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By

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Selected paper presented at the American Agricultural Economics Association Annual Meetings Denver, CO, August 1-4, 2004

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July 2004

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ABSTRACT

We examine the management of livestock diseases from the producers' perspective, incorporating information and incentive asymmetries between producers and regulators. Using a dynamic model, we examine responses to different policy options including indemnity payments, subsidies to report at-risk animals, monitoring, and regulatory approaches to decreasing infection risks when perverse incentives and multiple policies interact. This conceptual analysis illustrates the importance of designing efficient combinations of regulatory and incentive-based policies.

Key Words: livestock disease; asymmetric information; reporting; indemnities; risk management.

JEL Codes: C61; D82; Q12; Q18; Q28.

1. Introduction

Governments are under pressure to manage the threat of livestock diseases because of public health concerns and the negative impacts on livestock producers. Traditional policies for addressing livestock diseases include testing and monitoring activities, conducted by the government, and regulations imposed on livestock producers and processors. Such policies might have limited success, however, if producers do not cooperate with the government. Payments for reporting sick animals, indemnity payments for livestock destroyed for disease control, and other incentive-based policies could encourage producers to aid in disease detection. By creating a suboptimal mix of incentives, however, regulators could fail to reduce, and even exacerbate disease outbreaks. If indemnities are too high, producers could find it beneficial to submit low-grade (cheap) cattle for testing or increase the probability of disease outbreaks. With insufficient indemnity payments, producers may slaughter too many animals to avoid future losses; similarly, regulatory policies that ban the use of sick animals may promote early slaughter to avoid detection.

Designing policies to address animal diseases requires understanding the incentives faced by livestock owners. In this paper, we develop a model to examine livestock disease management when both the government and producers can affect disease risks. Economic studies of livestock diseases have focused on the effects of health concerns on prices (Piggott and Marsh 2004; Lloyd et al. 2001) and on estimating potential economic impacts (Matthews and Buzby 2001; Matthews and Perry 2003). Studies of livestock owner behavior and livestock populations focus chiefly on explaining cyclical patterns in cattle stocks (e.g. Aadland 2004). Bicknell, Wilen, and Howitt (1999) examined cattle owners' incentives to control bovine tuberculosis. In their model, producers select marketing levels, private testing, and eradication of wild animal vectors based upon prices, biological parameters, indemnity payments, the cost and efficacy of testing, and government monitoring of slaughterhouse activity. When government monitoring is 100% effective and producers have no private information about disease infection, they show that government policies reduce aggregated disease outbreaks, as well as private incentives to control disease.

Using a stochastic dynamic model, we examine the incentives of livestock producers to take private actions that can increase or decrease the potential for disease detection. We incorporate the asymmetry of information between producers and government regulators. Producers maximize expected economic benefits from livestock sales and government incentive payments in pre and post disease-detection scenarios. Livestock producers make decisions over harvest, reporting, and other activities that aid government monitoring efforts. They consider expected prices before and after disease detection, government incentive payments, and subjective probabilities of disease detection that can be influenced by federal monitoring and by producers themselves via reporting and other activities such disposal methods of sick animals. We use this model to examine producers' responses to a range of policy options including indemnity payments for destroyed livestock, subsidies to voluntarily aid in disease detection, government monitoring, and regulations that raise the cost of maintaining livestock. We also consider measures that reduce demand losses upon disease detection, such as improving animal tracking and identification systems.

The analysis characterizes the complex incentives produced by multiple related policies. Increased monitoring by the government, regulations to reduce disease transmission, payments to producers for reporting sick animals, indemnity payments for destroyed livestock, and policies to identify diseased animals may all potentially increase the stock of disease. However, perverse incentives may be mitigated in some cases through changes in payments for reporting, but whether payments should be raised or lowered may vary depending on the level of monitoring and other variables. We highlight the significance of designing the right combination of regulatory and incentive-based policies.

2. Model of Producer Behavior with Endogenous Risk of Detection

The livestock producer maximizes the expected economic benefits from livestock sales and from government payments before and after the disease is detected by the government. Upon detection, the presence of the disease becomes public knowledge, and prices fall due to lowered demand domestically and/or internationally. The model includes three state variables and two control variables. The first state variable is c_t, the stock of livestock at time t.

The second state variable is q_t, the stock of the disease in the population. We model the stock of disease directly rather than the stock of infected animals, as in Bicknell, Wilen and Howitt (1999). This formulation is general enough to include diseases that continue to spread infection after the death of the host animal.

The probability that the presence of the disease in the population is detected by the government is treated in the form of a third state variable, allowing us to endogenize the risk faced by producers. We model this endogenous risk using a survivor function, following previous work that examines behavior given risks from an environmental catastrophe (e.g. Clark and Reed 1994; Gjerde, Grepperud and Kverndokk 1999).

In each time period, the livestock producer faces a certain instantaneous probability of disease detection denoted as $\dot{\lambda}(t)$. For tractability, we specify a Poisson distribution, so the probability of detection in any interval dt is $\dot{\lambda}(t)dt$ where $\lambda(t) = \int_0^t \dot{\lambda}(t)ds$. A Poisson distribution is often used to represent counts of events across time and involves the assumptions that the probability of observing an event is approximately proportional to the size of a time interval; that there is virtually no probability of two events occurring within the same interval; that the process that determines the probability does not change over time; and that the probabilities are independent across intervals. While these assumptions will affect the exact results, the model is simply intended to illustrate potential producer behavior given endogenous risks of detection.

If T is a stochastic variable that represents the time of disease detection in the entire population of infected animals, the cumulative probability density function associated with detection is $F(t) = \Pr(T < t)$ and $F(t) = 1 - e^{-\lambda(t)}$ given the Poisson specification. The survivor function represents the probability of the livestock producers continuing to market cattle without disease detection up to each time period t and is given by $S(t) = \Pr(T > t) = 1 - F(t)$. Under the Poisson specification, $S(t) = e^{-\lambda(t)}$ and the probability of detection in a particular period t is $\dot{\lambda}(t)e^{-\lambda(t)}$. This equals the probability of detection at time t given survival up to time t without detection.

In our model, the livestock producer affects c_t as well as the stock disease and the risk of detection by choosing two variables at each point in time: h_t, the level of livestock harvested (and marketed), and d_t, the level of reporting. This reporting embodies the idea that livestock owners have certain private information regarding the likelihood that their animals are infected, which is not available to the government regulator. Thus, producers have the choice of reporting such information to the government or taking private actions that increase the chances that the disease is detected (if it is actually present in the population). For example, there might be reporting activities that entail some private costs and which will thus perhaps not be undertaken unless the producer receives an incentive.

Given this framework, the producer's problem is to maximize the present discounted value of an infinite stream of livestock harvests, net of carrying costs, plus government payments. In our base model, we consider only government payments associated with reporting activities. The producer's problem is thus to choose levels of h_t and d_t in each period to maximize:

$$J = \int_0^\infty \left\{ \pi_0 h - c(t)f + zd + \dot{\lambda}\, v(t) \right\} e^{-\lambda(t)} e^{-rt} \partial t \tag{1}$$

where π_0 is the price per unit of livestock prior to disease detection; h is the amount of livestock harvested (and sold); f is the cost of carrying (feeding) a unit of livestock; z is the reward (penalty) faced by a producer for each unit d of reporting activity; $v(t)$ denotes the value function in the post-detection scenario as of time t; and r is the instantaneous discount rate. The producer's choice in (1) is subject to the state equations for livestock, disease, and risk evolution, (2), (3), and (4) below.

