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ON LONG-RUN INDUSTRY BEHAVIOR UNDER HETEROGENEOUS FIRMS

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This paper presents a long run analysis of industry behavior allowing for entry and exit, and cost heterogeneity among firms. Treating the number of firms as endogenous provides linkages between firms' conduct (reflecting the exercise of market power) and market structure. In steady state equilibrium, the implications of cost structure for market equilibrium price, firms' conduct and industry concentration are investigated. We derive the firms' conduct that emerges in stationary equilibrium from evolutionary selection over time. We also show how globalization helps reduce the firms' exercise of market power, increase the responsiveness of aggregate supply, and reduce price sensitivity to shocks.

Keywords: oligopoly, heterogeneous firms, entry/exit, market equilibrium.

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On Long Run Industry Behavior under Heterogeneous Firms

Jean-Paul Chavas

1. Introduction

Market globalization generates significant gains in economic efficiency (e.g., Helpman and Krugman; Levinsohn). One argument is that globalization tends to increase competition. For example, domestic firms which may have reaped oligopoly profits in a protected domestic market are forced to behave more competitively in global markets. But that process can be complex and is still imperfectly understood. First, while globalization means that new firms enter a market, it is often associated with the exit of some old firms. This is especially relevant in the presence of cost heterogeneity among firms. This indicates a need to investigate the role of firm entry/exit and firm heterogeneity in the study of globalization. Second, with the rise in the number of firms active in a market, one can expect a decrease in oligopoly rents. But these effects may not be uniform across industries. This suggests a need to study the determinants of oligopoly behavior as markets become more global. The objective of this paper is to address these issues.

In general, firm and industry behavior depends on the nature of strategic interactions among firms. In an oligopoly, the nature of price competition varies between a Cournot game, a Bertrand game or a Stackelberg game (e.g., Tirole). In this context, the determination of firm conduct and industry behavior depends on the precise form of the underlying game (e.g., Dasgupta and Stiglitz; Shaked and Sutton; Tirole; Vickers). This does not generate clear predictions that would hold across industries. The analysis presented in this paper focuses on long run situations for an industry in stable steady sate equilibrium.[1] As we will show, this will help us obtain more positive results about industry behavior, and its relationship with cost and structure.

This paper develops a long run model of firm and industry behavior with a focus on the relationships between firm heterogeneity, entry/exit, firms' conduct (reflecting the exercise of market power), and market structure. We analyze entry/exit in the industry and treat the number of active firms as endogenous. This allows various market structures to arise, going from monopoly to oligopoly to competitive markets. We also allow for firm heterogeneity where each firm can face different cost. The cost difference can come from two sources: different production technology, and/or different access to market. The first source means that some firms have access to improved technology that reduces their cost of production and gives them some

[1] As shown by Friedman and Samuelson, there is close relationship between stable dynamic steady state equilibrium and Nash equilibrium.

comparative advantage. The second source means that transaction costs vary across firms. This can be due to location differences (e.g., generating different transportation cost), differential access to market information, and/or different regulation impacts (e.g., with quotas, taxes or tariffs/subsidies that vary across firms). The effects of changing transaction costs are particularly relevant in the context of studying the globalization of markets. Indeed, transaction costs reduce incentives to produce and trade. By reducing the number of market participants, they can contribute to the creation of "local markets" that fails to be integrated in a global economy. In this context, the development of global markets is supported by a reduction of transaction costs associated with lower transportation and information costs, and by a move toward market liberalization policies. Our analysis provides useful insights on the effects of changing cost structures, as they affect pricing and industry behavior in global markets. Finally, while there is some anecdotal evidence that price instability may increase in thin and concentrated markets, it remains unclear when such relationships may develop. The paper examines how market concentration can affect supply responsiveness and price sensitivity to shocks.

This paper makes several contributions. First, it develops a refined analysis of long run market equilibrium under entry/exit and heterogeneous firms. For example, it highlights the effects of fixed cost on industry behavior. Second, the paper investigates the determinants of oligopoly behavior in long run equilibrium. We show how firms' conduct (representing the exercise of market power) relates to the number of firms active in the market. In particular, we show which particular firms' conduct emerges in a stable steady state equilibrium from evolutionary selection over time. Third, by analyzing the joint determination of the number of active firms and their conduct, the paper provides useful insights in the economics of globalization. In particular, we show how globalization helps reduce the firms' exercise of market power, increase the responsiveness of aggregate supply, and reduce price sensitivity to shocks.

The paper is organized as follows. Section 2 presents our model. Section 3 analyzes the associated market equilibrium under entry/exit and fixed cost. Section 4 studies the long run equilibrium firms' conduct under various market structures. Section 5 investigates the properties of industry behavior when both the number of active firms and the exercise of market power are endogenous. Finally, section 6 presents concluding comments.

2. Firm Behavior

Consider an industry composed of firms producing a homogenous product with an aggregate demand given by the price dependent demand $p(Y_t)$, where Y_t denotes aggregate quantity consumed at time t, and $\partial p(Y_t)/\partial Y_t < 0$. The quantity Y_t can be produced by a set M of potential firms, $M = \{1, \ldots, m\}$. At time t, the i-th firm produces output $y_{it} \geq 0$ at cost $c_i(y_{it}, y_{i,t-1}) \geq 0$. Including both current and lagged output in the cost function $c_i(y_{it}, y_{i,t-1})$ captures the dynamics of the production process. Throughout, we assume that the cost function $c_i(y_{it}, y_{i,t-1})$ is twice continuously differentiable on R^2_{++} and satisfies $c_i(0, 0) = 0$, $i \in M$. Possible discontinuities of the cost function $c_i(y_{it}, y_{i,t-1})$ at (0, 0) allow for fixed cost as well as sunk cost. Since a subset of firms can be inactive at time t (when $y_{it} = 0$), we treat the number of active firms (with $y_{it} > 0$) as endogenous. By considering entry/exit among the m firms, this will allow us to address the determination of industry structure (see below). We also consider firm heterogeneity by allowing the cost function $c_i(y_{it}, y_{i,t-1})$ to vary across firms.

At time t, the i-th firm is observed choosing $y_{1i} \geq 0$, $i \in M$, with $Y_t = \sum_{i \in M} y_{it}$. We consider a general representation of industry behavior, allowing for any possible strategic interactions among firms. In this context, firm behavior can range from competition, to oligopoly behavior, and to monopoly pricing. However, as mentioned in the introduction, firm and industry behavior typically depends on the game played (e.g., a Cournot game, Bertrand game, Stackelberg game, etc.) and the nature of strategic interactions among firms (see Dasgupta and Stiglitz; Shaked and Sutton; Tirole; Vickers). This does not generate clear predictions that would hold across industries. One way to generate more positive results is to focus the analysis on long run situations for an industry in stable steady sate equilibrium. This is the approach explored in this paper.

Under a stable steady state equilibrium, $y_{it} = y_{i,t-1} \equiv y_i$,[2] and the long run cost function for the i-th firm can be written as $C_i(y_i) \equiv c_i(y_i, y_i)$, $i \in M$. The Lerner index associated with the i-th active firm (with long run output $y_i > 0$) is

$$L_i \equiv [p(Y) - \partial C_i(y_i)/\partial y_i]/p(Y). \qquad (1a)$$

Using competition as benchmark, the Lerner index L_i in (1) measures the relative price enhancement obtained in the long run due to the i-th firm's exercise of market power. $L_i = 0$ identifies a competitive firm where marginal cost pricing applies. $L_i > 0$ occurs when the i-th firm has market power as price exceeds its marginal cost. In general, a rise in L_i can be interpreted as an increase in the exercise of market power by the i-th firm. Below, we will work with the closely related representation

$$v_i \equiv L_i\, \varepsilon/s_i - 1, \qquad (1b)$$

where $\varepsilon \equiv -[\partial \ln p(Y)/\partial \ln(Y)]^{-1} > 0$ is the price elasticity of demand, and $s_i \equiv y_i/Y$ is the market share of the i-th firm. It follows that competitive markets (where $L_i = 0$) are associated with $v_i = -1$ for all active firms. Alternatively, under imperfect competition (where $L_i > 0$), we have $v_i > -1$. Equation (1b) has the advantage of introducing explicitly the role of the price elasticity of

[2] Throughout the paper, we focus our attention on stable steady state equilibrium. It means that we rule out possible situations of a limit cycle or chaotic attractor. However, we will show below (in Proposition 3) that unique stable steady state equilibrium exists under the particular specification of demand and cost investigated in this paper.

demand ε and of the market share s_i. Given $L_i \equiv (1 + v_i)\, s_i/\varepsilon$, an increase the Lerner index L_i can be associated with a rise in the market share s_i, a rise in v_i, and/or a more inelastic demand (i.e., a decline in ε). In general, v_i measures possible departures from marginal cost pricing for the i-th firm. Specific values of v_i correspond to well-known special cases (e.g., Dixit). First, $v_i = -1$ is equivalent to Bertrand competition, where there is no anticipated price response to the i-th firm supply. Second, $v_i = 0$ corresponds to a Cournot game, where the i-th firm expects no quantity response from other firms to its own supply decision. Third, $v_i = x_i/y_i$ corresponds to market collusion, where all firms behave as a cartel implementing monopoly pricing. Throughout the paper, we will interpret v_i as reflecting the long run conduct of the i-th active firm. Assuming $Y > 0$, combining (1a) and (1b) yields

$$p(Y) - \partial C_i(y_i)/\partial y_i + y_i\,(1 + v_i)\,[\partial p(Y)/\partial Y] = 0, \qquad (2)$$

Conditional on Y and v_i, equation (2) provides a characterization of the decision rule for the i-th active firm (with $y_i > 0$).[3] The determination of firms' conduct (represented by the v_i's) will be addressed in section 4 below.

Allowing for entry and exit, y_i also needs to satisfy the non-negative profit condition: $p(Y)\, y_i - C_i(y_i) \geq 0$ (otherwise the i-th firm would prefer to shut down). It follows that, given $Y = \sum_{i \in M} y_i$, the production decision of the i-th active firm must satisfy equation (2), $y_i \geq 0$, and $p(Y)\, y_i - C_i(y_i) \geq 0$, $i \in M$.

