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INFORMATIVE ADVERTISING IN CONCENTRATED, DIFFERENTIATED MARKETS

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***Abstract:** I examine the welfare implications of informative advertising in a differentiated product duopoly market. The analysis reconciles the apparently conflicting results in previous studies that find advertising to be undersupplied in homogeneous product markets and in differentiated markets with a limited number of firms, but oversupplied in differentiated markets with a large number of firms. In equilibrium, purely informative advertising is always overprovided when the degree of product differentiation exceeds a threshold level. The result is robust and obtains under conditions of both price and quantity competition. Product differentiation also has welfare implications for the effect of technological change in the advertising sector. In response to an advertising cost innovation (e.g., through the emergence of cable TV and internet media), the equilibrium prices and advertising levels converge to the social optimum when the products are sufficiently non-differentiated, whereas divergence occurs for more differentiated goods.*

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Informative Advertising in Concentrated, Differentiated Markets

Is advertising a socially wasteful activity? Proponents of this view argue that advertising is a pointless attempt to capture market share by manipulating consumer preferences. Others portray advertising as a beneficial activity that provides consumers with factual information about product attributes and promotes better matches between consumers and brands. Advertising is often classified along these lines according to its “persuasive” or “informative” content; however, identifying the role of advertising does not resolve its underlying welfare implications. Individual preferences must count in any reasonable criterion for an economic optimum, and to the extent that viewing advertisements influences consumer choices, advertising produces measurable economic rents.¹ Whether or not firms advertise excessively in the market equilibrium depends only on the degree to which firms view themselves capable of appropriating the advertising rents.

This paper examines the welfare implications of decentralized advertising choices in the case of truthful and purely informative advertising. Consumers have well-defined preferences for products with identifiable characteristics, but, prior to purchase, are genuinely unaware of the characteristics contained in individual brands. The role of advertising is to convey information to consumers about the product specifications of the advertised good. This process generates two types of economic rents. Advertising stimulates new consumption by reaching consumers who would otherwise not purchase the product, and advertising facilitates better matches between existing customers and the brands that suit their tastes.²

The basic message of the paper is that purely informative advertising tends to be oversupplied in the market equilibrium. The overadvertising result is quite robust and obtains as

¹ There is no obvious criterion on which to base the external judgment that economic rents represents “real” or “imaginary” values. This argument is made by Mark Stegeman (1991) and is embodied in the analysis of Gary Becker and Kevin Murphy (1993), who model advertisements as goods in stable metautility functions.

² Identifying the relative magnitude of each effect has long been a focus of the empirical advertising literature, and it is interesting to note that the outcome seems to depend on the industry studied. For example, Mark Roberts and Larry Samuelson (1988) find that cigarette advertising increases total market demand, but has no significant influence on market shares, whereas Jon Nelson (1999) finds that individual advertisements for beer, wine and distilled spirits alter market shares, but have little or no impact on total market demand.

long as the products are sufficiently differentiated.

To put the present work in context, it is helpful to consider the seminal treatment of informative advertising in differentiated markets by Gene Grossman and Carl Shapiro (1984) (henceforth GS). The GS model extends the informative advertising process first conceived by Gerard Butters (1977) to a circular-city model of product differentiation based on Steven Salop (1979).³ Their result is striking. Firms oversupply advertising relative to the socially optimum only when the number of firms is sufficiently large.

The surprising aspect of this result is that it contrasts sharply with the outcome of models with homogenous goods. Mark Stegeman (1991) and Dahl Stahl (1994) find purely informative advertising to be undersupplied in a homogeneous product equilibrium where firms select prices, and Robert Gary-Bobo and Philippe Michel (1991) demonstrate that advertising does not occur at all under quantity competition as the number of firms becomes large.

This paper reconciles these apparently conflicting results. GS's excessive advertising result rests on the "large-group" assumption of Avinash Dixit and Joseph Stiglitz (1977), which is particularly strong in the case of informative advertising, since it implies that aggregate output is exogenous. Hence, the welfare standard in their model becomes solely one of cost minimization. In Stegeman (1991) and Stahl (1994), advertising increases the total size of the market, but there is no product differentiation. With non-differentiated goods, consumers who are aware of more than a single brand always purchase the brand offered at the lowest price, and the capacity of consumers to costlessly switch between brands attenuates the ability of firms to extract informational rents. Informative advertising, for this reason, is always undersupplied for homogeneous goods, and I find this to be the case here as well. However, I also show the converse to be true --advertising is always excessive in the market equilibrium-- when the products are more highly differentiated.

³ The merit of this approach over the standard "linear city" model of Harold Hotelling (1927) is that a circular city permits a more reasonable characterization of market entry. The central welfare result of GS does not depend on entry, however, and the models are qualitatively equivalent when the number of firms is exogenous.

The equilibrium outcome for advertising differs from the socially optimal allocation because of three distinct effects that impact the ability of firms to acquire advertising rents. First, branded advertising draws new consumers to the product category and increases the total size of the market. The contribution of this new consumer to marginal social benefit, which includes consumer surplus, generally differs from the marginal contribution to the advertiser's profit. I refer to the failure of the advertising firm to fully appropriate this rent as the *market size effect*.⁴ The market size effect tends to cause advertising to be undersupplied in the market equilibrium.

Second, there is a *matching effect*. For a given market size, informative advertising improves the match between existing consumers and brands. Informative advertising aligns consumers better with their desired product characteristics, and this contributes to social surplus. However, the value of informing consumers about alternative brands depends on individual preferences, which are idiosyncratic. Absent price discrimination, the value created by the improved match between consumers and brands fails to capitalize fully into the market prices. This matching effect causes advertising to be undersupplied in the market equilibrium.

Finally, there is a *customer-poaching effect*. Firms advertise brands to increase their own sales and do not recognize the effect of their advertisements on the sales made by rival firms. The customer-poaching effect represents the neglected cost of advertising on the profit of rival brands and causes advertising to be oversupplied in the market equilibrium.

The goal of this paper is to construct a simple model to nest each of these welfare effects. The requirements of such a model are as follows. To produce the market size effect, total sales of the product must depend on the advertising intensities of firms. To produce the matching and poaching effects, it is necessary to have heterogeneous consumer preferences for product characteristics and at least two differentiated brands. A model with two differentiated brands is sufficient to encompass these welfare effects, moreover, because an increase in the (exogenous)

⁴ The market size effect embodies an externality familiar to search goods. The cost of search is borne entirely by the searcher (here the advertiser), while the benefits are divided between the searcher and the agent with whom he trades. This externality is the dominant force at work in homogeneous product markets.

number of firms has qualitatively equivalent effects as a decrease in the degree of product differentiation that distinguished them.

The analysis emphasizes the qualitative character of the market failures that occur in partial equilibrium consisting of small numbers of firms. The small numbers case introduces a new complication: the outcome under oligopoly now differs from the socially optimal resource allocation in terms of both the equilibrium output and advertising levels. In GS this problem is handled by calibrating output levels prior to optimization with the large groups assumption. In Stegeman (1991) and Stahl (1994) the issue is sidestepped by inferring the welfare implication indirectly from the advertising level selected by the highest-priced firm. There is no comparable shortcut available here, and I follow Michael Spence (1975) in comparing the equilibrium outcome and the optimal allocation at the social output level. Thus, the results refer to the outcome that would arise if independent instruments were available to control the pricing and advertising externalities in the market.⁵

The paper also considers advertising outcomes under both price-setting and quantity-setting forms of oligopoly. It is a stylized fact that advertising intensities are positively related to industry-average profitability (e.g., Richard Schmalensee, 1986). However, the role of market structure in influencing this outcome has remained largely unexplored. Equilibrium prices and profits are higher in models where firms compete in quantities (as opposed to prices), but this need not imply systematic differences in advertising levels. Nonetheless, I find this to be the case. The equilibrium advertising intensities are always greater under quantity competition than under price competition.

I. Informative Advertising and Consumer Demand

The market consists of two firms who sell differentiated products (i.e., brands) to a group of consumers. Each consumer is atomistic and has unit demand. Consumers rely on information

⁵ Put somewhat differently, I choose to calibrate the market output levels to the efficient level after deriving the oligopoly equilibrium conditions, rather than doing so prior to optimization as in GS.

received from ads to locate specific brands in the product space and are passive in the sense that they do not engage in any activities designed to acquire information, other than by viewing ads. Thus, I implicitly assume consumer search costs to be high relative to the value of the product.