Dropping the time notation for simplicity, livestock increase simply as a function of the existing stock, c, times a fixed growth rate, ρ, and decline with the level of harvest, h, and with the stock of disease, q, where u denotes the extent to which the stock of disease contributes to livestock mortality:

$$\dot{c} = \rho c - h - uq \tag{2}$$

The contagious disease stock, q, increases with c and an exogenous component θ:

$$\dot{q} = cq - \theta \tag{3}$$

Disease evolves in proportion to its existing level times the size of the livestock population net of spontaneous introduction or remission of the disease. Negative (positive) θ implies an increase (decay) over time due to exogenous effects. The multiplicative term captures the element of contagion so that the greater the stock of disease and the greater the size of the population, the greater the amount of infection over each time period. This reflects the case of a contagious disease which is directly transmitted across living animals. The formulation also applies to cases where transmission occurs through other pathways, such as feed contaminated with tissues from infected animals.

The change in the probability (risk) of disease detection is modeled as an additive function of the stock of disease, the amount of reporting, and a function that depends on the level of government monitoring activity m:

$$\dot{\lambda} = a_0 q + a_1 d - e^{-m} \tag{4}$$

The probability of disease detection depends positively on the disease stock, which will affect the probability of detection given some base level of surveillance activity.[1] Higher levels of government monitoring m also increase the chances of detection. However, monitoring and base-level surveillance alone may not be effective in detecting the disease. Producers can also directly affect the degree of detection through reporting actions d, which include measures that a producer can take to affect the detection probability given private information or behavior that is not observable by the monitoring agency. This formulation for the probability of detection highlights the importance of private participation in disease control. For simplicity, the marginal impacts of d or m on the detection probability are assumed independent of q. This may be realistic if producers or the monitoring agency can target testing or reporting in a manner that does not depend on the disease stock.

[1] Equation (4) only applies to positive and nonzero levels of disease.

For the base case, we specify a simple post-detection scenario, in which the price of livestock declines but the government is able to eradicate the disease completely and prevent its future introduction.[2] Thus in the post-detection scenario the stock of livestock grows as:

$$\dot{c} = \rho c - h \tag{5}$$

so livestock growth depends only on the growth rate and level of harvest, with no death from disease. A more realistic representation would be a scenario where the exogenous risks of disease evolution remains positive and the price of beef recovers over time. However, the results from the above formulation can be generalized in a straightforward fashion to incorporate this and the implications of more complex and realistic scenarios are discussed in section 3.3. In the base-case post-detection scenario, the livestock owner realizes returns from livestock sales at the reduced price of $\pi_1 < \pi_0$. The producer's objective is then simply to choose harvest levels to maximize the infinite stream of net returns from livestock sales starting at detection time *T*:

$$v(T) = \underset{h}{Max} \int_T^\infty (\pi_1 h - c(t) f) e^{-rt} \partial t \tag{6}$$

subject to (5).

Restricting attention to the steady-state level of livestock ($\dot{c} = 0$), producers receive an infinite stream of net benefits equal to $c(t)\left(\pi_1 \rho - f\right)$ and the value function can be rewritten as:

$$v(T) = c(t) \frac{\pi_1 \rho - f}{r} e^{-rt} \tag{7}$$

As discussed below, we focus on behavior in the steady state even though there is no guarantee that this equilibrium exists or will actually be reached. As discussed by Clark and Reed (1994), we assume that the steady state solution indicates the direction in which the system is headed. This will be true if the system converges rapidly towards the steady state behavior, even if the equilibrium is never actually attained.

[2] For simplicity, we assume that the number of cattle that need to be eradicated for disease eradication are minor and do not to affect the livestock owner's incentives.

Substituting equation (6) into equation (1) and using the result in (7), the producer's optimization problem can be solved using Pontryagin's maximum principle. The current value Hamiltonian is written as:

$$H = \left[\pi_0 h(t) - c(t) f + z d(t) + \dot{\lambda} v(t)\right] e^{-\lambda(t)} + l_1 \dot{c} + l_2 \dot{q} + l_3 \dot{\lambda} \tag{8}$$

where l_1, l_2, l_3 are, respectively, the shadow prices with respect to livestock c, disease q and the hazard rate λ. Substituting (2), (3), and (4), the first order necessary condition for an optimum with respect to the harvest level h is:

$$\frac{\partial H}{\partial h} = \pi_0 e^{-\lambda(t)} - l_1 = 0 \tag{9}$$

The first order condition with respect to reporting is:

$$\frac{\partial H}{\partial d} = z e^{-\lambda(t)} + a_1 v(t) e^{-\lambda(t)} + l_3 a_1 = 0 \tag{10}$$

Further, the rate of evolution of shadow prices is given by:

$$\dot{l}_1 = -\frac{\partial H}{\partial c} + r l_1 = \frac{\partial l_1}{\partial t} = -\left\{-f e^{-\lambda(t)} + \dot{\lambda}\left(\frac{\pi_1 \rho - f}{r}\right) e^{-rt} e^{-\lambda(t)} + l_1 \rho + l_2 q\right\} + r l_1 \tag{11}$$

$$\dot{l}_2 = -\frac{\partial H}{\partial q} + r l_2 = \frac{\partial l_2}{\partial t} = -\left\{v(t) a_0 e^{-\lambda(t)} - l_1 u + l_2 c(t) + l_3 a_0\right\} + r l_2 \tag{12}$$

$$\dot{l}_3 = -\frac{\partial H}{\partial \lambda} + r l_3 = \frac{\partial l_3}{\partial t} = \left\{\pi_0 h(t) - f c(t) + d(t) + \dot{\lambda} v(t)\right\} e^{-\lambda(t)} + r l_3 \tag{13}$$

These necessary conditions will also be sufficient conditions for maximization of the Hamiltonian if it is jointly concave in both the state and control variables (Mangasarian's theorem).[3] In this paper, we assume that the conditions for sufficiency are satisfied (see Kamien and Schwartz, 1981 for further details).

The steady state requires $\dot{l}_1 = 0$, $\dot{l}_2 = 0$, $\dot{l}_3 = 0$. Transforming le^{λ} into present value shadow prices μ, we obtain:

[3] This would require that the 5x5 Hessian matrix comprising the second order partial derivatives of the three state and two control variables is negative semi-definite. In order to establish negative semi-definiteness, it must be shown that all the principal minors have discriminants that alternate in sign, with the first one being negative.

$$f-\dot{\lambda}(\frac{\pi_1\rho-f}{r})e^{-rt}-\mu_1\rho-\mu_2 q+r\mu_1=0 \tag{14}$$

$$-v(t)a_0+\mu_1 u-\mu_2 c-\mu_3 a_0+r\mu_2=0 \tag{15}$$

$$\left\{\pi_0 h(t)-fc(t)+zd(t)+\dot{\lambda}\,v(t)\right\}+r\mu_3=0 \tag{16}$$

Further, $\frac{\partial c}{\partial t}=0$, $\frac{\partial q}{\partial t}=0$, and $\frac{\partial \lambda}{\partial t}=0$ imply:

$$h=\rho c-uq \tag{17}$$

$$q=\frac{\theta}{c} \tag{18}$$

In the steady state, harvest equals the growth in the stock of livestock net of death from disease. In the steady-state, disease growth from contagion equals cq which is perfectly offset by exogenous decay θ.[4] The risk of disease detection represented by the hazard function, λ, remains constant as the impact of reporting behavior, monitoring, and disease levels are balanced as follows:

$$d=\frac{e^{-m}}{a_1}-\frac{a_0}{a_1}q \tag{19}$$

Equations (14)-(19) and first order conditions given by (9) and (10) comprise eight equations in eight unknowns, namely: $c,q,d,h,\mu_1,\mu_2,\mu_3,$ and λ.

