A Dynamic Programming Framework for Using Weather Derivatives to Manage Dairy Profit Risk

Gang Chen and Matthew C. Roberts[*]

Department of Agricultural, Environmental, and
Development Economics
The Ohio State University
2120 Fyffe Road
Columbus, Ohio 43210

Contact: Chen.796@osu.edu

A Dynamic Programming Framework for Using Weather Derivatives to Manage Dairy Profit Risk

Abstract

Dairy farms confront unique risks from weather conditions. Hot and humid weather induces heat stress, which reduces both the quantity and quality of dairy production. Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling. Adoption of abatement equipment, however, is hindered by its high initial cost and possibly long payback period, especially for small- and medium-scale firms. Moreover, the abatement equipment is only seasonally useful as a fixed asset whose price rises with efficacy. Weather derivatives provide an alternative method of dairy farmers' risk management. Since abatement equipment can be used for many years once installed, and its maintenance costs will increase and efficacy will decrease with age, a decision that must regularly be made by a dairy farmer is when to maintain his abatement equipment and when to replace it with a new one. The decision affects both current and expected future revenues. Considering that weather derivatives can be purchased periodically, the objective of this study is twofold: first, to test the risk management value of weather derivatives for dairy plant operations; second, to examine how weather derivatives can affect dairy producers' abatement equipment decisions. In this study, we employ a dynamic programming framework to study the case that a representative dairy farmer maximizes his long-run utility using weather derivatives and abatement equipment.

Keywords: abatement technology, dynamic programming, mean-variance efficiency, profit risk, weather derivatives

Introduction

Dairy farms confront unique risks from weather conditions. Hot and humid weather induces heat stress, which reduces both the quantity and quality of dairy production (Barth; Thompson). Heat stress is measured by temperature-humidity index (THI, also commonly known as the 'heat index'). Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling. Adoption of abatement equipment, however, is hindered by its high initial cost and possibly long payback period, especially for small- and medium-scale firms. Moreover, the abatement equipment is only seasonally useful as a fixed asset whose price rises with efficacy. Weather derivatives provide an alternative method of dairy farmers' risk management. Instead of reducing production losses, weather derivatives make payments based upon observed weather conditions over a period of time so that they offer the potential to offset profit losses caused by adverse weather events.

Investment in abatement equipment is both irreversible[1] and deferrable.[2] So keeping the money on hand and deferring the investment is similar to keeping an American call option. These irreversible investment opportunities are typically called real options. They are valuable and the value is highly sensitive to the uncertainty of future cash flow of the investment. Once a farmer has made an investment in abatement equipment, the real option has been exercised and he gives up the possibility of waiting for new information to arrive that might affect the desirability or timing of the investment expenditure.[3] The value of the real option is an opportunity cost that must be included as part of the investment cost. So whether or not to investment will not only depend on whether the Net Present Value (NPV) of a unit of capital is positive, but also whether the positive NPV is larger than the value of the real option. Pindyck (1991) shows that this kind of investment decision making problem under ir-

---

[1] Due to, for example, prohibitively high sunk cost, the "lemons" problem or that the investment is firm-specific.

[2] "There may be a cost to delay - the risk of entry by other firms, or simply foregone cash flows - but this cost must be weighed against the benefits of waiting for new information." pp. 1111, Pindyck (1991).

[3] For example, see McDonald and Siegel (1986).

reversibility and uncertainty can be solved by using two qualitatively equivalent approaches: option pricing and dynamic programming.

Since abatement equipment can be used for many years once installed,[4] and its maintenance costs will increase and efficacy will decrease with age, a decision that must regularly be made by a dairy farmer is when to maintain his abatement equipment and when to replace it with a new one. The decision affects both current and expected future revenues. Many similar problems have been studied by using dynamic programming models in the literature. Examples include dairy cow replacement (Smith, 1972; Miranda and Schnitkey 1995), swine production (Chavas *et al.*, 1985), poultry production (McClelland *et al.*, 1989), livestock feeding (Burt, 1993), and bus engine replacement (Rust, 1987).

Considering that weather derivatives can be purchased periodically, the objective of this study is twofold: first, to test the risk management value of weather derivatives for dairy plant operations; second, to examine how weather derivatives can affect dairy producers' abatement equipment decisions. Dynamic programming approach is employed to determine the optimal actions to abatement equipment and optimal weather derivatives purchase. Specifically, we employ a dynamic programming framework to study the case that a representative dairy farmer maximizes his long-run utility using weather derivatives and abatement equipment. The actions that the dairy farmer need to determine at the start of each period include weather derivatives purchase amount and a choice from three alternative treatments to abatement equipment – no action, replacement, or maintenance, based upon the abatement equipment age and its maintenance history. A solution as the optimal risk management strategy for abatement equipment replacement and weather derivatives purchase is derived using policy iteration method.

The empirical analysis is performed using 35-year daily weather data from Summit County and Cleveland, Ohio. The results suggest that simultaneously using weather derivatives and

---

[4]Abatement equipment typically has a lifespan of 7-10 years depending on the maintenances.

abatement technologies will outperform using abatement technologies alone, even in presence of basis risk and transaction costs. Thus weather derivatives expand the portfolio choice set, and make more desirable profit distributions available. Moreover, using weather derivatives will reduce the initial investment in abatement equipment and postpone the asset replacement, which mitigates the problem of high initial installation cost of abatement equipment for small- and medium-size firms.

Background

Weather derivatives as relatively new financial products possess several unique properties distinguishing them from other derivatives. First, their underlying (i.e. weather) is not a tradable asset. Second, they hedge against volumetric risk instead of price risk. The indemnity is calculated based on a weather index (Cooling/Heating Degree-Days, rainfall, etc) rather than asset price. Third, they are not as liquid as traditional standard derivatives. If we assume away transaction costs, the traditional financial derivative markets are liquid. Weather, by its nature, is location-specific. Different locations have different weather conditions whether at the same time or across time.

Due to their properties, weather derivatives have advantages over traditional financial derivatives in the view of hedging against weather risk. Because there is no need to prove damage to claim payoffs, there is little moral hazard. Furthermore, as weather information is almost perfectly symmetric, adverse selection is eliminated. At the same time, the use of weather derivatives is accompanied by basis risk caused by the fact that an end-user's location often is not the same location as the reference location of the weather derivatives he holds.

Economic losses are induced in the dairy industry when effective temperature conditions are out of dairy cows' thermal comfort zone. According to St-Pierre, Cobanov and Schnitkey (SCS, 2003), heat stress in dairy cattle is a function of the Temperature-Humidity Index (THI). Johnson (1980) reports that a THI higher than 72 degrees is likely to have adverse ef-

fects on per-cow yield. In SCS, it is suggested that the threshold of THI to trigger heat stress should be lowered to 70 degrees accounting for the lower heat tolerance of the current selection of dairy cows. So 70 degrees is used as a threshold for risk from heat stress, *THI$_{threshold}$*. According to the National Oceanic Atmospheric Administration (NOAA, 1976), the standard formula of THI is: THI = T – (0.55 – 0.55 RH) (T – 58), where T is temperature in degrees Fahrenheit and RH is relative humidity in percent. Since RH is is expressed as a percentage, it is easy to see that THI is positively correlated with temperature.