Brands in the model differ according to their characteristics in the sense of Kevin Lancaster (1975), and the role of informative advertising is to match consumers to the products that best suit their tastes. The demand structure is represented by a Hotelling (1927) “linear city”, in which firms are located at the endpoints of a unit segment, and each consumer is identified by a point on the line segment that corresponds to her ideal product characteristics.

Each consumer purchases at most one unit and receives a gross value of v from consuming the good. Consumers incur transportation costs of t per unit of distance to acquire a brand from a given location, so that the net surplus enjoyed by a consumer who purchases a brand a distance x units away at a price of p is $v - p - tx$. Consumers only purchase the product if they are aware of a brand that offers positive net surplus, and a consumer who is aware of both brands selects the brand that offers the highest net surplus. Finally, for analytical convenience, I assume consumers are distributed uniformly on the unit interval with a density of n .

The role of advertising in the model is to convey information about a firm’s product to consumers. Advertisements are truthful and inform consumers about the characteristics and price of individual brands.⁶ Following Butters (1977), the analysis is simplified by assuming that firms send independent advertising messages and have no ability to target advertisements towards consumers located at particular points in the product space.

Let ϕ_i denote the advertising intensity of firm i . Advertising intensity is measured in terms of the reach of the ad campaign, so that ϕ_i is interpreted as the fraction of the consumer population that is exposed, at least once, to the advertising message of firm i . This divides consumers at each location in the product space into four types: With probability $\phi_i\phi_j$ a

⁶ Assuming a consumer can be aware of a product, but not of its price would require an explicit formulation of a consumer search process. For models of informative advertising and consumer search, see, e.g., Jacques Robert and Dale Stahl (1993) and Helmut Bester and Emmanuel Petrakis (1995).

consumer simultaneously receives advertising messages from both brand i and brand j ($j \neq i$); with probability $\phi_i(1-\phi_j)$ a consumer receives an ad from brand i but not from brand j ; with probability $(1-\phi_i)\phi_j$ a consumer receives an ad from brand j but not from brand i ; and with probability $(1-\phi_i)(1-\phi_j)$ a consumer fails to receive an advertising messages from either brand.

Aggregate demand facing each firm sums the demand functions for each consumer type. I examine only the case in which the demand regions of the firms overlap (a necessary condition for oligopolistic interaction), and this requires that the equilibrium prices be set at a level to induce all fully informed consumers to purchase. However, the equilibrium prices need not be set at a level to induce all partially informed consumers to purchase, and the resulting possibilities for the oligopoly equilibrium are classified by the following definitions.

Definition 1. Complete coverage. The market is completely covered if all consumers who receive at least one advertising message purchase a brand at prevailing prices: $t \leq (v - p_i)$, for $i = 0, 1$.

Definition 2. Incomplete coverage. The market is incompletely covered if at least one partially informed consumer does not make a purchase at prevailing prices: $t/2 \leq (v - p_i) < t$, for $i = 0, 1$.

GS consider the case of complete coverage. The equilibrium prices in their model are assumed to be sufficiently low that a partially informed consumer located at the most distant allowable location from the advertised brand still finds it worthwhile to consume it. I consider this possibility here as well, but also consider the second possibility of incomplete coverage. Under incomplete coverage, the equilibrium prices are such that consumers located at the endpoints of the line segment find it prohibitively costly to purchase the more distant brand.

Figure 1 depicts the case of incomplete coverage. Among the fully informed population of consumers, let $x_f^i(p)$ denote the distance from firm i to the consumer who is indifferent between purchasing brand i and the rival brand j ($j \neq i$) at prices $p = (p_0, p_1)$. $x_f^i(p)$ is depicted in Figure 1, and is given algebraically by $x_f^i(p) = (t + p_j - p_i)/2t$. All fully informed consumers at locations $x \leq x_f^i(p)$ prefer brand i to brand j , so that the demand facing brand i from its fully

informed consumers is

$$X_f^i(p) = n(t + p_j - p_i) / 2t. \quad (1)$$

Firms also face demand from partially informed consumers. In the case of complete coverage, this demand is assumed to be perfectly inelastic, but under incomplete coverage at least one partially informed consumer fails to purchase the advertised brand. Let $x_p^i(p_i)$ denote the location of the partially informed consumer who is indifferent between purchasing brand i at a price of p_i or purchasing nothing at all.⁷ $x_p^i(p_i)$ is depicted in Figure 1, and is given by $x_p^i(p_i) = (v - p_i) / t$. Consumers located between $x_f^i(p)$ and $x_p^i(p_i)$ would prefer brand j to brand i if they were fully informed, but nevertheless prefer brand i to consuming nothing at all. The demand facing brand i from its partially informed consumers is

$$X_p^i(p_i) = n(v - p_i) / t. \quad (2)$$

Demand for each firm depends on the equilibrium outcomes for advertising and for the market prices. For the remainder of this section, I describe the demand facing a representative firm under the two possible scenarios for oligopoly: (i) complete coverage ($t \leq v - p$); and (ii) incomplete coverage ($t/2 \leq v - p < t$).⁸

A. Complete Coverage ($t \leq v - p$).

Under complete coverage, the demand for brand i is given by

$$\tilde{X}^i(\tilde{p}, \tilde{\phi}) = \tilde{\phi}_i \tilde{\phi}_j X_f^i(\tilde{p}) + \tilde{\phi}_i (1 - \tilde{\phi}_j) n, \quad i \neq j, i = 0, 1,$$

where $\tilde{p} = (\tilde{p}_0, \tilde{p}_1)$ is the vector of brand prices and $\tilde{\phi} = (\tilde{\phi}_0, \tilde{\phi}_1)$ is the vector of advertising intensities. A notable feature of this demand function is that all partially informed consumers of brand i make a purchase; that is, $X_p^i(\tilde{p}_i) = n$.

Substituting (1) into this expression, the demand for brand i under complete coverage is

$$\tilde{X}^i(\tilde{p}, \tilde{\phi}) = n \tilde{\phi}_i \left(\tilde{\phi}_j \left(\frac{t + \tilde{p}_j - \tilde{p}_i}{2t} \right) + 1 - \tilde{\phi}_j \right). \quad (3)$$

⁷ By the definition of incomplete coverage, this consumer resides on the unit interval at permissible prices ($p_i < t$).

⁸ I ignore the remaining case of local monopoly ($v - p < t/2$). For an analysis of advertising outcomes under monopoly, see Carl Shapiro (1980).

Notice that the price elasticity of demand, $\tilde{\varepsilon}_i$, evaluated at the symmetric Nash equilibrium point, $\tilde{p}_i = \tilde{p}$, $\tilde{X}^i = \tilde{X}$, and $\tilde{\phi}_i = \tilde{\phi}$ for $i = 0,1$, is given by $\tilde{\varepsilon} = \tilde{p}\tilde{\phi}/t(2-\tilde{\phi})$. As in GS, an increase in the level of informative advertising increases demand elasticities (and reduces prices). The intuition for this is straightforward. Demand in (3) is a weighted average of the demand of fully informed and partially informed consumers, with weights given by the equilibrium advertising levels. For the subgroup of fully informed consumers, the elasticity of demand is $\tilde{\varepsilon}_f = \tilde{p}/t$. For the subgroup of partially informed consumers, demand is perfectly inelastic (up to the reservation level). Because the share of fully informed consumers in the market increases with the advertising intensities, advertising makes demand more price elastic.

B. Incomplete Coverage

Under incomplete coverage ($t/2 \leq v - p < t$), the number of partially informed consumers who purchase the brand depends endogenously on the firm's price. The demand for brand i is given by the weighted sum of the group demands (1) and (2),

$$X^i(p, \phi) = \phi_i \phi_j X_f^i(p) + \phi_i (1 - \phi_j) X_p^i(p_i) \quad i \neq j, i = 0,1,$$

where $p = (p_0, p_1)$ is the vector of prices and $\phi = (\phi_0, \phi_1)$ is the vector of advertising intensities.