3. Results

In this section, we examine how the steady-state levels of the state variables change with changes in the model parameters[5]. We emphasize the impacts of policy parameters on the livestock stock, which is inversely proportional to the stock of disease in the steady state as shown in (18). We first describe comparative static results and then illustrate the system's dynamics using numerical simulations.

[4] In examining the steady state solution, we assume the existence of a steady state in which monitoring and an exogenous decay of the disease stock lead to constant λ and q.

[5] Note that for diseases that do not expericence any exogenous decay, there may not be a steady state. However, a steady state analysis is only a comparison of relative values and it may still be possible to redefine variables in order to study their steady state behavior.

3.1 Comparative Static Analysis

Using (7) and (14)-(19), we derive an implicit function for the steady state level of μ_3, the rate of change of the shadow price of detection probability λ, in terms of the model parameters:

$$G = \dot{\mu}_3 = c^2(\pi_0\rho - f - a_1(\pi_1\rho - f)e^{-rt}) - c(\frac{z}{a_1})(r - e^{-m}) - (\pi_0 u\theta + \frac{za_0\theta}{a_1}) = 0 \quad (20)$$

This equation is quadratic in the steady state level of the livestock stock. We illustrate the shape of this function given purely hypothetical values for the model parameters. Figure 1 shows an example of how $\dot{\mu}_3$ varies with the stock of livestock for particular values that were selected for producing steady state livestock and disease levels, as discussed further in section 3.3 on the numerical simulations.[6] Because the parameter values are purely hypothetical, the livestock stock numbers are just an indicator of the total herd size and do not correspond to any particular physical units, such as number of cows. The U-shape of the function 1 is a function of the parameters selected and is simply intended to illustrate some possible results of the model. In the present case, rewards from extra livestock beyond a certain threshold exceed their impact on risk from increased growth of disease. Given other values, for example if the growth rate in disease is highly susceptible to livestock stock or if livestock mortality highly sensitive to stock of disease, the shadow price of λ could follow an inverted U-shaped curve with respect to livestock levels.

While the chosen values are hypothetical, the figure illustrates how the function depends on the stock level. The figure indicates that at high levels of livestock, the shadow price of increased risk of detection is positive while at lower (positive) levels of livestock, it is actually negative, with $\dot{\mu}_3$=0 at a level of c about equal to 11. Given our specification of the post-detection value function, λ is a "bad" from a producer's perspective. However, if rewards in the post-detection scenario exceed the pre-detection

[6] The steady-state cattle and disease levels are shown in figures 5 and 6, respectively, for $a_0 = 0.2,\ a_1 = 0.7,\ f = 0.4,\ m = 10,\ r = 0.5,\ t = 10,\ u = 0.6,\ z = 30,\ \pi_0 = 5,\ \pi_1 = 1,\ \rho = 0.1,\ \theta = 0.3.$

scenario, λ would be a "good" and it would pay to increase the probability of disease detection. Factors that may lead to an increase in the post-detection reward may be higher prices of livestock (or greater market share) for some producers or indemnity payments from the government. Given our specification, all else equal, the shadow price of λ should be negative as greater risks of detection imply lower expected profits. A positive shadow price indicates that higher levels of risks enable higher steady state levels of livestock sales or of reporting, which increase expected profits.

To examine possible policy impacts, we conduct a comparative static analysis of the steady-state stock of livestock with respect to key exogenous parameters in the model. We use the implicit function theorem to obtain partials of the stock of livestock with respect to the different parameters, with particular emphasis on those which the government can directly influence. Understanding the impacts of key variable on the stock of livestock helps in understanding the impacts on the steady-state level of the stock of disease. As indicated in (18), in the steady state, livestock and disease stock are inversely related in proportion to the exogenous disease decay parameter θ. Depending on this exogenous rate of disease evolution, livestock and disease must remain in a fixed proportion in order to maintain the steady state level of detection risk. As the steady state stock of livestock rises, the steady state stock of disease falls, and vice versa.

We first consider the change in the steady state livestock with respect to the pre-detection price (π_0):

$$\frac{\partial c}{\partial \pi_0} = -\frac{\frac{\partial G}{\partial c}}{\frac{\partial G}{\partial \pi_0}} = \frac{u\theta - c^2\rho}{2c\left\{\pi_0\rho - f - a_1(\pi_1\rho - f)e^{-rt}\right\} - \frac{z}{a_1}(r - e^{-m})} \tag{21}$$

This equation reflects the tradeoff in terms of balancing risk of detection and increased mortality due to disease from a marginal increase in livestock versus the increased benefits from that unit of livestock in terms of current and future harvests. The denominator (which is the same in all of the partials of *c*) equals the change in $\dot{\mu}_3$ with respect to the stock of livestock in the steady state. As such, it is the partial derivative of the instantaneous expected benefits with respect to a marginal change in livestock in the steady state. The sign of this term varies depending on whether the benefits of an additional unit of livestock in the pre-detection world exceed the costs of reducing reporting to compensate for the added risk from the additional livestock.

In the pre-detection scenario, the benefits from an additional unit of livestock are its contribution to the profits from harvest and additional stock growth minus the additional carrying costs f and the foregone benefits from the post-detection scenario. These effects comprise the terms $2c\left\{\pi_0\rho - f - a_1(\pi_1\rho - f)e^{-rt}\right\}$ in the denominator. The bracketed terms are magnified by the level of livestock. This dependence on the livestock level arises because both livestock growth and disease growth depends upon the livestock stock. Livestock growth (before harvest and mortality) equals the stock times the constant growth rate ($c\rho$) in (2) and (5). The dependence of disease on the livestock stock arises from the biological feature of disease contagion embodied in the term cq in equation (3). This term indicates that the chances of disease spread increase in proportion to the size of the population. Thus, for a given level of disease, contagion is greater for a higher animal population. As a result, when the steady-state livestock level is high (and disease levels are thus low), the added risk produced on the margin by an extra unit of disease is greater than when the livestock population is lower (and disease higher). As a result, the cost-benefit tradeoff in terms of added risk is relatively more favorable to livestock versus reporting when the livestock stock is higher. This relationship provides an essential feature of the comparative static results discussed further below.

