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DIVERSIFICATION AND SUSTAINABLE AGRICULTURAL PRODUCTION-
THE CASE OF SOIL EROSION

by

Renan U. Goetz*

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THE CASE OF SOIL EROSION

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Diversification and Sustainable Agricultural Production

- The Case of Soil Erosion -

Abstract

A dynamic economic model of soil erosion is presented where the choice of crops, each one associated with distinct erosion rates, allows the farmer to control soil losses. The results show that it is predominately optimal to approach the steady state equilibrium most rapidly by the cultivation of a single crop. At the steady state, however, a mix of crops is cultivated supporting the argument that diversification, as a prerequisite for sustainable agricultural production, is a necessary condition for long run profit maximizing behavior of the farmer. Myopic behavior of the farmer favors the cultivation of a single crop throughout the planning horizon irrespectively whether the farmer owns or leases the land. Decretion of the soil stock may occur if the land market is not efficient, if the price of a high erosion crop increases or if the social discount rate raises at the beginning of the planning horizon. At the steady state, however, an increase in the price of a low erosion crop may decrease the soil stock if the initiated intensification of production has a stronger negative effect on the soil stock than does the positive effect of an increase in the share of this crop in the crop mix.

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1 Introduction

Over the last thirty years the intensification of agricultural production has caused a substantial increase in soil erosion on cropland. If the rate of soil erosion exceeds the rate of soil genesis, then the soil productivity is expected to decline. Empirical studies on water erosion, usually classified as sheet or rill erosion, (Papendick, Young, Mc Cool and Krauss, 1985), (Troeh, Hobbs and Donahue, 1991) confirmed these deliberations and showed that crop yields may decrease progressively with soil depth. Moreover, they estimated that the national annual soil loss due to water erosion is about 11.2 tons per hectare (t/ha) compared to the goal of 4.5t/ha (Bentley, 1985). Economic studies for the U.S. suggest that the decline of soil productivity given the current rate of water erosion would result in a loss of \$ 500 million to \$ 1 billion per year. (Colacicco, Osborn and Alt, 1989). The resulting concern stimulated studies modeling the optimal private and social agricultural utilization of the soil. In particular, it was recognized that soil management is a dynamic process that has to be adjusted continually to changes in the soil depth.

Past research, including that of McConnell (1983) and Burt (1981), focused on the analysis of the optimal private and social intertemporal path of soil use when the farmer grows a single crop. Miranowski (1984), as well as Smith and Shaykewich (1990), considered the more realistic case where the farmer grows a variety of different crops. Based on dynamic linear programming models they determined optimal cropping and tillage practices. However, the mathematical models employed did not allow for a nonlinear relation for crop yields and soil losses as a function of the soil depth. Yet, the different soil characteristics of the distinct soil layers commonly found in many cultivated types of soils suggest nonlinear relationships. Recent research by LaFrance (1992) and Clarke (1992) incorporated nonlinear relationships, but their analysis is confined to the case of a single crop. Both papers examine the situation where the farmer has the opportunity to reduce soil losses. LaFrance (1992) considers the case where the farmer employs soil conserving inputs which decrease crop yields. However, common practices for the reduction of soil losses such as the utilization of twin tyres for tillage operations, or not leaving the land fallow over the winter period, do not have a direct effect on crop yields. Clarke (1992) focuses on soil conserving investments which do not affect crop yields directly such as water way constructions or terracing. Yet, the availability of economically viable soil conserving investments often depend crucially on the cooperation of individual farmers due to the size of the investment. Thus, both papers address specific soil conserving strategies which are an option for some farmers. The fact that crop yield reduction and the soil loss rate vary considerably for different types of crops, suggests that the crop itself is an important choice variable with respect to the rate of soil loss. Hence, it is argued that the choice of the crop is a soil conservation strategy open for nearly all farmers facing the problem of soil erosion. This paper analyzes the optimal private and social intertemporal path

of soil use when the farmer's crop mix is determined endogenously while accounting for nonlinear relationships within the dynamic process of soil losses caused by sheet and rill erosion.

Furthermore the paper provides an additional argument for diversification versus non-diversification. Traditionally, diversification is supported in the presence of uncertainty or in the case of abundance or lack of quasi-fixed production factors such as labor or capital goods. Provided that a natural resource is essential for the production process, diversification may increase the long run net returns. Non diversification, on the other hand, may lead to an accelerated exhaustion or complete deterioration of the resource. The management of natural resources such as soil or pests are examples where diversified cropping may increase the long run net returns and can be a prerequisite for sustainable agricultural production.

This paper shows that the optimal cropping strategy is predominately given by the most rapid approach path to a steady state. The steady state equilibrium can be characterized by a saddle point and it can be attained from any given initial value of the soil stock. The results of a comparative dynamic analysis show that an increase in the crop prices may not affect the optimal path towards the equilibrium. However, if it alters the optimal path then an increase in the price of a low erosion crop, for example, leads to an increase of the steady state value of the soil stock. The analysis suggests further that the private discount rate has a negative impact on the steady state value of the soil stock. However, a high private discount rate will not lead to a total depletion of the soil. On the contrary, depletion may occur if the land market does not account appropriately for the soil depth. The results of a comparative static analysis based on particular specifications of the relationship between the production and erosion functions for the different crops indicate that an increase in the prices of a low and high erosion crop leads to a decrease of the steady state value of the soil stock.

2 Soil erosion and the allocation of land

2.1 The Dynamic Economic Model

Let $u(t)$ be an index of inputs and $z(t)$ be the overall soil depth at instant t . The per hectare production function is then given by:

$$f(z(t), u(t)). \quad (1)$$

To simplify notation the argument t of the variables z, u and of the variables $x, \lambda, \phi_1, \phi_2, \phi_3, \phi_4$ to be introduced later will be suppressed unless it is required for an unambiguous notation. All our assumptions about the production function can formally be summarized as follows:

$$\begin{aligned}
f_z &> 0, \quad f_u > 0, \quad f_{zz} < 0, \quad f_{uu} < 0 \\
f_{zz}f_{uu} - (f_{zu})^2 &> 0 \\
f_{zu} &> 0, \quad f(z, u) = f(z, 0) = 0 \\
f_z(z, u) &= 0, \quad \forall z \geq \bar{z},
\end{aligned} \tag{2}$$

where the subscript indicates the partial derivative with respect to the variable. The production function is jointly strongly concave in z and u , and twice continuously differentiable. The inputs are complementary, and considered essential for production where \bar{z} denotes the minimum soil depth necessary for agricultural production. Beyond a certain soil depth \bar{z} the crop yields will not increase any longer with the soil depth.

The concavity of the production function is a result of the non-homogeneity of the soil. A vertical soil profile displays different layers or horizons with distinct physical and chemical properties. They provide the key for understanding the vulnerability of agricultural production to soil erosion. The A-horizon lies near or at the surface and is characterized by maximum accumulation of organic matter and maximum leaching of clay materials. The underlying subsoil, the B-horizon, consists of weathered material with maximum accumulation of iron and aluminium oxides and silicate clays. Generally the B-horizon is less favorable to plant growth than the A-horizon because of its accumulation of clay, its high density and strength, its low pH value and its high aluminium saturation (Lal, Pierce and Dowdy, 1983). When tilled or ploughed, the A-horizon of the eroding soil will increasingly be mixed with the B horizon and will gradually show more and more properties of the B-horizon. Hence, the deterioration of the soil leads to lower crop yields as a result of the reduction of the effective field capacity, the maximal rooting depth, the infiltration and permeability rate, the humus content, the hydraulic conductivity and the availability of plant nutrients (National Soil Erosion-Soil Productivity Research Planning Committee, 1981). This deterioration of the soil properties, however, can only be partly offset by an increase in inputs.

The proposed specification of the production function is in line with the work of LaFrance (1992), Clarke (1992) and Walker and Young (1986). Unlike other studies, such as McConnell (1983) or Barrett (1991), soil loss is not being considered as an argument of the function since it is a result of the choice of u as well as the development of the soil depth over time, rather than a choice variable for the farmer.

The soil depth, however, does not only affect crop yields, it also influences the magnitude of soil losses for a given amount of rain. Therefore a twice continuously differentiable per hectare erosion function is introduced given by:

$$h(z, u). \tag{3}$$

We assume the following properties for the erosion function:

$$\begin{aligned}
h_z &< 0, \quad h_u > 0, \quad h_{zz} > 0, \quad h_{uu} > 0 \\
h_{zz}h_{uu} - (h_{zu})^2 &> 0 \\
h_{zu} &< 0, \quad h(z, 0) = 0 \\
h_z(z, u) &= 0, \quad \forall z \geq \bar{z}.
\end{aligned} \tag{4}$$

The erosion function is jointly strongly convex in z and u . As the soil depth decreases the soil becomes finer textured and less friable. Furthermore, the low content of organic matter in the subsoil decreases the aggregate stability of the soil particles such that rain can destroy them more easily. Hence, reduced infiltration and permeability as well as the unstable structure of the soil aggregate results in increased runoff and erosion of the subsoil (Troeh et al., 1991), suggesting $h_z < 0$. An increase in the production intensity will result in higher erosion rates, implying $h_u > 0$. The structural changes of the soil due to water erosion help to determine the sign of h_{zz} . Rain drops disintegrate aggregates on the surface and produce a compact surface crust. Percolating rainwater dislocates suspended fine soil particles from the top soil to the subsoil. These changes amplify the magnitude of soil erosion as the soil layers decrease (Troeh et al., 1991) suggesting $h_{zz} > 0$. An augmentation of the production intensity leads to higher erosion rates. However, a simultaneous accretion of the soil adversely affects this increase which implies $h_{uz} < 0$. In case $u = 0$ no agricultural production takes place and the soil is naturally covered by plants. Finally, an accretion of the soil above the soil depth \bar{z} does not alter the magnitude of the soil loss.

Instead of an erosion function, Clarke (1992) proposed a particular proportion of the production function to capture soil losses. This specification, however, implies that the magnitude of soil losses is increasing with the soil depth, contradicting chemical and physical properties for most types of soils being cultivated. Another functional form for the erosion function was proposed by LaFrance (1992). Focusing on soil conserving inputs he assumed the erosion function to be independent of z . Yet, the decrement of the soil depth alters the properties of the soil as reflected by decreasing crop yields. Hence, one would expect a change in the erosion function as z changes.

In our analysis we consider two different kind of crops. The share of the land cultivated by crop one, say pasture or small grains, is denoted by x . The remaining share of the land, $1 - x$, is utilized for the cultivation of crop two, such as row crops. We hereby assume that the farmer does not leave any land fallow and the entire cultivated land is equal to one by an appropriate normalization procedure. The production and erosion functions for crop one and two are denoted by a corresponding superscript. To analyze a meaningful problem we assume that $f^1 \neq f^2$ and $h^1 \neq h^2$.