Simplifying this expression using (1) and (2) gives

$$X^i(p, \phi) = \frac{n\phi_i}{2t} \left(2(1 - \phi_j)v + \phi_j t - (2 - \phi_j)p_i + \phi_j p_j \right), \quad i \neq j, i = 0,1. \quad (4)$$

A notable property of the demand functions (4) is that the own-price effect is larger in magnitude than the cross-price effect, $|\partial X^i(\cdot)/\partial p_i| > |\partial X^i(\cdot)/\partial p_j|$. Thus, the inverse demand system is well defined. Defining $X = (X_0, X_1)$ to be the vector of output levels, the inverse demands are

$$p^i(X, \phi) = v + \frac{\phi_j t}{2 - \phi_i - \phi_j} - \frac{(2 - \phi_i)tX_i}{n\phi_i(2 - \phi_i - \phi_j)} - \frac{tX_j}{n(2 - \phi_i - \phi_j)}, \quad i \neq j, i = 0,1. \quad (5)$$

Let p_b and ϕ_b (p_c and ϕ_c) denote the equilibrium price and advertising level under price (quantity) competition. Demand conditions under oligopoly compare as follows:

Proposition 1. *Under incomplete coverage:*

- (i) *for given advertising levels ($\phi = \phi_b = \phi_c$), the price elasticities of demand are larger*

- under price competition than under quantity competition;*
- (ii) for given prices ($p = p_b = p_c$), the advertising elasticities of demand are larger under price competition than under quantity competition; and*
- (iii) in either case, informative advertising decreases the price elasticities of demand.*

Proof. See the appendix. \square

Residual demand is less (price) elastic when firms compete in quantities than when firms compete in prices. The intuition for this is best seen by noting that the selection of a price or quantity by firm i is isomorphic given its assumption that the quantity of rival firm j remains fixed.⁹ Now consider the effect of an implicit price decrease by firm i . Under quantity competition, an implicit price decrease by firm i reduces demand for the rival product j , so that firm j must respond by reducing its own price to maintain a constant output level. Relative to the case of price competition, this makes price-cutting less attractive.

Similar reasoning applies to the advertising elasticity of demand. Under quantity competition, firm i perceives both the output and the advertising level of firm j to be fixed in equation (4). Confining attention to the symmetric case, firm j responds to a change in advertising by firm i by making the implicit price adjustment

$$\partial p_j / \partial \phi_i = -(2(v - p) - t) / (2 - \phi) < 0, \quad (6)$$

where the inequality holds by the definition of incomplete coverage. If firm i increases its advertising intensity, consumers who would otherwise be partially informed only of brand j now become fully informed, so that, to maintain a constant output level, firm j must respond by reducing its price. Relative to the case of price competition, this makes advertising less attractive for quantity-setting firms.

It is interesting to note that advertising decreases the demand elasticities (and increases

⁹ Formally, for any invertible demand function $x^j(p_i, p_j)$, if the quantity of firm j is fixed at $\bar{x}_j = x^j(p_i, p_j)$, inverting yields the implicit price response $p_j = p^j(p_i, \bar{x}_j)$, so that $x^i(p_i, p_j) = x^i(p_i, \bar{x}_j)$ and the choice of price or quantity by firm i is isomorphic.

prices) under incomplete coverage. The reason for this is that partially informed consumers no longer have perfectly inelastic demand. The elasticity of demand of fully informed consumers is, as before, $\varepsilon_f = p/t$, but the elasticity of demand partially informed consumers is now given by $\varepsilon_p = p/(v-p)$ in (2). By the definition of incomplete coverage, it follows that $\varepsilon_f < \varepsilon_p$. Greater equilibrium advertising levels increase the share of fully informed consumers in the market, and the effect of this is to decrease the price elasticities of demand.

II. Oligopoly Outcomes

Each firm produces a single brand at marginal production cost of $c > 0$ and fixed cost of $F > 0$. The cost of sending an advertising message through a unit population density is $A(\phi; \alpha)$, where α is a shift parameter that reflects available advertising technology. Among other things, α captures the effect of the emergence of new media (e.g., the internet) that reduce the cost of achieving a given reach in an ad campaign. The advertising cost function has the properties $A_\phi > 0$, $A_\alpha > 0$, $A_{\phi\phi} > 0$ and $A_{\phi\alpha} > 0$.

The outcome under oligopoly depends on whether the parameters of the model support equilibrium prices that are consistent with complete or incomplete coverage. In each case, I confine attention to interior solutions of the model by suppressing all side constraints on the prices and characterize only the symmetric, pure strategy Nash equilibrium.

A. Complete Coverage ($t \leq v - \tilde{p}$)

Under complete coverage, the total number of consumers in the market is fixed (for given advertising levels), so that the outcome under quantity competition is not well defined. Under price competition, the profit of firm i is

$$\pi^i(\tilde{p}, \tilde{\phi}) = (\tilde{p}_i - c)\tilde{X}^i(\tilde{p}, \tilde{\phi}) - nA(\tilde{\phi}_i; \alpha) - F,$$

where $\tilde{X}^i(\tilde{p}, \tilde{\phi})$ is given by (3). The first-order conditions for profit maximization, evaluated in the symmetric equilibrium, $\tilde{p} = \tilde{p}_0 = \tilde{p}_1$ and $\tilde{\phi} = \tilde{\phi}_0 = \tilde{\phi}_1$, are

$$\pi_p(\tilde{p}, \tilde{\phi}) = t(2 - \tilde{\phi}) - \tilde{\phi}(\tilde{p} - c) = 0, \quad (7)$$

$$\pi_\phi(\tilde{p}, \tilde{\phi}) = (\tilde{p} - c)(1 - \tilde{\phi}/2) - A_\phi(\tilde{\phi}; \alpha) = 0. \quad (8)$$

The optimal advertising condition (8) is intuitive. Because $n\tilde{\phi}$ consumers receive the advertising message for each brand, a small increase in $\tilde{\phi}$ of $d\tilde{\phi} = 1/n$ informs (on average) one more consumer. With probability $\tilde{\phi}$, this consumer was previously informed of the rival brand and now becomes fully informed, in which case she makes a purchase exactly half the time. With probability $1 - \tilde{\phi}$, this consumer becomes partially informed, in which case she always purchases the brand. A small increase in $\tilde{\phi}$ therefore generates $1 - \tilde{\phi}/2$ sales and raises profits by $(\tilde{p} - c)(1 - \tilde{\phi}/2)$. The firm equates this marginal gain to the marginal cost of informing a consumer, $A_\phi(\tilde{\phi}; \alpha)$.

Let \tilde{p}^e and $\tilde{\phi}^e$ denote the equilibrium price and advertising intensity that solve (7) and (8). This solution can be expressed compactly by expressing (7) as

$$\tilde{p}^e - c = (2 - \tilde{\phi}^e)t / \tilde{\phi}^e, \quad (9)$$

then substituting this into (8) to get a single equation in $\tilde{\phi}^e$:

$$\frac{(2 - \tilde{\phi}^e)^2 t}{2\tilde{\phi}^e} = A_\phi(\tilde{\phi}^e; \alpha). \quad (10)$$

Notice that the equilibrium price in (9) is inversely related to the advertising intensity. Informative advertising increases the demand elasticities under complete coverage, and reduces prices. In (10), the left-hand side decreases monotonically in $\tilde{\phi}^e$ and increases monotonically in t . It follows that an increase in advertising costs decreases the advertising intensities, whereas an increase in product differentiation increases the advertising intensities.

B. Incomplete Coverage ($t/2 \leq v - p < t$)

In the case of incomplete coverage, a symmetric noncooperative oligopoly equilibrium exists under conditions of both price and quantity competition. I consider each case in turn.

¹⁰ It is straightforward to verify that the second-order conditions for profit maximization are satisfied for each firm in the neighborhood of the symmetric equilibrium.

Price Competition. Under price competition, the objective of firm i is to maximize profit,

$$\pi^i(p, \phi) = (p_i - c)X^i(p, \phi) - nA(\phi_i; \alpha) - F,$$

where $X^i(p, \phi)$ is given by (4). The first order conditions for a profit maximum, evaluated in the symmetric equilibrium, $p_b = p_0 = p_1$ and $\phi_b = \phi_0 = \phi_1$, are

$$\pi_p(p_b, \phi_b) = \frac{n\phi_b}{2t} [2(v - p_b)(1 - \phi_b) + \phi_b t - (p_b - c)(2 - \phi_b)] = 0, \quad (11)$$

$$\pi_\phi(p_b, \phi_b) = (p_b - c) \left(\frac{2(v - p_b)(1 - \phi_b) + \phi_b t}{2t} \right) - A_\phi(\phi_b; \alpha) = 0. \quad (12)$$

The optimal advertising level in (12) has the following interpretation. A small increase in advertising that informs a marginal consumer fully informs her with probability ϕ_b , in which case she makes a purchase exactly half the time. With probability $1 - \phi_b$, the advertising unit partially informs the marginal consumer, in which case she buys the brand with frequency $(v - p_b)/t$, the length of the segment served by the brand. A small increase in ϕ_b therefore generates $\phi_b/2 + (1 - \phi_b)(v - p_b)/t$ sales and raises profits by $(p_b - c)(2(v - p_b)(1 - \phi_b) + \phi_b t)/2t$. The firm equates this marginal gain to the marginal cost of informing a consumer, $A_\phi(\phi_b; \alpha)$.

