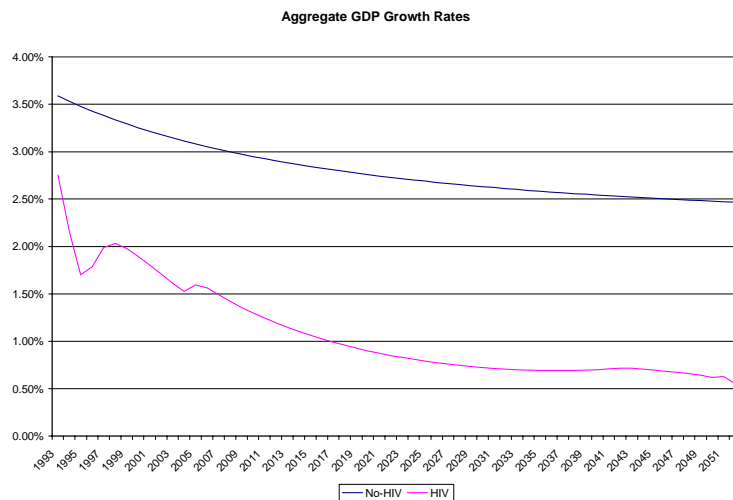

Aggregate GDP Growth Rates

Figure 3.a

The trajectory of actual and projected per capital growth rates are presented in Figure 3.b.

Per capital GDP growth without HIV is about 3.6% in 1993, and steadily declines to about 0.22% in 2053. As in the observed data, per capita GDP under the HIV scenario is erratic between 1993 and 2001. This pattern continues until about 2006, after which per capital GDP growth steadily declines, reaching about 0.4% three generations later in 2053. Relative to the no-HIV case, Arndt and Lewis predict per capita GDP will be about 8% lower in the presence of HIV.

Per Capita GDP Growth Rates

Figure 3.b

Observe that the difference in per capita growth rates is significant only during the first few years of the disease, when the morbidity effects, $H(t)$, decrease rapidly (see Figure 3.c). Once the morbidity effects begin leveling off, the rate of growth under both the no-HIV and HIV scenarios are similar.
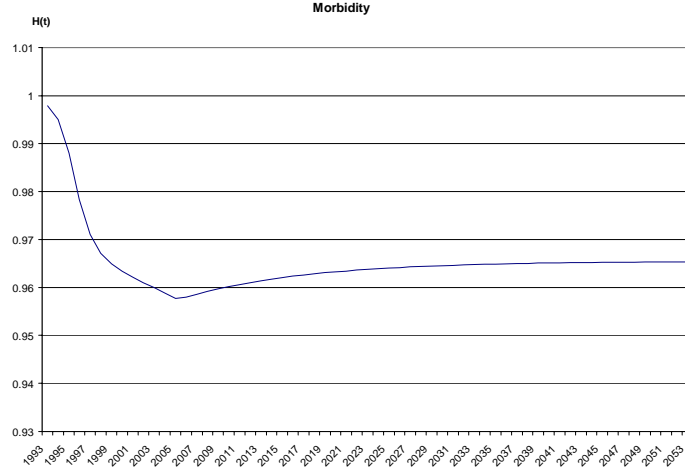
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Figure 3.c

The impact of HIV on economic growth is much more pronounced when viewed from the perspective of aggregate GDP. Figure 3.d presents the projected differences between aggregate and sectoral output under the HIV and no-HIV cases over the three generation period, 1993 – 2053. Unless effective intervention measures are implemented, by 2053, South African GDP levels are projected to be more than 60% lower than they could have been had there not been an HIV pandemic. Keep in mind that these projections include a modest morbidity effect and no impact on total factor productivity.
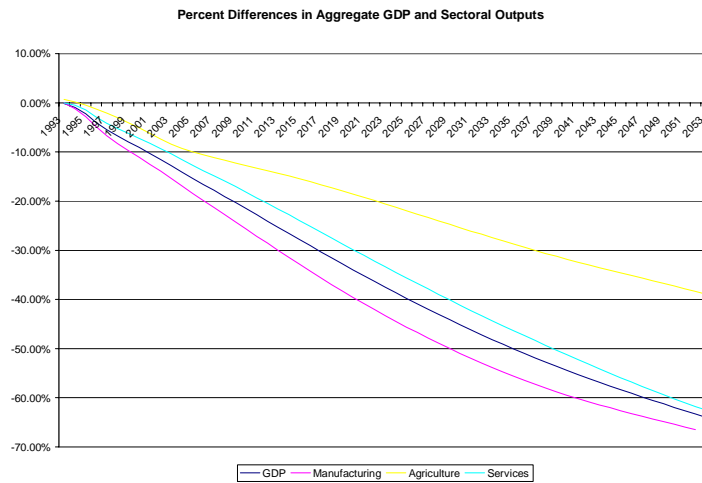

