ESTIMATING THE MVP AND OPTIMUM IRRIGATION LEVEL FOR GRAIN SORGHUM UTILIZING EVAPOTRANSPIRATION REQUIREMENTS FOR THE TEXAS PANHANDLE

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Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Denver, CO, August 1-4, 2004

Partial funding provided by the Dryland Agriculture Institute, and the Killgore Research Center West Texas A&M University, Canyon, Texas.

Abstract: Production data provided by commercial producers of grain sorghum is used to estimate response functions for three alternative management decision models. The evaluation of yield to the total water availability, irrigation, and water application as a percent of potential evapotranspiration. The three methods provide similar results, but each provides unique information and adds valuable information to the decision process. The value product functions and the energy cost function are used to determine the profit maximizing level of water application where adequate water to fully meet the crop needs is available and to determine the irrigation vs. no irrigation decision where water availability is limiting.

Key Words: irrigation efficiency, water response function, sorghum, input use optimization, ET.

Texas agriculture generated over $15 billion in receipts in 2001. Although the High Plains represents less than 15% of the area it accounts for over 40% of the value of agricultural production for the state. In addition to leading the state in the production of feed grain, wheat, and cotton; more than 6 million cattle are fed annually within 75 miles of Amarillo (Texas Agricultural Statistics Service 2002).

Irrigation is important to maintaining the agricultural productivity and the regional economy. The development of irrigation in the region is a recent phenomenon with virtually all of the development occurring since the end of World War II. Between 1950 and 1980 irrigated acres increased from 19,315 to 1,754,560. Since 1980 irrigated acres have declined to 1,363,438. The water availability in the Ogalalla aquifer has declined and pumping costs have increased (Table 1). The significance of irrigation to agricultural production is shown by the differential between the yield of irrigated and non-irrigated corn. In 1999, the yield on the 757,500 acres of irrigated corn averaged 180.4 bushels per acre, compared to an average of 40.0 bushels per acre on the 6,500 acres of non-irrigated corn (Texas Agricultural Statistics Service 2000). Irrigation increases yield by 2 to 7 times over non-irrigation. When risk is defined as a function of the variability in yield, irrigation reduces risk by 75% to 90%.

Precipitation is not only limiting but is also highly variable. At the Bushland agricultural research center near Amarillo the annual average precipitation over the 120-year period from 1880 through 2000 is 20.53 inches. However, the range in annual precipitation is from less than 9 inches to over 40 inches (Figure 1). In addition to the pronounced year-to-year variations with as much as 15 to 20 inch differences in consecutive years there also are major wet and dry cycles observed. Short periods of significantly above average precipitation are usually followed by long periods of below average-to-average precipitation. A seasonal pattern in which over 50% of the annual precipitation is received during the summer growing season from May through October adds to the variability. The months with the highest average rainfall are May, June and August.

Grain sorghum is an important feed grain crop in the Panhandle due to its drought resistance and ability to produce under limited precipitation. Dryland production has been important since the introduction of farming in the area in the late 1800s. Sorghum production expanded rapidly in the 1950s as a result of hybrid grain sorghum, irrigation and nitrogen fertilizer (Figure 2). Production peaked in the 1960s but after decreasing significantly appears to have stabilized in recent years. Dryland production of grain sorghum is becoming more important as the water level in the Ogallala declines and irrigation is reduced. Previous analyses of the profitability of irrigated and non-irrigated sorghum production have been based on simple budgets reflecting current or recommend practices (Bean 2000; Johnson and Falconer 2001; and Amosson et al 2003).

The economic importance of the development of irrigation from the Ogallal aquifer to the region has been a concern of many economists following the rapid expansion of irrigation in the 1950s and 1960s (Grubb 1966; Osborn and McCrary 1972; and Mathews et al 1984). Resource use and the optimal combination of fixed, renewable and non-renewable resources have been analyzed by various economists (The economic implications of the depletion of a fixed resource have been a concern of agricultural economics since the early 1970s (Osborne 1973; Osborne and Harris 1973; Musick et al 1990; Amosson et al 2001; Colette, Robinson, and Almas 2001).

The decline in the water level in the Ogalalla aquifer is an on-going concern. Wells that produced 1000 to 1200 gallons per minute in the 1960's often produced less than 200 gallons per minute in the 1990's. Since there is only limited recharge of the Ogalalla aquifer in this area, irrigation water is a fixed supply and excessive pumping results in shortening the economic life of the farming operation and reduces the returns to the resources held by the farmer (Amosson et al. 2001). This year fuel prices have more than doubled. Natural gas is the primary energy source used for pumping irrigation water in the Texas Panhandle.