An additional unit of livestock raises the growth rate of the disease, which in turn increases the risk of detection. Thus, reporting must be lowered as disease rises to maintain a steady state level of risk. The term $\frac{z}{a_1}(r - e^{-m})$ captures the cost of marginal livestock unit on forgone benefits from reporting. As the effectiveness (a_1) of reporting increases, reporting levels need to be reduced by less for the same reduction in risk. Thus, as a_1 increases, fewer benefits from reporting need to be foregone for each additional unit of livestock. Also, the level of monitoring augments the effect of the discount rate r as the term e^{-m} decreases with m. The effect of the monitoring level is to increase the importance of reporting benefits. When monitoring is lowered, the term

$(r - e^{-m})$ may actually turn negative, reversing the impact on livestock stock of the exogenous variable, if the denominator was negative. This dependence on the monitoring level is discussed further below.

The net effect on the sign of the partial in (21) depends on the sum of the two denominator terms, as well as the sign of the numerator. As long as the benefit of an added unit of livestock exceeds the opportunity cost in terms of foregone reporting, the denominator will be positive. For a given value of all exogenous variables, there will be a threshold level of livestock above which the sign of the partial will change. The switch in the signs of equation (21) depending on the livestock level is depicted in figure 2. For values that produce a negative numerator, there is a level of stock above which the partial is negative and, below which, it is positive. This change in sign depending on the livestock level is a feature of all the partials for the livestock stock in the steady state.

The denominator captures the benefits of livestock versus reporting. The numerator reflects the tradeoff between added livestock and a higher level of risk in the steady state. While higher levels of prices in the pre-detection period increase current benefits from livestock, higher levels of livestock add to risk by producing disease and higher risk levels increase the chances of detection and of transition into the post-detection scenario where producers will face a new level of prices. This tradeoff is captured in the numerator as the numerator is the marginal change in $\dot{\mu}_3$, the shadow price of λ, resulting from the marginal change in the exogenous variable, the pre-detection price. The numerator is also the partial derivative of the instantaneous expected benefits resulting from the change in risk with respect to a marginal change in the exogenous variable.

The numerator of (21) shows that the cost of the increased risk resulting from the higher stock of livestock after an increase in prices is mitigated by the death rate of livestock from the disease and the exogenous rate of disease decay, given $u\theta$, the first term in the numerator. The increment in risk is augmented by the growth rate of livestock as given in $c^2\rho$, the second term in the numerator. If the parameters and steady state levels are such that this second numerator term outweighs the first, then marginal livestock adds so much risk that the optimal response is actually to reduce livestock as prices rise to maintain current livestock benefits in the pre-detection state.

We now consider the impacts on the steady-state livestock stock of livestock carrying costs *f*. These costs will potentially be affected by government policies such as regulations banning certain types of feed, which presumably increase the costs of maintaining a unit of livestock. The change in steady state livestock with respect to *f* is:

$$\frac{\partial c}{\partial f}=\frac{c^2(1-a_1e^{-rt})}{2c\left\{\pi_0\rho-f-a_1(\pi_1\rho-f)e^{-rt}\right\}-\frac{z}{a_1}(r-e^{-m})} \tag{22}$$

This partial is illustrated in figure 3. This shows the change in sign depending on the level of stock, but in the opposite direction than (21), with a positive partial at high livestock levels and a negative partial at low livestock levels. The reverse direction makes sense because the impact of *f* is to decrease the value of livestock stock while higher prices serve to increase the value of this stock in terms of potential harvests. The instantaneous benefits from a change in risk resulting from higher *f* decrease in a_1 and the discount rate. This is because the value of pre-detection livestock (which is now costlier to maintain) increases in these variables relative to the alternatives of reporting and post-detection profits. As a result, as a_1 and *r* increase, the costs of decreasing livestock stock (and increasing reporting) in response to greater *f* are greater.

The partial with respect to the post-detection price-level is:

$$\frac{\partial c}{\partial \pi_1}=\frac{c^2a_1\rho e^{-rt}}{2c\left\{\pi_0\rho-f-a_1(\pi_1\rho-f)e^{-rt}\right\}-\frac{z}{a_1}(r-e^{-m})} \tag{23}$$

As long as the value of additional livestock sales exceeds the foregone benefits from reporting (the denominator is positive), an increase in post-detection prices will increase livestock. This is because now there are greater benefits from adding risk through increased livestock given that post-detection profits are greater. The benefit of adding risk through more livestock will be higher for higher levels of a_1 because this decreases the foregone benefits from reporting from higher livestock. Post-detection profits generate greater instantaneous benefits from a change in risk when the growth rate is higher and the discount rate is lower, as these raise the post-detection livestock and the value of future livestock harvests, respectively.

We now consider the effect of maintaining the livestock stock with respect to reporting subsidies (z):

$$\frac{\partial c}{\partial z} = \frac{\frac{c}{a_1}(r - e^{-m}) + \frac{a_0}{a_1}\theta}{2c\left\{\pi_0\rho - f - a_1(\pi_1\rho - f)e^{-rt}\right\} - \frac{z}{a_1}(r - e^{-m})} \tag{24}$$

Figure 4 shows the shift in this partial as stock of livestock increases. For the particular values of the exogenous variables, higher rewards for reporting imply the denominator is negative at low levels of livestock. Thus, when the stock of livestock is low (and rewards for reporting are sufficiently high), the denominator is negative and an increase in rewards would further reduce the relative benefits of livestock versus reporting, further lowering the steady-state level livestock. On the other hand beyond a threshold level of livestock (about $c=5$ in the example), the partial becomes positive and rewards would have a positive impact on livestock.

The numerator indicates that the instantaneous benefits from a change in risk in response to z increases in a_1, as reporting rewards can be obtained for less added risk, and increases in c, a_0 and θ as less livestock stock needs to be foregone to offset the added risk from reporting. The level of monitoring enters in both the numerator and denominator to adjust the discount rate for the change in the risk of detection. Both the magnitude and direction of the impact of rewards on livestock could depend critically on the monitoring level as this can potentially switch the sign of both the numerator and the denominator if $e^{-m} > r$. In order for this to happen, monitoring must fall below some critical level m^*. Consider the implications of a monitoring level below m^* combined with a high level of reporting rewards. Earlier we saw that the response to increasing reporting payments was to lower livestock at high levels of reward. However, the incentives are reversed for monitoring below m^*. This highlights the role of designing the optimal mix of public policies in order to reach the desired objectives.

Equation (25) indicates the relationship between livestock stock and monitoring:

$$\frac{\partial c}{\partial m} = \frac{\frac{cz}{a_1} e^{-m}}{2c\left\{\pi_0 \rho - f - a_1(\pi_1 \rho - f)e^{-rt}\right\} - \frac{z}{a_1}(r - e^{-m})} \tag{25}$$

An interesting feature of this equation is that the impact of monitoring will vary based on its level. Under high monitoring, chances of detection are higher, thus making increases in livestock more costly in terms of foregone current reporting benefits. This implies that higher monitoring will lower the livestock stock. This incentive is augmented at high levels of reporting rewards, adjusted for the contribution of reporting to risk in the term $\frac{z}{a_1}$. These reporting rewards will also be less important at higher levels of livestock because, simplifying further, c drops out except for the last term in the denominator which becomes $\frac{z}{ca_1}(r - e^{-m})$.