The last function to be introduced is the twice continuously differentiable soil genesis function $e(z)$. Troeh, Hobbs and Donahue (1980) reported on studies showing that soil genesis is decreasing with the soil depth, implying $e_z < 0$. Additionally, we assume that $e_{zz} < 0$, and $e(\bar{z}) = 0$ for all $z \geq \bar{z} > \bar{z}$, where \bar{z} is the soil depth beyond which the soil

does not grow anymore. The dynamics of the soil can now be stated as

$$\dot{z} = -h^1(z, u_1)x - h^2(z, u_2)(1 - x) + e(z), \quad (5)$$

where the dot over z denotes $\frac{d}{dt}$ and u_i is an index of inputs applied to crop i for $i = 1, 2$. For any instant in time the “dynamics” can be illustrated by Figure 1. The two highlighted lines correspond to the steady state of the soil dynamics, $\dot{z} = 0$, where the upper line represents the example $x = 1$ and the lower one $x = 0$. Below the $x = 0$ line, long term monoculture of crop one or two is feasible and the soil stock is replenished. Between the $x = 0$ and $x = 1$ lines monoculture of crop one or a crop rotation of crop one and two is consistent with sustainable agricultural production. Beyond the $x = 1$ line farming would lead to a complete exhaustion of the soil stock finally ending in the “desertification” of the land. Figure 2 depicts the possible choices of z, u_1 and u_2 supporting sustainable agricultural production. Above this three-dimensional hyperplane, satisfying $\dot{z} = 0$, desertification would occur, whereas the space below characterizes monoculture. Thus, crop diversification may enable the farmer to increase the long run farm net returns while sustaining the soil stock. The farmer’s options for sustainable agricultural production via crop diversification are illustrated by a combination of the graphs in the $(u_1, 0, \bar{z} - z)$ and $(0, u_2, \bar{z} - z)$ space for $\dot{z} = 0$.

Our discussion of the dynamic processes of the soil so far was framed in the context of soil erosion, which is usually relevant for sloping lands. However, in the case of plains lands, soil quality is a pressing problem. For the functions discussed so far, as well as the model to be introduced later, the variable z can also be interpreted as an index of soil quality. Hence, the results of our analysis apply to the case of soil degradation as well, given an appropriate rephrasing.

2.2 The Maximum Principle

It is assumed that the farmer wants to maximize the discounted net returns received over the planning horizon. Thus the farmer’s decision problem, (P) can be formulated as:

$$\max_{x, u_1, u_2} \int_0^{t_1} e^{-\delta t} ((p_1 f^1(z, u_1) - c_1 u_1)x + (p_2 f^2(z, u_2) - c_2 u_2)(1 - x)) dt + e^{-\delta t_1} s(z(t_1)), \quad (P)$$

subject to

$$\begin{aligned} \dot{z} &= -h^1(z, u_1)x - h^2(z, u_2)(1 - x) + e(z) \\ z(0) &= z_0, \quad u_i \geq 0, \quad i = 1, 2 \quad x \in [0, 1], \end{aligned}$$

where t_1 denotes the final point of time under consideration, $\delta > 0$ is the private rate of discount, c_i , $i = 1, 2$, are the constant costs per unit of the index of inputs, $s(z(t_1))$ is the value of the land at time t_1 , and $p_1 > 0$ and $p_2 > 0$ are the constant prices of crops one and two respectively.

Using Pontryagin's Maximum Principle in current value form, the Hamiltonian \mathcal{H} is given by:

$$\begin{aligned}\mathcal{H} = & p_2 f^2(z, u_2) - c_2 u_2 - \lambda(h^2(z, u_2) - e(z)) \\ & + (p_1 f^1(z, u_1) - c_1 u_1 - p_2 f^2(z, u_2) + c_2 u_2 - \lambda(h^1(z, u_1) - h^2(z, u_2)))x. \quad (6)\end{aligned}$$

Taking account of the restrictions on the control variables leads to the Lagrangian \mathcal{L} which is maximized with respect to the control variables. It reads as:

$$\begin{aligned}\mathcal{L} = & p_2 f^2(z, u_2) - c_2 u_2 - \lambda(h^2(z, u_2) - e(z)) + (p_1 f^1(z, u_1) - c_1 u_1 - p_2 f^2(z, u_2) + c_2 u_2 \\ & - \lambda(h^1(z, u_1) - h^2(z, u_2)))x + \phi_1(1 - x) + \phi_2 x + \phi_3 u_1 + \phi_4 u_2. \quad (7)\end{aligned}$$

The optimal values of the control variables are associated with the costate variable $\lambda(t)$ and the Lagrange multipliers $\phi_i(t)$, $i = 1, 2, 3, 4$. A solution of problem (P) has to satisfy the following necessary conditions, which are stated in accordance with propositions 2.3 and 6.1 of Feichtinger and Hartl (1986),

$$\begin{aligned}\mathcal{L}_x = & p_1 f^1(z, u_1) - c_1 u_1 - p_2 f^2(z, u_2) + c_2 u_2 - \lambda(h^1(z, u_1) - h^2(z, u_2)) \\ & - \phi_1 + \phi_2 = 0 \quad (8)\end{aligned}$$

$$\mathcal{L}_{u_1} = (p_1 f^1_{u_1}(z, u_1) - c_1 - \lambda h^1_{u_1}(z, u_1))x + \phi_3 = 0 \quad (9)$$

$$\mathcal{L}_{u_2} = (p_2 f^2_{u_2}(z, u_2) - c_2 - \lambda h^2_{u_2}(z, u_2))(1 - x) + \phi_4 = 0 \quad (10)$$

$$\begin{aligned}\dot{\lambda} = & -p_1 f^1_z(z, u_1)x - p_2 f^2_z(z, u_2)(1 - x) + \lambda(\delta - e_z(z) \\ & + h^1_z(z, u_1)x + h^2_z(z, u_2)(1 - x)) \quad (11)\end{aligned}$$

$$\dot{z} = -h^1(z, u_1)x - h^2(z, u_2)(1 - x) + e(z), \quad z(0) = z_0. \quad (12)$$

Moreover, the optimal values of the control variables and the Lagrange multipliers have to satisfy the Kuhn-Tucker conditions

$$\mathcal{L}_{\phi_i} \geq 0, \quad \phi_i \geq 0 \quad \text{and} \quad \phi_i \mathcal{L}_{\phi_i} = 0 \quad \text{for } i = 1, 2, 3, 4. \quad (13)$$

The constraint qualification, a prerequisite for the Lagrangian approach, will be satisfied due to the linearity of the restrictions in x and u_i , $i = 1, 2$ (Takayama, 1985). The transversality condition stated by Feichtinger and Hartl (1986) takes the form

$$\lambda(t_1) = s_z(z(t_1)). \quad (14)$$

The costate variable λ can be interpreted as the current value shadow price of the soil depth. Therefore, the necessary condition (8), for $\phi_1 = \phi_2 = 0$, indicates that the allocation of the land between crops one and two is optimal when their returns minus their costs for eroding the soil equal each other. In the case of a boundary solution, their returns minus their cost should differ by the additional shadow price for the binding constraint. The second and third necessary condition (9) and (10), for the case where u_1 and u_2 are strictly greater than zero, states that the value of the marginal productivity

of the index of inputs should equal the sum of the marginal costs of the index of inputs and of the erosion of the soil. In the case where u_1 or u_2 is zero these conditions state that the value of the marginal productivity of the index of inputs and of the Lagrange multiplier equal the sum of the marginal costs mentioned above.

The structure of the problem does not allow one to determine unambiguously the sign of \mathcal{H}_{xz} . However, it turns out that the description of the qualitative properties of the solution only requires that this cross derivative does not vanish over some positive time interval. $\mathcal{H}_{xz} = 0$ is given by $p_1 f_z^1 - \lambda h_z^1 = p_2 f_z^2 - \lambda h_z^2$. Over some time horizon this seems to be a rather special case and it is not further pursued here.

The linearity of \mathcal{H} in x suggests defining a switching function, σ , for the determination of the optimal trajectory of x . This function is given by:

$$\mathcal{H}_x \equiv \sigma \equiv p_1 f^1(z, u_1) - c_1 u_1 - p_2 f^2(z, u_2) + c_2 u_2 - \lambda(h^1(z, u_1) - h^2(z, u_2)) \stackrel{\leq}{\geq} 0, \quad (15)$$

and proposes that

$$x = \begin{pmatrix} 1 & , & \sigma > 0 \\ x \in [0, 1] & , & \sigma = 0 \\ 0 & , & \sigma < 0 \end{pmatrix}. \quad (16)$$

The economic interpretation suggests choosing only crop one if its return minus the costs for eroding the soil exceeds that of crop two. The maximal choice of crop two ($x = 0$) is similarly explained. If the returns of the two crops minus their costs for eroding the soil are equal to each other, x remains undetermined within the interval $[0, 1]$.

It is known from the theory of optimal control that the singular path ($\sigma = 0$), if it exists, will be attained by the most rapid approach path which suggests the first proposition.