The objective of this study is to estimate the marginal value product of irrigation, provide alternative water management decision making tools, and provide guidelines for determining water applications that will maximize profit and extend the productive life of the Ogallala aquifer.

Data included in this study represents production information collected from producers cooperating in the AgriPartners program. Cooperating producers recorded irrigation, rainfall, soil water, and other production information weekly. Final crop production data was provided following harvest. The date, number and amount of individual irrigations were recorded and calculated using well delivery gallons per minute and the number of acres irrigated. A rain gauge located at the site measured rainfall. Beginning and ending soil moisture readings were used to calculate net soil water depletion during the growing season. Total water availability was measured and tabulated in comparison to corresponding seasonal water use reported by the North Plains PET Network for fully irrigated crops (New 1999-2003).

The water response function for sorghum must be estimated before the marginal physical product and optimal water application rate can be determined. The response function shows the relationship between the yield and the amount of water used by the crop. One of the management tools available to producers is a measurement of water requirements for a given crop as indicated by potential evapotranspiration.

Jensen and Musick (1960) were among the first to recognize the relationship between evapotranspiration (ET) and sorghum grain production. ET is a measurement of the needs of the plant and is determined by biological and climatic factors. Since the producer has no control over the level of ET it may be used as a guide but cannot be considered a management factor. The ET requirement is based on Reference Evapotranspiration (ET_0) adjusted to reflect the demands of the specific crop. The reference evapotranspiration is adjusted by multiplying by the specific crop coefficient (K_C) which reflects biological factors such as the crop, maturity rating, and the stage of growth; and climatic conditions such as maximum and minimum temperatures, growing degree days (GDD-56^{o}F), humidity, solar radiation, wind speed and direction, etc. Three sources of water to meet the ET requirement include residual soil moisture, natural precipitation, and irrigation. A producer has control over only one of these, irrigation. ET can be an aid to management decision making by indicating the amount of water that is needed by the plant. Applying water so that the ET requirement is just satisfied minimizes excessive application and subsequent water loss.

Data: The data used in the study includes 61 observations of sorghum grain production, total water availability, supplemental irrigation, and percentage evapotranspiration obtained from producers in the Texas Panhandle during the period from 1998 through 2003. The 26 counties in the Texas Panhandle are divided into two areas based on the relative availability of water from the Ogallala aquifer (Figure 3). Area A, shown in red, represents the counties with the greatest saturated thickness of the aquifer and greater availability of irrigation water. Area B, shown in blue, includes the counties with the shallowest saturated thickness and least amount of available irrigation water. The number of producers reporting grain sorghum results in the AgriPartners program is shown in parentheses.

Three approaches to the estimation of the sorghum-water response function are evaluated. In the first sorghum grain production is defined as a function of total water available for the crop. In the second approach grain production is viewed as a function of supplemental irrigation to correct for the deficiency in natural precipitation. And, in the third approach, The application of water to the crop is based on the evapotranspiration requirements of the crop. The input cost for water is calculated by one uniform method for all three approaches.

Production costs: The cost of production is the sum of the fixed cost and the variable input cost incurred in the production process. In evaluating the optimum level of a single variable input, the levels of all of the other inputs are assumed constant. The costs associated with all other inputs are considered as a part of fixed cost and only the cost of the single variable input is included in variable cost. The fixed cost is a constant and independent of the amount of water applied. The variable input cost is directly associated with the level of variable input. Since all irrigation in the region uses groundwater, the variable cost associated with irrigation is limited to pumping and application cost. Therefore, the variable input cost associated with the level of irrigation is made up of the fuel cost; cost of lubrication, maintenance, and repairs; labor costs; and annual investment costs (Equation 1) (Almas et al. 2000).

$$TC = FC + (FULC + LMR + LC + AIC)W \tag{1}$$

Where:

TC is the total production cost,

FC is the fixed cost associated with the inputs at constant levels,

FULC is the fuel cost per acre inch of water,

LMR is the cost of lubrication, maintenance and repairs,

LC is labor cost per acre inch of water,

AIC is annual investment cost per acre inch of water, and

W is the amount of water available to meet ET requirements.