THI is varying in a day along with change of temperature and relative humidity. The maximum THI is in the afternoon, when the temperature is highest and relative humidity is lowest; and the minimum THI is in the night, when the temperature is lowest and relative humidity is highest. In this paper, daily THI refers to daily maximum THI. If maximum THI is lower than 70 degrees in a day, there is no heat stress for dairy cows.

One-period Setting

In a one-period setting, we develop a representative dairy farmer's net profit model with using weather derivatives and abatement equipment to reduce the profit risk induced by heat stress. And in next section, this one-period profit function will be used in dynamic programming analysis for determining optimal sequential risk management strategies. To reduce the visual complexity, we drop the time subscripts of model notations in this section.

Consider a dairy farmer who produces without using abatement equipment or weather derivatives. His profit is $\widetilde{y} = P \cdot \widetilde{Q} - TC$, where P is milk price, $\widetilde{Q}$ is the stochastic yield, and TC denotes a total cost. For analytical simplicity, it is assumed there is no price risk; therefore price is normalized to unity. The tilde ($\sim$) denotes a random variable.

Suppose expected profit of a farmer is his historical average, μ, so the difference between $\widetilde{y}$ and μ is his profit risk. The profit risk is orthogonally decomposed into two parts. One

is systematic risk which comes from weather conditions; the other is nonsystematic risk which reflects the individual's production variability not arising from weather and is assumed uncorrelated with weather conditions. Equations (1)-(10) give the daily models, because equation (3), by its nature, is for daily calculation. However, they can be easily transformed into yearly models by using yearly cumulative values of the variables.

$$(1) \qquad \widetilde{y} = \mu + \theta \cdot f(\widetilde{x}) + \widetilde{\varepsilon},$$

where

$$(2) \qquad \widetilde{x} = E(\widetilde{z}) - \widetilde{z}$$

$$(3) \qquad \widetilde{z} = \max(\widetilde{THI} - THI_{threshold},\ 0)$$

$$(4) \qquad \theta = \mathrm{cov}(\widetilde{y}, f(\widetilde{x}))/\mathrm{var}(f(\widetilde{x}))$$

$$(5) \qquad E(\widetilde{y}) = \mu,\ \ E(\widetilde{\varepsilon}) = 0,\ \ \mathrm{var}(\widetilde{\varepsilon}) = \sigma_{\widetilde{\varepsilon}}^2,\ \ \mathrm{cov}(\widetilde{z}, \widetilde{\varepsilon}) = 0,\ \ \mathrm{cov}(\widetilde{x}, \widetilde{\varepsilon}) = 0.$$

The coefficient θ quantifies the sensitivity of the farmer's individual profit to systematic risk. The factor $\widetilde{z}$, which is common to all producers in a region, measures the degree of heat stress, and the factor $\widetilde{x}$ denotes the weather condition compared to its expectation. If $\widetilde{z}$ is lower than $E(\widetilde{z})$, it means the heat stress is milder than its expectation. In this case, $\widetilde{x}$ is positive. And $f(\widetilde{x})$ captures systematic risk and increases with $\widetilde{x}$. The functional form of $f(\widetilde{x})$ is assumed to be linear, i.e. $f(\widetilde{x}) = \alpha \cdot \widetilde{x}$, where α is a positive parameter of the linear relationship. The final term $\widetilde{\varepsilon}$ is a nonsystematic risk component.

Then equation (1) becomes,

$$(6) \qquad \widetilde{y} = \mu + \theta \cdot \alpha \cdot \widetilde{x} + \widetilde{\varepsilon} = \mu + \beta \cdot \widetilde{x} + \widetilde{\varepsilon}$$

where

$$(7) \qquad \beta = \mathrm{cov}(\widetilde{y}, \widetilde{x})/\ \mathrm{var}(\widetilde{x}).$$

Suppose that weather derivatives are available for purchase. Since here the risk is from excessively high THI, weather derivatives that will be used are focused on weather call options. The underlying index is $\widetilde{THI}$, and the strike price is $THI_{threshold}$. The payoff from a weather call option is:

$$\widetilde{n} = \max(\widetilde{THI} - THI_{threshold},\ 0) = \widetilde{z}. \tag{8}$$

The hypothetical[5] option premium is calculated on the basis of actuarial fairness. So purchasing weather options cannot change the farmer's farmer's expected profit. The option premium equals the expected payoff:

$$\pi = E(\widetilde{n}) = E(\widetilde{z}). \tag{9}$$

The timing of using weather derivatives is as follows: at the start of the period, the farmer buys weather derivatives; then at the end of the period, when all the weather information of this period has been observed, the farmer receives payoff (if any) from weather derivatives.

Also suppose that the producer is free to choose his abatement equipment investment and maintenance strategy. By using abatement equipment, the production loss from heat stress can be reduced. The biological effectiveness of abatement equipment is a function of its initial investment, age, maintenance, and weather condition. Let $\widetilde{m}$ denote the reduced profit loss, i.e. the increased profit from using abatement equipment; and let λ denote the costs of abatement equipment operating strategy. The explicit functional form will be given in the next section.

Thus with weather options and abatement equipment, the producer's net profit equals:

$$\widetilde{y}^{net} = \widetilde{y} + \phi \cdot (\widetilde{n} - \pi) + \widetilde{m} - \lambda \tag{10}$$

where ϕ is weather options purchase amount. Therefore, there are two elements that the

---

[5]There has not been a weather derivative on THI in the security market yet.

producer need to determine: weather options purchase amount and abatement equipment operating strategy.

Dynamic Programming Model

This section presents a theoretical model of analyzing an asset replacement problem and examining how weather derivatives will affect the decisions. The assumptions include that first and second moments of accumulated weather conditions are constant and a representative farmer has mean-variance utility form. Thus the abatement investment and operating strategy will not depend on the current weather conditions, instead it will depend on the first and second moments of weather conditions.

Suppose a representative dairy farmer is seeking an optimal risk management strategy with the aim to maximize the present value of his long-run utility. Since abatement equipment is a kind of fixed asset, its investment level cannot be changed once installed. So the farmer need to decide the lump-sum initial installation investment. Then at the beginning of every following year, he observes the state, i.e. the status of his abatement equipment: (i) its age, and (ii) how many years since last maintenance – the state information affects the effectiveness of abatement equipment. Then the farmer needs to determine: (i) which action to take for his abatement equipment in three alternative choices: no action, replacement, and service, and (ii) weather derivatives purchase amount.

Formally, the representative farmer's decision problem is written as:

$$(11) \qquad \max \sum_{t=0}^{\infty} \delta^t U(s_t, x_t)$$

where t is time index in calendar year; $\delta \in (0, 1)$ is the farmer's discount factor reflecting interest rate and the individual's impatience; $U(s_t, x_t)$ is utility of year t; s_t is a vector of state

7

variables at t; and x_t is the farmer's decisions at t. This is an infinite horizon, deterministic model.