Below, we explore the properties of firm behavior and industry equilibrium. To obtain analytical results, we will focus our attention on the following specification. We assume that the demand function is linear: $p(Y) = \alpha_1 - \alpha_2 Y$, where $\alpha_1 > 0$ and $\alpha_2 > 0$. And we assume that the cost function of the i-th firm is quadratic: $C_i(y_i) = c_{0i} + c_{1i}\, y_i + ½\, c_2\, y_i^2$, where $c_{0i} \geq 0$ represents fixed cost, $c_{1i} > 0$, and $c_2 \geq 0$, $i \in M$. Firm heterogeneity is represented by the distribution function $F(\cdot, \cdot)$ of the cost parameters (c_{0i}, c_{1i}) among the m potential firms. While we treat m and $F(\cdot, \cdot)$ as given throughout the paper, we want to stress that only some the m firms may be active.

Note that our quadratic cost specification is flexible in its representation of returns to scale. To see that, note that the i-th firm technology exhibits increasing returns to scale (IRTS), constant returns to scale (CRTS), or decreasing returns to scale (DRTS) when average cost $C_i(y_i)/y_i$ is

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[3] Note that equation (2) is also the first-order condition for an interior solution to the profit maximization problem

$$\mathrm{Max}_{y_i}\ \{p(y_i + x_i(y_i))\, y_i - C_i(y_i) \colon y_i \geq 0\},$$

where $x_i \equiv \sum_{j \neq i} y_j$ is the aggregate production of all firms but the i-th one, $Y = y_i + x_i(y_i)$, and $v_i \equiv \partial x_i/\partial y_i \geq -1$. This suggests that v_i can be interpreted as the conjecture made by the i-th firm about the supply response of other firms to a marginal change in its own production (e.g., Breshanan; Perry; Dixit). However, our analysis doe not rely on this interpretation. Following Genesove and Mullin, we interpret v_i simply as reflecting the long run conduct of the i-th firm.

decreasing, constant, or increasing in output y_i, respectively. When $y_i > 0$, this implies that IRTS, CRTS, or DRTS corresponds to $c_2 < 2\, c_{0i}/y_i^2$, $c_2 = 2\, c_{0i}/y_i^2$, or $c_2 > 2\, c_{0i}/y_i^2$, respectively. It means that the i-th firm technology exhibits global IRTS when fixed cost c_{0i} is positive and $c_2 = 0$; it exhibits global CRTS when $c_{0i} = c_2 = 0$; and it exhibits global DRTS when $c_{0i} = 0$ and $c_2 > 0$. While c_2 is treated as constant, firm heterogeneity is captured by the cost parameters c_{0i} and c_{1i} that can vary across firms. In general, we interpret $C_i(y_i)$ to represent the total cost of operation, including both production cost and transaction cost. In this context, as discussed in the introduction, cost differences across firms can come from two sources: different technology, and/or different access to market. The first yields different cost of production. The second gives different transaction cost across firms due to location differences (e.g., different transportation cost), differential access to market information, and/or different regulation impacts (e.g., quotas, taxes or tariffs that vary across firms). To the extent that moves toward global markets are motivated in large part by a reduction in these costs, our analysis provides useful insights on pricing and industry behavior in such markets.

Below, we will assume that

$$c_2 + \alpha_2 (1 + v_i) > 0, \tag{3}$$

$i \in M$.[4] Condition (3) is required to be able to solve equation (2) for y_i, conditional on Y and v_i.[5] Given $p(Y) = \alpha_1 - \alpha_2 Y$, and $C_i(y_i) = c_{0i} + c_{1i}\, y_i + ½\, c_2\, y_i^2$, equation (2) yields the decision rule for $y_i \geq 0$:

$$y_i^{\#} = \frac{\alpha_1 - \alpha_2 Y - c_{1i}}{c_2 + \alpha_2 (1 + v_i)}, \text{ if } c_{1i} \leq \alpha_1 - \alpha_2 Y, \tag{4}$$

$= 0$, otherwise,

$i \in M$. However, in the presence of fixed cost (when $c_{0i} > 0$), the decision rule (4) can generate negative profit for the i-th firm. To guarantee non-negative profit, the following constraint must also be satisfied: $\pi_i^{\#} = p(Y)\, y_i^{\#} - C_i(y_i^{\#}) \geq 0$. Noting that $\pi_i^{\#} = -c_{0i} + \left[\frac{\alpha_1 - \alpha_2 Y - c_{1i}}{c_2 + \alpha_2 (1 + v_i)}\right]^2 [½\, c_2 + \alpha_2 (1 + v_i)]$, this implies

$$c_{1i} \leq \alpha_1 - \alpha_2\, Y - \sqrt{c_{0i}}\, \frac{c_2 + \alpha_2 (1 + v_i)}{\sqrt{½\, c_2 + \alpha_2 (1 + v_i)}}. \tag{5}$$

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[4] Given $c_2 \geq 0$, $\alpha_2 > 0$, and $v_i \geq -1$, note that condition (3) is very mild. However, it excludes the situation where both $c_2 = 0$ and $v_i = -1$.

With $v_i \geq -1$ and $\sqrt{c_{0i}}\,\dfrac{c_2 + \alpha_2(1+v_i)}{\sqrt{½\, c_2 + \alpha_2(1+v_i)}} \geq 0$, combining (4) and (5) yields the following decision rule for any of the m firm

$$y_i^*(Y, v_i) = \frac{\alpha_1 - \alpha_2 Y - c_{1i}}{c_2 + \alpha_2(1+v_i)}, \text{ if } c_{1i} \leq \alpha_1 - \alpha_2\, Y - \sqrt{c_{0i}}\,\frac{c_2 + \alpha_2(1+v_i)}{\sqrt{½\, c_2 + \alpha_2(1+v_i)}}, \qquad (6)$$

$$= 0, \text{ otherwise,}$$

$i \in M$. Equation (6) allows for active as well as inactive firms. It shows that the i-th firm would become inactive when c_{1i} is sufficiently large (corresponding to high marginal cost) and/or when fixed cost c_{0i} is sufficiently large. By evaluating when firms become active, this endogenizes the number of firms.[6]

3. Market Equilibrium

Starting from (6), whether the i-th firm is active or not can be represented by the indicator variable

$$I_i(Y, v_i) = 1 \text{ if } K_i(Y, v_i) - c_{1i} \geq 0, \qquad (7a)$$

$$= 0 \text{ otherwise,} \qquad (7b)$$

where $K_i(Y, v_i) \equiv \alpha_1 - \alpha_2\, Y - \sqrt{c_{0i}}\,\dfrac{c_2 + \alpha_2(1+v_i)}{\sqrt{½\, c_2 + \alpha_2(1+v_i)}}$, with $\partial K_i/\partial Y < 0$ and $\partial K_i/\partial v_i < 0$ (= 0) when $c_{0i} > 0$ (= 0), $i \in M$. Note that $I_i(Y, v_i)$ in (7) is a step function. It is non-increasing in Y, and non-increasing in (independent of) v_i when $c_{0i} > 0$ (= 0). And it is discontinuous at points where a firm either enters or exits the industry. In this context, $\sum_{i\in M} I_i(Y, v_i)$ represents the number of active firms in the industry as an integer. We make the following assumption:

Assumption A1: $\sum_{i\in M} I_i(Y, 0) \geq 1$ for some $Y > 0$.

Given equation (9c), assumption A1 states that the market is large enough (e.g., α_1 is large enough) so that it can sustain at least one active firm. Often, we will be interested in situations where there are multiple firms. Then, equation (7) can be used to define "marginal firms." Let $i^+(Y, \mathbf{v}) \in \text{argmin}_i\ \{K_i(Y, v_i) - c_i\colon K_i(Y, v_i) - c_i \geq 0,\ i \in M\}$ and $i^-(Y, \mathbf{v}) \in \text{argmax}_i\ \{K_i(Y, v_i) - c_i\colon$

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[5] Equation (3) is equivalent to the second order sufficiency condition for the profit maximization problem $\text{Max}_{y_i}\ \{p(y_i + x_i(y_i))\, y_i - C_i(y_i)\colon y_i \geq 0\}$ discussed in footnote 3, when $\partial^2 x_i(y_i)/\partial y_i^2 = 0$. We will show below (in the proof of Proposition 3) that this condition holds in steady state equilibrium.

[6] Tanaka also considered the number of firms in the industry as endogenous. However, in contrast with Tanaka, we allow for cost heterogeneity among firms.

$K_i(Y, v_i) - c_i < 0, i \in M\}$, where the vector $\mathbf{v} = (v_1, \ldots, v_m)$ represents the conduct of all firms. Given Y and $\mathbf{v}$, $i^+(Y, \mathbf{v})$ defines the <u>marginal active firm</u>, i.e. the active firm (which may not be unique) that is the closest from exiting the industry. And $i^-(Y, \mathbf{v})$ defines the <u>marginal inactive firm</u>, i.e. the inactive firm (which again may not be unique) that is the closest from entering the industry.

Given Y and $\mathbf{v}$, from equation (6) and (7), aggregate production is

$$S(Y, \mathbf{v}) \equiv \sum_{i \in M} I_i(Y, v_i) \cdot \frac{\alpha_1 - \alpha_2 Y - c_{1i}}{c_2 + \alpha_2(1 + v_i)}.$$