The impact of a change in the price of fuel is observed in the change in the cost of fuel. Since natural gas is the predominate source of energy for pumping irrigation water in the area, natural gas is used in the calculations. The fuel cost (FULC) is equal to the product of the amount of fuel used (NG) multiplied by the price of the fuel (P_{NG}) (Equation 2).

$$FULC = NG*P_{NG} \tag{2}$$

In turn the amount of natural gas needed to pump and deliver one inch of water depends on the efficiency of the system, the lift required to get the water from below the ground to the delivery system, and the pressure of the delivery system (Equation 3).

$$NG = 0.0038*L + 0.088*PSI - ((7.623E\text{-}6)*PSI)*(L) - (3.3E\text{-}6)*L^2 \qquad (3)$$

Where:

NG is the mcf of natural gas

L is the system lift in feet

PSI is the system pressure per square inch

The NG, LMR, LC and AIC are known constants for a given irrigation system. (Almas 2000). For example, the Total Cost function for a typical Low Elevation Spray Application (LESA) system with a 350 foot system lift can be expressed as Equation 4.

$$TC = FC + (1.018P_{NG} + 2.03 + 0.68 + 1.92)W \qquad (4)$$

The Marginal Factor Cost of water (MFC_W) can now be calculated from the cost function. The MFC_W is the first derivative of the cost function with respect to the input, water (W) (Equation 5).

$$MFC_W \ ? \ \frac{dC_W}{dW} \qquad (5)$$

$$MFC_W \ ? \ 1.018P_{NG} \ ? \ 2.03 \ ? \ 0.68 \ ? \ 1.92$$

$$MFC_W \ ? \ 1.018P_{NG} \ ? \ 4.63$$

Estimation of response function, marginal value product, and economic optimum level of irrigation: Three approaches to the estimation of the sorghum-water response function are evaluated. The first approach is the traditional approach in which grain production is defined as a function of the total water available during the growing season. In Area A the mean yield for the 35 producers reporting is 7,145.2 lbs per acre. The quadratic form produces the best explanation of the relationship between sorghum yield and water available with a $Pr>F_{(2,32)}=0.0043$ for the model and an R^2 of 0.2882. The estimated coefficients for the terms representing water application are shown in Equation 6. The $Pr>t_{(32)}$ is in parentheses below the coefficients.

$$\begin{array}{l} Y_A \ ? \ ?1416.44 \ ? \ 373.08W \ ? \ 5.28W^2 \\ \qquad\quad 0.5609 \quad 0.0729 \qquad\quad 0.2042 \end{array} \tag{6}$$

The Marginal Physical Product of Water in Area A (MPP_{WA}) is equal to the derivative of the response function with respect to the input water (Equation 7).

$$MPP_{WA} \ ? \ \frac{dY_A}{dW} \ ? \ 373.08 \ ? \ 10.55W \tag{7}$$

The Marginal Value Product of water in Area A (MVP_{WA}) is obtained by multiplying the Marginal Physical Product of water in Area A (MPP_{WA}) by the price of the product (P_Y) (Equation 8).

$$\begin{array}{l} MVP_{WA} \ ? \ MPP_{WA} * P_Y \\ MVP_{WA} \ ? \ (373.08 \ ? \ 10.55W)P_Y \end{array} \tag{8}$$

The optimal economic level of a productive input is based on the principle of profit maximization (Heady and Canler 1961; and Beattie and Taylor 1985). Profit is maximized at that input level where the increase in value from using an additional unit of input, Marginal Value Product, is equal to the increase in cost associated with the use of that same unit of input, Marginal Factor Cost. The MVP is equal to the increase in output obtained from the use of an additional unit of input, Marginal Physical Product (MPP), multiplied by the price of the output (P_Y). The Optimum level of the input water application in Area A is determined by equating the Marginal Value Product of water (MVP_{WA}) from Equation 8 and the Marginal Factor Cost of water (MFC_W) from Equation 5.

$$MVP_{WA} = MFC_W \qquad (9)$$

$$(373.08 - 10.55W)\, P_Y = 1.018P_{NG} + 4.63$$

Solving for the level of water availability (W) produces a function in the price of natural gas (P_{NG}) and the price of the output (P_Y) (Equation 10).

$$W \; ? \; \frac{373.08 \; ? \; \dfrac{1.018P_{NG} \; ? \; 4.63}{P_Y}}{10.55} \qquad (10)$$

Profit maximizing levels of water availability derived from Equation 10 for sorghum prices between \$3 and \$6 and natural gas prices between \$2 and \$11 are in Table 2.