Let p_b^e , X_b^e and ϕ_b^e denote the equilibrium price, quantity and advertising level when oligopoly firms compete in prices.

Quantity Competition. Under quantity competition, the objective function of firm i is

$$\pi^i(X, \phi) = p^i(X, \phi)X_i - cX_i - nA(\phi_i; \alpha) - F,$$

where $p^i(X, \phi)$ is given by (5). The first order conditions for a profit maximum, evaluated in the symmetric equilibrium, $X_c = X_0 = X_1$ and $\phi_c = \phi_0 = \phi_1$, are

$$\pi_X(X_c, \phi_c) = v - c + \frac{\phi_c t}{2(1 - \phi_c)} - \frac{(4 - \phi_c)tX_c}{2n\phi_c(1 - \phi_c)} - \frac{(2 - \phi_c)}{2n\phi_c(1 - \phi_c)} = 0, \quad (13)$$

$$\pi_\phi(X_c, \phi_c) = \frac{tX_c(n\phi_c^3 + 2(2 - 3\phi_c)X_c)}{4n\phi_c^2(1 - \phi_c)^2} - nA_\phi(\phi_c; \alpha) = 0. \quad (14)$$

For the comparison to follow, it is helpful to convert (13) and (14) into terms of the market prices using (4). Demand in the symmetric case is given by $X(p, \phi) = (\phi n / 2t)(2(v - p)(1 - \phi) + \phi t)$, and making this substitution into (13) and (14) yields

$$p_c - c = \frac{(2 - \phi_c)(2(v - p_c)(1 - \phi_c) + \phi_c t)}{4(1 - \phi_c)}, \quad (15)$$

$$(p_c - c) \left(\frac{((2 - 3\phi_c)(v - p_c) + \phi_c t)}{t(2 - \phi_c)} \right) = A_\phi(\phi_c; \alpha). \quad (16)$$

The intuition for the optimal advertising level in (16) can be seen by writing this expression as

$$(p_c - c) \left((1 - \phi_c) \frac{(v - p_c)}{t} + \frac{\phi_c}{2} - \frac{\phi_c^2 (v - p_c - t/2)}{(2 - \phi_c)t} \right) = A_\phi(\phi_c; \alpha).$$

A small increase in ϕ_c of $d\phi_c = 1/n$ informs (on average) one more consumer. With probability $1 - \phi_c$, this consumer becomes partially informed, in which case she buys the brand with frequency $(v - p_c)/t$. With probability ϕ_c , the consumer becomes fully informed, in which case she buys the advertised brand half the time. These first two terms effects coincide precisely with the effects described in (12) under price competition. Under quantity competition, however, advertising has an additional effect. An increase in advertising by firm i causes rival firm j to reduce its price by the amount in (6). From (initially) symmetric prices, this price decrease alters the location of the indifferent fully informed consumer in (1) by the amount $(\partial p_j / \partial \phi_i) / 2t$. The price decrease by firm j reduces firm i sales to fully informed consumers, but has no effect on firm i sales to partially informed consumers, so that the implication of this price change on the total sales level of firm i is $\frac{-\phi_c^2 (v - p_c - t/2)}{(2 - \phi_c)t}$ by (6). In equilibrium, the firm equates the value of the increased sales from a greater advertising intensity to its marginal cost, $A_\phi(\phi_c; \alpha)$.

Let p_c^e , X_c^e and ϕ_c^e denote the equilibrium price, quantity and advertising level when oligopoly firms compete in quantities.

III. Welfare Analysis

The goal is to compare the equilibrium outcomes of the model with the socially optimal resource allocation. A difficulty in doing so is that the socially optimal advertising intensity depends on the extent of market coverage at the efficient market prices ($p^* = c$), and, for given model primitives, the level of coverage under equilibrium prices and socially optimal prices generally does not coincide. I compare the socially optimal advertising allocation and the equilibrium

advertising levels by evaluating these values at the efficient prices. This is the “partial view” attributed to Spence (1975), which posits that the magnitude of each distortion (here advertising and price) be evaluated as if independent instruments were available to control each externality.

It is helpful at this point to distinguish between social and private costs of production and advertising. Let $A^s(\phi; \alpha)$ denote the social cost of advertising and let c_s denote the unit social cost of production. The socially optimal resource allocation can now be derived in each case.

A. Complete Coverage ($v - c_s \geq t$)

Under complete coverage, all consumers who receive at least one advertising message purchase the good. The gross value of consumption is $v - c_s$ for each consumer, regardless of her location in the product space. Total transportation costs, however, depend on the extent to which consumers at each location are aware of the existence of brands.

With probability $\tilde{\phi}^2$, a consumer receives both advertising messages. A consumer who receives both advertising messages travels (on average) $1/4$ the length of the unit segment to make a purchase and incurs transportation cost of $t/4$ to obtain her desired brand. This results in the social net benefit, $v - c_s - t/4$. With probability $2\tilde{\phi}(1 - \tilde{\phi})$, a consumer receives exactly one advertising message. This message is equally likely to be received from the nearest brand as from the farthest brand, so that a partially informed consumer (on average) incurs transportation cost of $t/2$ to obtain the good. This results in the social net benefit, $v - c_s - t/2$.

Aggregate welfare is given by

$$\tilde{W}(\tilde{\phi}) = n\tilde{\phi} \left(\tilde{\phi}(v - c_s - t/4) + 2(1 - \tilde{\phi})(v - c_s - t/2) \right) - 2nA^s(\tilde{\phi}; \alpha) - 2F .$$

It is helpful to write this expression as

$$\tilde{W}(\tilde{\phi}) = n\tilde{\phi}(2 - \tilde{\phi})(v - c_s) - n\tilde{\phi}t(1 - 3\tilde{\phi}/4) - 2nA^s(\tilde{\phi}; \alpha) - 2F , \quad (17)$$

where the first term represents consumption benefits (net of production costs but gross of transportation costs) and the second term is total transportation costs. The welfare measure (17) differs from that employed by GS in two ways. First, in GS consumption benefits are given by $n(v - c_s)(1 - (1 - \phi)^m)$, where m is the number of firms. This coincides precisely with the

consumption benefits in (17) in the case of $m = 2$.¹¹ Prior to optimization, however, GS approximate this welfare measure by setting the term $(1-\phi)^m$ to zero. The consequence of this is that total consumption is fixed at $n(v - c_s)$ and does not vary with the advertising intensity. Hence, their welfare measure is equivalent to one that seeks to minimize the sum of total transportation costs and advertising expenditures in the market.

Second, GS's assumption of fixed total consumption also modifies the definition of total transportation costs in the welfare standard. To see this, notice that total transportation cost in (17) is non-monotonic in the advertising intensity. There are two effects at work here. On the one hand, informative advertising improves the match between consumers and brands, which reduces total transportation costs, but on the other hand, advertising increases the number of consumers who purchase the good, which increases total transportation costs. In GS, this latter effect is suppressed, so that transportation cost always decreases monotonically with advertising.

The first-order condition for a welfare maximum, after rearranging, can be expressed as

$$(1-\tilde{\phi})(v-c_s-t/2)+\tilde{\phi}t/4=A_{\phi}^s(\tilde{\phi};\alpha). \quad (18)$$

A small increase in $\tilde{\phi}$ that informs a marginal consumer has the following welfare effects. With probability $(1-\tilde{\phi})$, the ad informs a previously uninformed consumer. This creates a new consumption unit in the economy that has a social value (on average) of $v-c_s-t/2$. With probability $\tilde{\phi}$, the ad fully informs a consumer who was previously only partially informed. This creates no new consumption value, but economizes on transportation cost (on average) by a factor of $t/4$. The sum of these marginal gains is set equal to the marginal social cost of advertising in (18). Let $\tilde{\phi}^*$ denote the solution to this equation.