More explicitly, the discrete time dynamic programming model consists of the following objects: a state space, an action space, transition rule, a utility function, and a discount factor δ. We now turn to the detailed description of the model.

State and Action Variables

The *state vector* $s_t = (a_t, h_t)$ consists of two variables:

$a_t = \{1, 2, ..., \text{lifespan}\}$: age of abatement equipment;

$h_t = \{1, 2, ..., \text{lifespan}\}$: maintenance history (i.e. how many years since last maintenance).

The state variables, observed at the start of period t, represent the abatement equipment's current status that affects its effectiveness and maintenance costs in period t.

And the *action vector* $x_t = (i_t, \phi_t)$ also consists of two variables:

$$i_t = \begin{cases} 1, & \text{no action} \\ 2, & \text{maintenance} \\ 3, & \text{replace} \end{cases}$$

is abatement equipment operating decision at time t;

$\phi_t \in \mathbb{R}^+$: weather derivatives purchase amount at time t.

Once observing the state values, the farmer's decision problem is to choose the abatement equipment operating action, which will affect the current and future utility; and weather derivatives purchase amount, which will only affect utility of period t since the payoff (if any) will be claimed at the end of that period.

The abatement status *transition function* is:

$$(a_{t+1}, h_{t+1}) = \begin{cases} (a_t + 1, h_t + 1), & \text{if } i_t = 1 \text{ (no action)} \\ (a_t + 1, 1), & \text{if } i_t = 2 \text{ (maintenance)} \\ (1, 1), & \text{if } i_t = 3 \text{ (replace)}. \end{cases}$$

Formulating Profit and Utility Functions

In the one-period setting, net profit is given by:

$$(12) \qquad \widetilde{y}_t^{net} = \widetilde{y}_t + \phi_t \cdot (\widetilde{n}_t - \pi) + \widetilde{m}_t - \lambda_t,$$

where $\widetilde{y}_t = \mu + \beta \cdot \widetilde{x}_t + \widetilde{\varepsilon}_t$ and $\pi = E(\widetilde{n}_t)$.

In (12), the biological functional form of the effectiveness of abatement equipment is formulated as:

$$(13) \qquad \widetilde{m}_t = \kappa^{(h_t + a_t/n)} [(b + c \cdot \widetilde{THI}_t)\sqrt{\eta}],$$

where $\widetilde{m}_t$ is the reduced profit loss, i.e. the increased profit from using abatement equipment; $\widetilde{THI}_t$ is cumulative THI of period t; η is initial abatement equipment installation investment; $\kappa \in (0,1)$ reflects the declining effectiveness with aging and less frequent maintenance; and b and c are parameters reflecting the effect of weather condition.

Suppose that the producer is free to choose his initial abatement equipment investment η ($\eta \geq 0$; and $\eta = 0$ means he does not install abatement equipment). It is easy to see that $\widetilde{m}_t$ is increasing with η and $\widetilde{THI}_t$. When $\eta = 0$, $\widetilde{m}_t$ is also equal to 0. And with fixed η, $\widetilde{m}_t$ is increasing with $\widetilde{THI}_t$. That is because although the profit is low when $\widetilde{THI}_t$ is high, the reduced profit loss will be high with using abatement equipment; on the other hand, when $\widetilde{THI}_t$ is low (i.e. weather is good in period t), the abatement equipment is not of much use, so the reduced loss is low. Thus the parameter c is positive.

In (12), the costs of abatement equipment operating strategy, λ_t are specified as:

$$(14) \qquad \lambda_t = \begin{cases} 0, & i_t = 1 \\ (k_1 a_t + k_2 h_t) \cdot \eta, & i_t = 2 \\ \eta, & i_t = 3. \end{cases}$$

Here, k_1 and k_2 are positive parameters. So maintenance costs are increasing with age a_t and maintenance history h_t. Replacement costs are equal to initial abatement investment by assuming the residual value of the old equipment equals its uninstall costs.

In summary, the *profit function* is specified as:

$$(15) \qquad \widetilde{y}_t^{net} = \begin{cases} \widetilde{y}_t + \phi_t \cdot (\widetilde{n}_t - \pi) + \kappa^{(h_t + a_t/n)}[(b + c \cdot \widetilde{THI}_t)\sqrt{\eta}], & i_t = 1 \\ \widetilde{y}_t + \phi_t \cdot (\widetilde{n}_t - \pi) + \kappa^{(a_t/n)}[(b + c \cdot \widetilde{THI}_t)\sqrt{\eta}] - (k_1 a_t + k_2 h_t), & i_t = 2 \\ \widetilde{y}_t + \phi_t \cdot (\widetilde{n}_t - \pi) + [(b + c \cdot \widetilde{THI}_t)\sqrt{\eta}] - \eta, & i_t = 3. \end{cases}$$

Note that at the beginning of a period, if the farmer chooses to replace his abatement equipment with a new one, a_t and h_t turn to zero in that period; if he chooses to have maintenance service, h_t turns to zero in that period.

The producer is assumed to have a mean-variance *utility function*[6] of

$$(16) \qquad U = E(\bullet) - \frac{1}{2} A \cdot \text{var}(\bullet)$$

where A is an index of agents' aversion to taking on risk. Then the representative producer's objective is to choose initial abatement investment η, optimal option purchase ϕ_t, and abatement equipment operating strategy i_t to maximize his long-run utility:

$$(17) \qquad \max_{\eta, \phi_t, i_t} \sum_{t=0}^{\infty} \delta^t U_t \equiv \max_{\eta, \phi_t, i_t} \sum_{t=0}^{\infty} \delta^t [E(\widetilde{y}_t^{net}) - \frac{1}{2} A \cdot \text{var}(\widetilde{y}_t^{net})].$$

---

[6]This framework is equivalent to expected utility maximization if (net) profit is distributed normally and producers' utility function is exponential. But Meyer has shown that the mean-variance model is consistent with expected utility model under much weaker restrictions. See Pratt (1964) and Meyer (1987).

Thus far, the dynamic risk management problem is summarized by equations (15) and (17). In order to examine how weather derivatives will affect the decisions, we will compare the optimization solutions of the cases with and without using weather derivatives. Then the next task is to find the solution to this dynamic optimization problem.

Deriving Optimization Solution

We first derive the optimal weather options purchase within one period setting taking the state and action variables as given. Then a dynamic programming problem across periods will be solved numerically. And the optimal initial abatement investment is chosen by examining a series of different investment levels and picking up the one with maximum Bellman value.

Optimization Within One Period

Because weather derivatives purchase amount in each period will only affect utility at that period and will not affect state variables, the optimal weather derivatives purchase, ϕ_t, can be decided within one period setting.