In Area B the mean yield for the 26 producers reporting is 3,255.0 lbs per acre. The linear form produces the best fit between sorghum yield and water available to meet crop requirements with a $Pr>F_{(2,24)}=0.0092$ for the model. The R^2 is 0.2496. The estimated coefficients for the terms representing water application are shown in Equation 11. The $Pr>t_{(24)}$ is in parentheses below the coefficients.

$$Y_B \; ? \; \underset{0.3341}{892.94} \; ? \; \underset{0.0094}{145.76}W \qquad (11)$$

The Marginal Physical Product of Water in Area B (MPP_{WB}) is equal to the derivative of the response function with respect to the input water (Equation 4).

$$MPP_{WB} ? \frac{dY_B}{dW} ? 145.76 \qquad (12)$$

The Marginal Value Product of water in Area B (MVP_{WB}) is obtained by multiplying the Marginal Physical Product of water in Area B (MPP_{WB}) by the price of the product (P_Y) (Equation 13).

$$MVP_{WB} ? MPP_{WB} * P_Y \qquad (13)$$
$$MVP_{WB} ? (145.76) P_Y$$

The Optimum level of the input water application in Area B is determined by equating the Marginal Value Product of water (MVP_{WB}) from Equation 13 and the Marginal Factor Cost of water (MFC_W) from Equation 5.

$$MVP_{WB} = MFC_W \qquad (14)$$

$$145.76*P_Y = 1.018*P_{NG} + 4.63$$

Since both the MVP_{WB} and the MFC_W are both linear and independent of the level of water use, the decision becomes a simple irrigation vs. no irrigation. If the MVP_{WB} is greater or equal to the MFC then irrigate. If the MFC_W is less than the MVP then no irrigation is the optimal decision (Table 3).

Optimization of irrigation supplementing natural precipitation: The second approach is to define the production function of sorghum grain production as a function of the irrigation water added to the natural precipitation available during the growing season.. The best response function relating the production of sorghum to the water available through natural precipitation and supplemental irrigation is linear in natural precipitation and quadratic with respect to the supplemental water added through irrigation. The model has a $Pr>F_{(2,31)}=0.0021$ with an R^2 of 0.3720. The estimated coefficients for the terms representing water application are shown in Equation 15. The $Pr>t_{(31)}$ is in parentheses below the coefficients.

$$Y_A \ ? \ \underset{0.0234}{2667.41} \ ? \ \underset{0.0055}{159.60P} \ ? \ \underset{0.0170}{379.76I} \ ? \ \underset{0.0500}{11.18I^2} \qquad (15)$$

Where: Y_A is the production of sorghum grain in lbs per acre,

P is natural precipitation in inches; and

I is inches of supplemental irrigation.

The Marginal Physical Product of Water in Area A (MPP_{WA}) is equal to the derivative of the response function with respect to the input water (Equation 16).

$$MPP_{IA} \ ? \ \frac{dY_A}{dI} \ ? \ 379.76 \ ? \ 22.36I \qquad (16)$$

The Marginal Value Product of water in Area A (MVP_{WA}) is obtained by multiplying the Marginal Physical Product of water in Area A (MPP_{WA}) by the price of the product (P_Y) (Equation 17).

$$MVP_{IA} \ ? \ MPP_{IA} * P_Y \qquad (17)$$
$$MVP_{IA} \ ? \ (379.76 \ ? \ 22.36I)P_Y$$

The Optimum level of the input water application in Area A is determined by equating the Marginal Value Product of water (MVP_{IA}) from Equation 17 and the Marginal Factor Cost of water (MFC_W) from Equation 5.

$$MVP_{IA} = MFC_W \quad (18)$$

$$(379.08 - 22.36I)\, P_Y = 1.018P_{NG} + 4.63$$

Solving for the level of irrigation (I) produces a function in the price of natural gas (P_{NG}) and the price of the output (P_Y) (Equation 18).

$$I \; ? \; \frac{379.76 \; ? \; \dfrac{1.018P_{NG} \; ? \; 4.63}{P_Y}}{22.36} \quad (19)$$

Optimal water availability for natural gas prices between $2 and $10 per mcf and sorghum prices between $3 and $6 per cwt are shown in Table 4.

In Area B the linear model in natural precipitation and supplemental irrigation produces the best fit between sorghum yield and water available to meet ET requirements with a $Pr>F_{(2,23)}=0.0355$ for the model. The R^2 is 0.2519. The estimated coefficients for the terms representing water application are shown in Equation 3. The $Pr>t_{(23)}$ is in parentheses below the coefficients.

$$Y_B \; ? \; \underset{0.3600}{1071.52} \; ? \; \underset{0.2247}{123.68P} \; ? \; \underset{0.0137}{151.29I} \quad (20)$$

Where: Y_B is the production of sorghum grain in lbs per acre,

P is natural precipitation in inches; and

I is inches of supplemental irrigation.