B. Incomplete Coverage ($t/2 \leq v - c_s < t$)

Under incomplete coverage, a consumer receives both advertising messages with probability ϕ^2 and becomes fully informed. Each fully informed consumer purchases the nearest brand,

¹¹ Setting $m = 2$ entails no loss of generality, moreover, provided the number of firms is exogenous and the level of product differentiation is allowed to vary, as it is here.

generating net surplus (on average) of $v - c_s - t/4$. With probability $2\phi(1-\phi)$, a consumer receives an advertising message from only one of the two brands, and among these partially informed consumers, some purchase the advertised brand and some do not. The conditional probability that a consumer makes a purchase given that she receives a single advertising message is $(v - c_s) / t$. Thus, the probability that a consumer receives a single ad *and* finds it worthwhile to consume the brand is $2\phi(1-\phi)(v - c_s)/t$. The consumer who makes a purchase travels (on average) $(v - c_s)/2t$ the unit distance, and this generates net surplus of $(v - c_s)/2$.

Aggregate welfare is given by

$$W = n\phi \left[\phi \left(v - c_s - \frac{t}{4} \right) + \frac{(1-\phi)(v - c_s)^2}{t} \right] - 2nA^s(\phi; \alpha) - 2F.$$

The first-order necessary condition for ϕ^* , after rearranging, is

$$(1-\phi) \left(\frac{(v - c_s)^2}{2t} \right) + \phi \left(v - c_s - \frac{t}{4} \right) - \phi \frac{(v - c_s)^2}{2t} = A_\phi^s(\phi; \alpha). \quad (19)$$

This condition has an intuitive interpretation. A marginal increase in ϕ informs a previously uninformed customer with probability $(1-\phi)$, and with probability $(v - c_s)/t$, this customer also purchases the product, generating social surplus (on average) of $(v - c_s)/2$ when so doing. The expected social value of informing a previously uninformed consumer is thus $(v - c_s)^2/2t$. With probability ϕ , the marginal consumer becomes fully informed. The resulting consumption generates social surplus (on average) of $v - c_s - t/4$. Finally, the opportunity cost of fully informing a consumer is the social surplus that the economy would have received (on average) by leaving the consumer partially informed, which is $(v - c_s)^2/2t$. At the socially optimal advertising level, the sum of these three terms is set equal to the marginal cost of advertising. Let ϕ^* denote the solution to (19).

IV. Optimal Advertising Policy

Now consider a social planner who has the ability to independently control production and advertising levels in the market. This at least implicitly assumes that the regulator has two

policy tools at his disposal, and for ease in exposition, I model these instruments explicitly.¹²

It is well known that a production subsidy can be used to internalize the pricing distortion under oligopoly. Let s denote a unit subsidy on production. Social unit production cost then relates to private unit production cost as $c = c_s - s$.

To the extent that the equilibrium advertising levels diverge from the optimal advertising allocation at the efficient market prices, an advertising tax (or subsidy) can similarly internalize this distortion. Let τ denote a unit tax on advertising. The social cost of advertising then relates to private costs as $A(\phi; \alpha) = A^s(\phi; \alpha) + \tau\phi$. Firms in the market equilibrium either oversupply advertising or fail to provide enough according to whether the optimal advertising policy involves a tax, $\tau > 0$, or a subsidy, $\tau < 0$, respectively.

Under oligopoly, the equilibrium outcome for advertising differs from the socially optimal allocation according to four effects. First, oligopoly firms ignore consumer surplus, so that the social benefits of advertising tend to exceed the private benefits (the market-size effect). Second, oligopoly firms do not internalize the effect of advertising on better matching consumers to their desired brands (the matching effect). This tends to reduce the equilibrium advertising levels relative to the social optimum. Third, an individual firm does not consider the profit reduction his advertising creates for other firms (the customer-poaching effect), which tends to increase advertising levels relative to the social optimum. Finally, an increase in the advertising intensity alters the demand elasticities facing firms. Purely informative advertising has a socially beneficial effect on prices under complete coverage, but a socially undesirable effect under incomplete coverage. This elasticity effect has no direct welfare bearing on advertising policy in the present context, however, since I evaluate the market equilibrium at the socially optimal price levels. Nevertheless, the elasticity effect alters the level of the production subsidy necessary to obtain social pricing, which indirectly impacts the advertising outcome by altering the private

¹² It should be noted that without an independent instrument on output, the socially optimal price level would be lower than the equilibrium price level under oligopoly, which would bias the equilibrium advertising levels downward under complete coverage (because advertising and prices are inversely related), but bias the equilibrium advertising levels upward under incomplete coverage (because advertising and prices are positively related).

benefit firms receive from sales generated through advertising.

In general, the relationship between these effects is unclear and depends, among other things, on the extent of market coverage.

A. Complete Coverage ($v - c_s \geq t$)

The optimal policy under complete coverage can be described as follows. Substituting $c = c_s - s$ into (9), the subsidy necessary to induce firms to price at the socially optimal level, $\tilde{p}^* = c_s$, is¹³

$$\tilde{s}^* = (2 - \tilde{\phi}^*)t / \tilde{\phi}^* > 0. \quad (20)$$

The optimal tax on advertising is found similarly by substituting $A_\phi(\tilde{\phi}; \alpha) = A_\phi^s(\tilde{\phi}; \alpha) + \tilde{\tau}$ into (10). Evaluating this expression at the optimal advertising level in (18) gives

$$\tilde{\tau}^* = \frac{(2 - \tilde{\phi}^*)^2 t}{2\tilde{\phi}^*} - (1 - \tilde{\phi}^*) \left(v - c_s - \frac{t}{2} \right) - \frac{\tilde{\phi}^* t}{4}. \quad (21)$$

The optimal advertising policy in (21) can be positive (a tax) or negative (a subsidy) depending on the relative magnitude of each of the four effects noted above. The first term on the right-hand side of (21) is the marginal private benefit of advertising in (10) evaluated at $\tilde{\phi}^*$, which can be equivalently expressed as the value of the production subsidy (20) multiplied by the anticipated output increase from a marginal unit of advertising ($1 - \tilde{\phi}^*/2$). The remaining terms subtract from this the marginal social benefit of advertising in (18).

Proposition 2. Under complete coverage, the optimal advertising policy is a subsidy for sufficiently homogenous brands, but a tax for more differentiated brands.

Proof. See the appendix. \square

In homogenous product markets, purely informative advertising is always undersupplied in the oligopoly equilibrium, as in Stegeman (1991) and Stahl (1994). This result contrasts sharply with that of GS, who find the equilibrium advertising levels to be excessive in highly competitive

¹³ It should be noted that the production subsidy potentially has no effect on total output. As long as the market is completely covered, a positive price-cost margin in (9) has purely redistributive effects on welfare. Nonetheless, a production subsidy is needed to ensure the equilibrium prices are set at a level consistent with complete coverage.

markets, but otherwise inefficiently low. The distinction arises because the approximation of GS suppresses the market size effect. In the present context, advertising reaches consumers who otherwise would be uninformed, and this creates social value by increasing total consumption. The additional sales generate consumer surplus in excess of their marginal contribution to profit, so that firms tend to undersupply purely informative advertising. The magnitude of the market size effect depends on the degree of product differentiation, moreover, and takes its largest value in the case of homogeneous products.

Consider the case of homogenous brands (i.e., $t \rightarrow 0$). In this case, three outcomes lead advertising to be underprovided in the market: (i) the contribution of a marginal customer to social surplus (on average), $v - c_s - t/2$, approaches its maximum value; (ii) the equilibrium price-cost margins tend to zero in (9) (equivalently in (20)); and (iii) total transportation costs tend to zero in the market. Without transportation costs, matching consumers better to brands produces no social return, and accordingly, there are no matching effects in the market. Without positive price-cost margins, attracting customers from rival firms by increasing the advertising intensity makes no contribution to profit, so that there is no customer-poaching effect in the market. Finally, with marginal social surplus at its maximum value and price-cost margins at zero, the market-size effect obtains its maximum value, $v - c_s$. As a consequence, purely informative advertising is always undersupplied for homogeneous goods.