(18)
$$\max_{\phi_t} U_t = \max_{\phi_t}[E(\widetilde{y}_t^{net}) - \frac{1}{2}A \cdot \mathrm{var}(\widetilde{y}_t^{net})].$$

Specifically,

$$
\begin{aligned}
(19) \quad U_t &= E(\widetilde{y}_t) + \phi_t E(\widetilde{n}_t - \pi) + E(\widetilde{m}_t - \lambda_t) - \frac{1}{2}A \cdot [\mathrm{var}(\widetilde{y}_t) + \phi_t^2 \mathrm{var}(\widetilde{n}_t) \\
&\quad + \mathrm{var}(\widetilde{m}_t) + 2\phi_t \mathrm{cov}(\widetilde{y}_t, \widetilde{n}_t) + 2\mathrm{cov}(\widetilde{y}_t, \widetilde{m}_t) + 2\phi_t \mathrm{cov}(\widetilde{n}_t, \widetilde{m}_t)] \\
&= \mu + \kappa^{(h_t + a_t/n)}(b + c\mu_{\widetilde{THI}})\sqrt{\eta} - \lambda_t - \frac{1}{2}A \cdot [\beta^2 \sigma_{\widetilde{z}}^2 + \sigma_{\widetilde{\varepsilon}}^2 + \phi_t^2 \sigma_{\widetilde{n}}^2 - 2\beta\phi_t \sigma_{\widetilde{z},\widetilde{n}} \\
&\quad + \kappa^{(2h_t + 2a_t/n)} c^2 \eta \sigma_{\widetilde{THI}}^2 - 2\beta\kappa^{(h_t + a_t/n)} c\sqrt{\eta}\sigma_{\widetilde{THI},\widetilde{z}} + 2\kappa^{(h_t + a_t/n)}\phi_t c\sqrt{\eta}\sigma_{\widetilde{THI},\widetilde{n}}]
\end{aligned}
$$

11

where $\mu = E(\widetilde{y}_t)$, $\mu_{\widetilde{THI}} = E(\widetilde{THI}_t)$; $\sigma_{\widetilde{z}}^2 = \text{var}(\widetilde{z}_t)$, $\sigma_{\widetilde{n}}^2 = \text{var}(\widetilde{n}_t)$, $\sigma_{\widetilde{THI}}^2 = \text{var}(\widetilde{THI}_t)$; $\sigma_{\widetilde{z},\widetilde{n}} = \text{cov}(\widetilde{z}_t, \widetilde{n}_t)$, $\sigma_{\widetilde{THI},\widetilde{z}} = \text{cov}(\widetilde{THI}_t, \widetilde{z}_t)$, $\sigma_{\widetilde{THI},\widetilde{n}} = \text{cov}(\widetilde{THI}_t, \widetilde{n}_t)$. From equation (9), we know that $\widetilde{n}_t = \widetilde{z}_t$. Therefor $\sigma_{\widetilde{n}}^2 = \sigma_{\widetilde{z}}^2 = \sigma_{\widetilde{z},\widetilde{n}}$ and $\sigma_{\widetilde{THI},\widetilde{z}} = \sigma_{\widetilde{THI},\widetilde{n}}$. And all these are positive numbers.

Take first order condition with respect to ϕ_t,

$$(20) \qquad \phi_t \sigma_{\widetilde{n}}^2 - \beta \sigma_{\widetilde{z},\widetilde{n}} + \kappa^{(h_t + a_t/n)} c \sqrt{\eta} \sigma_{\widetilde{THI},\widetilde{n}} = 0 \, .$$

From (20), the optimal weather options purchase amount is

$$(21) \qquad \phi_t^* = \beta - \kappa^{(h_t + a_t/n)} c \frac{\sigma_{\widetilde{THI},\widetilde{z}}}{\sigma_{\widetilde{z}}^2} \sqrt{\eta} \, .$$

The following three propositions are derived from equation (21):

Proposition 1 . *The optimal option purchase amount is increasing with β. It means that the more the producer's profit is sensitive to weather risk, the more options he should purchase, ceteris paribus.*

Proposition 2 . *The optimal weather option purchase amount is decreasing with initial abatement equipment investment η. Thus it indicates that weather options can act as a substitute for abatement equipment.*

Proposition 3 . *The optimal weather options purchase amount is increasing with age a_t and maintenance history h_t.*

Propositions 2 and 3 say, for the best risk management results, if the farmer's initial abatement investment is relatively low or the abatement equipment is old and less-frequently maintained, he should buy more weather options to hedge against risk from excessive heat stress.

Optimization Across Periods

Except in rare and highly specialized cases, it is impossible to derive analytical solution for dynamic programming problems. Here we employ numerical method to find the optimization

12

solution.

The Bellman's "principle of optimality" formally is expressed in the form of *Bellman's equation:*

$$(22) \qquad V(a_t, h_t) = \max_{\substack{\phi_t \in \mathbb{R}^+; \\ i_t \in \{1,2,3\}}} \{U(a_t, h_t; \phi_t, i_t) + \delta V(a_{t+1}, h_{t+1})\}$$

where the *Bellman value*, $V(a_t, h_t)$, is the maximum discounted sum of current and future utility starting from period t.

Because the dynamic problem has an infinite horizon, the value function in equation (22) will not depend on time t.[7] We may drop the time subscripts for the sake of visual clearness. Let v and x denote the vectors of Bellman values and optimal decisions corresponding to all possible states. And the Bellman's equation is rewritten as a vector fixed-point equation:

$$(23) \qquad v = \max_x \{u(x) + \delta v\}.$$

The fixed-point equation can be solved to obtain unique and exact v and x by the *policy iteration* method. Specifically, the policy iteration algorithm consists of two steps: *policy evaluation* and *policy improvement*. Suppose that at iteration j, we have candidate decision rule x_j. Then the value function v_j can be recovered via *policy evaluation*, i.e.

$$(24) \qquad v_j = \{u(x_j) + \delta v_j\} = (1 - \delta)^{-1} u(x_j).$$

Next, at iteration $j + 1$, with x_j and v_j, a new policy x_{j+1} is formed via a *policy improvement* step:

$$(25) \qquad x_{j+1} = \arg\max\{u(x_j) + \delta v_j\}.$$

---

[7]Our empirical analysis shows that there is not an obvious difference in optimal decisions and Bellman values between problems of infinite horizon and of a long finite horizon (say, 40 years), because after these many years the present discounted value of future utility is negligible.

Then x_{j+1} is put into equation (24) to update the Bellman's value. The loop of (24) and (25) will be iterated until it converges. Therefore, the policy iteration algorithm converts the dynamic programming problem into the problem of computing a fixed point to a certain contraction mapping. The contraction property guarantees that the unique fixed point solution exists and is insensitive to rounding errors as long as discount rate δ is less than 1.