Proposition 1 *The steady state of the system of equations (11) and (12) subject to (8) - (10) is identical with a singular path. No other singular path exists if $\frac{\mathcal{H}_{xz}}{\mathcal{H}_{x\lambda}} = -\frac{\dot{\lambda}}{\dot{z}}$ does not hold for any positive interval of time.*

Proof:

To analyze the properties of a singular path we start out by assuming that a singular path exists where x lies in the interior of $[0, 1]$. Equation (16) provides the condition

$$\dot{\lambda} = \frac{(h^1 - h^2)(\frac{d}{dt}(p_1 f^1 - c_1 u_1 - p_2 f^2 + c_2 u_2)) - (\frac{d}{dt}(h^1 - h^2))(p_1 f^1 - c_1 u_1 - p_2 f^2 + c_2 u_2)}{(h^1 - h^2)^2}. \quad (17)$$

Taking the derivatives and evaluating z along its time path results in:

$$\begin{aligned} \dot{\lambda} = & \frac{\mathcal{H}_{xu_1} \dot{u}_1 + \mathcal{H}_{xu_2} \dot{u}_2}{h^1 - h^2} + \left[\frac{(h^1 - h^2)(p_1 f_z^1 - p_2 f_z^2)}{(h^1 - h^2)^2} \right. \\ & \left. - \left(\frac{(p_1 f^1 - c_1 u_1 - p_2 f^2 + c_2 u_2)(h_z^1 - h_z^2)}{(h^1 - h^2)^2} \right) \right] (- (h^1 - h^2)x - h^2 + e). \end{aligned} \quad (18)$$

Next, (18) will be equated with (11), and an expression for λ obtained from (16) is utilized. As a result, the x will cancel and equation (18) can be written as

$$\begin{aligned} \frac{\mathcal{H}_{xu_1}\dot{u}_1 + \mathcal{H}_{xu_2}\dot{u}_2}{h^1 - h^2} &= \frac{h^2 - e}{h^1 - h^2} \left[p_1 f_z^1 - p_2 f_z^2 - \left(\frac{p_1 f^1 - c_1 u_1 - p_2 f^2 + c_2 u_2}{(h^1 - h^2)} \right) (h_z^1 - h_z^2) \right] \\ &+ \left(\frac{p_1 f^1 - c_1 u_1 - p_2 f^2 + c_2 u_2}{(h^1 - h^2)} \right) (\delta - e_z + h_z^2) - p_2 f_z^2. \end{aligned} \quad (19)$$

Algebraic manipulation and utilizing (16) yields

$$\begin{aligned} \mathcal{H}_{xu_1}\dot{u}_1 + \mathcal{H}_{xu_2}\dot{u}_2 &= p_1 f_z^1 h^2 - p_2 f_z^2 h^1 + \lambda (h_z^1 h^2 - h_z^2 h^1) - \lambda (\delta - e_z) (h^1 - h^2) \\ &- e (p_1 f_z^1 - p_2 f_z^2 - \lambda (h_z^1 - h_z^2)). \end{aligned} \quad (20)$$

Next we utilize $\mathcal{H}_{xu_1} = \mathcal{H}_{xu_2} = 0$ and $\phi_3 = \phi_4 = 0$ as a result of (9), (10) and the fact that $x \in (0, 1)$ and obtain

$$0 = p_1 f_z^1 \frac{h^2 - e}{h^2 - h^1} + p_2 f_z^2 \frac{e - h^1}{h^2 - h^1} - \lambda \left(\delta + h_z^1 \frac{h^2 - e}{h^2 - h^1} + h_z^2 \frac{e - h^1}{h^2 - h^1} - e_z \right). \quad (21)$$

For any singular path equation (21) suggests that the value of a marginal increase in the soil depth represented by the first two terms equals the marginal costs. These costs are given by the private rate of discount and the marginal erosion minus the marginal soil genesis all valuated with the shadow price of the soil. Thus, the marginal value of delayed extraction of the soil does not only need to cover the marginal forgone profits by not using the soil but also the capital costs of these profits. To find conditions when the right side of (21) vanishes consider the steady state $\dot{z} = 0$. Thus, we obtain

$$x = \frac{h^2 - e}{h^2 - h^1}, \quad (1 - x) = \frac{e - h^1}{h^2 - h^1}, \quad (22)$$

and equation (21) reads as

$$p_1 f_z^1 x + p_2 f_z^2 (1 - x) = \lambda (\delta - e_z + h_z^1 x + h_z^2 (1 - x)). \quad (23)$$

Hence, the singular path coincides with the steady state. To analyze the qualitative properties of a singular path which is not identical with the steady state we assume again that such a singular path exists and therefore it has to hold that

$$\dot{\sigma} = \mathcal{H}_{xz}\dot{z} + \mathcal{H}_{xu_1}\dot{u}_1 + \mathcal{H}_{xu_2}\dot{u}_2 + \mathcal{H}_{x\lambda}\dot{\lambda} = \mathcal{H}_{xz}\dot{z} + \mathcal{H}_{x\lambda}\dot{\lambda} = 0 \quad (24)$$

For any singular path off the steady state $\dot{\lambda}$ and \dot{z} does not vanish. Hence, for a positive interval of time $\mathcal{H}_{xz} = \mathcal{H}_{x\lambda} = 0$ are sufficient conditions for the existence of singular path off the steady state. However, this contradicts our previous assumption, and it can therefore be concluded that a singular path is identical with a steady state provided that for any time interval of positive length the following condition, obtained from (24), does not hold:

$$\frac{(p_2 f_z^2 - \lambda h_z^2) - (p_1 f_z^1 - \lambda h_z^1)}{h^2 - h^1} = \frac{\dot{\lambda}}{\dot{z}} \quad (25)$$

However, if equation (25) holds for some time then the optimal trajectory of x will take on an interior value for this period which is equivalent to diversification of agricultural production. If this singular path is not a steady state one expects that equation (25) holds only for a finite amount of time¹. Thereafter, the next singular path is approached most rapidly. This process comes to an end when the singular path is given by a steady state. Hence, the optimal trajectory of x approaches the steady state most rapidly if not interrupted for an overall finite amount of time where it is characterized by an interior value. Moreover, the most rapid approach path indicates that the steady state is reached within finite time provided that the planning horizon is long enough.

□

After completing this proposition we are now able to illustrate the optimal trajectories for x, u_1, u_2 and z as shown in Figure 3. We illustrate the case where $h^1 < e < h^2$. Depending on the initial soil stock in relation to the closest steady state stock of the soil, it is either optimal to deplete or to build up the soil stock. For $\sigma > 0$ and $z_0 < z^*$, where the superscript $*$ denotes the steady state value of the variable², it is optimal to build up the soil with the cultivation of crop one in monoculture until z^* is attained. For $\sigma < 0$ and $z_0 > z^*$ it is optimal to deplete the soil by cultivating only crop two. The most rapid approach path may be interrupted for a finite amount of time when $\dot{\sigma}$ vanishes. During this time period the graph of $x(t)$ may be off the boundary and off the steady state equilibrium. In Figure 3 we depict the case where $\sigma > 0$ and $z_0 < z^*$. The sign for $\frac{\partial u_1}{\partial z} = \frac{-\mathcal{H}_{u_1 z}}{\mathcal{H}_{u_1 u_1}} > 0$ is obtained by employing the implicit function for equation (9). As a result u_1 is increasing over time. After a certain time x^*, u_1^*, u_2^* and z^* is reached. The steady state equilibrium is characterized by a mix of crop one and two. If for instance $h^1, h^2 > e$ then agricultural production would lead to the depletion of the soil and the abandoning of the land thereafter. In this case $z_0 > z^*$ and the crop with the highest returns minus the costs for eroding the soil will be grown in monoculture until the soil is depleted. Likewise, if $h^1, h^2 < e$ for $z < \tilde{z}$ and $h^i(\tilde{z}) = e(\tilde{z}) = 0, i = 1$ or 2 , the optimal cropping strategy results in the cultivation of the crop with the highest returns minus the costs for eroding the soil until the maximum stock of the soil is attained. From there on crop i , satisfying the steady state requirement $h^i(\tilde{z}) = e(\tilde{z}) = 0, i = 1$ or 2 , will be grown.

Comparative dynamics allows one to analyze the effect of initial changes in the parameters on the optimal trajectories of the control, state and costate variables. However, the variable x is not a continuous function of the state and costate variable and thus, a comparative dynamic analysis based of calculus is not possible. Yet, equation (16) provides some insight in the adjusted optimal trajectories of x, u_1, u_2 and z for initial changes of the crop prices and the private discount rate. In this paragraph we assume that no singular path besides the steady state exists and $h^2 > e > h^1$. Equation (16) indicates that a rise of p_2 , provided that σ switches its signs, leads to $x = u_1 = 0$

¹With respect to the case where equation (25) holds for an infinite amount of time our main conclusion that agricultural production will be diversified along a singular path remain valid. Hence, we do not consider this case in the subsequent part of the paper.

²In this context the superscript $*$ refers in particular to the closest steady state value of the soil stock in relation to the initial soil stock.

and to the most rapid approach of the nearest steady state value of z below the initial value of z . Likewise, an increase of p_1 , switching σ , suggests $x = 1$ and $u_2 = 0$ and increases the attained steady state value of z . For an increase in the discount rate one expects a decrease in the shadow price of the soil stock over time. Consequently, a switching σ , leads to a more intensive use of the resource and to a lower steady state value of z . If σ does not change its sign as a result of an increase in p_1, p_2 or δ then the optimal trajectory of x remains unchanged and the adjusted optimal trajectories u_1 or u_2 can be determined by comparative dynamic analysis based on a fixed x . In particular, the analysis showed that even in the presence of a high discount rate, a steady state is approached most rapidly and the soil is not completely eroded. This finding seems somehow contradictory to well known results of the economics of renewable resources like fish or forest. However, soil itself is an input for agriculture and as such it can only be capitalized by agricultural production. At the same time soil is an important characteristic of the tradable good land which can be sold at any point of time. Therefore, a high private discount rate reflecting the opportunity costs of holding land as an asset may favour the sale of land or the farm, but it does not lead to an exhaustion of the soil stock. It is commonly argued that a social discount rate reflecting the rate of time preferences between generations, should be lower than the private discount rate (Zilberman, Wetzstein and Marra, 1993). Thus, the social optimal cropping strategy would result in a higher steady state stock of the soil and there would be less incentives to sell land causing the share of land utilized for agricultural production to decline.

Comparative dynamic analysis was not included in the work by LaFrance (1992) and Clarke (1992) and as such it is not possible to compare their results with the one of this study. In section 2.3.3, however, the comparative static results obtained in this paper will be discussed in comparison with the results obtained by LaFrance (1992) and Clarke (1992), along with some policy implications.

Until now we have not addressed the question of the existence and uniqueness of a singular path. The identity of a singular path with a steady state, however, suggest to discuss this question in the following section 2.3.1 of the paper, where the existence of the steady state is analyzed.

2.3 Stability analysis

2.3.1 Existence of a steady state

So far we have shown that it is optimal to approach the steady state most rapidly. However, we do not know whether a steady state exists. The complexity as well as the non concavity of the model unfortunately does not allow to show the existence of the steady state directly through algebraic manipulations of the steady state equations which are given by:

$$p_1 f^1 - c_1 u_1^* - p_2 f^2 + c_2 u_2^* - \lambda^*(h^1 - h^2) = 0 \quad (26)$$

$$(p_1 f_{u_1}^1 - c_1 - \lambda^* h_{u_1}^1) x^* = 0 \quad (27)$$

$$(p_2 f_{u_2}^2 - c_2 - \lambda^* h_{u_2}^2)(1 - x^*) = 0 \quad (28)$$

$$-p_1 f_z^1 x^* - p_2 f_z^2 (1 - x^*) + \lambda^* (\delta - e_z + h_z^1 x^* + h_z^2 (1 - x^*)) = 0 \quad (29)$$

$$-h^1 x^* - h^2 (1 - x^*) + e = 0. \quad (30)$$

Therefore we invoke an existence theorem proposed by Filippov-Cesari, given in Theorem 8, page 132, Seierstad and Sydsæter (1987).