Under low monitoring, the sign of the denominator could switch from negative to positive. Thus, when the benefits from reporting are relatively high, greater monitoring can switch the tradeoff towards increasing livestock and away from reporting. At low enough levels of monitoring risk, producers are willing to raise livestock despite high reporting rewards. The monitoring level in this equation serves to augment the market rate of discount by increasing the risks of detection.

The comparative static relationships described above illustrate the risk management tradeoffs that govern producer's responses to different possible policies. The results suggest potentially perverse policy outcomes given the public health implications of livestock diseases. Policies that increase benefits from livestock (such as subsidies to beef or dairy industries or other livestock producers), that increase costs of carrying livestock (such as regulations on feed), that reduce post-detection livestock losses (through improved tracking and surveillance), that pay producers for reporting, or that increase monitoring can each lead to either increases or decreases in the livestock stock with opposite implications for the level of the disease. Both the magnitude and direction on the steady-state disease stock will depend on the value of all of the exogenous parameters, as well as the steady-state level of the livestock stock itself. For example, equation (21) shows that for the case of $r > e^{-m}$ policies that increase livestock prices will increase stock and reduce steady-state level of disease only when the livestock level (and growth rate) is relatively low compared to the degree of lethality and decay of the disease. This effect could be reversed at low levels of stock.

The comparative static results underscore the importance of selecting an efficient mix of incentive-based policies and monitoring. The importance of sufficient monitoring is evident from the incentives induced by a scheme in which payments are increased but monitoring is low. These will be the reverse of the incentives induced by higher payments and high monitoring. Policies that lead producers to increase disease stock might increase the livestock sector's profits, but may not necessarily increase the overall public good. Understanding the risk calculus of producers is thus essential for making policy adjustments and developing an efficient portfolio of government interventions.

3.2 Alternative Scenarios for Post-Detection Values

Our analysis illustrates certain elements of optimizing behavior under simple assumptions about the nature of disease spread and livestock dynamics. Several additional complexities are worth considering. One case is that of high sensitivity of import demand to an outbreak of the disease. In this case, the fall in world prices after detection may be related to the extent of the disease in the environment. This is reflected in the following post-detection value function:

$$v(T,q,c,r,\pi(q)) = \int_t^\infty (\pi_1(1-lq)h - c(t)f)e^{-rt}\partial t \tag{26}$$

where l is the parameter measuring the impact of the disease stock on prices upon detection. In this case, producers will have additional incentives to reduce the disease. Similarly, if indemnities i are provided in case of disease detection and indemnities are based upon the level of q, then the post-detection value function becomes:

$$v(T,q,c,r,\pi,\rho) = \int_t^\infty (\pi_1 h - c(t)f)e^{-rt}\partial t + i(q)e^{-rt} \tag{27}$$

Given (27), producers would face greater incentives to report but also would potentially face perverse incentives to raise the level of q in order to increase detection risk and thus obtain indemnities. Both the cases represented in (26) and (27) could be present, with the effect of indemnities and price declines correspondingly decreasing and increasing the costs (incentives to avoid) disease detection.

Another potentially relevant scenario is one in which the disease cannot be eliminated completely and recurs after detection. For simplicity, consider a case where the second detection leads to complete destruction of the cattle stock and thus a total shutdown of the industry. The value function after the second detection is:

$$v(T,q,c,r,\pi,\rho)=0 \tag{28}$$

For the period between the first and the second detections, the current value Hamiltonian is:

$$H=\left[\pi_1 h(t)-c(t)f+zd(t)\right]e^{-\lambda t}+l_1\dot{c}+l_2\dot{q}+l_3\dot{\lambda} \tag{29}$$

The owner's objective is to maximize the sum of the discounted value of his returns from cattle and reporting rewards net of the costs of carrying the stock. In contrast to equation (8), λ in this equation serves only as an additional discounting term because the livestock owner receives no benefits in the post-detection scenario. Lower prices after the first detection reduce the optimal steady state level of cattle as shown by equation (23).[7] Assuming that the amount of cattle stock eradication after the first detection is trivial, we can derive the steady state level of cattle as a solution to the equation below:

$$c^2-rc-\frac{\pi_1 u\theta+z\theta\dfrac{a_0}{a_1}}{f+\pi_1(r-\rho)}=0 \tag{30}$$

Equation (30) is a quadratic form whose roots are given by:

$$c=\frac{r}{2}\pm\sqrt{r^2+4\frac{\pi_1 u\theta+z\theta\dfrac{a_0}{a_1}}{f+\pi_1(r-\rho)}} \tag{31}$$

The value function after the first detection is now:

$$v=\int_t^{\infty}\{\pi_1\rho c-cf+z(\frac{e^{-m}}{a_1}-\frac{a_0}{a_1}\frac{\theta}{c})\}e^{-rt} \tag{32}$$

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[7] The owner will raise the cattle stock as compared to the steady state if the post-detection prices increase or if the indemnities paid by the government ex-post are sufficiently high. To model these cases, the value from cattle after the first detection would have to be broken into two parts. The first part would equal the stream of benefits from cattle until the cattle stock reaches its steady state value and the second part would equal the stream of benefits at the steady state value.

where the first term in the integral is the steady-state benefits from cattle harvest, and the third term is the benefits from steady-state reporting activities. Using these equations, the Hamiltonian is:

$$H = \left[\pi_0 h(t) - c(t) f + zd(t) + \lambda v\right] e^{-\lambda t} + l_1 c^{\bullet} + l_2 q^{\bullet} + l_3 \lambda^{\bullet} \tag{33}$$

This Hamiltonian differs from (8) in that there is now a constant reward to be had after the first detection. The steady state value of cattle satisfies this implicit function:

$$c^2 - rc - \frac{\pi_0 u\theta + z\theta \dfrac{(a_0 + v)}{a_1}}{f + \pi_0 (r - \rho)} = 0 \tag{34}$$

The roots of this equation are:

$$c = \frac{r}{2} \pm \sqrt{r^2 + 4 \frac{\pi_0 u\theta + z\theta \dfrac{(a_0 + v)}{a_1}}{f + \pi_0 (r - \rho)}} \tag{35}$$

It is interesting to compare the steady state values of cattle before the first and second detections. The steady state value of cattle after the first detection, given by (31), is higher than the steady state value before the first detection, given by (35), as long as the effect of lower profits π_0 in equation (35) dominates the extra term (v) within the roots, which could be negative. As the number of detections increase and the value from cattle falls with additional detections, it pays more to anticipate these possible revenue losses in the future and to adjust the current cattle stock accordingly. Solving for the steady state levels of cattle using the same set of parameters as before, we find that the value function (v) after the first detection is actually negative and that the steady state level of cattle equals about 3.2 units after the first detection. The cattle stock does not converge to a steady state for the period before the first detection, implying that the risks from cattle increase at a much faster rate than can be compensated for by the exogenous parameters. In contrast, the steady state level of cattle in the single detection scenario is equal to 13, as presented in figure 1. This confirms the intuition that multiple detection scenarios imply a lower optimal cattle stock. This suggests that expectations about continuing government efforts—and how they will affect future livestock profits—will be important in shaping livestock owners' risk mitigation decisions.

So far we have focused on comparisons of state variables. However, it is also important to examine the dynamics involved with the non-linear nature of disease evolution. We explore these issues in the next section.