Basis Risk and Transaction Costs

Basis risk exists in incomplete risk-world markets. Basis risk in this study comes from the difference between the underlying index of weather derivatives and the weather factor that affects the dairy profit (namely, THI). Two kinds of basis risk are investigated. One is geographical basis risk, which occurs from the difference in location between the reference site of weather derivatives and the actual production area. The other is reference-index basis risk, which occurs because weather derivatives are typically based upon temperature, yet biological stresses occur as a function of THI. Accordingly, the payoff from a weather call option, equation (8), is changed into:

$$(26) \qquad\qquad \widetilde{n} = \max(\widetilde{I} - I_{threshold},\ 0)$$

where $\widetilde{I}$ is the stochastic value of a weather index, and $I_{threshold}$ is the strike level. Equation (26) captures the presence of both reference-index and geographical basis risk. If the reference index $\widetilde{I}$ is temperature rather than THI, it reflects index basis risk. If the reference index $\widetilde{I}$ is weather condition of a location other than the production area, then geographical basis risk exists. Note that if $\widetilde{I}$ is THI of the production area, there is no basis risk.

Transaction costs in weather options are imposed by setting the option premium as the expected payoff plus proportional transaction costs. Then equation (9) is revised into:

$$(27) \qquad\qquad \pi = (1+\gamma)E(\widetilde{n})$$

where the loading rate $\gamma > 0$ reflects transaction costs related to administrative and implementation fees and the desirability to the issuers. If γ is zero, the weather options are actuarially-fairly priced.

The presence of basis risk and transaction costs changes the optimal weather derivatives purchase amount, i.e. equation (21), into:

$$(28) \qquad \phi_t^* = -\frac{\gamma \mu_{\widetilde{n}}}{A \sigma_{\widetilde{n}}^2} + \beta \frac{\sigma_{\widetilde{z},\widetilde{n}}}{\sigma_{\widetilde{n}}^2} - \kappa^{(h_t + a_t/n)} c \frac{\sigma_{\widetilde{THI},\widetilde{n}}}{\sigma_{\widetilde{n}}^2} \sqrt{\eta}.$$

Propositions 1-3 still hold in equation (28). In addition, equation (28) shows that the optimal weather derivative purchase amount is negatively related to the loading rate γ. The intuition is that due to transaction costs, purchasing weather derivatives lowers expected profit, and thus higher transaction costs make weather derivatives less attractive.

Risk Management Effectiveness

In order to derive the increased utility from using weather options and abatement equipment, we define the utility with using none of these two instruments as a benchmark.

$$(29) \qquad U_t^0 = E(\widetilde{y}_t) - \frac{1}{2} A \cdot \mathrm{var}(\widetilde{y}_t),$$

And the increased utility in certainty equivalent at t is:

$$(30) \qquad \begin{aligned} \Delta U_t &= U_t - U_t^0 \\ &= [E(\widetilde{y}_t^{net}) - E(\widetilde{y}_t)] - \frac{1}{2} A \cdot [\mathrm{var}(\widetilde{y}_t^{net}) - \mathrm{var}(\widetilde{y}_t)]. \end{aligned}$$

Clearly, the optimal decision rule satisfies:

$$(31) \qquad \underset{\eta, \phi_t, i_t}{\arg \max} \{ \sum_{t=0}^{\infty} \delta^t U_t \} \equiv \underset{\eta, \phi_t, i_t}{\arg \max} (\sum_{t=0}^{\infty} \delta^t \Delta U_t).$$

It is also viable to study the cases in which only one of the two instruments is used by

15

imposing the value of the other instrument to be zero. Therefore, risk management value of weather derivatives can be examined by comparing the cases of using both two instruments and using abatement equipment alone.

Data

For the empirical illustration, we use 35-year (1949 to 1964 and 1984 to 2002) weather data of Summit County and Cleveland, Ohio collected from the National Climate Data Center (NCDC), a subsidiary of the National Oceanic Atmospheric Administration (NOAA).[8] Summit County is treated as the real production area. And Cleveland, as a metropolitan city in the same state as the production area, is selected as the weather derivatives reference city for the purpose of investigating the effect of basis risk.

The weather data include daily maximum and minimum temperature and daily maximum and minimum relative humidity. Daily temperature and dew point[9] both follow routinely seasonal patterns each year. So the "burn-rate" method works well with them for pricing weather options. Daily maximum temperature-humidity index (THI) can be derived from daily maximum temperature and minimum relative humidity.[10] Note in the models, $\widetilde{THI}$ corresponds to maximum THI. When maximum THI is lower than 70 degrees in a day, there is no heat stress for dairy cows.

Corresponding to the weather data of Summit County, a representative producer's milk loss from heat stress and reduced loss from using abatement equipment are generated by employing the results in SCS.[11]Abatement investment cannot change in a relatively long

---

[8]It is a quite common phenomenon that daily relative humidity data from 1965 to 1983 are missing across weather stations in NCDC database.

[9]Dew point measures how much water vapor is in the air. In many places, the air's total vapor content varies only slightly during an entire day, and so it is the changing air temperature that primarily regulates the full variation in relative humidity. Related information can be found at: http://www.usatoday.com/weather .

[10]In a day, the maximum THI is in the afternoon, when the temperature is highest and relative humidity is lowest; and the minimum THI is in the night, when the temperature is lowest and relative humidity is highest.

[11]See the Appendix for detail.

period once fixed. Also weather options are assumed to be written on summer basis, i.e. the payoff is cumulative $\widetilde{n}$ of a summer. Thus, Equations (6) and (13) will be estimated based on cumulative summer data. The summer period is set from May 1$^{\text{st}}$ to Oct. 31$^{\text{st}}$ every year, because 97% of heat stress occurs in this period.

Empirical Illustration

Note that all results of the empirical illustration in this paper is for one dairy cow, namely, the herd size is normalized into unity.

We need values of the parameters in the models in order to solve the dynamic programming problem. Some of the parameters can be estimated from our data. Unfortunately, others such as the abatement maintenance cost parameters cannot be estimated due to lack of data. As a reference point for the empirical illustration, a base set of parameter values which cannot be estimated is established based upon discussions with experienced people and reasonable calibrations. Deviations from the base values provide insight into how changes in abatement equipment parameters and investors' preference affect the risk management strategies and their corresponding results.

A more favorable case would be to have data of dairy farmers' abatement maintenance and replacement records, and effectiveness of abatement equipment in different states. Assuming the farmers' asset maintenance/replacement decisions coincide with the dynamic optimization principle, several alternative functional forms of abatement effectiveness and maintenance costs can then be estimated and tested. With the estimated parameters, the optimal dynamic decision with simultaneously using weather derivatives and abatement equipment can be given.

Table 1 gives the summary of the parameters, which are either estimated from our data or assigned. By and large, the empirical part of this paper is to provide an illustration on

17

how weather derivatives can be used together with abatement equipment to enhance a dairy farmer's risk management effectiveness.