The Marginal Physical Product of Irrigation in Area B (MPP_{WB}) is equal to the derivative of the response function with respect to the input irrigation (Equation 21).

$$MPP_{IB} ? \frac{dY_B}{dI} ? 151.29 \quad (21)$$

The Marginal Value Product of irrigation in Area B (MVP_{IB}) is obtained by multiplying the Marginal Physical Product of water in Area B (MPP_{IB}) by the price of the product (P_Y) (Equation 22).

$$MVP_{IB} ? MPP_{IB} * P_Y \quad (22)$$
$$MVP_{IB} ? (151.29)P_Y$$

The Optimum level of the input water application in Area B is determined by equating the Marginal Value Product of water (MVP_{IB}) from Equation 22 and the Marginal Factor Cost of water (MFC_W) from Equation 5.

$$MVP_{IB} = MFC_W \quad (23)$$

$$151.29*P_Y = 1.018*P_{NG} + 4.63$$

Since both the MVP_{WB} and the MFC_W are both linear and independent of the level of water use, the decision becomes a simple irrigation vs. no irrigation. If the MVP_{IB} is greater or equal to the MFC then irrigate. If the MFC_W is less than the MVP_{IB} then no irrigation is the optimal decision Table 5.

Optimization based on Potential Evapotranspiration: The third approach is to determine the application of an input based on the physiological requirement of the crop. In this case, basing the application of water on the physiological requirements of the crop as determined by Potential Evapotranspiration (ET). In the third method the production of sorghum grain is defined as a function of the relationship between the amount of water available and the amount of water required for the growing plant as indicated by the Percent of Potential Evapotranspiration (PET).

In Area A the quadratic form produces the best explanation of the relationship between sorghum yield and water available to meet ET requirements with a $Pr>F_{(2,32)}=0.0080$ for the model. The R^2 is 0.2607. The estimated coefficients for the terms representing water application are shown in Equation 24. The $Pr>t_{(32)}$ is in parentheses below the coefficients.

$$\underset{}{Y_A} \;?\; \underset{0.5989}{?1855.87} \;?\; \underset{0.0302}{179.11PET} \;?\; \underset{0.0625}{0.83PET^2} \tag{24}$$

$$MPP_{PETA} \;?\; \frac{dY_A}{dPET} \;?\; 179.11 \;?\; 1.66PET \tag{25}$$

Since PET is a measurement instead of an input, the productivity of the PET must reflect the relationship between PET and water availability. In Area A the best estimate is a linear model (Equation 26)

$$PET_A \;?\; \underset{0.0399}{13.99} \;?\; \underset{<0.0001}{3.13W} \tag{26}$$

Since PET does not refer to units of water or price the chain rule is utilized to determine the Marginal Physical Product of water based on PET.

$$\frac{dPET_A}{dW_A} \;?\; 3.13 \tag{27}$$

The marginal physical product of water applied to meet evapotranspiration requirements as reflected by the PET is shown in Equation 28.

$$MPP_{PETA_w} \; ? \; ?\frac{dY_A}{dPET_A}??\frac{dPET_A}{dW_A}? \qquad (28)$$

$$MPP_{PETA_w} \; ?? \; (179.11 \; ? \; 1.66PET) * 3.13$$

The marginal value product is shown in Equation 29.

$$MVP_{PETA_w} \; ? \; (560.61 \; ? \; 5.10PET) * P_Y \qquad (29)$$

Solving for the level of irrigation (I) produces a function in the price of natural gas (P_{NG}) and the price of the output (P_Y) (Equation 5).

$$PET_A \; ? \; \frac{560.61 \; ? \; \dfrac{1.018P_{NG} \; ? \; 4.63}{P_Y}}{5.10} \qquad (30)$$

Optimal water availability for natural gas prices between $2 and $10 per mcf and sorghum prices between $3 and $6 per cwt are shown in Table 6.