The function S(Y, v) is illustrated in Figure 1. It is decreasing in Y (which is intuitive since price p declines as Y rises). It is continuous from the left in Y. However, it exhibits points of discontinuity in Y when a marginal active firm producing a positive output exits the industry as Y rises (or alternatively, when a marginal inactive firm enters the industry to produce a positive output as Y declines). In this context, defining market equilibrium is problematic. There are situations where there does not exist an aggregate consumption Y (with associated price $p = \alpha_1 - \alpha_2 Y$) which clears the market and satisfies $Y = S(Y, \mathbf{v})$. This is illustrated in Figure 1, where point A is the largest aggregate production that satisfies the feasibility condition $Y \leq S(Y, \mathbf{v})$. Under assumption A1, it corresponds to the point $Y^*(\mathbf{v}) \equiv \text{Max}_Y \{Y: Y \leq S(Y, \mathbf{v}); Y \in R_+\}$. But at that point, there is an excess supply: $Y < S(Y, \mathbf{v})$. This situation arises due the discontinuity of the function $S(Y, \mathbf{v})$ between point A and point C in Figure 1. This occurs when $\lim_{Z \downarrow Y^*(\mathbf{v})} S(Z, \mathbf{v}) < Y^*(\mathbf{v}) < S(Y^*(\mathbf{v}), \mathbf{v})$. To deal with this problem, consider the case where firm heterogeneity is such that there is a unique marginal active firm. We propose to allow this marginal active firm to become only partially active. This is done by modifying equation (7) as follows

$$I_i'(Y, \mathbf{v}) = 1 - \frac{S(Y, \mathbf{v}) - Y}{S(Y, \mathbf{v}) - \lim_{Z \downarrow Y} S(Z, \mathbf{v})}, \text{ if } i = i^+(Y, \mathbf{v}) \text{ and } \lim_{Z \downarrow Y} S(Z, \mathbf{v}) < Y < S(Y, \mathbf{v}), \quad (7a')$$

$$= I_i(Y, v_i) \text{ otherwise,} \quad (7b')$$

$i \in M$. Equations (7) and (7') differ only for the marginal active firm $i^+(Y, \mathbf{v})$ as given in (7a'). The difference is relevant only if two conditions hold: 1) $\lim_{Z \downarrow Y} S(Z, \mathbf{v}) < S(Y, \mathbf{v})$, i.e. Y is a point of discontinuity of $S(Y, \cdot)$; and 2) $\lim_{Z \downarrow Y} S(Z, \mathbf{v}) < Y < S(Y, \mathbf{v})$, i.e. Y is located between $\lim_{Z \downarrow Y} S(Z, \mathbf{v})$ and $S(Y, \mathbf{v})$. Then, equation (7a') defines $I_{i^+}'(Y, \mathbf{v})$ as a real number reflecting the relative distance between $S(Z, \mathbf{v})$ and Y. It satisfies $I_{i^+}(Y, \mathbf{v}) \in (0, 1]$. In this context, $I_{i^+}(Y, \mathbf{v})$ in (7a') is the proportion of output the marginal active firm must produce to satisfy the market equilibrium condition $Y = S(Y, \mathbf{v})$. Indeed, firm $i^+(Y, \mathbf{v})$ now produces $[I_{i^+}'(Y, \mathbf{v}) \cdot \frac{\alpha_1 - \alpha_2 Y - c_{1i^+}}{c_2 + \alpha_2(1 + v_{i^+})}]$. If $I_{i^+}'(Y, \mathbf{v}) = 1$, then the marginal active firm $i^+(Y, \mathbf{v})$ produces its full output (as before). However, if I_{i^+}'(Y, **v**) < 1, then it produces only a fraction of its full output: I_{i^+}'(Y, **v**) · [S(Y, **v**) - Y]. This corresponds to the quantity AB in Figure 1. It is exactly the quantity required to breach the gap between S(Y^*(**v**), **v**) and Y^*(**v**). Define aggregate production under (7') as

$$S'(Y, \mathbf{v}) \equiv \sum_{i\in M} I_i'(Y, \mathbf{v}) \cdot \frac{\alpha_1 - \alpha_2 Y - c_{1i}}{c_2 + \alpha_2(1+v_i)}. \tag{8}$$

By allowing the marginal active firm i^+(Y, **v**) to be "only partially active," equation (7') implies that the market clearing condition Y = S'(Y, **v**) always holds at Y^*(**v**). We will rely on equation (7') and (8) through the rest of the paper. This amounts to assuming that the marginal firm i^+ can be active during only a fraction of the time period being analyzed, and thus producing only fraction of its "full time output." Then, the number of active firms in the industry is represented by the real number n where n(Y, **v**) ≡ $\sum_{i\in M} I_i'$(Y, **v**).

Since active firms have an incentive to produce, we must have $K_i(Y^*(\mathbf{v}), v_i) - c_{1i} \geq 0$ for all active firms. Under (7'), when $I_{i^+}'(Y^*(\mathbf{v}), \mathbf{v}) < 1$, then $K_{i^+}(Y^*(\mathbf{v}), v_{i^+}) - c_{1i^+} = 0$ as firm $i^+(Y^*(\mathbf{v}), \mathbf{v})$ is indifferent between producing and exiting the industry. In Figure 1, it means that $K_{i^+}(Y^*(\mathbf{v}), v_{i^+}) - c_{1i^+} = 0$ at any point between A and C. Thus, $K_{i^+}(Y^*(\mathbf{v}), v_{i^+}) - c_{1i^+} = 0$ holds when $I_{i^+}'(Y^*(\mathbf{v}), \mathbf{v}) \in (0, 1]$. In other words, under (7'), the production incentive condition, $K_i(Y^*(\mathbf{v}), v_i) - c_{1i} \geq 0$, continues to hold for all active firms, including the marginal active firm $i^+(Y^*(\mathbf{v}), \mathbf{v})$.

The above results are summarized next.

<u>Proposition 1</u>: Under assumption A1, for given firms' conduct **v**, a unique market equilibrium exists and satisfies

$$Y^*(\mathbf{v}) \equiv \{Y\colon Y = S'(Y, \mathbf{v}), Y \in R_+\}, \tag{9a}$$

where the market equilibrium price is

$$p^*(\mathbf{v}) = \alpha_1 - \alpha_2 Y^*(\mathbf{v}), \tag{9b}$$

and the market equilibrium number of active firms is

$$n^*(\mathbf{v}) = n(Y^*(\mathbf{v}), \mathbf{v}) \equiv \sum_{i\in M} I_i'(Y^*(\mathbf{v}), \mathbf{v}). \tag{9c}$$

With I_{i^+}(Y, **v**) being defined as a real number between 0 and 1 in (7a'), the marginal firm can enter or exit the industry "slowly" and the number n of active firms is a real number. The functions Y^*(**v**) in (9a) and n^*(**v**) in (9c) are each continuous and differentiable almost everywhere.[7] Thus, conditional on firms' conduct **v,** equations (9a)-(9c) provide a basis for investigating industry behavior under heterogeneous firms and allowing for firm entry/exit. By being conditional on **v**, this shows how the firms' conduct affects market equilibrium.

What are the implications of the firms' conduct **v** for aggregate welfare? Consider the case where aggregate welfare is measured by the total surplus: $W = \int_0^Y p(z)\, dz - \sum_{i\in M} C_i(y_i)$. Then, the welfare impact of a change in the conduct v_i of the i-th active firm is given by

$$\partial W/\partial v_i = \sum_{j\in M} [p(Y) - \partial C_j(y_j)/\partial y_j]\, (dy_j^*/dv_i),$$

$$= 0 \ \text{ if } v_j = -1 \text{ for all active firms,}$$

$$< 0 \ \text{ if } v_i > -1,\ dy_i^*/dv_i < 0, \text{ and } v_j \geq -1,\ dy_j^*/dv_i \leq 0 \text{ for all } j \neq i.$$

This gives the well known result that, to the extent that it contributes to a reduction in supply,[8] any increase in the exercise of market power (as reflected by a rise in v_i from -1) has adverse effects on aggregate welfare.

4. The determination of firms' conduct

The previous section has investigated how firms' conduct **v** affects market equilibrium. This section explores the reverse linkages: how industry structure affects firms' conduct. Note that such linkages are at the core of the traditional Structure-Conduct-Performance approach to industrial organization (e.g., Scherer). Below, we analyze how the number n of active firms in the industry influences the firms' ability to exercise market power (as represented by **v**). In other words, this section treats the set of active firms as given and studies how changing n affects **v**. (The issue of the joint determination of n and **v** will be addressed in section 5 below.)

Much research has investigated the determinants of oligopoly behavior (e.g., Dasgupta and Stiglitz; Dixit; Hahn; Kreps and Scheinkman; Seade; Shaked and Sutton; Tirole; Vickers). Such behavior becomes complex under entry/exit and firm heterogeneity. This section analyzes the determination of firms' conduct. We assume that the firms behave non-cooperatively, where each firm chooses its own conduct independently of others.[9] While each firm can choose

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[7] These functions are not differentiable at a finite number of points where entry/exit takes place. However, at these points, the directional derivatives still exist. Below, we will use derivatives of a function to mean directional derivatives when evaluated at points that are not differentiable.

[8] This issue is examined in Proposition 5 below.

[9] Assuming that each firm chooses its own conduct independently of others rules out collusion. In the case of collusion, note that our analysis could still apply by treating the colluding firms as if they were a single

alternative strategies in the short run, our focus on steady state equilibrium means that we want to investigate what happens to such strategies in the long run. Would each industry see the behavior of its firms converge to a particular conduct? Under some weak regularity conditions, we show that it does, as firms eventually "discover" what works better for each one. And in the process of analyzing the properties of long run equilibrium conduct, we obtain useful insights into the linkages between industry structure and firms' conduct.

In presenting our arguments, we will make use of the properties of "reactions functions" representing interactions among firms in the industry. Since this section treats the set of active firms as given, we start with some values of the indicator variables $\{I_j': j \in M\}$ representing a given industry structure. In a way consistent with (7'), let $I_j' = 0$ identifies an inactive firm, $I_j' = 1$ identifies a fully active firm, and $0 < I_j' < 1$ corresponds to a "partially active" marginal firm. Let $N \equiv \{j: I_j' > 0, j \in M\}$ denote the set of active firms, the number of active firms in the industry being $n \equiv \sum_{j \in M} I_j'$. To derive the firms' reaction functions, note from (6) and (7') that the production decision for the i-th firm is $y_i^* = I_i' \cdot \dfrac{\alpha_1 - \alpha_2 Y - c_{1i}}{c_2 + \alpha_2(1+v_i)}$, $i \in N$. Under market equilibrium where $Y = y_i + x_i$ (with $x_i \equiv \sum_{j \neq i} y_j$ denoting the production of all firms but the i-th one), it follows that $x_i = \sum_{j \neq i} \left(I_j' \cdot \dfrac{\alpha_1 - \alpha_2(y_i + x_i) - c_{1j}}{c_2 + \alpha_2(1+v_j)} \right)$. Solving for x_i gives

$$x_i = x_i^r(y_i, \mathbf{v}_N) \equiv \frac{\sum_{j \neq i} \left(I_j' \cdot \dfrac{\alpha_1 - \alpha_2 y_i - c_{1j}}{c_2 + \alpha_2(1+v_j)} \right)}{1 + \sum_{j \neq i} \left(I_j' \cdot \dfrac{\alpha_2}{c_2 + \alpha_2(1+v_j)} \right)}, \; i \in N, \tag{10}$$

where $\mathbf{v}_N = \{v_i: i \in N\}$ denotes the conduct of the active firms.[10] For a given industry structure N, equation (10) gives "reaction functions" to the decision of the i-th active firm, y_i, $i \in N$. It measures the aggregate production response of other firms, x_i, to changes in the output level y_i. In general, the reaction function $x_i^r(y_i, \mathbf{v}_N)$ depends on y_i and $\mathbf{v}_N$. The slope of the reaction function $\partial x_i^r(y_i, \mathbf{v}_N)/\partial y_i$ gives the marginal response of other firms to the i-th firm production, $i \in N$.[11] Equation (10) shows that $x_i^r(y_i, \mathbf{v}_N)$ is linear in y_i and independent of y_j, $j \neq i$. Thus, $\partial x_i^r(y_i,$

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firm. However, given our focus on long run equilibrium, this would require the collusion to be sustained over an extended period of time. On this issue see Friedman and Thisse, MacLeod et al., and Rothschild.