Proposition 2 *An optimal control exist if:*

$$\text{There exist an admissible pair } (z(t), u_1(t), u_2(t), x(t)) \quad (31)$$

$$\text{The set } N(z, U, t) = \{(g_0 e^{-\delta t} + \gamma, g_1) : \gamma \leq 0, x, u_1, u_2 \in U\}$$

$$\text{is convex for each } (z, t) \quad (32)$$

$$U \text{ is closed and bounded} \quad (33)$$

$$\text{There exist a number } b \text{ such that } |z(t)| \leq b$$

$$\text{for all } t \in [0, t_1] \text{ and all admissible pairs } (z(t), u_1(t), u_2(t), x(t)), \quad (34)$$

where $g_0 = p_1 f^1(z, u_1) - c_1 u_1 x + (p_2 f^2(z, u_2) - c_2 u_2)(1 - x)$; $g_1 = -h^1(z, u_1)x - h^2(z, u_2)(1 - x) + e(z)$, and $U = \{x, u_i : 0 \leq x \leq 1, 0 \leq u_i \leq \bar{u}_i, i = 1, 2\}$

Proof:

In appendix I it is shown that these conditions are met and the existence of an optimal solution can be concluded. \square

2.3.2 Stability analysis in the state-costate phase plane

The equations (8) - (10) do not allow one to solve for x, u_1, u_2 as functions of z, λ . Thus, the usual procedure by inserting the obtained functions $\hat{x}, \hat{u}_1, \hat{u}_2$ in the differential equations (11) and (12) for a qualitative analysis of the steady state does not work. However, equation (16) and proposition 1 show that x is a piecewise constant function and we can study the set of differential equations (11), (12) for constant x (Hartl, 1982).

Proposition 3 : *The equilibrium point of the system of equations (11) and (12) subject to (8) - (10) can be characterized for a constant x by a saddle point.*

Proof:

Only one element of the Jacobian matrix for the system of differential equations (11) and (12) evaluated at the equilibrium point $\dot{z} = \dot{\lambda} = 0$ can be signed. However, in our case it suffices to evaluate the trace of the Jacobian matrix. The elements of the trace are given by:

$$\frac{\partial \dot{z}}{\partial z} = \mathcal{H}_{\lambda z} + \mathcal{H}_{\lambda u_1} \frac{\partial \hat{u}_1}{\partial z} + \mathcal{H}_{\lambda u_2} \frac{\partial \hat{u}_2}{\partial z} \quad (35)$$

$$\frac{\partial \dot{\lambda}}{\partial \lambda} = \delta - \mathcal{H}_{z\lambda} - \mathcal{H}_{zu_1} \frac{\partial \hat{u}_1}{\partial \lambda} - \mathcal{H}_{zu_2} \frac{\partial \hat{u}_2}{\partial \lambda}. \quad (36)$$

The calculation of the trace shows that

$$\frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{\lambda}}{\partial \lambda} = \delta > 0. \quad (37)$$

where the terms $\frac{\partial \dot{u}_i}{\partial z}$ and $\frac{\partial \dot{u}_i}{\partial \lambda}$ were replaced by $\frac{-\mathcal{H}_{zu_i}}{\mathcal{H}_{u_i u_i}}$ and $\frac{-\mathcal{H}_{\lambda u_i}}{\mathcal{H}_{u_i u_i}}$, $i = 1, 2$ respectively, obtained from (27) and (28) by the implicit function theorem. Next, employing the fact that the trace of the Jacobian matrix equals the sum of its eigenvalues assures that at least one eigenvalue is positive. Both eigenvalues positive implies that

$$\lim_{t \rightarrow \infty} z = \pm \infty. \quad (38)$$

However, the stock of the soil cannot grow infinitely large or small, and thus one eigenvalue has to be negative. \square

This result shows that the steady state is characterized locally by a saddle point and we can additionally conclude that the equilibrium point is unique. Moreover, the most rapid approach path towards the equilibrium guarantees that the equilibrium point can be attained from any given initial value of the soil stock within finite time.

2.3.3 Comparative statics of the steady state

Now we turn to a sensitivity analysis of the steady state with respect to changes in the parameters of the model. In the general formulation, however, an analysis of the comparative statics of the steady state does not yield unambiguous results. In order to drive refutable results, we maintain the general functional form of the production and erosion functions but we will specify the relationship between them. This specification is considered an approximation, but it captures the essence of the relationship of the production, erosion and soil genesis functions for the studied problem. The loss of generality, however, is compensated by a gain in unequivocalness of the comparative statics results.

In particular, we consider the case where $f^1(z, u_1) = f(z, u)$ and $f^2(z, u_2) = \gamma f(z, u)$, $\gamma > 0$. Not distinguishing between the inputs applied to crop one or crop two implies the interest in the overall level of production intensity, and less in crop specific production intensity levels. This can be motivated by the observation that farmers usually either cultivate crops intensively or extensively, but not both at the same time. The multiplicative relationship of the two production functions is supported by studies which demonstrate that the relationship is not constant as the soil erodes (Reid, 1985). We also take account of the fact that the erosion functions for crop one and two vary to the greatest extent as a result of their different capabilities to cover the ground during time periods where erosive precipitation are likely to occur³. This capability, however, is independent from z and u which suggests $h^1(z, u_1) = h(z, u)$ and $h^2(z, u_2) = h(z, u) + \eta$, where the constant term η reflects the lower ground cover capability of crop two, (Troeh et al., 1991), (Mosimann,

³We hereby refer to the cover management factor C of the Universal Loss Equation which reflects the ground cover capability and the rainfall-runoff erosivity factor for particular time periods of the year.

Crole-Rees, Neyroud, Thöni, Musy and Rohr, 1990). Furthermore we propose $p_1 < \gamma p_2$ which implies that the returns of crop two are higher than the one of crop one. In case $p_1 > \gamma p_2$ only crop one would be cultivated since it yields the higher returns and shows a lower erosion rate than crop two. Finally we propose $e(z) = e$ to be a constant. We are hereby following the concept of soil loss tolerance as promoted by the U.S. Soil Conservation Service. Depending on the type of soil, tolerable soil losses with respect to the soil productivity ranging from 4 - 11 t/ha and year are defined.

With these specifictions at hand (26) - (30) reads as:

$$(p_1 - \gamma p_2)f + \lambda^* \eta = 0 \quad (39)$$

$$((p_1 - \gamma p_2)x^* + \gamma p_2)f_u - c - \lambda^* h_u = 0 \quad (40)$$

$$- ((p_1 - \gamma p_2)x^* + \gamma p_2)f_z + \lambda^*(\delta + h_z) = 0. \quad (41)$$

$$e - h - \eta(1 - x^*) = 0 \quad (42)$$

Note that we can now utilize the maximum conditions (39) - (40) for an interior solution to solve for $(x, u) = (\hat{x}(z, \lambda), \hat{u}(z, \lambda))$. Applying the implicit function theorem to (39) and (40) yields the following two equations.

$$\begin{pmatrix} \mathcal{H}_{xz} & \mathcal{H}_{x\lambda} \\ \mathcal{H}_{uz} & \mathcal{H}_{u\lambda} \end{pmatrix} + M \begin{pmatrix} \frac{\partial x}{\partial z} & \frac{\partial x}{\partial \lambda} \\ \frac{\partial u}{\partial z} & \frac{\partial u}{\partial \lambda} \end{pmatrix} = 0, \quad M \equiv \begin{pmatrix} \mathcal{H}_{xx} & \mathcal{H}_{xu} \\ \mathcal{H}_{ux} & \mathcal{H}_{uu} \end{pmatrix} \quad (43)$$

Since $\Delta \equiv \det M \equiv -(\mathcal{H}_{ux})^2 < 0$ does not vanish (43) can be solved by using Cramer's rule, which yields.

$$\frac{\partial \hat{x}}{\partial z} = \frac{-1}{\Delta} (\mathcal{H}_{xz} \mathcal{H}_{uu} - \mathcal{H}_{uz} \mathcal{H}_{xu}) > 0 \quad (44)$$

$$\frac{\partial \hat{x}}{\partial \lambda} = \frac{-1}{\Delta} (\mathcal{H}_{x\lambda} \mathcal{H}_{uu} - \mathcal{H}_{u\lambda} \mathcal{H}_{xu}) < 0 \quad (45)$$

$$\frac{\partial \hat{u}}{\partial z} = \frac{1}{\Delta} \mathcal{H}_{ux} \mathcal{H}_{xz} < 0 \quad (46)$$

$$\frac{\partial \hat{u}}{\partial \lambda} = \frac{1}{\Delta} \mathcal{H}_{ux} \mathcal{H}_{x\lambda} > 0 \quad (47)$$

Using (44) - (47) the Jacobian matrix, \tilde{J} , of the system (11) and (12) with $x = x(z, \lambda)$ and $u = u(z, \lambda)$ can be calculated.

$$\frac{\partial \dot{z}}{\partial z} = \eta \frac{\partial \hat{x}}{\partial z} - h_z - h_u \frac{\partial \hat{u}}{\partial z} > 0 \quad (48)$$

$$\frac{\partial \dot{z}}{\partial \lambda} = \eta \frac{\partial \hat{x}}{\partial \lambda} - h_u \frac{\partial \hat{u}}{\partial \lambda} < 0 \quad (49)$$

$$\begin{aligned} \frac{\partial \dot{\lambda}}{\partial z} &= -(p_1 - \gamma p_2)f_z \frac{\partial \hat{x}}{\partial z} - ((p_1 - \gamma p_2)x + \gamma p_2)f_{zz} + \lambda h_{zz} \\ &\quad - (((p_1 - \gamma p_2)x + \gamma p_2)f_{zu} - \lambda h_{zu}) \frac{\partial \hat{u}}{\partial z} > 0 \end{aligned} \quad (50)$$

$$\frac{\partial \dot{\lambda}}{\partial \lambda} = (\delta + h_z) - (p_1 - \gamma p_2)f_z \frac{\partial \hat{x}}{\partial \lambda} - (((p_1 - \gamma p_2)x + \gamma p_2)f_{zu} - \lambda h_{zu}) \frac{\partial \hat{u}}{\partial \lambda} < 0 \quad (51)$$

where the sign of (51) can be concluded from the fact that

$$\det \tilde{J} = \frac{\partial \dot{z}}{\partial z} \frac{\partial \dot{\lambda}}{\partial \lambda} - \frac{\partial \dot{\lambda}}{\partial z} \frac{\partial \dot{z}}{\partial \lambda} < 0 \quad (52)$$

which in turn is a direct consequence of the saddle point property of the steady state as established in section 2.3.2.