3.3 Numerical Analysis of the Dynamics

Given the non-linear nature of the state equations, examining the time path of the key policy variables such as livestock, disease, and reporting may provide some insights into potential policy effects. In this section, we briefly explore the role of some key parameters on the system dynamics. We use numerical simulations to examine the impacts of key parameters on the time path of the state variables. Figure 5 shows the time path of the stock of livestock under various situations. The base case reflects a set of hypothetical parameter values that were selected for producing steady state livestock and disease levels, as shown in figures 5 and 6 respectively.

While the stock of livestock falls to a steady state level in the base case, when the growth rate of disease--affected by the stocks of both livestock and disease---is lowered exogenously, the stock of livestock falls. This parameter, termed a_3, lowers the disease transmission mechanism in equation (3) and could reflect regulatory measures, such as restrictions on livestock feed, that could reduce disease spread.[8] A lower impact of the stock of disease and livestock to the growth rate of disease would allow for a larger stock of livestock. The stock of livestock falls initially along an optimal path up to a certain point and then increases beyond it. This ability to raise the stock of livestock at later stages is made possible after the exogenous rate of decay of disease has a higher (negative) impact on the growth rate of disease as compared to the much lower (but positive) impact from the combined effect of increased livestock but decreased disease. Stock of livestock is also lower when the disease-induced death of livestock, given by the parameter *u*, is higher. The state stock of livestock, however, later rises above the base case even though the death rate is higher. This is again made possible by the high reduction in the stock of livestock in the beginning stages, which has a significant

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[8] We redefine equation (3) as $\dot{q} = a_3(cq) - \theta$.

lowering impact on the growth rate of disease in the later stages, thus allowing for a higher stock of livestock. This reveals the complex nature of possible disease dynamics.

Finally, we examine the impact of discounting. Initially, a higher discount rate lowers the stock of cow livestock but eventually the steady state level of livestock is higher than the base case. While this may seem counter-intuitive, a larger stock of livestock adds to the death rate and risk of disease detection, as well as providing revenues from livestock sales. The negative impacts are reduced once the stock of livestock is significantly lowered in the initial stages. As a return, later stages allow for a higher stock of livestock. Although the livestock stock is higher in the later periods, the disease is growing at a lower rate compared to the base case.

Figure 6 depicts the time path of disease under similar conditions. In the base case, disease increases at the same time as the stock of livestock. When the growth rate of disease (parameter a_3) is lower, disease actually falls in the later stages as the exogenous impact of the decay parameter takes over. This fall in the disease is driving the counter intuitive results above. Disease falls under the case of higher disease-induced death of livestock as livestock levels are reduced. Finally, the impact of discounting is to stabilize the stock of disease above the base case levels.

Figure 7 illustrates the impact on reporting behavior under similar scenarios. There is no reporting in the base case. When a_3 is lower, stockowners avail of the benefits of reporting rewards by considering the costs and benefits of increased detection risks. Higher disease-induced death rate increase also allows for reporting due to a reduction in the growth of disease due to the reduction in livestock stock. All the reporting actions take place at a later stage when the discounted value of the costs of reporting in terms of livestock sales is lower.

This examination of the dynamic aspects of the model reveals that the time path of disease may follow highly counter-intuitive patterns. These responses would be difficult to explain without understanding the underlying patterns of private optimizing behavior.

4. Conclusion

This paper examined the behavioral aspects of livestock disease management from the livestock owner's perspective. We developed a stochastic, dynamic model of livestock levels and disease for a representative producer who can take private actions to increase the government's chances of disease detection. In this model, the producer maximizes expected revenues from the optimal management of livestock sales and any behavior that increases the chances of disease detection.

Several insights arise from the comparative statics and the numerical dynamic analysis. The comparative statics indicated that it is critical for the regulator to use the efficient mix of available options, lest they should lead to perverse incentives. The dynamic analysis further revealed complex interactions of the biological and economic processes that may lead to counter-intuitive behavior on the part of the private stock owners faced with various sources of risk. Next steps in this work will focus on determining the existence of steady-state equilibria under different modeling assumptions.

Future research would benefit from a better understanding of the biological processes and their relationship to the potential economic and policy responses. Additional insights could potentially be gained from modeling the variation in individual producer behavior and the relationship to the livestock industry at the national level. Operations of different sizes and types could also respond differently to prices, costs, disease, and government policies. The level and nature of disease in the national herd or in different subpopulations might also affect the risk calculus of individual producers given different levels of contagion as well as market segmentation and traceability. Realistic estimates for key parameters would also enable comparisons of producer responses to different policies in the context of actual economic and biological scenarios.

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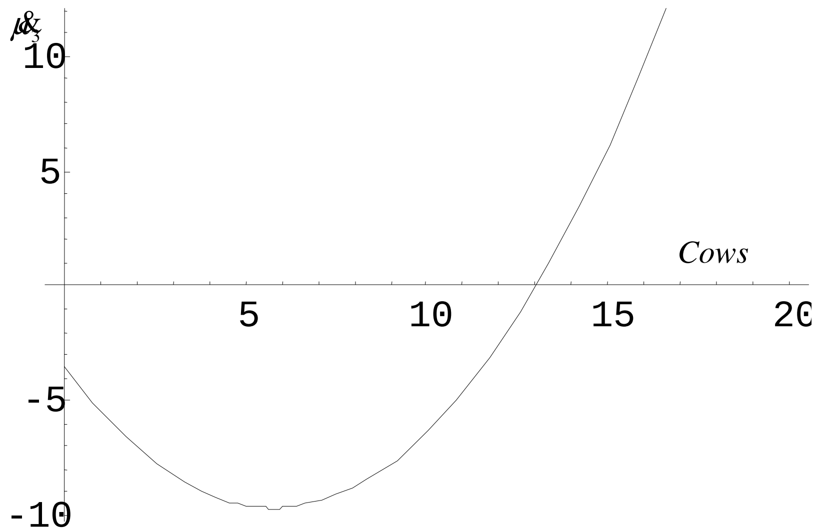
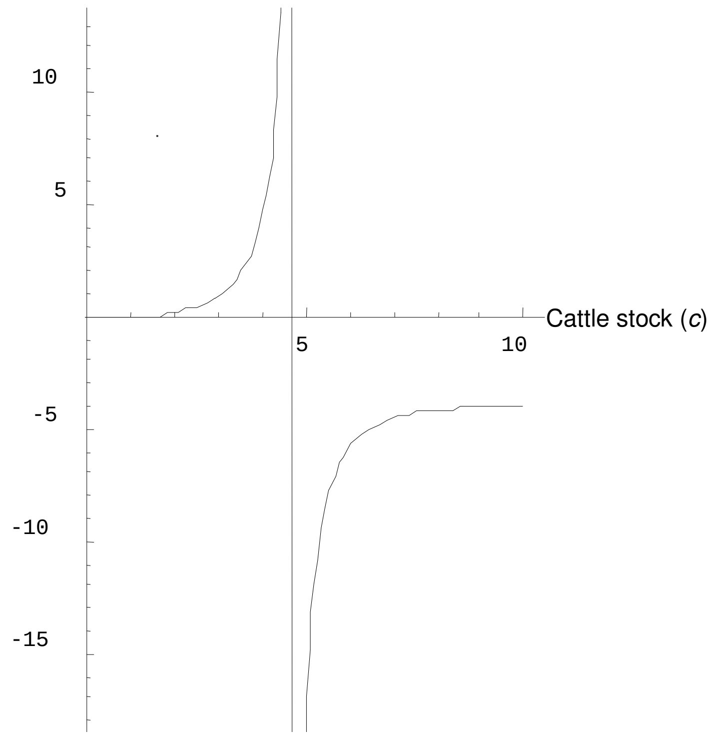
Figure 1:
Rate of Change of Shadow Price of Risk with respect to Steady-State Cattle Stock