[10] In this section, we take the industry structure N as given. It follows that inactive firms stay inactive. This means that the relevant conducts v_i's involve only active firms.

[11] In this context, the literature has defined "consistent conjectures" as conjectures satisfying $v_i = \partial x_i^r(y_i, \mathbf{v}_N)/\partial y_i$, i.e. conjectures that are locally consistent with firms' interactions (e.g., Perry; Bresnahan; Dixit). Note that consistent conjectures have been criticized for lacking proper motivation from a game theory

$\mathbf{v}_N)/\partial y_i$ is independent of the production levels of all firms. It follows that, in our case, for a given industry structure N, $\partial x_i^r(y_i, \mathbf{v}_N)/\partial y_i$ is constant and provides a global characterization of the properties of reaction functions, $i \in N$.

We now analyze the determinants of conduct $\mathbf{v}_N$. We focus our attention on the long term evolution of firms' conduct in steady state equilibrium. The questions are: How does firms' conduct $\mathbf{v}_N$ change over time? And to what values might it converge in the long run? Denote the i-th firm profit by: $\pi_i(y_i, x_i) \equiv p(y_i + x_i)\, y_i - C_i(y_i) \equiv [\alpha_1 - \alpha_2 (y_i + x_i)]\, y_i - c_{0i} - c_{1i}\, y_i - ½\, c_2\, y_i^2$. When the i-th firm is active ($y_i > 0$) and using equations (6) and (7'), let $y_i^+(\mathbf{v}_N) \equiv I_j' \cdot \frac{\alpha_1 - \alpha_2 Y^+(v_N) - c_{1i}}{c_2 + \alpha_2(1+v_i)}$ denote the production decision of the i-th firm, where $Y^+(\mathbf{v}_N) = \mathrm{Max}_Y \{Y\colon Y \le \sum_{i\in M} I_j' \cdot \frac{\alpha_1 - \alpha_2 Y^+(v_N) - c_{1i}}{c_2 + \alpha_2(1+v_i)},\ Y \in R_+\}$ is aggregate quantity.[12] Denote by $V = \{v_i\colon v_i \ge -1, i \in N\}$ the feasible set for the conduct of active firms. We make the following assumption:

Assumption A2: At time t, consider the firms' conduct $\mathbf{v}_N = (v_1, \ldots, v_n) \in V$. If there exist $\mathbf{v}_N' = (v_1', \ldots, v_n') \in V$ satisfying $v_i' \neq v_i$, $v_j' = v_j$ for $j \neq i$, and $\pi_i(y_i^+(\mathbf{v}_N'), x_i^r(y_i^+(\mathbf{v}_N'), \mathbf{v}_N')) > \pi_i(y_i^+(\mathbf{v}), x_i^r(y_i^+(\mathbf{v}), \mathbf{v}))$ for some active firm $i \in N$, then the i-th firm will change its conduct v_i at time t+1.

Assumption A2 states that each active firm will modify its conduct if its current conduct gives it a lower profit. This seems intuitively reasonable. It is quite general. Under non-cooperative behavior, it lets each active firm choose its own conduct. And it allows for complex interaction effects among firms. While it does not provide insights on short term conduct and its evolution over time, it does generate an important prediction: any conduct v_i that does not maximize the profit of the i-th active firm, $\pi_i(y_i^+(\mathbf{v}_N), x_i^r(y_i^+(\mathbf{v}_N), \mathbf{v}_N))$, is not sustainable in the long term. This gives the following result:

Proposition 2: Under assumption A2 and for a given set N of active firms, a long run stable steady state equilibrium must necessarily satisfy

$$v_i^* \in \mathrm{argmax}_{v_i} \{\pi_i(y_i^+(\mathbf{v}_N), x_i^r(y_i^+(\mathbf{v}_N), \mathbf{v}_N))\colon \mathbf{v}_N \in V\},\ i \in N. \qquad (11)$$

We show next that equation (11) has strong implications for the firms' conduct in steady state equilibrium. This is given in the following proposition. (See the proof in the Appendix).

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viewpoint (e.g., Lindh; Makowsky). We just want to stress that our analysis does not apply to short term strategic interactions among firms. Rather, it is developed in the context of long run equilibrium of an industry in stable steady state.

[12] Note that $Y^+(\mathbf{v})$ becomes identical to $Y^*(\mathbf{v})$ in (9a) when the (I_j')'s are consistent with equation (7'). This will occur in the analysis presented in section 5 below.

Proposition 3: For a given number set N of active firms, assume that A2 holds and that $\partial y_i^+/\partial v_i \neq 0$ for all active firms. Then:

a) A long run stable steady state equilibrium for conduct $\mathbf{v}_N$ exists and is unique.

b) The long run steady state equilibrium for $\mathbf{v}_N$ is given by $v_i^* = \frac{\partial x_i^r}{\partial y_i}(y_i, \mathbf{v}_N)$, $i \in N$.

c) The v_i^*'s are constant across firms and satisfy

$$v_i^* = v^*(n) = -\tfrac{1}{2}(n + c_2/\alpha_2) + \tfrac{1}{2}\sqrt{(n + c_2/\alpha_2)^2 - 4(n-1)}, \tag{12}$$

$i \in N$, with

$$\partial v^*/\partial n = -\tfrac{1}{2} + \frac{\tfrac{1}{2}(n + c_2/\alpha_2) - 1}{\sqrt{(n + c_2/\alpha_2)^2 - 4(n-1)}}. \tag{13}$$

Propositions 2 and 3 establish under general conditions the long run equilibrium of firms' conduct in steady state. This establishes a formal linkage between market structure (as represented by n) and firms' conduct (as represented by v^*).

Note that $v_i^* = \frac{\partial x_i^r}{\partial y_i}(y_i, \mathbf{v}_N)$ implies that the v_i^*'s are also the consistent conjectures discussed by Bresnahan, Perry, and Dixit (see footnotes 3 and 11). Thus, Propositions 2 and 3 show that the long run equilibrium leads to consistent conjectures among active firms. Dixon and Somma, and Müeller and Normann obtained similar results in the context of duopoly (where n = 2). Thus, Propositions 2 and 3 generalize their results to oligopoly situations, with an arbitrary number of firms, $n \geq 2$. It provides a possible economic rational for consistent conjectures. Indeed, Propositions 2 and 3 show that, if more profitable conjectures tend to become more common, identical consistent conjecture is the unique evolutionary stable strategy. As a result, under bounded rationality, identical consistent conjectures simply emerge from selection over time.

Equations (12) and (13) show analytically how the firms' conduct v^* varies with the structural parameters (c_2/α_2) and the number of active firms in the industry (n). The relationship between v^* and n is of particular interest as it makes firms' conduct depend on industry structure.

Proposition 4: Given (12)-(13) and $n \geq 1$,

a) the firms' conduct $v^*(n)$ satisfies $-1 \leq v^* \leq 0$, with

- $v^* = 0$ if $n = 1$,
- for a finite c_2/α_2, $v^* \to -1$ as $n \to \infty$,
- for a finite n, $v^* \to 0$ as $c_2/\alpha_2 \to \infty$;
- for $n \geq 2$, $v^* \to -1$ as $c_2/\alpha_2 \to 0$.

b) In addition, $\partial v^*/\partial n$ satisfies $-1 \leq \partial v^*/\partial n \leq 0$, with

- $\partial v^*/\partial n = -1/(1 + c_2/\alpha_2)$ if $n = 1$,
- for a finite c_2/α_2, $\partial v^*/\partial n \to 0$ as $n \to \infty$,
- for a finite n, $\partial v^*/\partial n \to 0$ as $c_2/\alpha_2 \to \infty$,
- $c_2/\alpha_2 \to 0$ implies $\partial v^*/\partial n \to -½ + ½\ (n - 2)/|n - 2|$ for $n \neq 2$,

$\to -1$ if $1 \leq n < 2$,

$\to -½$ if $n = 2$

$\to 0$ if $n > 2$.

Proposition 4 establishes the properties of conduct $v^*(n)$ as the number n of active firms changes. From a), v^* is in general non-positive and bounded between -1 and 0. It attains its lower bound ($v^* = -1$) when the number n of active firms is large. This corresponds to Bertrand competition, where firms anticipate no price response from changing supply. And the conduct $v^*(n)$ attains its upper bound ($v = 0$) under monopoly (with $n = 1$). In between these two extremes, increasing the number n of active firms tends to reduce v^* ($\partial v^*/\partial n \leq 0$ from b)). This is intuitive: the potential to exercise market power becomes stronger (weaker) when the number of active firms is smaller (larger).

In addition, from proposition 4a, the firms' conduct v^* tends to 0 when c_2/α_2 becomes large. This corresponds to cases where marginal cost is rising sharply (c_2 = large) and/or where demand is very price-responsive (with $|\partial Y/\partial p| = 1/\alpha_2$ = large). In such situations, as long as the number n of active firms is finite, Cournot pricing is (approximately) satisfied irrespective of industry structure.[13] Alternatively, given $n \geq 2$, firms' conduct $v^*(n)$ tends to -1 when c_2/α_2 becomes close to 0. This includes cases where marginal cost is constant ($c_2 = 0$, with large supply response),[14] and/or where demand exhibits little price responsiveness (with $|\partial Y/\partial p| = 1/\alpha_2$ = small). In such situations, as long as $n \geq 2$, Bertrand competition is (approximately) satisfied irrespective of the industry structure. Finally, from Proposition 4b, the marginal effect of n on v^* becomes small ($\partial v^*/\partial n \to 0$) when c_2/α_2 is close to zero and $n > 2$. And $\partial v^*/\partial n \to 0$ when c_2/α_2 becomes very large. It means that, when $n > 2$, changes in industry structure (i.e., changes in n) affect firms' conduct only in situations where c_2/α_2 takes on moderate values (i.e., neither too small nor too large). For example, assuming constant marginal cost (with $c_2 \to 0$) would basically remove the possibility for firms to exercise market power when $n > 2$ (Kamien and Schwartz). This stresses the importance of the cost structure in the study of oligopoly behavior.