The sign of the slopes of the two isoclines ($\dot{z} = 0$ and $\dot{\lambda} = 0$) can be specified by applying the implicit function theorem

$$\left. \frac{d\lambda}{dz} \right|_{\dot{z}=0} = - \frac{\frac{\partial \dot{z}}{\partial z}}{\frac{\partial \dot{z}}{\partial \lambda}} > 0 \quad (53)$$

$$\left. \frac{d\lambda}{dz} \right|_{\dot{\lambda}=0} = - \frac{\frac{\partial \dot{\lambda}}{\partial z}}{\frac{\partial \dot{\lambda}}{\partial \lambda}} > 0 \quad (54)$$

Utilizing (52) yields

$$\frac{-\frac{\partial \dot{z}}{\partial z}}{\frac{\partial \dot{z}}{\partial \lambda}} - \frac{-\frac{\partial \dot{\lambda}}{\partial z}}{\frac{\partial \dot{\lambda}}{\partial \lambda}} > 0 \quad (55)$$

Hence, the slope of the ($\dot{z} = 0$) isocline is steeper than the slope of the ($\dot{\lambda} = 0$) isocline. The corresponding phase plane is presented in Figure 4 indicating that the stable branch is upward sloping and the unstable branch is downward sloping. Hence an increase in z , for $z < z^*$, is accompanied by an increase in the shadow price of the resource. At first sight this result seems to be counterintuitive. However, an increase in the soil depth allows the reduction of the index of inputs making the soil more valuable. Besides the phase plane, Figure 4 also indicates the optimal values of $z(t_1)$ and $\lambda(t_1)$. The endpoint of the optimal trajectory is determined by the transversality condition (14). For simplicity the case where $s(z)$ is linear in z is depicted, for example, $s(z) = \alpha_1 + \alpha_2 z$, $\alpha_i > 0, i = 1, 2$. The optimal trajectory depends on the initial stock of the soil and on the length of the farmer's planning horizon. In the case where the steady state can be reached within the time interval $[0, t_1]$, the optimal trajectories of z and λ coincide with the steady state values of z and λ as long as possible. Equivalent to our previous discussion of the most rapid approach path, the steady state will be left at the latest point in time and the transversality condition will be approached most rapidly (Feichtinger and Hartl, 1986) via the unstable branch. If the time interval $[0, t_1]$ does not allow one to reach the equilibrium, the optimal trajectory follows the stable branch until it is necessary to cross the $\dot{z} = 0$ isocline where, for the case $h^1 < e < h^2$, the variable x changes from 0 to 1 or vice versa, in order to approach $z(t_1)$ and $\lambda(t_1)$. This may be particular the case for myopic farmers who grow just one crop for some time and then switch to the other crop for the rest of their planning horizon. As a result these farmers do not diversify their agricultural production. In the case that the farmer leases the land the function $s(z(t_1))$ does not exist and the transversality condition for problem (P) is given by:

$$\lambda(t_1) \geq 0 \quad z(t_1) - \hat{z}(t_1) \geq 0 \quad \lambda(t_1)[z(t_1) - \hat{z}(t_1)] = 0, \quad (56)$$

where we assume that a minimum soil depth $\hat{z}(t_1)$ is required by a lease contract. Farmers with a short term lease and/or a high initial soil depth may be unable to deplete the soil stock up to $\hat{z}(t_1)$, and equation (56) will be satisfied for $\lambda(t_1) = 0$. In this case the optimal cropping strategy is given by the permanent cultivation of the crop with the highest returns minus the costs for eroding the soil. However, if $\hat{z}(t_1)$ can be attained during the lease then the optimal cropping strategies are identical for farsighted farmers either owning or leasing the land up to the steady state equilibrium. Thereafter, the different transversality conditions are approached most rapidly. The consideration of the ownership shows that the myopic farmers as well as the short term tenant are likely to grow a crop in monoculture. In particular, not the ownership, but the planning horizon determines whether the agricultural production will be diversified or not. Moreover, the tenant will deplete the soil more than the owner if the specification of $\hat{z}(t_1)$ is below the market outcome due to equation (14).

The function $s(z(t))$ will be determined by the land market. If the land market does not account appropriately for the soil depth, farmers are encouraged to deplete their soil up to a certain extent while approaching the end of their planning horizon⁴. Even if the land market were efficient by taking into account the soil depth, farmers may find it difficult to determine the optimal private intertemporal utilization of the soil since technical progress covers the 'true' productivity impacts of soil losses. This information can be considered as a public good and as such it will be supplied below its optimum. Data, supplied by a particular state run agency, about the 'true' productivity impact would help to restore an efficient land market. However, the implementation of this measurement should be preceded by a cost-benefit analyses which proves its efficiency.

After determining all the signs of the entries of the Jacobian matrix we are now able to conduct a comparative static analysis. Applying again the implicit function theorem to the system (12) and (11) where $\dot{\lambda} = \dot{z} = 0$, and (x, u) is given by $(\hat{x}(z, \lambda), \hat{u}(z, \lambda))$. Thus, we obtain a system of two equations.

$$\begin{pmatrix} 0 & 0 & 0 & x^* - 1 & 0 \\ -f_z x^* & -\gamma f_z (1 - x^*) & \lambda^* & 0 & -p_2 f_z (1 - x^*) \end{pmatrix} + \tilde{J} \begin{pmatrix} \frac{\partial z^*}{\partial p_1} & \frac{\partial z^*}{\partial p_2} & \frac{\partial z^*}{\partial \delta} & \frac{\partial z^*}{\partial \eta} & \frac{\partial z^*}{\partial \gamma} \\ \frac{\partial \lambda^*}{\partial p_1} & \frac{\partial \lambda^*}{\partial p_2} & \frac{\partial \lambda^*}{\partial \delta} & \frac{\partial \lambda^*}{\partial \eta} & \frac{\partial \lambda^*}{\partial \gamma} \end{pmatrix} = 0 \quad (57)$$

Solving by Cramer's rule yields:

$$\frac{\partial z^*}{\partial p_1} = \frac{-1}{\det J} f_z x^* \frac{\partial \dot{z}}{\partial \lambda} < 0 \quad (58)$$

$$\frac{\partial z^*}{\partial p_2} = \frac{-1}{\det J} f_z \gamma (1 - x^*) \frac{\partial \dot{z}}{\partial \lambda} < 0 \quad (59)$$

$$\frac{\partial \lambda^*}{\partial p_1} = \frac{1}{\det J} f_z x^* \frac{\partial \dot{z}}{\partial z} < 0 \quad (60)$$

$$\frac{\partial \lambda^*}{\partial p_2} = \frac{1}{\det J} f_z \gamma (1 - x^*) \frac{\partial \dot{z}}{\partial z} < 0 \quad (61)$$

⁴Clarke (1992) and LaFrance (1992) consider the case of an infinite planning horizon. As such they assume that the land market is efficient and they do not pay attention to the possibility of soil depletion in case of an finite planning horizon

$$\frac{\partial z^*}{\partial \delta} = \frac{1}{\det J} \lambda^* \frac{\partial \dot{z}}{\partial \lambda} > 0 \quad (62)$$

$$\frac{\partial \lambda^*}{\partial \delta} = \frac{-1}{\det J} \lambda^* \frac{\partial \dot{z}}{\partial z} > 0 \quad (63)$$

$$\frac{\partial z^*}{\partial \eta} = \frac{-1}{\det J} (x^* - 1) \frac{\partial \dot{\lambda}}{\partial \lambda} > 0 \quad (64)$$

$$\frac{\partial \lambda^*}{\partial \eta} = \frac{-1}{\det J} (1 - x^*) \frac{\partial \dot{\lambda}}{\partial z} > 0 \quad (65)$$

$$\frac{\partial z^*}{\partial \gamma} = \frac{-1}{\det J} p_2 (1 - x^*) f_z \frac{\partial \dot{z}}{\partial \lambda} < 0 \quad (66)$$

$$\frac{\partial \lambda^*}{\partial \gamma} = \frac{1}{\det J} p_2 (1 - x^*) f_z \frac{\partial \dot{z}}{\partial z} < 0, \quad (67)$$

where $\frac{\partial \dot{z}}{\partial z}$, $\frac{\partial \dot{z}}{\partial \lambda}$, $\frac{\partial \dot{\lambda}}{\partial z}$ and $\frac{\partial \dot{\lambda}}{\partial \lambda}$ are given by (48) - (51).

Additionally, we are now able to analyze changes of the control variables, given by $x^* = \hat{x}(z^*, \lambda^*)$ and $u^* = \hat{u}(z^*, \lambda^*)$, resulting from variations of the parameters. However, appendix II, where we focus on the parameters p_1, p_2 and δ , shows that the signs of $\frac{\partial x^*}{\partial p_1}$, $\frac{\partial x^*}{\partial p_2}$, $\frac{\partial u^*}{\partial p_1}$, $\frac{\partial u^*}{\partial p_2}$, $\frac{\partial x^*}{\partial \delta}$ and $\frac{\partial u^*}{\partial \delta}$ remain undetermined. Thus, in contrast to the comparative dynamic analysis for the general model we do not obtain the signs of the changes in x^* and u^* as a result of variations in the parameters. Yet, we derive uniequivocal results for the signs of the changes in z^* and λ^* as a result of variations in the parameters. For the following interpretation of the results we assume that $\frac{\partial x^*}{\partial p_1} > 0$, $\frac{\partial x^*}{\partial p_2} < 0$, $\frac{\partial x^*}{\partial \delta} < 0$, $\frac{\partial x^*}{\partial \eta} > 0$, $\frac{\partial x^*}{\partial \gamma} < 0$ (allocation effect), while the signs of the corresponding partial derivatives of u^* with respect to the parameters (cultivation effect) remain undetermined. The consideration of the reversed case would not yield additional insight and thus, we refrain from it.

The steady state values of z and λ always change in the same direction when one of the parameters is varied. The soil depth as well as the shadow price decrease due to an increase in either p_1 or p_2 . For example, increasing the price of the more erosive crop (crop two) could lead to an increase in the share of crop two in the crop rotation which in turn decreases the steady state value of the soil stock. On the other hand, one would expect that a rise of p_1 leads to an increase in the steady state value of the soil stock. However, an increase in p_1 not only induces a rise of the share of crop one in the crop rotation but also an intensification of production. Yet, the effect of an increase in u on the steady state value of z outweighs the effect of an increase in x . Thus the overall effect of an increase in p_1 on z is negative. Similarly, if p_1 rises, the share of crop one in the crop rotation increases, which in turn devaluates the soil stock, attributing a negative sign to equation (60). An increase in p_2 increases the share of the more erosive crop in the crop rotation. In order to keep the value of z constant over time the production intensity has to be reduced. This effect on the value of the soil stock is stronger than the effect from the extension of the area cultivated with crop two. Overall, the steady state value of the shadow price decreases with an increase in p_2 .