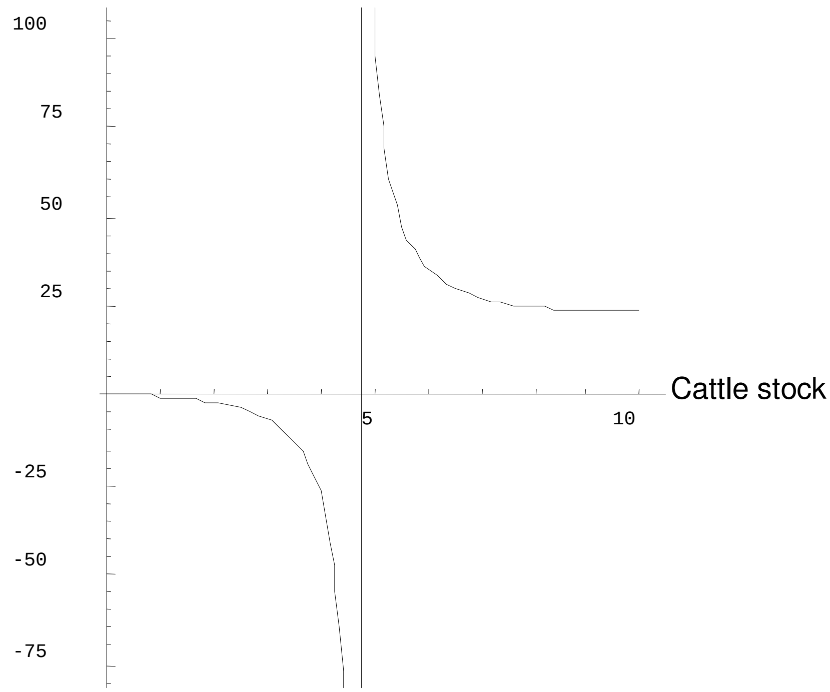
Parameters: $a_0 = .2, a_1 = .7, f = .4, m = 10, r = .05, t = 10, u = .6, z = 30, \pi_0 = 5, \pi_1 = 1, \rho = .1, \theta = .3$
Results for Solutions with negative cattle stock are omitted.

Figure 2: Change in Steady-State Cattle Stock with Respect to Current Prices ($\frac{\partial c}{\partial \pi_0}$)



Parameters: $a_0 = .2, a_1 = .7, f = .4, m = 10, r = .05, t = 10, u = .6, z = 30, \pi_0 = 5, \pi_1 = 1, \rho = .1, \theta = .3$
Solutions with negative cattle stock are omitted.

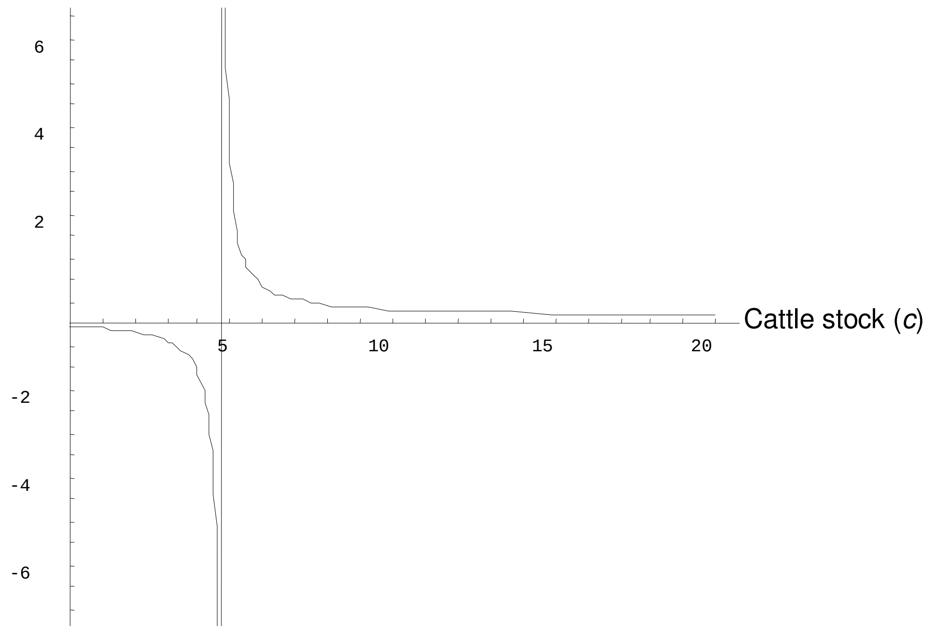
Figure 3: Change in Steady-State Cattle Stock with Respect to Carrying Costs ($\frac{\partial c}{\partial f}$)



Parameters: $a_0 = .2, a_1 = .7, f = .4, m = 10, r = .05, t = 10, u = .6, z = 30, \pi_0 = 5, \pi_1 = 1, \rho = .1, \theta = .3$
Solutions with negative cattle stock are omitted.

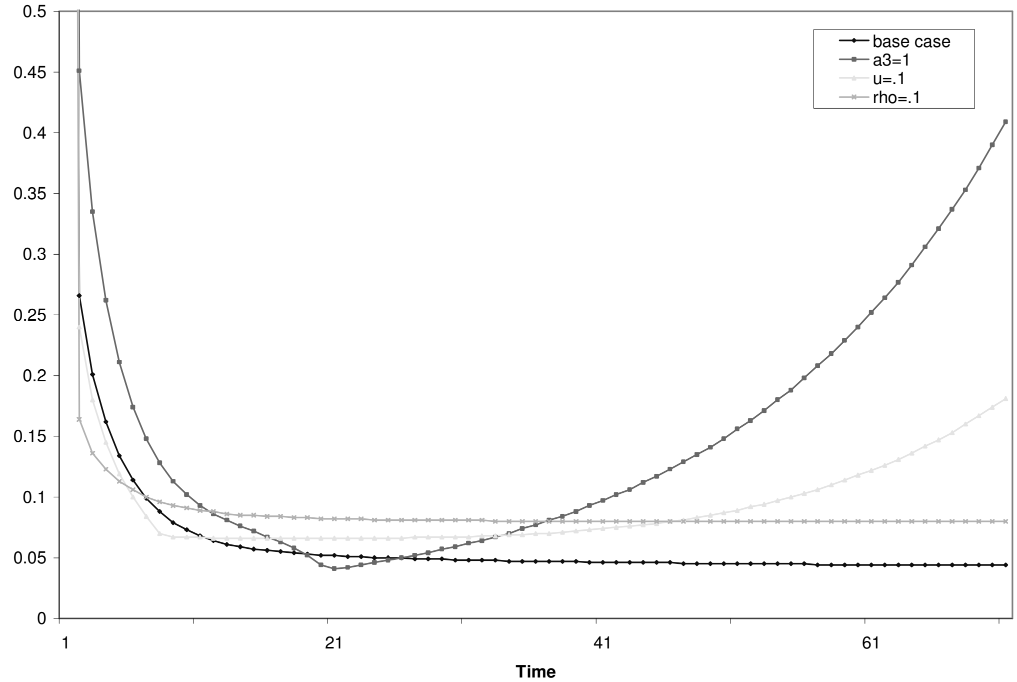
Figure 4: Change in Steady-State Cattle Stock with Respect to Reporting Rewards