Parameter Estimation

Following SCS, *THI$_{threshold}$* is set as 70 degrees. From the weather data of Summit County and the SCS milk loss model, we calculate the daily milk loss during summers of the 35 years and the corresponding daily $\widetilde{THI}$. Then by accumulating the milk loss and $\widetilde{z} = \max(\widetilde{THI} - THI_{threshold}, 0)$ during each summer in the 35 years, we have 35 observations of cumulative profit loss and $\widetilde{x} = E(\widetilde{z}) - \widetilde{z}$. From a least squares regression, β is estimated, which is 0.5635 kg milk per cow with Student's t-value and R-squared of 16.47 and 0.89. That is to say each degree of $\widetilde{z}$ beyond its mean will induce 0.5635 kg milk loss. The milk price is set as \$ 0.287/kg as in SCS, so the milk loss is \$ 0.1617 per degree of $\widetilde{x}$.

We put the daily summer weather data into the SCS abatement effect model[12] to calculate the daily reduced THI corresponding to seven abatement levels. Because the abatement levels in SCS are expressed as yearly investment with an annualization rate of 15%, we calibrate these levels into initial abatement investment levels η with the assigned parameters κ and n in equation (13) and with the assumption that the states of equipment in SCS is $a_t = 6$ and $h_t = 0$, or $a_t = 2$ and $h_t = 1$. Multiplying the estimated β and milk price, we calculate daily reduced profit loss (in dollars) due to abatement investment (in dollars). The reduced profit loss and THI are accumulated for each summer. Thus there are 35 observations of cumulative reduced profit loss and cumulative THI for each of the seven abatement investment levels. By a least squares regression, the estimates of b and c are -32.0920 and 0.0029 with Student's t-values of 16.66 and 19.06 and R-squared of 0.91.

---

[12]In SCS there are three abatement effect models corresponding to three abatement intensity levels. The first model is for only using fans or sprinklers; the second model is for a combination of fans and sprinklers; and the third model is for a specific system, the Korral Cool system, which is used in the Southwest and other dry and hot areas. In the research, we use the second model, and based on this model, we linearly simulate another six abatement effect functions corresponding to six different fixed cost levels. See Appendix B.

Dynamic Optimization Results

With the estimated and assigned parameter values can we solve the dynamic problem using *Policy Iteration* Method.[13] For the purpose of examining how weather derivatives can affect abatement equipment decisions, we study the dynamic optimization problem under two scenarios: (i) using abatement equipment alone; and (ii) simultaneously using abatement equipment and weather derivatives. In this subsection, basis risk and transaction costs are not investigated, and therefore weather data used here are merely from Summit County, Ohio.

The optimal initial abatement investment levels, η^*, for scenarios (i) and (ii) are found by examining the dynamic optimization solutions corresponding to a series of different investment levels and picking up the levels with maximum Bellman values. The optimal levels are $105.6 and $67.2 for the two scenarios, respectively. Hence, using weather derivatives can mitigate the problem of high initial installation cost of abatement equipment.

Table 2 gives the optimal abatement equipment maintenance/replacement decisions, i_t^*, corresponding to all possible states. It can be seen that the decisions for the cases with and without using weather derivatives are quite similar. The only three different decisions show that scenario (i) is a bit inclined to replace the equipment earlier. However, for an equipment no more than 6-year-old, the best strategy is to have maintenance.

Figure 1 (a) and (b) give the optimal abatement equipment replacement rules for scenarios (i) and (ii). Without using weather derivatives, a set of new abatement equipment can be used for 9 years before replaced; while with weather derivatives, it can be used for 10 years. Therefore, using weather derivatives can postpone the replacement of abatement equipment. Figure 2 gives the optimal abatement maintenance policies. Under scenario (i), maintenance services should be given each year except the first and last years during the life of abatement equipment; while under scenario (ii), maintenance will not be given in the 8th year besides

---

[13]CompEcon Toolbox in Miranda and Fackler (2002) is used to perform policy iteration.

the first and last years. Thus, using weather derivatives reduces the frequency of maintenance.

The optimal purchase amount of weather derivatives ϕ_t^* under scenario (ii) is displayed in figure 3. As can be seen, a farmer should buy more weather derivatives as his abatement equipment turns older, especially in the 8th and 10th years when no maintenance is given. It means that as the equipment turns old, its effectiveness is declining, and therefore more weather derivatives should be used as compensation.

The risk management value of weather derivatives and abatement equipment is investigated using equations (29)-(31). First, as a benchmark, suppose this representative farmer employs neither weather derivatives nor abatement equipment. According to our data and estimated β, the mean and variance of his annual revenue loss due to heat stress are \$49.69 and 411.19. By the mean-variance utility model, the annual utility loss of the farmer with risk aversion A of 0.20 is $(-49.69 - \frac{1}{2} \cdot 0.2 \cdot 411.19) = -90.81$ dollars in certainty equivalent. Thus the long-run utility loss from heat stress is $\sum_{t=0}^{\infty} \delta^t U_{loss} = \sum_{t=0}^{\infty} 0.9^t \cdot 90.81 = 908.1$ dollars. Under scenario (i), by using abatement equipment alone, the maximized increased utility in the dynamic optimization, i.e. $\max_{\eta, i_t} \sum_{t=0}^{\infty} \delta^t \Delta U_t$, is \$248.4 in certainty equivalent. So the optimal use of abatement equipment can reduce the utility loss by 27.35%. And under scenario (ii), the maximized increased utility, i.e. $\max_{\eta, i_t, \phi_t} \sum_{t=0}^{\infty} \delta^t \Delta U_t$, is \$499.8 in certainty equivalent. The optimal use of these two instruments can reduce utility loss by 55.04%. Therefore, using weather derivatives together with abatement equipment can significantly enhance the risk management effectiveness over using abatement equipment alone.

Effect of Basis Risk and Transaction Costs

In this subsection, Cleveland serves as the weather option reference city. Therefore, weather data of Cleveland are used to calculate the payoff and premium of weather options in equations (26) and (27). The option strike level, $I_{threshold}$ in (26), is assumed to be chosen by the buyers. The optimal strike levels and optimal initial abatement investment levels are

determined by examining the dynamic optimization solutions corresponding to a series of combinations of different strike levels and initial investment levels and then picking up the combinations with highest Bellmans values. And transaction cost loading rate γ is set as 5%.

Besides the two scenarios in the last subsection, another two scenarios are analyzed here: (iii) simultaneously using abatement equipment and weather options, but in the presence of geographical basis risk and transaction costs; and (iv) simultaneously using abatement equipment and weather options, but in the presence of both kinds of basis risk and transaction costs.

Under scenario (iii), the optimal strike level is 75 degrees of THI, and the optimal initial abatement investment level is $74. The maximized increased utility is $425.4 in certainty equivalent. Under scenario (iv), the optimal strike level is 81 Fahrenheit degrees of temperature, and the optimal initial abatement investment level is $77. The maximized increased utility is $391.3 in certainty equivalent. So although basis risk and transaction costs reduce the risk management effectiveness of weather derivatives, compared with using abatement equipment alone, it still is a significant improvement to use weather derivatives together with abatement equipment in dairy profit risk management (figure 4).

The optimal maintenance/replacement decisions under scenarios (iii) and (iv) are the same as those of using abatement equipment alone, i.e. scenario (i). And the optimal weather option purchase amount is displayed in figure 3. Presence of basis risk and transaction costs lowers the optimal purchase amount of weather options.