For Area B the linear form produces the best fit between sorghum yield and water available to meet ET requirements with a $Pr>F_{(2,24)}=0.0092$ for the model. The R^2 is 0.2496. The estimated coefficients for the terms representing water application are shown in Equation 3. The $Pr>t_{(24)}$ is in parentheses below the coefficients.

$$Y_B \; ? \; \underset{0.6322}{497.90} \; ? \; \underset{0.0088}{44.06}PET \qquad (31)$$

The Marginal Physical Product of Water as a Percent of Evapotranspiration in Area B (MPP_{PETB}) is equal to the derivative of the response function with respect to the input PET (Equation 32).

$$MPP_{PETB} = \frac{dY_B}{dPET} = 44.06 \qquad (32)$$

The Marginal Value Product of water in Area B (MVP_{WB}) is obtained by multiplying the Marginal Physical Product of water in Area B (MPP_{WB}) by the price of the product (P_Y) (Equation 5).

$$\begin{aligned} MVP_{PETB} &= MPP_{PETB} * P_Y \\ MVP_{PETB} &= (44.06)P_Y \end{aligned} \qquad (33)$$

Since PET is a measurement instead of an input, the productivity of the PET must reflect the relationship between PET and water availability. In Area A the best estimate is a linear model (Equation 34)

$$PET_B = \underset{0.0122}{13.41} \; ? \; \underset{<0.0001}{3.03W} \qquad (34)$$

Since PET does not refer to units of water or price the chain rule is utilized to determine the Marginal Physical Product of water based on PET.

$$\frac{dPET_B}{dW_B} = 3.03 \qquad (35)$$

The marginal physical product of water applied to meet evapotranspiration requirements as reflected by the PET is shown in Equation 36.

$$MPP_{PETB_W} = \left(\frac{dY_B}{dPET_B}\right)\left(\frac{dPET_B}{dW_B}\right) \qquad (36)$$

$$MPP_{PETB_W} \;?? \;(44.06)*(3.13) \;?\; 137.91$$

The marginal value product is shown in Equation 37.

$$MVP_{PETA_W} \;?\; (560.61 \;?\; 5.10PET)*P_Y$$

$$MVP_{PETB_W} \;?\; (137.91)*P_Y \qquad (37)$$

Solving for the level of irrigation (I) produces a function in the price of natural gas (P_{NG}) and the price of the output (P_Y) (Equation 38).

$$MVP_{PETB} = MFC_W \qquad (38)$$

$$137.91*P_Y = 1.018*P_{NG} + 4.63$$

Since both the MVP_{PETB} and the MFC_W are both linear and independent of the level of water use, the decision becomes a simple irrigation vs. no irrigation. If the MVP_{PETB} is greater or equal to the MFC_W then irrigate. If the MFC_W is less than the MVP_{PETB} then no irrigation is the optimal decision Table 7.

Summary: Often the answers to management decision problems cannot be found in individual controlled experiments but must be developed under commercial management conditions. Collecting adequate observations to estimate management decision functions for commercial producers is often difficult. Fortunately the participation of progressive producers in the Texas Panhandle in the AgriPartners Irrigation Demonstration Project allows access to the information needed to estimate a response function relating sorghum yield as a function of water availability and irrigation.

Although production cost will vary for different types of delivery systems and with different water lifts, for a given delivery system, such as LESA and a known lift the cost function can be expressed in terms of the energy cost. The response and cost functions are used to determine the profit maximizing level of water availability for various price levels for sorghum and natural gas.

Three approaches to making the management decision on the amount of water to apply to maximize profits and returns to resources from grain sorghum production are evaluated. The traditional approach of determining the optimum level of water application based on the total availability without regard for the origin of the water provides a response function indication the total water needs but only indirectly addressing the management decision of irrigation levels.

In the second approach, irrigation is viewed as a supplementation to natural precipitation. Irrigation becomes a management decision variable. The response function indicates that grain production increases as both natural precipitation and irrigation increase. The response is linear with respect to natural precipitation and quadratic with respect to irrigation. This may be due to the fact that natural precipitation is in the Panhandle is never sufficient to meet the total evapotranspiration needs of the crop. Therefore, we only observe response in the linear portion of the production function. On the other hand, irrigation moves the total water availability into the range where efficiency declines rapidly and the response per unit of input declines. This approach provides a measurement of the actual irrigation levels that would be relevant to the management decision.

The third approach is to base irrigation management decisions on the needs of the crop as indicated by potential evapotranspiration for a crop that is not limited by water availability. This method would be more valuable if a dynamic model which could account for the timing of irrigation application were available instead of a static model. It is interesting to note how low the optimal percent evapotranspiration levels are compared to the 100 percent PET level that would provide a water stress free environment for the crop.

The estimation of separate response functions for the two areas based on the availability of water in the Ogallala aquifer provides insight into the different management decisions that are faced by producers in those areas. In Area A where water is still readily available the decision is still selecting the optimum level of irrigation of water availability. Optimization has a unique solution. In Area B where water availability is limited and the decision becomes one of irrigation vs. no irrigation. A unique optimal level is not defined as producers do not have sufficient water to move into the range of application with rapidly declining marginal productivity.