A contestable market has been associated with free entry and exit, identical producers, and potential entrants exhibiting Bertrand pricing (Baumol et al.). Note that identical firms are obtained as a special case of our model when c_{0i} and c_{1i} are the same for all firms. Proposition 4 shows how firms' conduct can generate Bertrand competition. Bertrand pricing (with $v = -1$) can be obtained under at least two scenarios. First, from Proposition 4a, $v^* = -1$ if the number n of active firms is sufficiently large. Second, Bertrand competition ($v = -1$) is is generated under constant marginal cost (where $c_2 \to 0$) when $n \geq 2$. In either scenario, under entry and exit, a contestable market would arise in a steady state long run equilibrium. The first scenario (n = large) is the classical case of a competitive market. The second scenario arises under more general conditions: as long as marginal cost is constant, it applies under various industry structures exhibiting at least two active firms ($n \geq 2$). In contrast with Baumol et al. approach, it is worth emphasizing that our approach does not assume Bertrand pricing. Rather it shows how Bertrand strategies can arise from the evolution of firms' conduct in a long run steady state equilibrium.

5. Industry Behavior

In this section, we explore long run industry behavior with a focus on the joint determination of firms' conduct and firm entry/exit. Heterogeneity among firms is represented by the cost parameters (c_{0i}, c_{1i}) that have a non-degenerate distribution function $F(\cdot\,,\cdot)$. Below, we let c_0 be the mean fixed cost c_{0i} in the industry, and c_1 be the mean value of c_{1i}. In this context, we analyze long run industry behavior under three scenarios: 1) the case where the firms' conduct v is exogenous; 2) the long run equilibrium case when v is endogenous under long run equilibrium,

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[13] This is consistent with Kreps and Scheinkman's finding that capacity constraints generate Cournot pricing.

but in the absence of fixed cost; and 3) the general case of endogenous firms' conduct in the presence of fixed cost.

a. Case 1: The case of exogenous firms' conduct

First, we consider long run industry behavior when firms' conduct is treated as given. Using the results of Proposition 3, we focus our attention on the case where $v_i = v$ for all active firms. Then, conditional on v, the market equilibrium conditions are given by equation (9a) for aggregate quantity $Y^*(v)$, equation (9b) for price $p^*(v)$, and equation (9c) for the number of active firms $n^*(v)$.

From (9a), the aggregate quantity $Y^*(v)$ is given by the value Y that satisfies the market equilibrium condition: $Y = S'(Y, v)$. The properties of Y^* are presented next. They include the effects of changing conduct v, mean fixed cost c_0, mean variable cost c_1, as well as the demand shifter α_1. See the proof in the Appendix.

<u>Proposition 5</u>: The aggregate quantity $Y^*(v)$ in (9a) satisfies

a) $\partial Y^*/\partial v < 0$,

b) $\partial Y^*/\partial c_0 < 0$,

c) $\partial Y^*/\partial c_1 < 0$,

d) $\partial Y^*/\partial \alpha_1 \in (0, 1/\alpha_2)$.

Proposition 5 shows the factors influencing the market equilibrium aggregate quantity Y^* conditional on conduct v. Result a) shows that increasing v reduces industry supply. Interpreting a rise in v as an increase in market power gives the intuitive result that the exercise of market power implies a reduction in aggregate supply. Results b) and c) imply that increasing either fixed cost (c_0) or marginal cost (c_1) provides a disincentive to produce at the industry level. However, the sources of these adjustments differ: higher marginal cost reduces the supply from incumbent firms, while higher fixed cost stimulates exit by reducing the number of active firms (see equations (B7) and (B8) in the Appendix). Finally, result d) shows that an increase in demand (represented by a rise in α_1) tends to stimulate the market equilibrium aggregate quantity $Y^*(v)$, the marginal impact being bounded between 0 and $1/\alpha_2$.

The price equilibrium $p^*(v)$ is given in equation (9b). With $p^*(v) = \alpha_1 - \alpha_2 Y^*(\alpha)$ and using Proposition 5, we obtain the following results.

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[14] As shown by Perry, consistent conjectures generate Bertrand competition if marginal costs are constant.

Proposition 6: The market equilibrium price $p^*(v)$ in (9b) satisfies

a) $\partial p^*/\partial v > 0$,

b) $\partial p/\partial c_0 > 0$,

c) $\partial p^*/\partial c_1 > 0$,

d) $\partial p^*/\partial \alpha_1 \in (0, 1)$.

Proposition 6 shows the factors influencing the market equilibrium price p^* conditional on conduct v. Result a) shows that increasing v increases price. Intuitively, a rise in market power tends to increase price. Results b) and c) imply that increasing either fixed cost (c_0) or marginal cost (c_1) contributes to a higher price. Finally, result d) shows an increase in demand (represented by a rise in α_1) tends to increase price, although the marginal price increase is bounded between 0 and 1.

The market equilibrium number of active firms $n^*(v)$ is given in equation (9c). Equation (9c) states that $n^*(v) = n(Y^*(v), v)$, where $n(Y, v) = \sum_{i\in M} I_i'(Y, v)$. The properties of $n(Y, v)$ and $n^*(v)$ are presented next. See the proof in the Appendix.

Proposition 7: The number of active firms given by $n(Y, v)$ and $n^*(v)$ in (9c) satisfies

$$\partial n^*/\partial(v, c_0, c_1, \alpha_1) = \partial n/\partial(v, c_0, c_1, \alpha_1) + (\partial n/\partial Y)(\partial Y^*/\partial(v, c_0, c_1, \alpha_1)), \qquad (14)$$

where

a) $\partial n/\partial Y < 0$,

b) $\partial n/\partial v$ $= 0$ in the absence of fixed cost (where $c_{0i} = 0$ for all firms),

< 0 in the presence of fixed cost (where $c_{0i} > 0$ for all firms),

c) $\partial n/\partial c_0 < 0$,

d) $\partial n/\partial c_1 < 0$,

e) $\partial n/\partial \alpha_1 = -(\partial n/\partial Y)/\alpha_2 > 0$, and $\partial n^*/\partial \alpha_1 \in (0, -[\partial n/\partial Y]/\alpha_2)$.

By identifying the factors influencing the number of active firms, Proposition 7 provides useful information on the determinants of entry and exit in the industry (conditional on v). Result b) illustrates the importance of fixed cost. It shows that, for a given Y, increasing v adversely affects the number n of active firms, but only in the presence of fixed cost. Result b) and equation (14) mean that, in the absence of fixed cost, $\partial n^*/\partial v = (\partial n/\partial Y)(\partial Y^*/\partial v) > 0$ from a) and Proposition 5a: a higher v simulating increased market power reduces aggregate supply ($\partial Y^*/\partial v < 0$) and increases price, which in turn stimulates entry. In general, the net effect of cost on the number of active firms n^* is found to be ambiguous. Indeed, $\partial n^*/\partial(c_0, c_1) = \partial n/\partial(c_0, c_1) + (\partial n/\partial Y)(\partial Y^*/\partial(c_0, c_1))$ from (14). Then, the direct effect $\partial n/\partial(c_0, c_1)$ is negative from c) and d): for a given Y, increasing cost (either fixed or marginal) provides an incentive for firms to exit the industry. But the indirect effect is positive: $(\partial n/\partial Y)(\partial Y^*/\partial(c_0, c_1)) > 0$ from a) and Proposition 5b and 5c: a higher cost tends to decrease aggregate supply and increase price, which in turn provides an incentive for firms to enter. As a result the net effect of changing c_0 or c_1 on the number of active firms n^* can be either negative or positive depending upon whether the direct effect dominates or not. Finally, result e) shows that the net effect of expanding demand (as represented by the parameter α_1) on the number of firms $n^*(v)$ is unambiguously positive. And its marginal effect is bounded between 0 and $-[\partial n/\partial Y]/\alpha_2$.

b. Case 2: The case of endogenous firms' conduct without fixed cost

Case 1 has examined the properties of industry behavior holding the firms' conduct v constant. Under entry/exit, it has allowed the number of firms in the industry to adjust in response to changes in cost or demand. But holding v constant may appear unsatisfactory. As discussed in section 4, firms' conduct can change with the structure of the industry. As a result, we now extend our analysis to the case where conduct v is endogenous. Relying on Proposition 3, we will focus our attention on the long run equilibrium conduct, i.e., on the firms' conduct given by $v^*(n)$ in equation (12).

In case 2, we restrict our analysis to situations where there is no fixed cost. This provides an important simplification. From Proposition 7b, in the absence of fixed cost, the number of active firms n(Y, v) in (9c) no longer depends on v. Then, market equilibrium is given by equations (9a), (9c) and (12), where $Y = S'(Y, v^*(n(Y)))$. Define $S^*(Y) \equiv S'(Y, v^*(n(Y)))$. It follows that the market equilibrium aggregate quantity is the solution Y^e to the equation $Y = S^*(Y)$. Note that S'(Y, v) is decreasing in Y and v. In addition, $v^*(n)$ is non-increasing in n (from Propositions 4b) and n(Y) is decreasing in Y (from Proposition 7a). It follows that $S^*(Y)$ is decreasing in Y. Under assumption A1, this implies that $Y = S^*(Y)$ has a unique solution Y^e. Then, the market price is $p^e = \alpha_1 - \alpha_2 Y^e$, the equilibrium number of firms is $n^e = n(Y^e)$, and the long run equilibrium conduct is $v^e = v(n^e)$.