Figure 3.d

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Table 4 shows from 1993 through 2002, the difference in GDP levels were relatively small, with aggregate GDP under HIV being about 3% smaller than aggregate GDP with no HIV. Within three generations (by 2053), projections suggest aggregate GDP could be over 60% smaller with HIV than it might have been if there were no HIV. This result appears to be related to both the significantly smaller size of the aggregate capital stock and labor force, where both are about 60% smaller in the HIV scenario than in the no-HIV scenario. Figure 4 also portends of the three sectors, manufacturing loses the most as a result of the disease, followed by the service sector, and then agriculture.

Table 4. Percent differences in aggregate GDP. capital stock, labor, and sectoral output

	GDP	Capital	Labor	Morbidity	Manufacturing	Agriculture	Services
93 - 97	-3.18%	-3.51%	-1.30%	-1.15%	-3.91%	-0.94%	-2.27%
98 - 02	-9.12%	-9.86%	-5.12%	-2.73%	-11.09%	-5.18%	-7.24%
03 - 07	-15.34%	-16.38%	-10.82%	-3.90%	-18.38%	-9.87%	-12.66%
08 - 12	-21.55%	-22.77%	-17.26%	-4.66%	-25.68%	-12.77%	-18.18%
13 - 17	-27.82%	-29.14%	-23.88%	-5.16%	-32.83%	-15.50%	-23.98%
18 - 22	-33.96%	-35.30%	-30.36%	-5.48%	-39.53%	-18.59%	-29.86%
23 - 27	-39.77%	-41.05%	-36.51%	-5.69%	-45.59%	-21.97%	-35.63%
28 - 32	-45.15%	-46.29%	-42.25%	-5.83%	-50.95%	-25.41%	-41.14%
33 - 37	-50.05%	-50.98%	-47.55%	-5.92%	-55.59%	-28.73%	-46.35%
38 - 42	-54.45%	-55.07%	-52.42%	-5.97%	-59.47%	-31.82%	-51.27%
43 - 47	-58.35%	-58.57%	-56.85%	-6.01%	-62.61%	-34.59%	-55.90%
48 - 52	-61.93%	-61.78%	-60.90%	-6.04%	-65.40%	-37.29%	-60.20%

5   Conclusion

The main analytical contribution of this paper is incorporating epidemiological projections of mortality and morbidity into a neoclassical growth model of a small, open and competive economy – yielding an economy characterized by a system of nonautonomous differential

equations. In the theoretical development we use a simple, structural, epidemiological model of HIV prevalence dynamics to illustrate how such information can be linked to a dynamic neoclassical growth model. In the empirical model, we incorporate the projections from a "richer" epidemiological model developed by South African epidemiologists. We fit to 1993 South Africa data, two versions of the model – one without and one with HIV prevalence dynamics – and empirically solve them to obtain transition path equilibria over a period of three generations (about 60 years). Model projections are compared to data for the period 1993-2001, and we conclude that the HIV model fits the in sample data reasonablly well.

A summary of our findings include: (i) GDP growth without HIV declines steadily from 3.59% in 1993 to a long run rate of growth of about 2.5%. In the presence of HIV and AIDS, aggregate GDP growth rates would be 0.23% percentage points smaller than the no-HIV rates during 1997 – 2002, and over the intermediate run about the same, and over the longer run about 0.18 percentage points smaller. and long run were projected to be about equal0.9 percentage points less (e.g., drop from 2.5% to 1.6%); (ii) Per capital growth in GDP with no HIV begins at about 1.4% in 1993 and steadily declines to about 0.22% in 2053. As in the observed data, per capita GDP under the HIV scenario is erradic between 1993 and 2001, but then steadily declines to about 0.4% in 2053; (iii) The long run impact of the disease on aggregate GDP can be staggering – within three generations (by 2053), aggregate GDP could be over 60% smaller with HIV, with the negative impact of the disease impacting manufacturing the most.