The analysis for natural gas prices between $2 and $10 per mcf and sorghum prices between $3 and $6 per cwt indicate that the amount of water to apply increases as the price of sorghum increases. Conversely, for a fixed price of sorghum the optimal water application rate declines as the price of natural gas increases. Where water availability is severely limited, it is interesting to note that in none of the approaches is irrigation indicated at any sorghum price when the natural gas price is above $4.50

Table 1. Irrigated acres in Region A of the High Plains, by method of application, 1950 through 1997.

Year	Furrow Irrigated	Sprinkler Irrigated	Total Acres
1950	19,315	0	19,315
1960	549,884	20,397	570,281
1970	1,379,878	137,139	1,517,017
1980	1,353,443	401,117	1,754,560
1990	676,051	515,195	1,191,246
1997	509,267	854,171	1,363,438

Figure 1. Annual precipitation and growing season precipitation reported at the Amarillo weather station from 1880 to 2000.

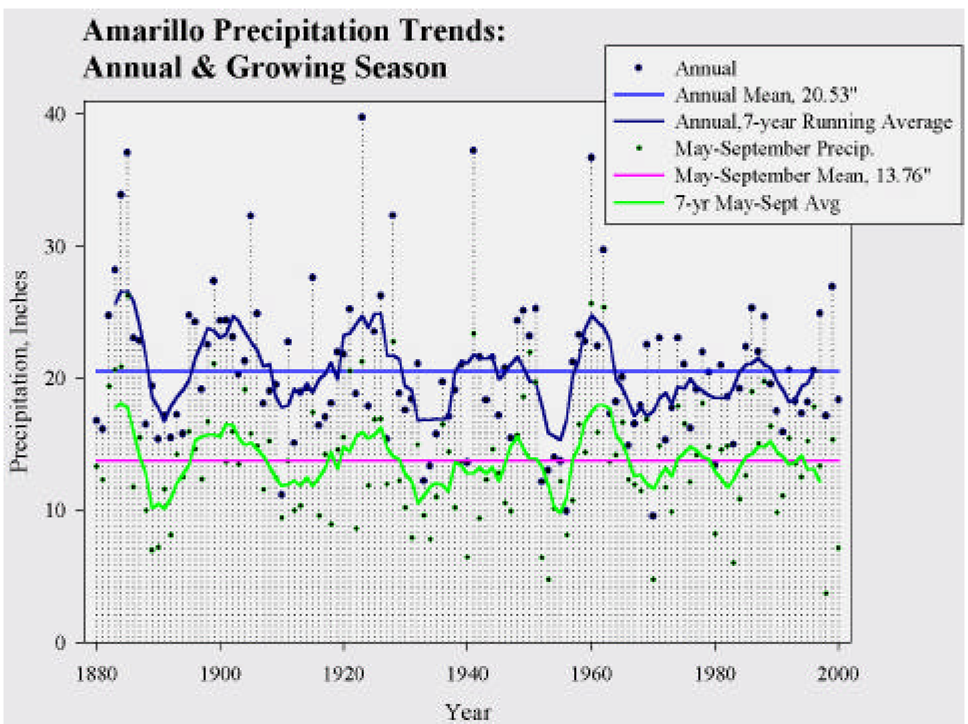


Figure 2. Acreage of Grain Sorghum in the 26 Counties in the Texas Panhandle, 1920-1997.