To illustrate the implications of our analysis, we compare the market equilibrium conditions between case 1 and case 2. They are: Y = S'(Y, v), where S'(Y, v) is the equilibrium aggregate supply in case 1, v being treated as exogenous; and $Y = S'(Y, v^*(n(Y))) \equiv S^*(Y)$ in case 2 (i.e., in the absence of fixed cost, and with $v^*(n)$ being given in (12)). The properties of v^* and n have been examined in Propositions 4 and 7, respectively. We have[15]

$$\partial S^*/\partial Y = \partial S'/\partial Y + (\partial S'/\partial v)(\partial v^*/\partial n)(\partial n/\partial Y), \tag{15}$$

where $\partial S'/\partial Y < 0$ from equation (B7a), $\partial S'/\partial v < 0$ from equation (B7b), $\partial v^*/\partial n \leq 0$ from Proposition 4b, and $\partial n/\partial Y < 0$ from Proposition 7a. This generates the following result.

Proposition 8: In the absence of fixed cost ($c_{0i} = 0$), the equilibrium aggregate supply functions $S(Y, v)$ and $S^*(Y)$ satisfy

$$\partial S^*/\partial Y \leq \partial S'/\partial Y < 0.$$

With Y being the aggregate quantity demanded, proposition 8 can be interpreted in terms of the responsiveness of aggregate supply to changing demand conditions. It implies that aggregate supply is more responsive (in absolute value) to changing demand conditions under case 2 than under case 1. It shows that allowing for adjustments in market structure (as represented by n(Y)) and in firms' conduct (as represented by $v^*(n)$ in equation (12)) tends to stimulate aggregate supply response. Alternatively, it indicates that neglecting the entry/exit process and the changing structure of an industry, along with the associated changes in firms' conduct, can result in underestimating the magnitude of supply adjustments to changing demand conditions. This stresses the importance of a proper understanding of the entry/exit process and its linkages with firms' conduct.

In addition, under case 2 (where $c_{0i} = 0$ in the absence of fixed cost), we have $\partial S'/\partial Y = -\dfrac{n}{c_2/\alpha_2 + (1+v)}$ from (B7a), $\partial S'/\partial v = -\dfrac{S'}{c_2/\alpha_2 + (1+v)}$ from (B7b), and $\partial n/\partial Y = -\alpha_2\, m\, f(0, \alpha_1 - \alpha_2 Y)$ from (B9a). Then, using (13), equation (15) becomes

$$\partial S^*/\partial Y = -\frac{n}{c_2/\alpha_2 + (1+v)}$$

$$+ f(0, \alpha_1 - \alpha_2 Y)\, \frac{\alpha_2 S' m}{c_2/\alpha_2 + (1+v)}\, \left[-½ + \frac{½(n + c_2/\alpha_2) - 1}{\sqrt{(n + c_2/\alpha_2)^2 - 4(n-1)}}\right]. \tag{16}$$

This shows how the number of active firms n relates to the responsiveness of aggregate supply $\partial S^*/\partial Y$. It illustrates the presence of interactions effects between industry structure (as

[15] Again, derivatives of a function should be interpreted as directional derivatives when evaluated at points that are not differentiable.

represented by n) and supply response. The first term in (16) indicates that an increase in n reduces $\partial S'/\partial Y$, and thus increases the responsiveness of aggregate supply. In addition, when $n > 2$ and c_2/α_2 becomes small, then v approaches -1 (from Proposition 4a) and the first term in (16) becomes large (in absolute value). In this case, any increase in n always stimulates the responsiveness of supply to changing demand conditions, $\partial S^*/\partial Y$. This result is summarized next.

<u>Proposition 9:</u> In the absence of fixed cost ($c_{0i} = 0$), when $n > 2$ and c_2/α_2 is small, then more competitive industry structures (corresponding to an increase in n) contribute to stimulating supply response.

Next, we consider the effects of the demand shifter α_1. In the absence of fixed cost, the market equilibrium is given by the value Y^e that solves $Y = S'(Y, v^*(n(Y))) \equiv S^*(Y)$. Applying the implicit function theorem yields

$$
\begin{aligned}
\partial Y^e/\partial \alpha_1 &= [1 - \partial S^*/\partial Y]^{-1} (\partial S^*/\partial \alpha_1), \\
&= [1 - \partial S^*/\partial Y]^{-1} (-\partial S^*/\partial Y)/\alpha_2 \in (0, 1/\alpha_2), \qquad (17)
\end{aligned}
$$

where $\partial S^*/\partial \alpha_1 = -(\partial S^*/\partial Y)/\alpha_2$. Equation (7) shows that the marginal impact of the demand shifter α_1 on the market equilibrium aggregate quantity Y^e, $\partial Y^e/\partial \alpha_1$, is positive and bounded between 0 and $(1/\alpha_2)$. With $p = \alpha_1 - \alpha_2 Y$, this implies $\partial p^e/\partial \alpha_1 = 1 - \alpha_2 (\partial Y^e/\partial \alpha_1) \in (0, 1)$. This is intuitive: any increase in demand (represented by a rise in α_1) tends to increase the equilibrium price p^e. From (16), equation (17) also shows how $\partial Y^e/\partial \alpha_1$ depends on the structure of the industry (through n) and on firms' conduct (through v). In the absence of fixed cost, and when $n \geq 2$ and c_2/α_2 is small, we have seen that $\partial S^*/\partial Y < 0$ tends to decrease with n (from proposition 9). In this case, it follows that $\partial Y^e/\partial \alpha_1$ in (17) increases with n. With $p^e = \alpha_1 - \alpha_2 Y^e$ where p^e is the equilibrium price, it follows that $\partial p^e/\partial \alpha_1 = 1 - \alpha_2\, \partial Y^e/\partial \alpha_1$ decreases with n. These results are summarized next.

<u>Proposition 10:</u> In the absence of fixed cost ($c_{0i} = 0$),

a) $0 < \partial p^e/\partial \alpha_1 \leq 1$,

b) when $n > 2$ and c_2/α_2 is small, then $\partial p^e/\partial \alpha_1$ tends to decrease with n.

In the absence of fixed cost, result a) shows that, the equilibrium price p^e increases with an exogenous rise in demand (represented by an increase in α_1). However, the supply response is such that the induced price increase tends to be less than the original shift in demand. Result b) shows that, if in addition $n > 2$ and c_2/α_2 is small, then the marginal price effect $\partial p^e/\partial \alpha_1$

decreases as the number n of active firms rises. Alternatively, as n declines, this price responsiveness would increase. This shows that the responsiveness of price adjustments to exogenous shocks is inversely related to the number of active firms in the market. Thus, a decreasing market concentration would contribute to reducing the price effect of a change in α_1. Alternatively, thin or concentrated markets (where n is low) would be characterized by greater price sensitivity to market changes. This result is summarized next.

Proposition 11: In the absence of fixed cost ($c_{0i} = 0$), and when $n > 2$ and c_2/α_2 is small, ceteris paribus, increasing (decreasing) market concentration tends to be associated with a higher (lower) price sensitivity to exogenous shocks.

c. Case 3: The case of endogenous firms' conduct under fixed cost

What happens if we introduce fixed costs in case 2? In the presence of fixed costs, the number of active firms n(Y, v) depends in general on the firms' conduct v (from Proposition 7b). In this case, the determination of the equilibrium number of active firms becomes more complex. This illustrates the presence of important interactions between fixed costs, market structure, and industry behavior. To see that, given $Y^*(v)$ in (9a), n(Y, v) in (9c) and $v^*(n)$ in (12), let $g(n) \equiv n(Y^*(v^*(n)), v^*(n))$. Then, the market equilibrium solution for the number of active firms n^e must satisfy $n^e = g(n^e)$. Under assumption A1, a sufficient condition for the equation $n = g(n)$ to have a unique solution for n is that $\partial g/\partial n \equiv [(\partial n/\partial Y)(\partial Y^*/\partial v) + \partial n/\partial v](\partial v^*/\partial n) < 1$. We know from Proposition 4b that $\partial v^*/\partial n \in [-1, 0]$. Thus, the condition $\partial g/\partial n < 1$ is always satisfied if $\partial v^*/\partial n = 0$. This corresponds to "case 1" above. In addition, given $n^*(v) = n(Y^*(n), v)$, the condition $\partial g/\partial n < 1$ is satisfied if $\partial v^*/\partial n \in [-1, 0)$ and $\partial n^*/\partial v \equiv (\partial n/\partial Y)(\partial Y^*/\partial v) + \partial n/\partial v > -1$. Thus, in the presence of fixed costs, a sufficient condition to have a unique solution for the equilibrium number of firms n^e is that $\partial n^*/\partial v > -1$, i.e. that $\partial n/\partial v > -1 - (\partial n/\partial Y)(\partial Y^*/\partial v)$.[16] Given $\partial n/\partial v \leq 0$ (from proposition 7b), note that this restricts the marginal effect $\partial n/\partial v$ from being "too negative" under fixed costs.

Applying the implicit function theorem to $n = g(n)$, we obtain the following properties of market equilibrium number of firms n^e with respect to mean fixed cost c_0, mean variable cost c_1, and demand shifter α_1.

Proposition 12: Assume that $\partial g/\partial n < 1$. In the presence of fixed costs, the market equilibrium number of firms n^e satisfies

$$\partial n^e/\partial(c_0, c_1, \alpha_1) = [1 - \partial g/\partial n]^{-1}\ \partial n^*/\partial(c_0, c_1, \alpha_1), \qquad (18)$$

[16] Note that this condition is always satisfied in the absence of fixed cost, i.e. under "case 2" above. Indeed, in the absence of fixed cost, we have $\partial n/\partial v = 0$ (from Proposition 7b), $\partial n/\partial Y < 0$ (from proposition 7a) and $\partial Y^*/\partial v < 0$ (from Proposition 5a).

where

a) $\partial n^e/\partial\alpha_1 = \text{sign}\{\partial n^*/\partial\alpha_1\} > 0$,

b) $\partial n^e/\partial(c_0, c_1) = \text{sign}\{\partial n^*/\partial(c_0, c_1)\}$.

Assuming $\partial g/\partial n < 1$, equation (18) implies that the sign of $\partial n^e/\partial(c_0, c_1, \alpha_1)$ is the same as the sign of $\partial n^*/\partial(c_0, c_1, \alpha_1)$. Noting that $\partial n^*/\partial\alpha_1 \in (0, -[\partial n/\partial Y]/\alpha_2)$ (from Proposition 7e), this generates "result a": $\partial n^e/\partial\alpha_1 > 0$. Thus, expanding demand (as represented by a rise in the parameter α_1) has always a positive effect on the equilibrium number of active firms n^e. Proposition 7 also showed that changing cost (c_0, c_1) has ambiguous effects on n^* (depending on whether the negative direct effects $\partial n/(c_0, c_1)$ dominate the positive indirect effects $(\partial n/\partial Y)(\partial Y^*/\partial(c_0, c_1)))$. Thus, from Proposition 12b, the effects of changing mean costs (c_0, c_1) on the equilibrium number of firms n^e are also ambiguous.