Advertising is oversupplied in the market when the brands are sufficiently differentiated. There are two reasons for this. First, because the equilibrium price-cost margins rise and the social value of consumption (net of transportation cost) falls as the products become more highly differentiated, the market size decreases monotonically with product differentiation. Second, the customer poaching effect becomes larger in value relative to the matching effect at higher levels of product differentiation. To see this, recall that the poaching effect arises because the anticipated increase in output from a marginal unit of advertising, $1 - \tilde{\phi}^*/2$, exceeds the equilibrium output increase, $1 - \tilde{\phi}^*$. The customer-poaching effect, which values this output difference ($\tilde{\phi}^*/2$) according to the price-cost margin in (9) (equivalently (20)), is $(2 - \tilde{\phi}^*)t/2$.

The matching effect is the reduction in aggregate transportation costs due to improved information, $\tilde{\phi}^* t/4$, so that the poaching effect exceeds the matching effect by the factor $(4-3\tilde{\phi}^*)t/4$. If the market size effect is suppressed as in GS, advertising is always oversupplied. Here, in contrast, the outcome depends on the magnitude of the market size effect relative to this composite of poaching and matching effects. Because the composite effect increases monotonically with t , whereas the market size effect decreases monotonically with t , advertising ultimately becomes excessive for sufficiently differentiated brands.

The basic outcomes of the model can be confirmed through numerical simulation. To do so, I simulate the private and social outcomes of the model using the advertising cost specialization of Butters (1977), $A^s(\phi; \alpha) = -\alpha \ln(1-\phi)$. For the baseline simulation, I select parameters such that both the social allocation and the unregulated oligopoly equilibrium (i.e., one with $\tilde{s}^* = \tilde{\tau}^* = 0$) are characterized by complete coverage. For the baseline parameter values ($v = 250$, $c_s = 50$, $t = 75$ and $\alpha = 25$), the socially optimal resource allocation obtains at $\tilde{p}^* = 50$ and $\tilde{\phi}^* = 0.64$. The unregulated equilibrium involves the prices $\tilde{p}^e = 185.38$ and the advertising levels $\tilde{\phi}^e = 0.71$. Advertising is oversupplied in the baseline case (with both distortions working together), and the optimal regulatory policy is $\tilde{s}^* = 158.25$ and $\tilde{\tau}^* = 37.30$.¹⁴

Figure 2 considers optimal policy for variations in the degree of product differentiation. Both the production subsidy and the advertising tax increase monotonically with product differentiation. The optimal subsidy on production increases smoothly from zero with the degree of product differentiation. The optimal advertising policy is a subsidy for relatively homogenous products, but a tax for sufficiently differentiated brands.

It is also useful to examine the implication of changes in advertising costs. Advertising costs may decrease over time through the introduction of new media. Figure 3 considers optimal policy for variations in advertising costs. Notice that the level of both the production subsidy

¹⁴ Notice that the optimal value of the subsidy exceeds the difference between the unregulated price and the optimal one. This is because the subsidy must compensate firms for the price-cost margin at the social allocation, which involves higher price-cost margins than the private market equilibrium (advertising and prices are inversely-related).

and the advertising tax increase with advertising costs. Reductions in advertising costs produce socially beneficially spillovers in oligopoly markets for advertised goods, as a decrease in advertising costs increases the advertising levels of firms, which increases the demand elasticities (and decreases the price-cost margins).

B. Incomplete Coverage ($t/2 \leq v - c_s < t$)

Under price competition, the optimal production subsidy is found by substituting $c = c_s - s$ into (11). The socially optimal price level, $p_b^* = c_s$, obtains in this expression when

$$s_b^* = \frac{2(v - c_s)(1 - \phi^*) + \phi^* t}{2 - \phi^*} > 0. \quad (22)$$

The optimal instrument on advertising is found by substituting $A_\phi(\phi; \alpha) = A_\phi^s(\phi; \alpha) + \tau_b$ into (12) and evaluating this expression at the optimal advertising level implied by (19). Doing so yields

$$\tau_b^* = \frac{(2(v - c_s)(1 - \phi^*) + \phi^* t)^2}{2t(2 - \phi^*)} - (1 - 2\phi^*) \frac{(v - c_s)^2}{2t} - \phi^* \left(v - c_s - \frac{t}{4} \right). \quad (23)$$

The first term on the right-hand side of (23) is the marginal private benefit of advertising under price competition, which is the subsidy rate (22) multiplied by the expected change in output from a marginal increase in advertising. The remaining terms subtract from this the marginal social benefit of advertising in (19).

The optimal policy under quantity competition is similarly obtained as

$$s_c^* = \frac{(2 - \phi^*)(2(v - c_s)(1 - \phi^*) + \phi^* t)}{4(1 - \phi^*)} > 0, \quad (24)$$

$$\tau_c^* = \frac{(2(v - c_s)(1 - \phi^*) + \phi^* t)((v - c_s)(2 - 3\phi^*) + \phi^* t)}{4t(1 - \phi^*)} - (1 - 2\phi^*) \frac{(v - c_s)^2}{2t} - \phi^* \left(v - c_s - \frac{t}{4} \right). \quad (25)$$

Two features distinguish the optimal advertising policies under price competition (23) and quantity competition (25). First, the equilibrium price-cost margins (evaluated at the optimal advertising level) are larger under quantity competition than under price competition. The higher price-cost margins under quantity competition induce larger advertising levels in the oligopoly equilibrium, which tends to increase advertising taxes. That is, the market-size effect is always smaller when firms compete in quantities than when firms compete in prices. Second, the

expected change in output following a marginal change in advertising is smaller under quantity competition than under price competition. The marginal product of advertising to a quantity-setting firm is less than its average product due to the anticipated price adjustment of the rival firm in (6). The poaching effect is therefore larger under price competition than under quantity competition, and this offsets the difference in market-size effects.

Proposition 3. Irrespective of the mode of competition (prices or quantities), purely informative advertising is always oversupplied in the market equilibrium. Relative to the outcome under price competition,

- (i) the optimal output subsidy is always higher under quantity competition; and**
- (ii) the optimal advertising tax is always higher under quantity competition.**

Proof. See the appendix. \square

Informative advertising is always oversupplied under incomplete coverage. The reason for this is that the equilibrium price-cost margins tend to be high. Indeed, it turns out that the effect of advertising on the price elasticity of demand always generates a sufficiently high production subsidy to produce a negative market size effect.

To see why a negative market size effect can occur, it is helpful to note that this effect would necessarily be positive if the market had only a single (advertising) distortion. The oligopoly output level in this case would coincide with the social output level without the need for policy correction, and the marginal social benefit of advertising, $v - c - t/2$, would necessarily exceed the marginal contribution of advertising to profit, $p - c$, by the amount of consumer surplus, $v - p - t/2 > 0$. With an output distortion in the market, however, the subsidy level in (22) (or (24)) must compensate each firm for the price-cost margin it would earn at the oligopoly output level associated with ϕ^* . This magnitude of this profit margin has no clear ranking relative to marginal social benefit. Put differently, private and social production costs must differ on the margin for the market output level to coincide with the efficient level, and this unit

cost difference can exceed the marginal contribution of a new customer to consumer surplus.

Under incomplete coverage, the equilibrium price-cost margins are sufficiently high that purely informative advertising is always overprovided in the oligopoly equilibrium. Because the equilibrium price-cost margins are higher when firms compete in quantities than when firms compete in prices, the degree of overadvertising is always larger under quantity competition.

Figure 4 presents a numerical simulation of the optimal advertising tax under price competition and quantity competition for variations in product differentiation. As in the case of complete coverage, the level of the advertising tax increases with the degree of product differentiation in the market.

Figure 5 presents the simulated outcomes of the model for variations in advertising cost. Unlike the outcome under complete coverage, notice that the optimal advertising tax decreases with increases in advertising cost. This is because the equilibrium price-cost margins are now positively-correlated with the advertising intensities in the market. A decrease in advertising costs under incomplete coverage now intensifies both forms of distortion (advertising and price), and this leads to a divergence between the private market equilibrium and the social optimum.

V. Concluding Remarks

This paper has considered a model of purely informative advertising with differentiated products. Advertising informs consumers about the characteristics of products and improves the matching between consumers and brands. Nonetheless, informative advertising does not necessarily make the demand for individual brands more elastic. For sufficiently differentiated products, purely informative advertising decreases the demand elasticities facing brands (and increases prices).

Relative to the social optimum, advertising levels under oligopoly can be either inefficiently low or excessive. For homogeneous products, advertising is always undersupplied in the market equilibrium, whereas advertising is overprovided for sufficiently differentiated brands.