Conclusion and Discussion

In this study, we employ a dynamic programming framework to study the case that a representative dairy farmer maximizes his long-run utility using weather derivatives and abatement equipment to reduce profit loss and risk from heat stress. An exact solution as the optimal

risk management strategy for weather derivatives purchase and abatement equipment replacement is derived numerically using policy iteration method. The empirical results suggest that simultaneously using weather derivatives and abatement technologies will outperform using abatement technologies alone, even in the presence of basis risk and transaction costs. And using weather derivatives reduces the investment in abatement equipment, which mitigates the problem of high initial installation cost of abatement equipment faced by small- and medium-size firms. Besides, the optimal purchase of weather derivatives depends on age and maintenance status of abatement equipment.

This study introduces a new application of weather derivatives while also exploring the possibilities for hedging a heretofore unhedgable risk. This study yields an analytical solution for a dynamic programming model, which, we believe, provides an applicable guide for the dairy producers. In addition, this research raises many questions of relevance to the economic community, such as the optimal contract design, whether the existence of these contracts reinforces economies of scale in dairy production, what level of sophistication is required to effectively utilize these tools, and finally, what size of a dairy is required to use weather derivatives. These questions may be of interest for further research.

References

Barth, C. L. "State-of-the-art for Summer Cooling for Dairy Cows." 52-61, in *Livestock Environment II, Proceedings of the Second International Livestock Environment Symposium*, Scheman Center, Iowa State Univeristy, April 20-23, 1982.

Burt, O. R. "Decision Rules for the Dynamic Animal Feeding Problem." *American Journal of Agricultural Economics* 75 (1993): 190-202.

Chavas, J.-P., Kliebenstein, J. and Crenshaw, T. D. "Modeling Dynamic Agricultural Production Response: The Case of Swine Production." *American Journal of Agricultural Economics* 67 (1985): 636-646.

Johnson, H. D. "Bioclimate Effects on Growth, Reproduction and Milk Production." p. 35, in Johnson, H. D. (ed.) *Bioclimatology and the Adaptation of Livestock*. Elsevier, Amsterdam, The Netherlands, 1980.

McDonald, R. and Siegel, D. "The Value of Waiting to Invest." *Quarterly Journal of Economics* 101 (1986): 707-728.

McClelland, J. W., Wetzstein, M. E. and Noles, R. K. "Optimal Replacement Policies for Rejuvenated Assets." *American Journal of Agricultural Economics* 71 (1989): 147-157.

Meyer, J. "Two-Moment Decision Models and Expected Utility." *American Economic Review* 77(1987): 421-430.

Miranda, M. J. and Schnitkey, G. "An Empirical Model of Asset Replacement In Dairy Production." *Journal of Applied Econometrics* 10 (1995): S41-S55.

Miranda, M. J. and Fackler, P. *Applied Computational Economics and Finance.* Cambridge, MA: MIT Press, 2002

National Oceanic and Atomospheric Adiministration, "Livestock Hot Weather Stress." Reg. Operations Letter C-31-76. US Department of Commerce, National Weather Service, Central Region, Kansas City, Missouri, 1976.

Pindyck, R. S. "Irreversibility, Uncertainty, and Investment." *Journal of Economic Literature* 29 (1991): 1110-1148.

Pratt, J. "Risk Aversion in the Small and in the Large." *Econometrica* 32(1964): 122-136.

Rust, J. "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher." *Econometrica* 55 (1987): 999-1033.

Smith, B. J. "Dynamic Programming of the Dairy Cow Replacement Problem." *American Journal of Agricultural Economics* 55 (1972): 100-104.

St-Pierre, N. R., Cobanov, B. and Schnitkey, G. "Economic Cost of Heat Stress: Economic Losses from Heat Stress by U.S. Livestock Industries." *Journal of Dairy Science* 86 (2003): E52-77.

Thompson, G. E. "Review of the Progress of Dairy Science Climatic Physiology of Cattle." *Journal of Dairy Research* 49(1973): 441-473.

Appendix

Milk Loss Function

The milk loss model in SCS (2003) is: $MILK_{loss} = 0.0695 * (THI_{max} - THI_{threshold})^2 * Duration$, where $MILK_{loss}$ is in kilogram, and $Duration$ is the proportion of a day where heat stress occurs (i.e. $THI_{max} > THI_{threshold}$).

The process to calculate the $Duration$ of heat stress:

$$THI_{mean} = (THI_{max} + THI_{min})/2$$

if $THI_{max} < THI_{threshold}$

$$Duration = 0$$

elseif $THI_{min} >= THI_{threshold}$

$$Duration = 24$$

elseif $THI_{mean} > THI_{threshold}$

$$Duration = (PI - 2 * \arcsin(\tfrac{THI_{threshold} - THI_{mean}}{THI_{max} - THI_{mean}}))/PI * 12$$

else $Duration = (PI + 2 * \arcsin(\tfrac{THI_{mean} - THI_{threshold}}{THI_{max} - THI_{mean}}))/PI * 12$

end

where $PI = 3.1415...$

Abatement Effect Function

In SCS, for a 50 m^2 cow pen, which can hold 7.1759 dairy cows, when the annualized fixed costs are \$310, the corresponding operating costs are \$0.0685/hour of operation. And the abatement effect is: $\Delta THI = -17.6 + (0.36 * T) + (0.04 * H)$, where ΔTHI is the change in apparent THI, T is ambient temperature ($^\circ$C), and H is ambient relative humidity in percent.

Based on the above specifications, we linearly simulate another six abatement effect func-

tions corresponding to six fixed cost levels. The six fixed cost levels are 130, 190, 250, 370, 430, 490 dollars respectively. That is, all the parameters in a simulated model are proportional to those in the SCS model, with the proportion equal to the ratio of fixed cost levels.

We define the reduced profit loss by:

$$\widetilde{m} = \max(\min(THI_{max} - THI_{threshold},\ \Delta THI),\ 0) * \beta * MILKprice.$$

Table 1: Summary of Parameters

Symbol	Value		Interpretation	Equation
	Estimated	Assigned		
β	0.1617***		Sensitivity of profit to weather	(6)
δ		0.9	Intertemporal discount factor	(11)
κ		0.8	Abatement effectiveness declining factor	(13)
n		4	Age's effect on abatement equipment	(13)
b	-32.0920***		Abatement effectiveness parameter	(13)
c	0.0029***		Abatement effectiveness parameter	(13)
k_1		0.012	Age's effect on maintenance cost	(14)
k_2		0.02	History's effect on maintenance cost	(14)
A		0.2	Pratt's Absolute Risk Aversion	(16)