With n^e denoting the equilibrium number of firms, the equilibrium aggregate quantity is then given by $Y^e = Y^*(v^*(n^e))$. This generates the following properties of market equilibrium quantity Y^e with respect to mean fixed cost c_0, mean variable cost c_1, and demand shifter α_1.

<u>Proposition 13</u>: Assume that $\partial g/\partial n < 1$. In the presence of fixed costs, the market equilibrium aggregate quantity Y^e satisfies

$$\partial Y^e/\partial(c_0, c_1, \alpha_1) = \partial Y^*/\partial(c_0, c_1, \alpha_1) + (\partial Y^*/\partial v)(\partial v^*/\partial n)(\partial n^e/\partial(c_0, c_1, \alpha_1)), \qquad (19)$$

Equation (19) decomposes the effects of (c_0, c_1, α_1) on equilibrium aggregate quantity Y^e into two effects: a direct effect, $\partial Y^*/\partial(c_0, c_1, \alpha_1)$; and an indirect effect, $(\partial Y^*/\partial v)(\partial v^*/\partial n)$ $(\partial n^e/\partial(c_0, c_1, \alpha_1))$, capturing the influence of entry/exit on industry structure and firms' conduct. First, consider the effects of expanding demand, as captured by a rise in α_1. Note that $\partial Y^*/\partial\alpha_1 > 0$ (from Proposition 5d) and $(\partial Y^*/\partial v)(\partial v^*/\partial n)(\partial n^e/\partial\alpha_1) \geq 0$ (from Propositions 5a, 4b and 12a). From equation (19), this gives the intuitive result that increasing demand always stimulates the equilibrium aggregate quantity: $\partial Y^e/\partial\alpha_1 > 0$. Equation (19) also implies that $\partial Y^e/\partial\alpha_1 \geq \partial Y^*/\partial\alpha_1$ in general, and that $\partial Y^e/\partial\alpha_1 > \partial Y^*/\partial\alpha_1$ if the indirect effect of α_1 is positive. This indicates that neglecting the role of entry/exit and firms' conduct tends to <u>underestimate</u> the effects of a demand shifter on aggregate quantity. In addition, with $p^e = \alpha_1 - \alpha_2 Y^e$, the equilibrium price p^e satisfies the following two properties: $\partial p^e/\partial\alpha_1 = 1 - \alpha_2\, \partial Y^e/\partial\alpha_1 < 1$; and $\partial p^e/\partial\alpha_1 \leq \partial p^*/\partial\alpha_1$ (where $p^* = \alpha_1 - \alpha_2 Y^*$). The first property means that, due to the supply response, the induced price increase tends to be less than the original shift in demand. The second property shows that neglecting the role of entry/exit and firms' conduct tends to <u>overestimate</u> the effects of a demand shifter on the equilibrium price p^e. This stresses the importance of properly accounting for changing industry structure in market analysis.

Next, consider the effects of changing mean costs (c_0, c_1). Again, equation (19) provides a decomposition of the effects of (c_0, c_1) on Y^e into direct and indirect effects. From proposition 5b and 5c, the direct effects are always negative: $\partial Y^*/\partial(c_0, c_1) < 0$. But, in the presence of fixed costs, the indirect effects cannot be signed. This follows from Proposition 12b, which found that the effects of (c_0, c_1) on n^e are ambiguous in sign. It means that, under fixed costs, it is not clear whether the marginal effects $\partial Y^e/\partial(c_0, c_1)$ are positive or negative. And with $p^e = \alpha_1 - \alpha_2 Y^e$, similar ambiguous results apply to the effects of mean costs (c_0, c_1) on equilibrium price p^e.

The above results point to analytical difficulties in evaluating the effects of changing cost structures on industry equilibrium under entry/exit and fixed costs. However, there are scenarios where a simple characterization of market equilibrium applies even in the presence of fixed costs. They involve the condition $\partial v^*/\partial n = 0$. Note that $\partial v^*/\partial n = 0$ means that v^* does not depend on n at least locally. This has an important implication: when $\partial v^*/\partial n = 0$, then the analysis developed in "case 1" above holds locally. It follows that under scenarios where $\partial v^*/\partial n = 0$, all our market equilibrium results obtained under "case 1" apply, with or without fixed costs. These scenarios are briefly discussed below.

In proposition 4, we have investigated the determinants of v^* and $\partial v^*/\partial n$ in long run equilibrium. First, from proposition 4, we have shown that $v^* \to 0$ (Cournot pricing) and $\partial v^*/\partial n \to 0$ when n is finite and $c_2/\alpha_2 \to \infty$. It follows that, under sharply increasing marginal cost ($c_2 \to \infty$) and/or a very elastic demand ($|\partial Y/\partial p| = 1/\alpha_2 \to \infty$), the effect of n on v^* also vanishes: $\partial v^*/\partial n \to 0$. Under such circumstances, the market equilibrium is obtained by solving $Y = S'(Y, 0)$ for Y^e, with $n^e = n^*(0) = n(Y^e, 0)$. This result applies with or without fixed cost.

Second, Bertrand pricing is obtained from Proposition 4 when n is large. Indeed, from Proposition 4, $n \to \infty$ implies that $v^* \to 1$ and $\partial v^*/\partial n \to 0$. This holds in the presence of fixed cost and for any finite c_2/α_2. This is the classical case where competitive behavior is obtained when the number of firms is sufficiently large. Under such circumstances, the market equilibrium quantity Y^e is obtained by solving $Y = S'(Y, -1)$ from (9a), and the market equilibrium number of firms is $n^e = n^*(-1) = n(Y^e, -1)$ from (9c).

Third, from Proposition 4, we have shown that v^* converges to Bertrand competition ($v \to -1$) and $\partial v^*/\partial n \to 0$ when $c_2/\alpha_2 \to 0$ and $n > 2$. It means that, when there are more than two active firms in the industry, and marginal cost is constant ($c_2 \to 0$) or demand is very inelastic ($|\partial Y/\partial p| = 1/\alpha_2 \to 0$), then the effect of n on v^* vanishes: $\partial v^*/\partial n \to 0$. Under such circumstances, market equilibrium is simple. Again, it is obtained by solving $Y = S'(Y, -1)$ from (9a), yielding Y^e as the market equilibrium aggregate quantity. The associated equilibrium number of firms is $n^e = n^*(-1) = n(Y^e, -1)$. This result applies with or without fixed costs. And with $n > 2$, it does not require the number of firms to be large. This scenario represents a situation where Bertrand competition arises even if the number of firms is relatively small. By identifying an alternative way of generating competitive behavior, it provides useful insights on how globalization (leading to an increase in n) can lead an industry to behave more competitively in the long run.

6. CONCLUDING REMARKS

We have investigated firm behavior, pricing, and market equilibrium in the long run. We considered the case of heterogeneous firms facing different costs (due to either different technology or different transaction cost). The analysis treats the number of active firms as endogenous, allowing industry concentration to depend on the underlying cost structure. In this context, we explored linkages between cost, industry structure (number of active firms), firms' conduct, and market equilibrium. We characterize the long run evaluation of firms' conduct and show that it converges toward consistent conjectures. This provides a basis for analyzing firms' conduct, establishing a formal linkage between industry structure and the exercise of market power. Our results show how different cost structures can support alternative market structures (going from monopoly, to oligopoly, to competition), and alternative firms' conduct (including monopoly pricing, Cournot pricing, and Bertrand competition). They indicate how price behavior can vary with the underlying cost structure of the industry. We show how changes in cost structures can affect entry, the exercise of market power, and the responsiveness of supply. This provides useful information on the economics of globalization. As a result of technological progress, reduced trade barriers, and the new information technology, markets have become more global. With global markets, the number of competing firms increases as markets become more integrated. Besides generating gains from trade, this affects firms' conduct and market behavior in the long run. Our analysis shows how globalization can help reduce the firms' exercise of market power, improve supply responsiveness, and reduce the price sensitivity to exogenous shocks.

While our long run analysis provides useful linkages between cost, industry structure, firms' conduct, pricing, and industry equilibrium, it also suggests some directions for future research. For example, we focused on a homogeneous product. There is a need to explore further the relationships between structure and conduct in the context of oligopoly under differentiated products. Finally, it is hoped that our analysis will help stimulate empirical research on both firm and industry behavior under changing cost and industry structures.

APPENDIX

<u>Proof of Proposition 3:</u> Under assumption A2 and using Proposition 2, the steady state conduct of the i-th active firm must satisfy (11). The associated first order necessary conditions for an interior solution are

$$[\partial\pi_i(y_i, x_i)/\partial y_i + (\partial\pi_i(y_i, x_i)/\partial x_i)(\partial x_i^r(y_i, \mathbf{v}_N)/\partial y_i)]\ (\partial y_i^+(\mathbf{v}_N)/\partial v_i)$$
$$+ (\partial\pi_i(y_i, x_i)/\partial x_i)(\partial x_i^r(y_i, \mathbf{v}_N)/\partial v_i) = 0, \qquad \text{(B1)}$$

$i \in N$. Note that $\partial x_i^r(y_i, \mathbf{v}_N)/\partial v_i = 0$ from (10). Equation (2) can be written as $\partial\pi_i(y_i, x_i)/\partial y_i = -v_i\ \partial\pi_i(y_i, x_i)/\partial x_i$, $i \in N$. Substituting into (B1) yields

$$F_i \equiv (\partial\pi_i(y_i, x_i)/\partial x_i)\ [-v_i + \partial x_i^r(y_i, \mathbf{v}_N)/\partial y_i]\ (\partial y_i^+(\mathbf{v}_N)/\partial v_i) = 0, \qquad \text{(B2)}$$

$i \in N$. Note that $\partial\pi_i(y_i, x_i)/\partial x_i = -\alpha_2\, y_i^* < 0$ when the i-th firm is active. If $(\partial y_i^+(\mathbf{v}_N)/\partial v_i) \neq 0$, then (B2) implies that the steady state conduct must satisfy: $v_i^* = \partial x_i^r(y_i, \mathbf{v}_N)/\partial y_i$, $i \in N$.