The effect of changes in advertising technology on the market equilibrium also depends

fundamentally on the degree of product differentiation. This result has particular relevance as the availability of new media (e.g., the internet) reduces the cost of achieving a given reach in an ad campaign. If the degree of product differentiation is sufficiently small relative to the intrinsic value of the good, then a decrease in advertising costs causes equilibrium prices and advertising levels to converge towards the socially optimal levels. For higher levels of product differentiation, however, a decrease in advertising costs causes the private market equilibrium to diverge from the socially optimal resource allocation.

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Appendix

Proof of Proposition 1. (i) In the symmetric case, $p_i = p$, $X^i = X$, and $\phi_i = \phi$ for $i = 0, 1$, the price elasticity of demand facing firm i , $\varepsilon_i = (\partial X^i / \partial p_i) p_i / x_i$, under price competition is given (in absolute terms) by

$$\varepsilon_b = \frac{(2-\phi)p}{2(v-p)(1-\phi) + \phi t}. \quad (\text{A.1})$$

Under quantity competition, the price elasticity of demand facing firm i is

$$\varepsilon_c = \frac{4(1-\phi)p}{(2-\phi)(2(v-p)(1-\phi) + \phi t)}. \quad (\text{A.2})$$

Comparison of (A.1) and (A.2) reveals

$$\varepsilon_c = \left(\frac{4(1-\phi)}{(2-\phi)^2} \right) \varepsilon_b, \quad (\text{A.3})$$

where the term in brackets is strictly greater than one.

For part (ii), notice that the demand function (4) is linear with respect to the advertising level of firm i . Letting $\eta_i = (\partial X^i / \partial \phi_i) \phi_i / x_i$ denote the advertising elasticity of demand, it follows immediately that $\eta_b = 1$ under price-setting oligopoly. Under quantity competition, the advertising elasticity of demand facing firm i is $\eta_i = \varepsilon_i (\partial p_i / \partial \phi_i) \phi_i / p_i$. Evaluating this elasticity at $p^i = p$, $X_i = X$, and $\phi_i = \phi$ for $i = 0, 1$, yields

$$\eta_c = \frac{2((2-3\phi)(v-p) + \phi t)}{(2-\phi)(2(v-p)(1-\phi) + \phi t)}.$$

It is straightforward to verify that $\eta_c \leq 1$ for $v-p \geq t/2$. Hence, $\eta_c \leq \eta_b$ by the definition of incomplete coverage.

For part (iii), differentiate equation (A.1) to get

$$\frac{\partial \varepsilon_b}{\partial \phi} = \frac{2p(v-p-t)}{[2(v-p)(1-\phi) + \phi t]^2} < 0, \quad (\text{A.4})$$

where the inequality follows by the definition of incomplete coverage. Next, differentiate (A.3),

$$\frac{\partial \varepsilon_c}{\partial \phi} = \frac{-4\phi \varepsilon_b}{(2-\phi)^3} + \frac{4(1-\phi)}{(2-\phi)^2} \left(\frac{\partial \varepsilon_b}{\partial \phi} \right),$$

which is negative by (A.4). \square

Proof of Proposition 2. The optimal advertising policy (21) can be written as

$$\tilde{\tau}^* = \frac{(8 - 6\tilde{\phi}^* - \tilde{\phi}^{*2})t}{4\tilde{\phi}^*} - (1 - \tilde{\phi}^*)(v - c_s), \quad (\text{A.5})$$

where $8 - 6\tilde{\phi}^* - \tilde{\phi}^{*2} > 0$ for all $\tilde{\phi}^* \leq 1$. The advertising tax increases monotonically in t . It follows that the necessary and sufficient condition for the advertising policy to be a subsidy is

$$t < \frac{4\tilde{\phi}^*(1 - \tilde{\phi}^*)(v - c_s)}{8 - 6\tilde{\phi}^* - \tilde{\phi}^{*2}}.$$

For any $\tilde{\phi}^* > 0$, this is clearly satisfied for homogenous brands (i.e., $t \rightarrow 0$). To complete the proof, it is only necessary to demonstrate that a complete coverage equilibrium exists for which an advertising tax is optimal. To see this, consider $v - c_s = t$. Making this substitution into (A.5),

$$\tilde{\tau}^* = \frac{(2 - \tilde{\phi}^*)(4 - 3\tilde{\phi}^*)}{4\tilde{\phi}^*},$$

which is positive for all $\tilde{\phi}^* \leq 1$. \square

Proof of Proposition 3. Factoring (23) yields

$$\tau_b^* = \frac{\left[(v - c_s)^2 ((2 - \phi^*)(1 - \phi^*)) + (\phi(v - c_s))^2 - 2\phi^{*2}(v - c_s)t + \phi^*t^2 + (t\phi^*)^2 / 2 \right]}{2(2 - \phi^*)t} \quad (\text{A.5})$$

The sign of τ_b^* is given by the sign of the numerator of (A.5). Adding and subtracting the terms $\phi^{*2}t(v - c_s)/2$ and $3(\phi^*t)^2/2$ from this expression, the numerator of (A.5) can be written

$$(2 - \phi^*)(1 - \phi^*)(v - c_s)^2 + \phi^{*2}(v - c_s)(v - c_s - t/2) - 3\phi^{*2}t(v - c_s - t)/2 + \phi^*(1 - \phi^*)t^2,$$

which is positive by the definition of incomplete coverage. Therefore, $\tau_b^* > 0$. $\tau_c^* > 0$ must also hold whenever $\tau_c^* > \tau_b^*$, which I show below in the proof of part (ii).

For part (i), a comparison of instruments in (22) and (24) reveals that $s_c^* > s_b^*$ whenever $\frac{2 - \phi^*}{4(1 - \phi^*)} > \frac{1}{2 - \phi^*}$. This clearly holds for all $\phi^* < 1$.

For part (ii), a comparison of instruments in (23) and (25) reveals that $\tau_c^* > \tau_b^*$ whenever

$$\frac{2(v - c_s)(1 - \phi^*) + \phi^*t}{2 - \phi^*} < \frac{(v - c_s)(2 - 3\phi^*) + \phi^*t}{2(1 - \phi^*)}. \quad (\text{A.6})$$

Factoring expression (A.6), this inequality is satisfied whenever $v - c_s - t < 0$. It follows by the definition of incomplete coverage that $\tau_c^* > \tau_b^*$. \square

FIGURE 1. Surplus offered by firms i and j under incomplete coverage.

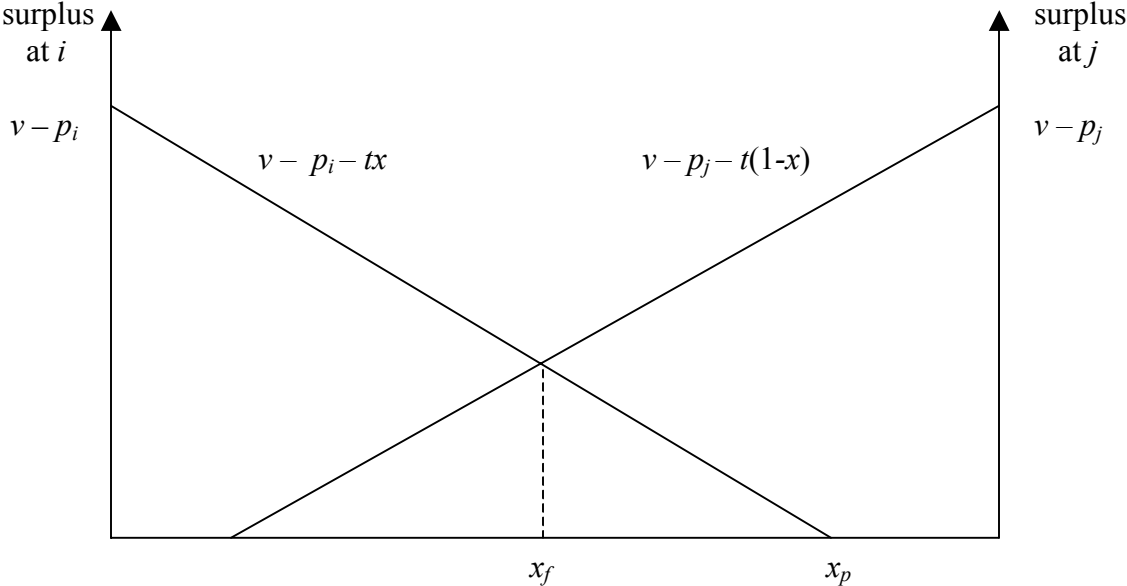


FIGURE 2. Simulation of the optimal advertising policy under complete coverage for variations in product differentiation ($v = 250, c_s = 50, \alpha = 25$).