An increase in the discount rate makes production today more valuable in relation to

future production, and an increase in crop two along with an increase in the production intensity is expected. Thus, the steady state value of the shadow price will rise for an increase in the discount rate as is indicated in equation (62). However, evaluating the effect of an increase in the discount rate on the steady state value of the soil stock requires keeping the shadow price constant over time, which entails a decrease in the production intensity. The latter effect on z^* surpasses the effect of an increase in crop two on z^* . Hence the steady state stock increases from a rise of the discount rate. A rise in the ground cover capability would lead to an increase in the less erosive crop, which implies an increase in the steady state value of the soil stock. Interpreting equation (65), one has to keep in mind that z has to remain constant over time. Thus, an increase in the ground cover capability would lead to an increase in crop one and of the production intensity. The latter effect on the shadow price of the soil stock, however, is stronger, attributing a positive sign to $\frac{\partial \lambda^*}{\partial \eta}$. Finally the arguments for the interpretation of equations (59) and (61) are invoked to attribute negative signs to $\frac{\partial z^*}{\partial \gamma}$ and $\frac{\partial \lambda^*}{\partial \gamma}$.

In essence, the discussion of the results of the comparative static analysis shows that the steady state value of the soil stock as well as the steady value of the shadow price is influenced by the sum of the cultivation and allocation effect. Depending on crop one or two, and the parameter variation in question, these effects point either in the same or in the opposite direction.

Clarke (1992) showed that the product price is positively related to the soil depth given that some economically viable investment exists for preserving the soil. The assumption of a perfectly divisible investment which does not impede agricultural production, however, does not correspond to the situation which is analyzed here or in the paper by LaFrance (1992). As such the results obtained by Clarke (1992) are difficult to compare with those presented here or by LaFrance (1992).

LaFrance (1992) obtained a negative relationship between the output price and the steady state value of the soil stock if the cultivation effect dominates the conservation effect. This paper supports the findings of LaFrance (1992), but in terms of the cultivation and allocation effects instead of the cultivation and conservation effects. However, we like to emphasize that in our model the two effects (cultivation and allocation effects) are not always working in the opposite directions. For example, a price decrease of an erosive crop leads unambiguously to an increase of the steady state value of the soil stock. In particular, this opens up the possibility of placing a tax on an erosive crop in order to enhance the soil stock. Yet the reversed proposal, to subsidize a low erosion crop, cannot be supported since the allocation and cultivation effects are working in opposite directions and their sum will lead to a decrease in the steady state stock.

The result of Clarke (1992), that a high discount rate makes it less likely to approach a steady state and may lead to a depletion of the soil, cannot be confirmed. LaFrance (1992) showed that an increase in the discount rate will always decrease the steady state value of the soil stock. While this result holds for the optimal path towards the steady state, it cannot be confirmed for the comparative static analysis of this study. As before, the sum of the allocation and cultivation effects is crucial.

3 Summary and conclusions

This paper analyzes the private and social optimal intertemporal utilization of the soil for agricultural production. It extends previous work by considering both crop yields and soil losses as a nonlinear function of the soil depth within the decision problem of the optimal allocation of land to a mix of crops. The choice of crops is considered the key element for controlling soil erosion. If farmers recognize the productivity impacts of soil loss and maximize their long run net returns, their optimal cropping strategy is predominately characterized by the most rapid approach path to a long term equilibrium by the cultivation of just one crop. A unique equilibrium exists and can be characterized by a saddle point. The steady state is reached within finite time and is depicted by the cultivation of a mix of crops. Thus, the maximization of the long run net returns in the presence of a renewable resource essential for production as well as the call for sustainable agricultural production both require diversification. Myopic farmers owing the land as well as farmers with a short term or not secure lease are likely not to reach the steady state equilibrium within their planning horizon. Their optimal cropping plan can be characterized by non diversification. They cultivate one crop for some time and the other crop for the rest of their planning horizon. Farsighted behavior of the farmer likely leads to diversification irrespectively of the question of the ownership. However, the amount of stock left at the end of the planning horizon is determined differently by the farmer owing the land or leasing the land.

A comparative static analysis of the steady state for a particular specification of the relationship between the production and erosion functions of crop one and two indicates that the change of the crop mix (allocation effect) and the production intensity (cultivation effect) as a result of variations in the parameters is not determined. However, their common effect, in contrast to their individual effect, on the steady state value of the soil stock and the shadow price can be signed unequivocally. For example, an increase in the price of any crop decreases the steady state value of the soil stock. This result seems to be contradictory given that the production and erosion functions are not identical for the different crops. However, an increase in the price of a low erosion crop may lead to a rise of the share of this crop within the crop mix as well as to an increase in the production intensity. Thus, the allocation and cultivation effect have an opposite impact on the steady state value of the soil stock. Their sum determines the result of the comparative static analysis with respect to the steady state value of the soil stock and the shadow price.

Without some knowledge of the relationship between the production and erosion functions of crop one and two, it is not possible to decide on theoretical grounds whether the allocation or cultivation effect is stronger with respect to the steady state stock of the soil. This fact indicates in particular that an increase in the price of a low erosion crop, via a subsidy on the basis of a government intervention to correct the market outcome of the optimal soil stock, may have an ambiguous effect on the steady state value of the soil stock. Yet, it is conjectured that a decrease in the price of a high erosion crop via a tax leads to an increase in the steady state value of the soil stock since the allocation and

cultivation effects are pointing in the same direction.

In contrast to the results of the comparative static analysis are those of the comparative dynamic analyses obtained for the initial model without further specification of the relationship between the production and erosion functions of crop one and two. For example, an increase in the price of a low erosion crop up to a certain point has no effect on the optimal cropping strategy. Beyond this point, however, the equilibrium is characterized by an attained higher steady state stock of the soil. Likewise, an increase in the private discount rate may not have any effect on the optimal cropping strategy. For a sufficiently strong decrease in the shadow price as a result of an increase in the private discount rate, however, the soil will be used more intensively and a lower steady state value of the resource is attained. Yet the most rapid approach path towards a steady state equilibrium within finite time guarantees, for any discount rate, that the resource will not be exhausted completely given an efficient land market. This finding, in contrast to the results of the economics of renewable resources, is explained by the fact that soil can be sold directly with the land. A high private discount rate reflecting the returns from capital or financial assets may lead to the sale of the land or farm, but not to the depletion of the soil stock. If the land values do not reflect the soil depth, farmers are discouraged from conserving their soil, and they will exhaust the stock most rapidly up to a certain point towards the end of their planning horizon.

Appendix

I)

(31): A pair (z, u_1, u_2) is admissible if it satisfies the restriction stated in problem (P). Consider the pair $(z_0, 0, 0)$. In this case the soil is not used for agricultural production and $\dot{z} > 0$ until the soil depth reaches \bar{z} and $\dot{z} = 0$ thereafter.

(32): We need to show that $N(z, U, t)$ is a convex set which requires that:

$$\lambda(g_0^1 e^{-\delta t} + \gamma^1, g_1^1) + (1 - \lambda)(g_0^2 e^{-\delta t} + \gamma^2, g_1^2) \in N(z, U, t), \quad (68)$$

where

$$\begin{aligned} g_0^1 &\equiv p_1 f^1(z, u_1^1) - c_1 u_1^1 x^1 + (p_2 f^2(z, u_2^1) - c_2 u_2^1)(1 - x^1) \\ g_0^2 &\equiv p_1 f^1(z, u_1^2) - c_1 u_1^2 x^2 + (p_2 f^2(z, u_2^2) - c_2 u_2^2)(1 - x^2) \\ g_1^1 &\equiv -h^1(z, u_1^1)x^1 - h^2(z, u_2^1)(1 - x^1) + e(z) \\ g_1^2 &\equiv -h^1(z, u_1^2)x^2 - h^2(z, u_2^2)(1 - x^2) + e(z) \end{aligned}$$

are elements of $N(z, U, t)$. Since any nonpositive γ is sufficient we can write

$$g_0 e^{-\delta t} + \gamma = x((p_1 f^1(z, u_1) - c_1 u_1)e^{-\delta t} + \gamma_1) + (1 - x)((p_2 f^2(z, u_2) - c_2 u_2)e^{-\delta t} + \gamma_2) \quad (69)$$

As a first step we analyze the following set

$$Y_i \equiv (p_i f^i(z, u_i) - c_i u_i)e^{-\delta t} + \gamma_i, \text{ for } \gamma_i \leq 0 \text{ and } u_i \in [0, \bar{u}_i], i = 1, 2 \quad (70)$$

Please note that we introduced an upper bound for the variables u_1 and u_2 . This bound is arbitrarily large but finite. It facilitates the proof of the existence but it has no impact at all on the economic results of this paper. For ease of notation we just consider the case of u and do not distinguish between u_1 and u_2 for the evaluation of the set $Y_i, i = 1, 2$. The result, however, is not affected by this simplification. Let

$$y^1 = (p f(z, u^1) - c u^1)e^{-\delta t} + \gamma^1 \quad \text{and} \quad y^2 = (p f(z, u^2) - c u^2)e^{-\delta t} + \gamma^2 \quad (71)$$

be elements of Y . Hence we need to show that

$$\lambda y^1 + (1 - \lambda)y^2 \in Y \quad (72)$$

Define

$$w = \lambda(p f(z, u^1) - c u^1)e^{-\delta t} + \gamma^1 + (1 - \lambda)(p f(z, u^2) - c u^2)e^{-\delta t} + \gamma^2 \quad (73)$$

Utilizing the concavity of f in u we obtain

$$\begin{aligned} w &\leq (p f(z, \lambda u^1 + (1 - \lambda)u^2) - \lambda c u^1 + (1 - \lambda)c u^2)e^{-\delta t} + \lambda \gamma^1 + (1 - \lambda)\gamma^2 \\ &= (p f(z, u^3) - c u^3)e^{-\delta t} + \gamma^3, \end{aligned} \quad (74)$$

where $u_3 \in U$. Now let

$$\gamma^3 = w - (pf(z, u^3) - cu^3)e^{-\delta t}, \quad (75)$$

which implies $\gamma^3 \leq \lambda\gamma^1 + (1 - \lambda)\gamma^2 \leq 0$. Thus, we obtain

$$\lambda y^1 + (1 - \lambda)y^2 = (pf(z, u^3) - cu^3)e^{-\delta t} + \gamma^3 \in Y, \quad (76)$$

which shows that Y and therefore Y_1 and Y_2 are convex sets. As a result we can write

$$g_0 e^{-\delta t} + \gamma = xY_1 + (1 - x)Y_2 \quad (77)$$

Next, we employ a result given by Sydsaeter and Hammond (1995), page 621, which states that: if S and T are convex set in \mathcal{R}^n then $S \times T$ is also convex. Hence, we can conclude that each summand on the right hand side of (77) is a convex set. To analyze the sum of these two summands we need the following lemma

Lemma 1 *If S and T are convex sets in \mathcal{R}^n then $S + T$ is also convex.*

Proof:

Let $s_1, s_2 \in S$ and $t_1, t_2 \in T$. With $\lambda \in [0, 1]$ we have $\lambda s_1 + (1 - \lambda)s_2 \in S$ and $\lambda t_1 + (1 - \lambda)t_2 \in T$. Hence, $\lambda(s_1 + t_1) + (1 - \lambda)(s_2 + t_2) \in S + T$, demonstrating that $S + T$ is convex. \square

Employing lemma 1 allows us to conclude that the first component, $g_0 e^{-\delta t} + \gamma$, of the set $N(z, U, t)$ is convex for each (z, t) . The convexity of the second component, g_1 , of this set can be verified in exactly the same way as the first component since $-h^i$ is also a concave function in $u_i, i = 1, 2$. Finally, we conclude that (32) is satisfied.