$$(\frac{\partial c}{\partial z})$$



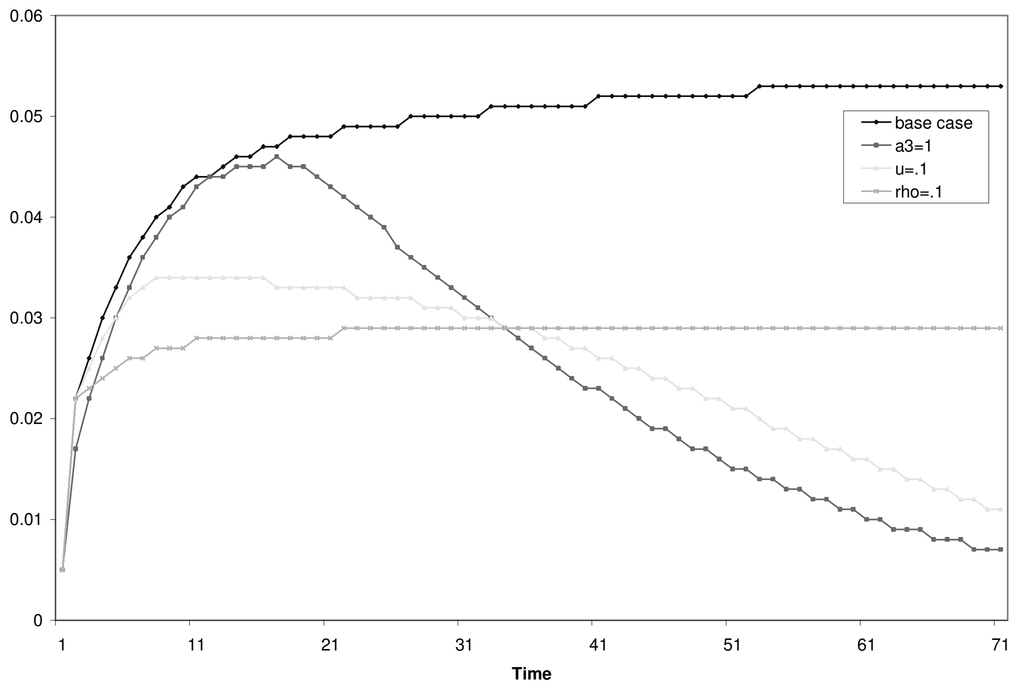
Parameters: $a_0 = .2, a_1 = .7, f = .4, m = 10, r = .05, t = 10, u = .6, z = 30, \pi_0 = 5, \pi_1 = 1, \rho = .1, \theta = .3$
Solutions with negative cattle stock are omitted.

Figure 5: Evolution of Cattle Stock (c) under Alternative Parameter Values



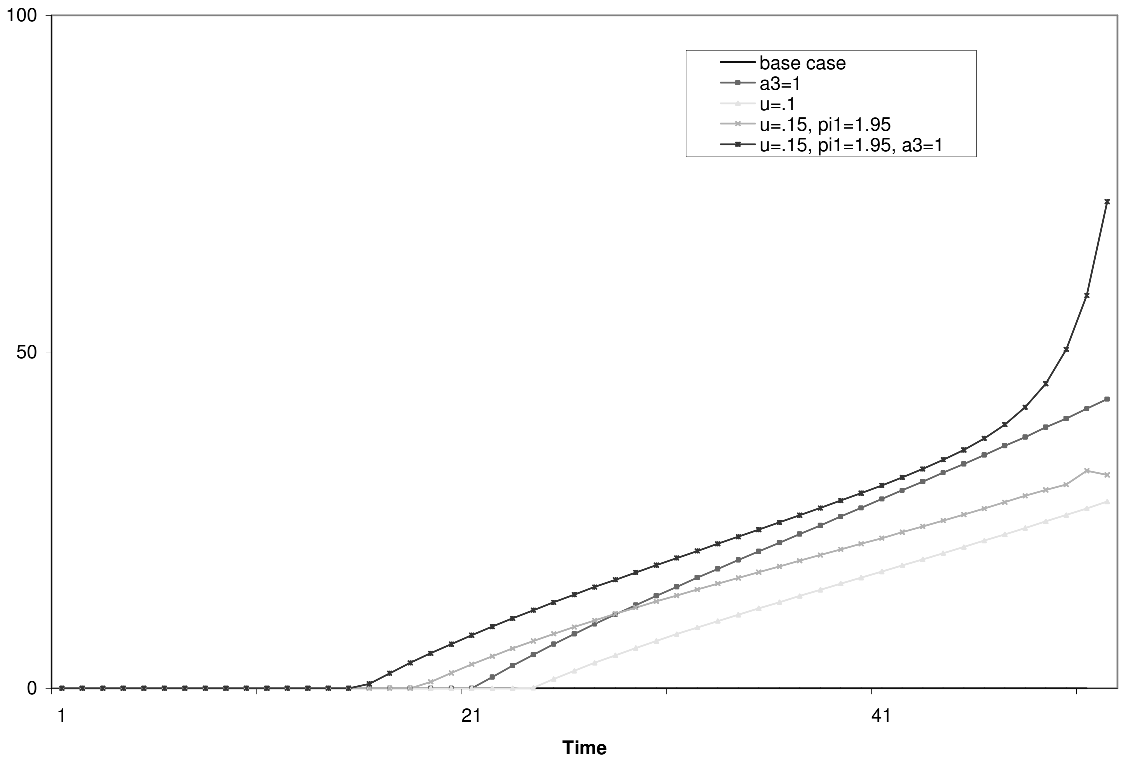
Parameters: $m = 100, r = .05, \rho = .05, a_0 = .2, a_1 = .01, u = .01, z = .005, \theta = .003, f = .681,$ $\pi_0 = 2.5, \pi_1 = .95, a_3 = 1.3, q_0 = .005, c_0 = 3, \lambda_0 = .01.$

Figure 6: Evolution of Disease Stock (*q*) under Alternative Parameter Values



Parameters: $m = 100, r = .05, \rho = .05, a_0 = .2, a_1 = .01, u = .01, z = .005, \theta = .003, f = .681,$ $\pi_0 = 2.5, \pi_1 = .95, a_3 = 1.3, q_0 = .005, c_0 = 3, \lambda_0 = .01.$

Figure 7: Evolution of Reporting Actions (d) under Alternative Parameter Values



Parameters: $m = 100, r = .05, \rho = .05, a_0 = .2, a_1 = .01, u = .01, z = .005, \theta = .003, f = .681,$ $\pi_0 = 2.5, \pi_1 = .95, a_3 = 1.3, q_0 = .005, c_0 = 3, \lambda_0 = .01.$