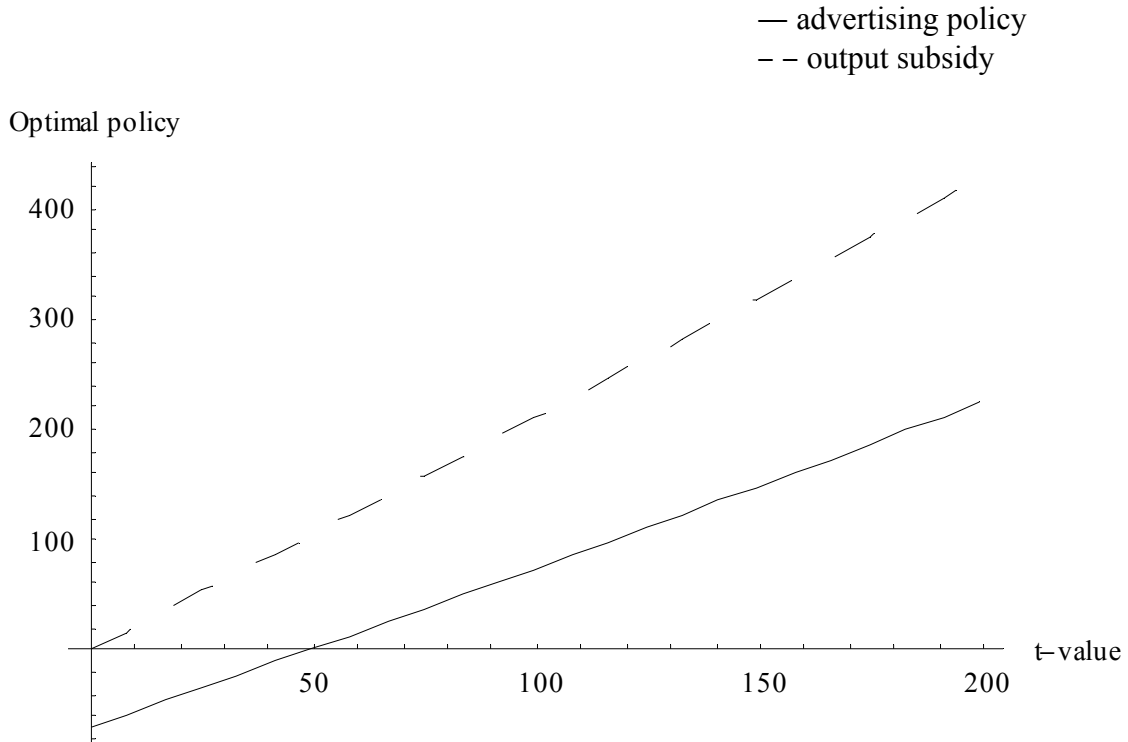


FIGURE 3. Simulation of the optimal advertising policy under complete coverage for variations in advertising cost ($v = 250, c_s = 50, t = 100$).

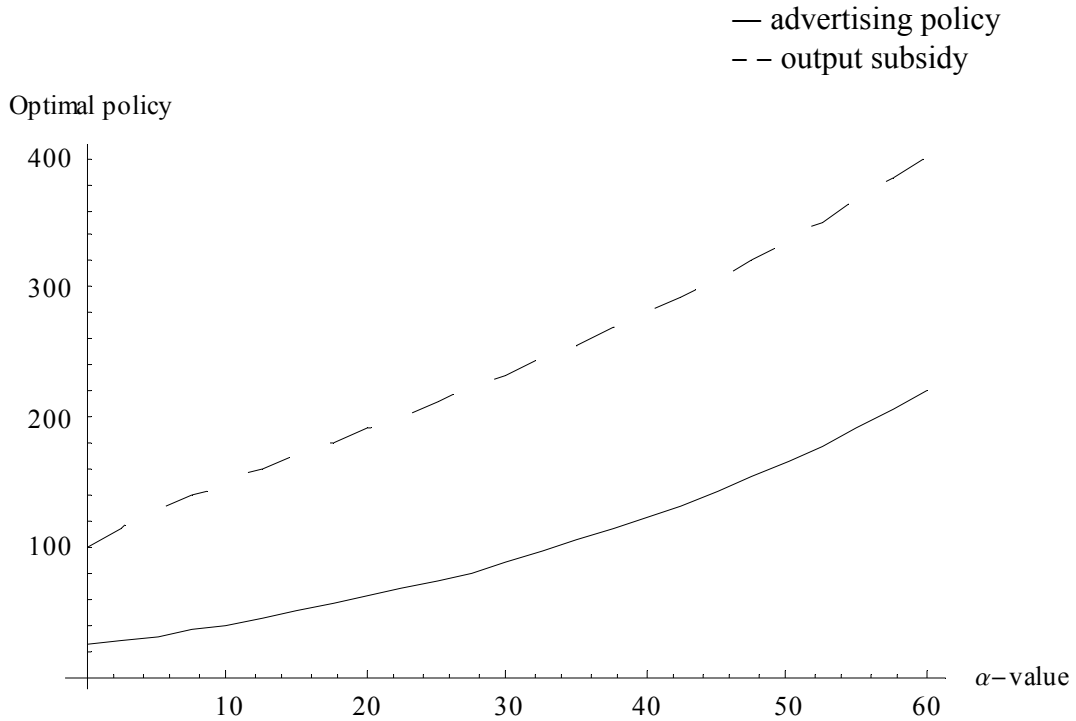


FIGURE 4. Simulation of the optimal advertising policy under incomplete coverage for variations in product differentiation ($v = 250, c_s = 50, \alpha = 25$).

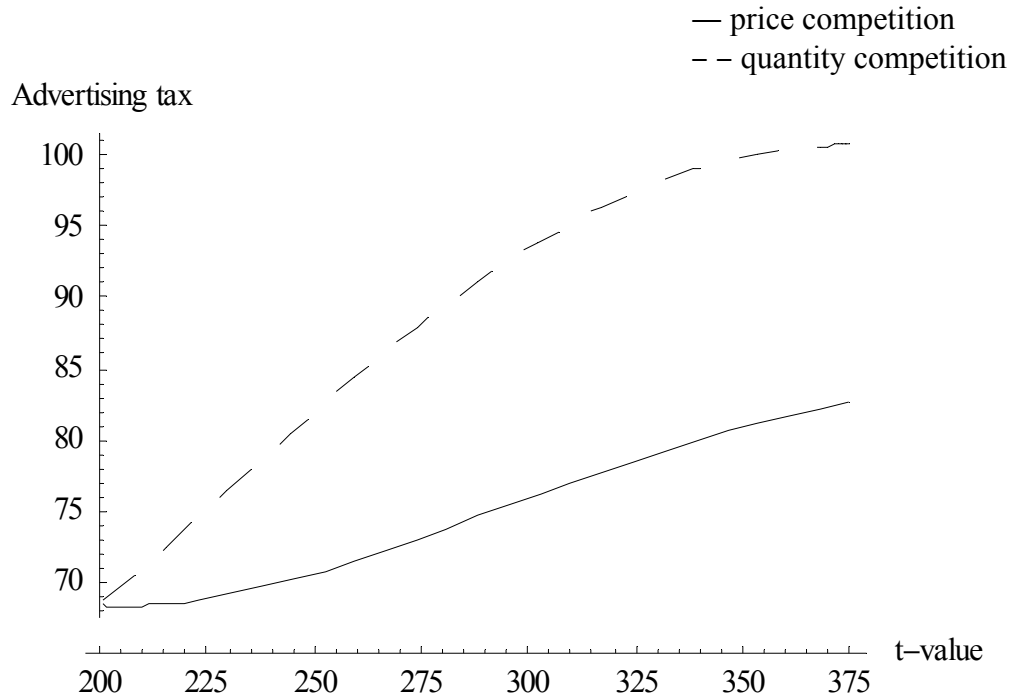


FIGURE 5. Simulation of the optimal advertising policy under incomplete coverage for variations in advertising cost ($v = 250, c_s = 50, t = 300$).