(33): This requirement holds by definition of the set U .

(34): The soil depth z lies in the interval $[\underline{z}, \bar{z}]$ for all admissible pairs (z, u_1, u_2) and thus for $b = \bar{z} - \underline{z}$ equation (34) is satisfied.

II)

Differentiating the functions $x^* = \hat{x}(z^*, \lambda^*)$ and $u^* = \hat{u}(z^*, \lambda^*)$ with respect to the parameters p_1, p_2 and δ yields:

$$\frac{\partial x^*}{\partial p_1} = \frac{\partial \hat{x}}{\partial z} \frac{\partial z^*}{\partial p_1} + \frac{\partial \hat{x}}{\partial \lambda} \frac{\partial \lambda^*}{\partial p_1} \lesseqgtr 0 \quad (78)$$

$$\frac{\partial x^*}{\partial p_2} = \frac{\partial \hat{x}}{\partial z} \frac{\partial z^*}{\partial p_2} + \frac{\partial \hat{x}}{\partial \lambda} \frac{\partial \lambda^*}{\partial p_2} \lesseqgtr 0 \quad (79)$$

$$\frac{\partial u^*}{\partial p_1} = \frac{\partial \hat{u}}{\partial z} \frac{\partial z^*}{\partial p_1} + \frac{\partial \hat{u}}{\partial \lambda} \frac{\partial \lambda^*}{\partial p_1} \lesseqgtr 0 \quad (80)$$

$$\frac{\partial u^*}{\partial p_2} = \frac{\partial \hat{u}}{\partial z} \frac{\partial z^*}{\partial p_2} + \frac{\partial \hat{u}}{\partial \lambda} \frac{\partial \lambda^*}{\partial p_2} \lesseqgtr 0 \quad (81)$$

$$\frac{\partial x^*}{\partial \delta} = \frac{\partial \hat{x}}{\partial z} \frac{\partial z^*}{\partial \delta} + \frac{\partial \hat{x}}{\partial \lambda} \frac{\partial \lambda^*}{\partial \delta} \lesseqgtr 0 \quad (82)$$

$$\frac{\partial u^*}{\partial \delta} = \frac{\partial \hat{u}}{\partial z} \frac{\partial z^*}{\partial \delta} + \frac{\partial \hat{u}}{\partial \lambda} \frac{\partial \lambda^*}{\partial \delta} \lesseqgtr 0, \quad (83)$$

where (58) - (63) were used to evaluate these terms. To analyse these expressions further we utilize (58) - (63) together with (48) - (51) and (44) - (47). Unfortunately, the signs of (78) - (83) can still not be determined as we will show for the case of

$$\begin{aligned} \frac{\partial x^*}{\partial p_1} &= \frac{\partial \hat{x}}{\partial z} \frac{\partial z^*}{\partial p_1} + \frac{\partial \hat{x}}{\partial \lambda} \frac{\partial \lambda^*}{\partial p_1} \lesseqgtr 0 \\ &= \frac{1}{\det J} f_z x^* \left[\left(\eta \frac{\partial \hat{x}}{\partial z} - h_z - h_u \frac{\partial \hat{u}}{\partial z} \right) \frac{\partial \hat{x}}{\partial \lambda} - \left(\eta \frac{\partial \hat{x}}{\partial \lambda} - h_u \frac{\partial \hat{u}}{\partial \lambda} \right) \frac{\partial \hat{x}}{\partial z} \right] \\ &= \frac{1}{\det J} f_z x^* \left[-h_z \frac{\partial \hat{x}}{\partial \lambda} + h_u \left(\frac{\partial \hat{u}}{\partial \lambda} \frac{\partial \hat{x}}{\partial z} - \frac{\partial \hat{u}}{\partial z} \frac{\partial \hat{x}}{\partial \lambda} \right) \right] \lesseqgtr 0. \end{aligned} \quad (84)$$

Analyzing the term in the inner brackets shows that

$$\begin{aligned} \frac{\partial \hat{u}}{\partial \lambda} \frac{\partial \hat{x}}{\partial z} - \frac{\partial \hat{u}}{\partial z} \frac{\partial \hat{x}}{\partial \lambda} &= \frac{-1}{\Delta^2} \left((\mathcal{H}_{xz} \mathcal{H}_{uu} - \mathcal{H}_{uz} \mathcal{H}_{xu}) (\mathcal{H}_{ux} \mathcal{H}_{x\lambda}) \right. \\ &\quad \left. - (\mathcal{H}_{x\lambda} \mathcal{H}_{uu} - \mathcal{H}_{u\lambda} \mathcal{H}_{xu}) (\mathcal{H}_{ux} \mathcal{H}_{xz}) \right) \\ &= \frac{-(\mathcal{H}_{ux})^2}{\Delta^2} (\mathcal{H}_{u\lambda} \mathcal{H}_{xz} - \mathcal{H}_{uz} \mathcal{H}_{x\lambda}) \\ &= \frac{-1}{\Delta} \left[-h_u (p_1 - \gamma p_2) f_z \right. \\ &\quad \left. - \eta \left(((p_1 - \gamma p_2) x^* + \gamma p_2) - \lambda^* h_{uz} \right) \right] \lesseqgtr 0. \end{aligned} \quad (85)$$