Note: "Equation" in the last column gives the numbers of equations where the parameters show up for the first time. The symbol *** indicates the estimates are at the 1% significance level.
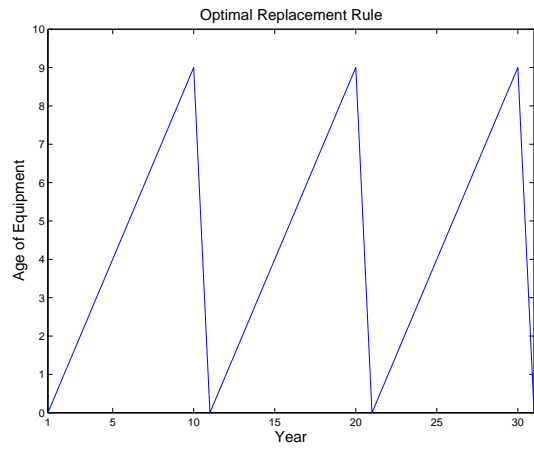
Table 2: Optimal Maintenance/Replacement Decisions

		Maintenance History									
		1	2	3	4	5	6	7	8	9	10
	1	2									
	2	2	2								
	3	2	2	2							
	4	2	2	2	2						
age	5	2	2	2	2	2					
	6	2	2	2	2	2	2				
	7	1^2	2	2	2	2^3	3	3			
	8	1	2	2^3	3	3	3	3	3		
	9	1	3	3	3	3	3	3	3	3	
	10	1	3	3	3	3	3	3	3	3	3

Note: Inside the decision matrix, numbers 1, 2, and 3 denote "no action", "maintenance", and "replacement" respectively. These numbers are the optimal decisions with using weather derivatives. Numbers in superscripts are different decisions in the case of not using weather derivatives.
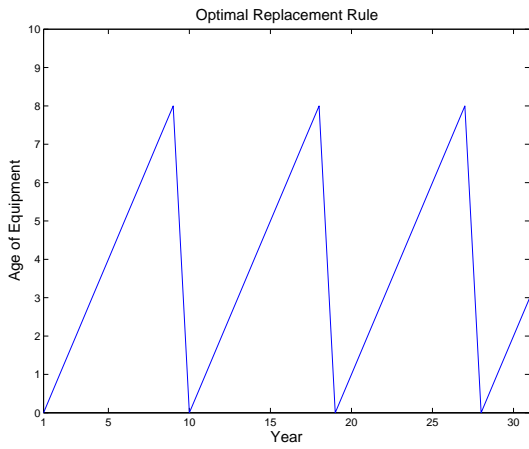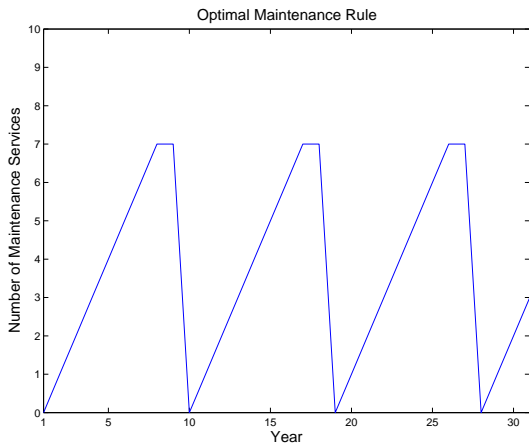
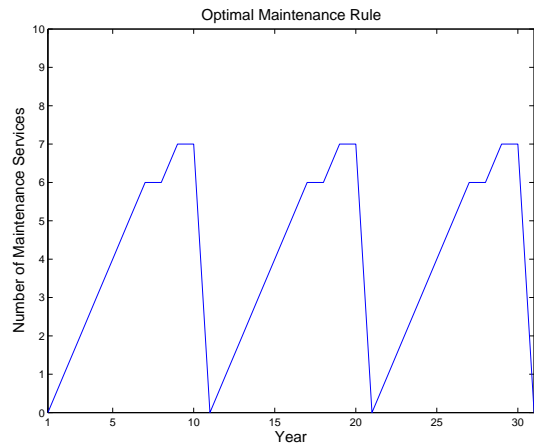(a) Not using weather derivatives   (b) Using weather derivatives

Figure 1: Optimal replacement rule

(a) Not using weather derivatives   (b) Using weather derivatives
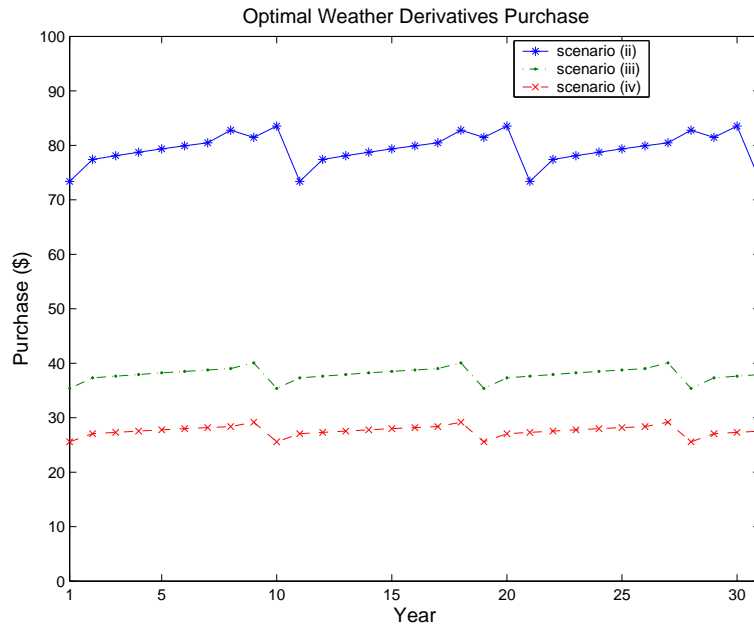
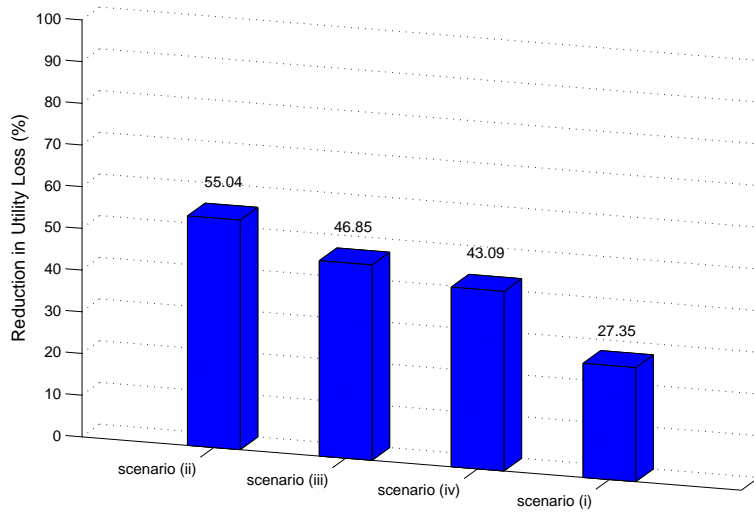Figure 2: Optimal maintenance rule

Figure 3: Optimal weather derivatives purchase

Figure 4: Comparison of risk management effectiveness