To understand our results, consider first the economic intuition underlying the no HIV model, as these forces are common to both models: As capital accumulates at a higher rate than the growth in labor, labor productivity rises to a larger extent in the capital intensive sector relative to the least capital intensive sector (services). Thus, at the period $t = 0$ wage rate, this accumulation gives rise to an excess demand for labor in the more capital intensive sectors. If the price of the non-traded (service) good were to remain constant, labor would be pulled from this sector. However, growth in income induces households to increase their demand for non-traded goods. The service sector responds by raising the price of non-traded goods. The rise in the price of non-traded goods causes an increase in effective wages, thus

dampening the demand for labor in agriculture and manufacturing. Diminishing returns in agriculture cause its' output per worker to fall as wage increases eventually dominate the decline in the rental rate of capital.

The effect of HIV causes agricultural production per worker to rise, in contrast to the no HIV case, because the slower growth in wages has a smaller negative effect on agricultural production. The negative effect of HIV on manufacturing output per worker is caused by the small positive Rybczynski like effect from a slower growth in capital stock compared to the no HIV model. The production of the non-traded good, on a per worker basis, is least affected by the presence of HIV. This result occurs because the slower rate of growth in the capital stock has a smaller negative Rybczynski like effect than in the no HIV model, and this smaller negative effect is almost balanced by the slower growth in labor.

HIV and AIDS adds to the basic model, the effects of morbidity and mortality. Since the morbidity term $H(t)$ appears in the Euler condition, (9), morbidity has a direct short-run effect on household savings per worker. The stock of capital per worker *without* HIV averages about 5% larger than the stock of capital in the economy with HIV. Effectively, the productivity of capital is less in the HIV model due to the smaller supply of effective labor, thus decreasing, at the margin, household incentives to save. The presence of HIV causes wages to evolve more slowly, and over the 1993-2027 period, averages about 4.2% less than wages in the no HIV model. The effect of HIV on morbidity is particularly pronounced from 1993 to about 2010, and then tapers off from about 2020 onward. The effect on mortality is especially damaging, with South Africa's population during 2023-27 being 35% smaller than which would have prevailed absent HIV. Correspondingly, aggregate GDP is almost 40% smaller in the HIV case.

As with most research, there are several potential improvements to the current model. One improvement is to estimate the statistical relationship between HIV prevalence rates and morbidity. The current model assumes that, on average, an HIV infected individual will miss 3/4 days work each week, and as a result will yield provide only 85% efficiency units of labor. Another improvement would be to introduce HIV related health expenditures, and then examine the impact of health expenditures on GDP levels and growth rates. Health

expenditures would likely affect the rate at which capital accumulates, and hence, exacerbate the decrease in long run GDP levels. On the other hand, health expenditures would likely decrease morbidity rates and mortality. These two opposing forces suggest there might be an optimal level of investment in AIDS treatment. Hence, linking (endogenizing) mortality with health expenditures is a natural next step in which to take the above research.

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Notes

[1]The *prevalence rate* of HIV for a population is defined as the percent of the population infected with the disease.

[2]Of course, ascertaining the "proper" relationship between γ and h requires empirical work, but unfortunately to the authors' knowledge such work has not yet become available.

[3]An alternative derivation is to simply derive the supply functions from the respective price gradient of the GDP function.

[4]For the case of Cobb-Douglas technologies,

$$p_3 = \left(A_m \alpha^\alpha \left(1-\alpha\right)^{(1-\alpha)} \right)^{\frac{\delta}{\alpha}} \left(\rho + x\right)^{\frac{\alpha-\delta}{\alpha}} \left(A_s^{-1} \delta^{-\delta} \left(1-\delta\right)^{(\delta-1)} \right)$$

where α and δ are production elasticities of labor in manufacturing and services respectively, and A_m and A_s are scale parameters.

[5]This presumes the other two sectors remain open and hence, the zero profit conditions hold as an equality.

[6]This result can be derived from the gradient of the economy's gross domestic product function with respect to capital.