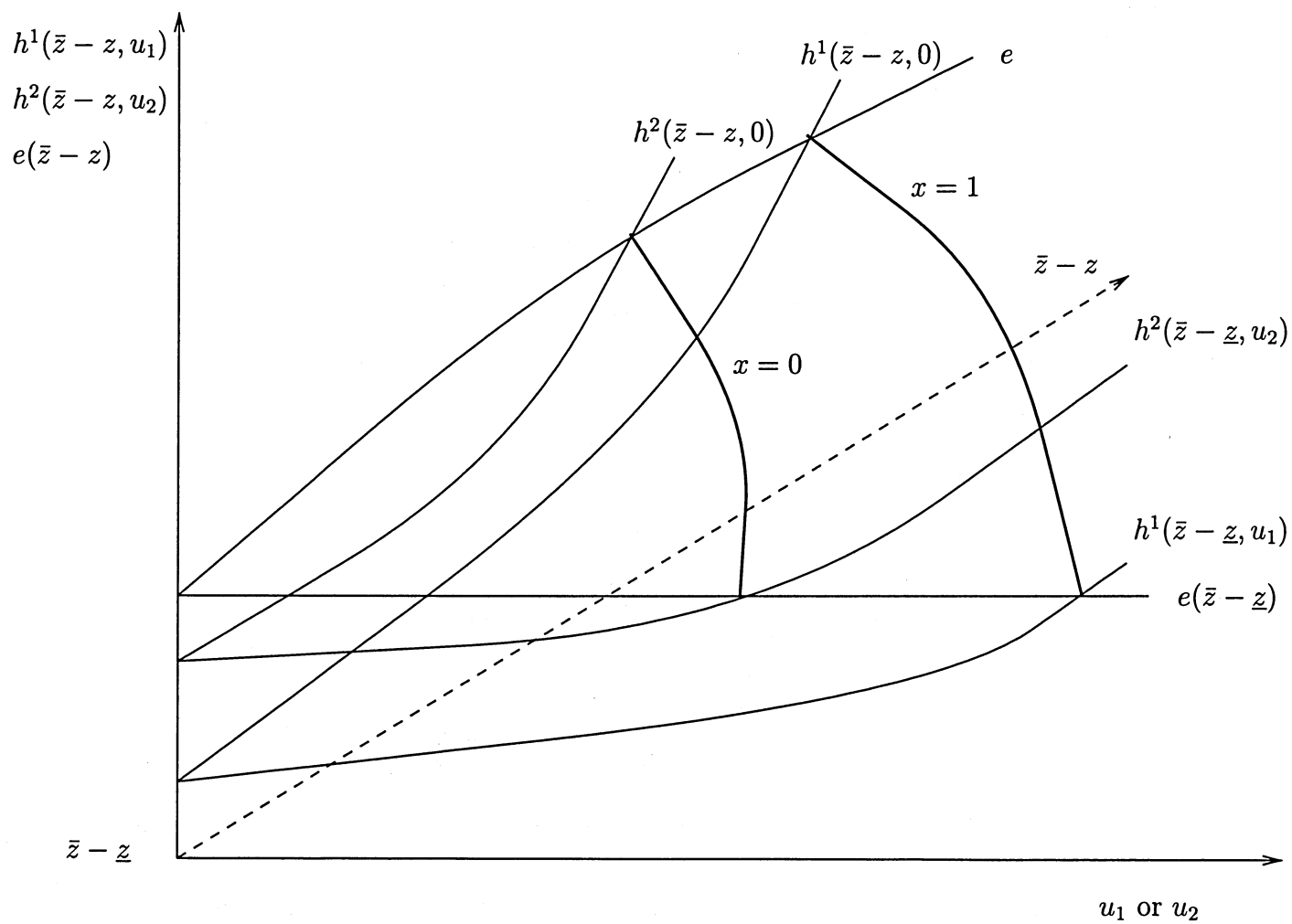


Figure 1: The erosion and soil genesis functions

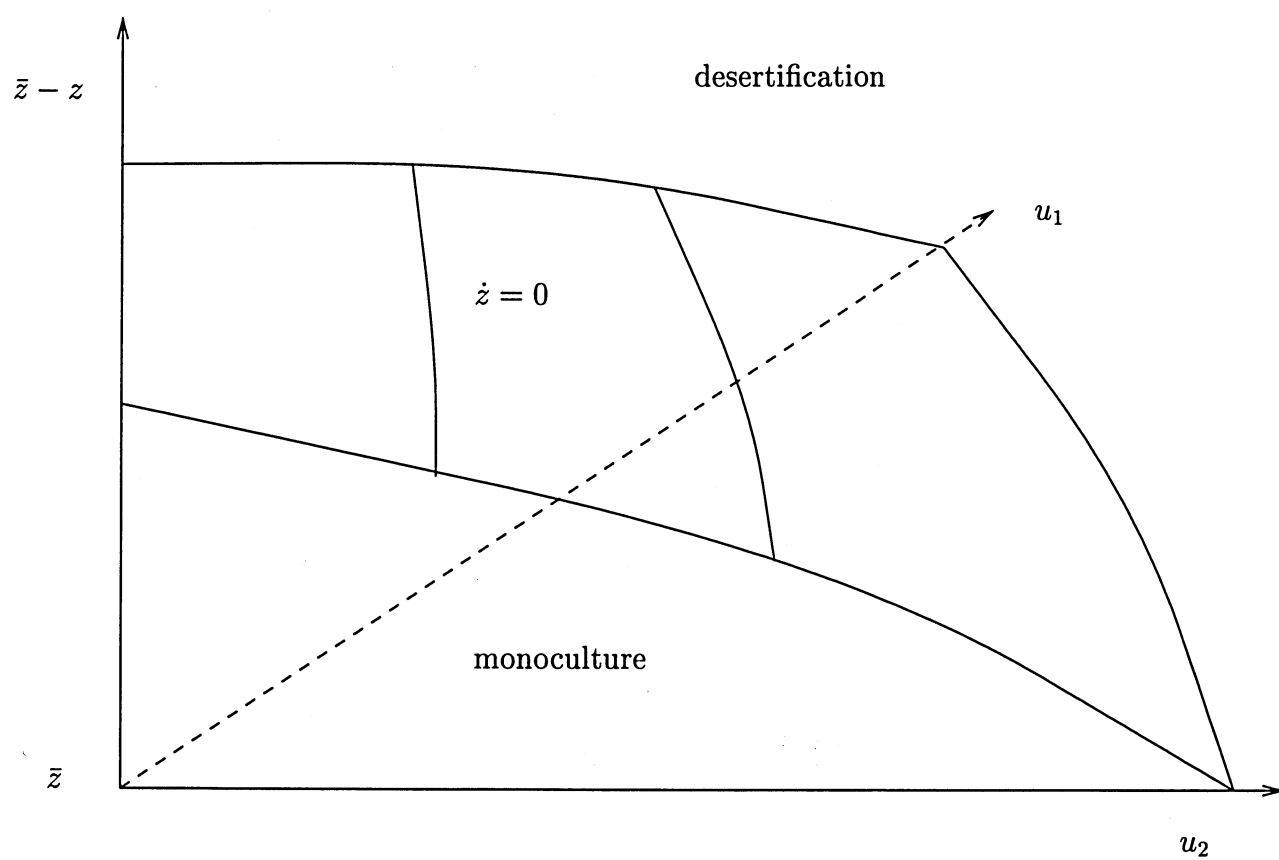


Figure 2: Choices of input combinations supporting sustainable agricultural production

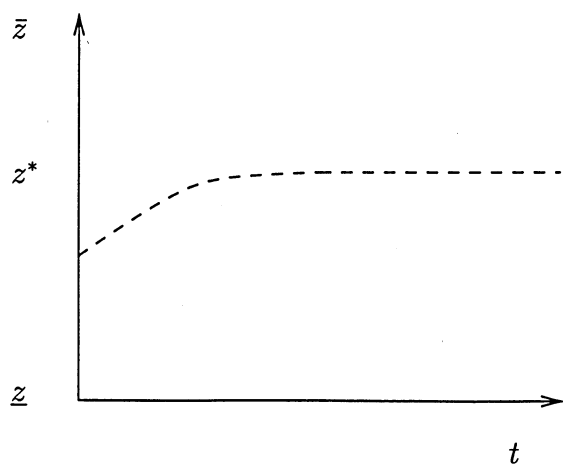
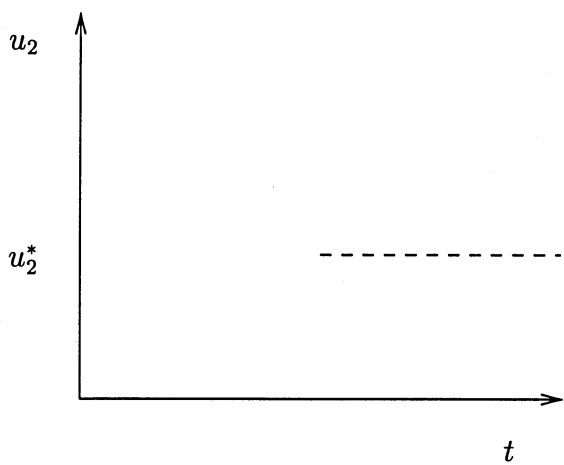
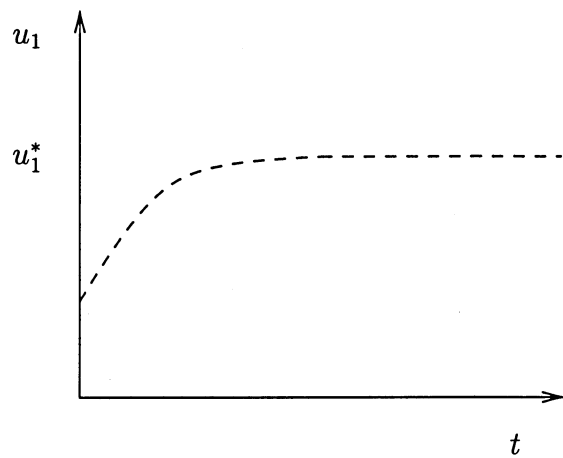
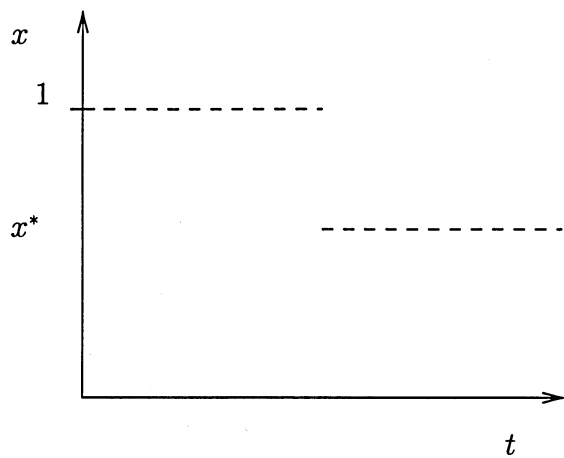


Figure 3: Graphs in the (t, x) ; (t, u_1) ; (t, u_2) and (t, z) space

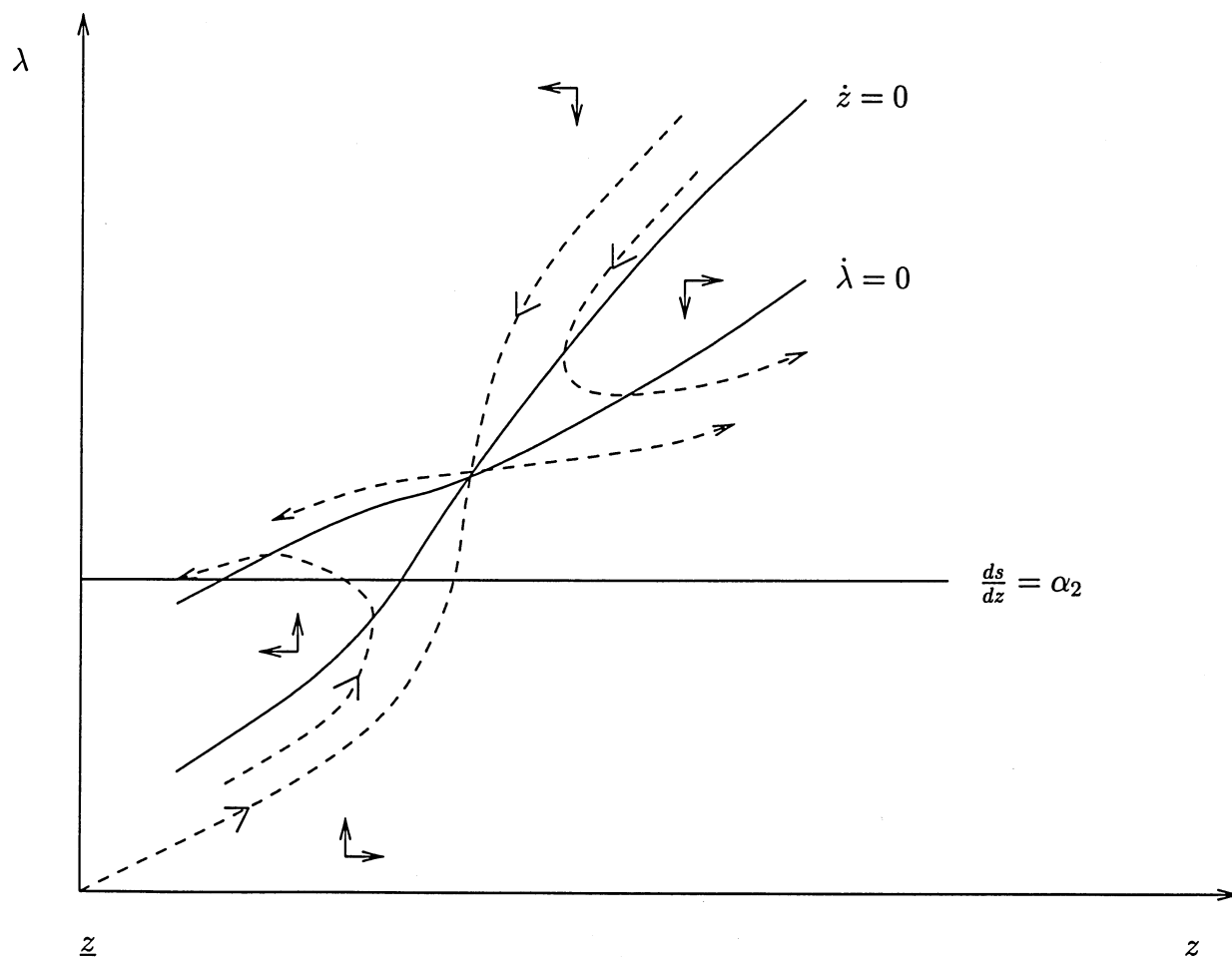


Figure 4: Phase diagram in the z, λ plane

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