Using equation (10), we have $\partial x_i^r(y_i, \mathbf{v}_N)/\partial y_i = \dfrac{\sum_{j\neq i}\left(I_j' \cdot \dfrac{-\alpha_2}{c_2+\alpha_2(1+v_j)}\right)}{1+\sum_{j\neq i}\left(I_j' \cdot \dfrac{\alpha_2}{c_2+\alpha_2(1+v_j)}\right)}$. It follows that the solution for the v_i^*'s is interior, $i \in N$. And noting from (10) that $x_i^r(y_i, \mathbf{v}_N)$ is independent of v_i, the second order condition for a maximum in (11) is satisfied: $\partial F_i/\partial v_i < 0$. The first order condition (B2) can then be written as

$$v_i^* = f_i(\mathbf{v}_N, n) \equiv \frac{\sum_{j\neq i}\left(I_j' \cdot \dfrac{-\alpha_2}{c_2+\alpha_2(1+v_j)}\right)}{1+\sum_{j\neq i}\left(I_j' \cdot \dfrac{\alpha_2}{c_2+\alpha_2(1+v_j)}\right)},\ i \in N, \qquad \text{(B3)}$$

This is a system of equations with $\mathbf{v}_N = \{v_i: i \in N\}$ as unknowns. Differentiating the right-hand side of (B3) with respect to v_k yields, for $k \in N$, $k \neq i$,

$$\frac{\partial f_i(\mathbf{v}_N, n)}{\partial v_k} = \left(\frac{\alpha_2}{c_2+\alpha_2(1+v_k)}\right)^2 \left(\frac{1}{1+\sum_{j\neq i}\left(I_i' \cdot \dfrac{\alpha_2}{c_2+\alpha_2(1+v_j)}\right)}\right)^2.$$

Noting that $\sum_{k\neq i}\left(I_k' \cdot \frac{\alpha_2}{c_2+\alpha_2(1+v_k)}\right)^2 < \left(1+\sum_{j\neq i}\left(I_j' \cdot \frac{\alpha_2}{c_2+\alpha_2(1+v_j)}\right)\right)^2$ and that $\frac{\partial f_i(\mathbf{v}_N, n)}{\partial v_i} = 0$, it follows that

$$\sum_{k\in N}\left(\frac{\partial f_i(\mathbf{v}_N, n)}{\partial v_k}\right) < 1,\ i \in N. \qquad \text{(B4)}$$

Condition (B4) implies that the system of equations (B3) is a contraction mapping in $\mathbf{v}_N$ (Ortegas, p. 154). From the contraction mapping theorem, it follows that equation (B3) has a unique solution $\mathbf{v}_N^*$. To find the solution, consider the case where the v_i's are constant across active firms: $v_i = v$ for $i \in N$. Then (B3) becomes $v = -(n-1)/(v + n + c_2/\alpha_2)$. This generates the quadratic equation $v_2 + (n + c_2/\alpha_2)\, v + (n - 1)$, which has for solutions

$$v^* = -\tfrac{1}{2}(n + c_2/\alpha_2) \pm \tfrac{1}{2}\sqrt{(n + c_2/\alpha_2)^2 - 4(n-1)}\,.$$

But only the positive root satisfies $v^* \in [-1, 0]$. Thus, the (unique) long run equilibrium conduct is given by equation (12).

<u>Proof of Proposition 5</u>: The joint distribution function for (c_{0i}, c_{1i}) across all m firms is $F(\cdot\,, \cdot)$, where $F(a, b) = \int_{c_{0i}\leq a}\int_{c_{1i}\leq b} f(c_{0i}, c_{1i})\, dc_{1i}\, dc_{0i}$, $f(c_{0i}, c_{1i})$ being the joint probability function of (c_{0i}, c_{1i}) for all firms. Given $v_i = v$ for active firms, the aggregate production $S'(Y, v) \equiv \sum_{i\in M} y_i^*(Y, v)$ in (8) is

$$S'(Y, v) = \frac{m}{c_2+\alpha_2(1+v)} \int_{c_{0i}}\int_{c_{1i}\leq K_i} [\alpha_1 - \alpha_2 Y - c_{1i}]\, dF(c_{0i}, c_{1i}), \qquad \text{(B5)}$$

where $K_i(Y, v) = \alpha_1 - \alpha_2 Y - \sqrt{c_{0i}}\,\frac{c_2+\alpha_2(1+v)}{\sqrt{\tfrac{1}{2}c_2+\alpha_2(1+v)}}$, $i \in M$. The inequality $c_i \leq K_i(Y, v)$ guarantees non-negative profit and determines whether or not the i-th firm is active. In addition, from (9c), the number of active firms is

$$n(Y, v) = m \int_{c_{0i}}\int_{c_{1i}\leq K_i} dF(c_{0i}, c_{1i}), \qquad \text{(B6)}$$

where $n(Y, v) \leq m$. Using Leibniz's rule, differentiating S' in (B5) gives

$$\partial S'/\partial Y = -\alpha_2\left[\frac{n}{c_2+\alpha_2(1+v)} + \frac{m}{\sqrt{\tfrac{1}{2}c_2+\alpha_2(1+v)}}\int_{c_{0i}}\sqrt{c_{0i}}\, f(c_{0i}, K_i)\, dc_{0i}\right] < 0, \quad \text{(B7a)}$$

using (B6),

$$\partial S'/\partial v = -\frac{\alpha_2 S'}{c_2 + \alpha_2(1+v)} + m \int_{c_{0i}} \frac{\sqrt{c_{0i}}}{\sqrt{\tfrac{1}{2} c_2 + \alpha_2(1+v)}} \; f(c_{0i}, K_i)\,(\partial K_i/\partial v)\, dc_{0i},$$

$$= -\frac{\alpha_2 S'}{c_2 + \alpha_2(1+v)} - \frac{m\,\alpha_2^2(1+v)}{2\,[\tfrac{1}{2} c_2 + \alpha_2(1+v)]^2} \int_{c_{0i}} c_{0i}\, f(c_{0i}, K_i)\, dc_{0i} < 0, \quad \text{(B7b)}$$

since $\partial K_i/\partial v = -\sqrt{c_{0i}}\,\dfrac{\alpha_2^2(1+v)}{2\,[\tfrac{1}{2} c_2 + \alpha_2(1+v)]^{3/2}}$,

$$\partial S'/\partial c_0 = -\frac{\tfrac{1}{2} m\,[c_2 + \alpha_2(1+v)]}{\tfrac{1}{2} c_2 + \alpha_2(1+v)} \int_{c_{0i}} f(c_{0i}, K_i)\, dc_{0i}, < 0, \quad \text{(B7c)}$$

$$\partial S'/\partial c_1 = -\frac{n}{c_2 + \alpha_2(1+v)} < 0, \quad \text{(B7d)}$$

using (B6), and

$$\partial S'/\partial \alpha_1 = -(\partial S'/\partial Y)/\alpha_2 < 0. \quad \text{(B7e)}$$

Note that the right-hand side of equations (B7a) and (B7b) involve two additive terms. The first term is associated with production adjustments by incumbent firms. The second term is associated with the entry/exit process of marginal firms. This term vanishes in the absence of fixed cost (where $c_{0i} = 0$), illustrating the importance of fixed cost in the entry/exit process.

With Y^* solving $Y = S'(Y, v)$, applying the implicit function theorem and using (B7) yield

$$\partial Y^*/\partial(v, c_0, c_1) = [1 - \partial S'/\partial Y]^{-1}\, \partial S'/\partial(v, c_0, c_1) < 0, \quad \text{(B8a)}$$

$$\partial Y^*/\partial \alpha_1 = -[1 - \partial S'/\partial Y]^{-1}\, (\partial S'/\partial Y)/\alpha_2 \in (0, 1/\alpha_2). \quad \text{(B8b)}$$

<u>Proof of Proposition 7</u>: The number of active firms n(Y, v) is given in (B6). Using Leibniz's rule, differentiating n in (B6) gives

$$\partial n/\partial Y = m \int_{c_{0i}} -\alpha_2\, f(c_{0i}, K_i)\, dc_{0i} < 0, \quad \text{(B9a)}$$

$$\partial n/\partial v \;= m \int_{c_{0i}} f(c_{0i}, K_i)\,(\partial K_i/\partial v)\,dc_{0i},$$

$$= -\frac{m\,\alpha_2^2(1+v)}{2\,[\tfrac{1}{2}c_2 + \alpha_2(1+v)]^{3/2}} \int_{c_{0i}} \sqrt{c_{0i}}\;\; f(c_{0i}, K_i)\,dc_{0i} \le 0, \qquad \text{(B9b)}$$

$$\partial n/\partial c_0 \;= m \int_{c_{0i}} f(c_{0i}, K_i)\,(\partial K_i/\partial c_0)\,dc_{0i},$$

$$= -\frac{\tfrac{1}{2}\,m\,[c_2 + \alpha_2(1+v)]}{\sqrt{\tfrac{1}{2}c_2 + \alpha_2(1+v)}} \int_{c_{0i}} f(c_{0i}, K_i)\,\frac{1}{\sqrt{c_{0i}}}\,dc_{0i} < 0, \qquad \text{(B9c)}$$

since $\partial K_i/\partial c_0 = -\dfrac{\tfrac{1}{2}}{\sqrt{c_{0i}}}\dfrac{c_2 + \alpha_2(1+v)}{\sqrt{\tfrac{1}{2}c_2 + \alpha_2(1+v)}}$,

$$\partial n/\partial c_1 = m \int_{c_{0i}} -f(c_{0i}, K_i)\,dc_{0i} < 0, \qquad \text{(B9d)}$$

$$\partial n/\partial \alpha_1 = -(\partial n/\partial Y)/\alpha_2 > 0. \qquad \text{(B9e)}$$

With $n^*(v) = n(Y^*(v), v)$, it follows from (B9e) that $\partial n^*/\partial\alpha_1 = \partial n/\partial\alpha_1 + (\partial n/\partial Y)(\partial Y/\partial\alpha_1) = -(\partial n/\partial Y)[1/\alpha_2 + (\partial Y/\partial\alpha_1)] \in (0, -[\partial n/\partial Y]/\alpha_2)$ from Proposition 5d.

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Figure 1: Market Equilibrium