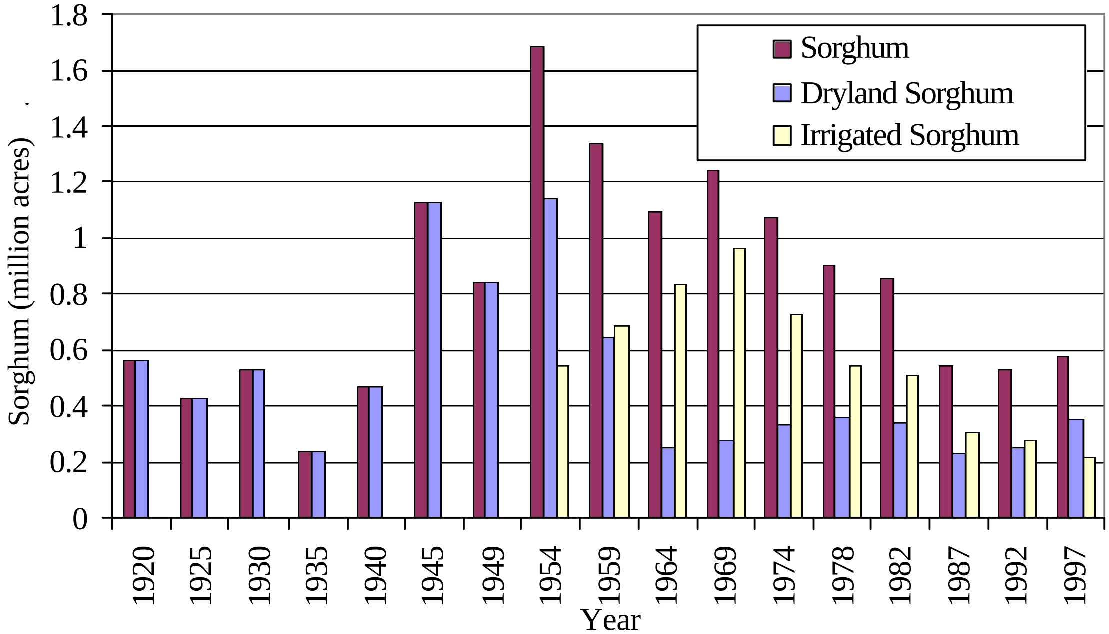
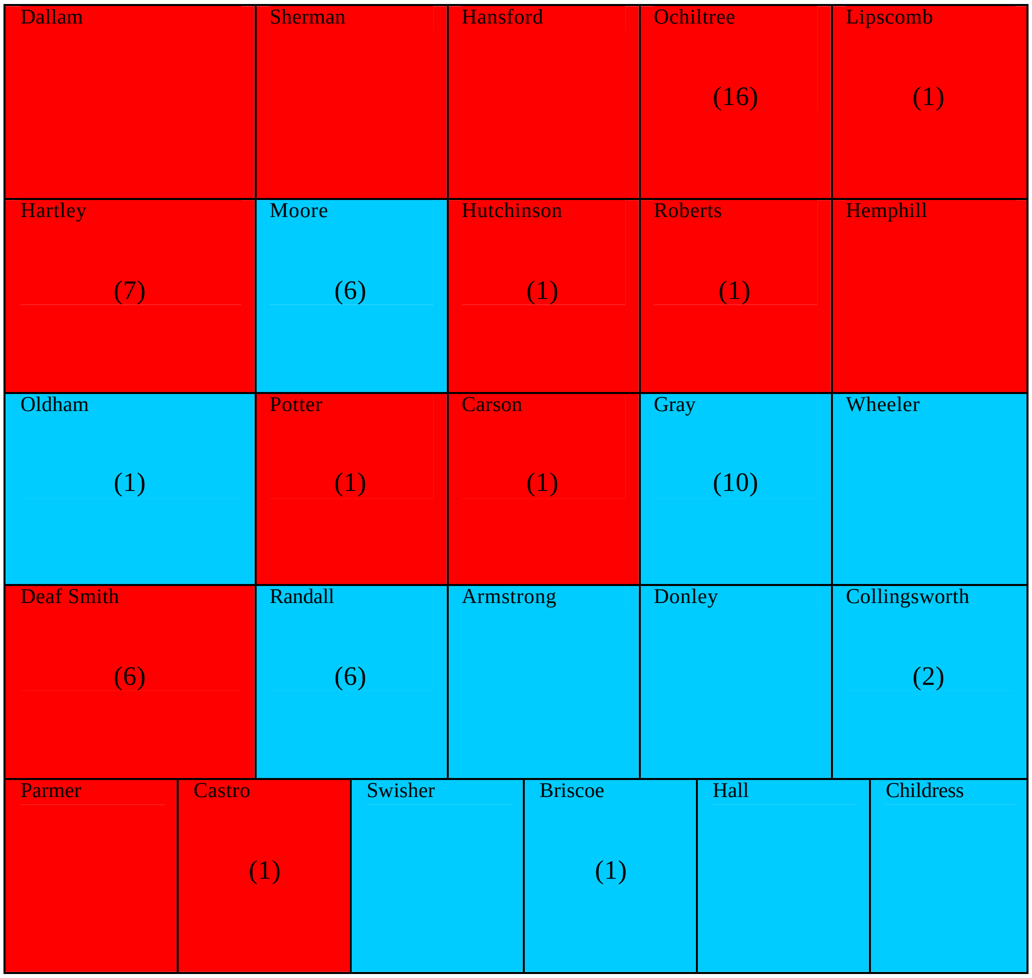


Figure 3. Counties in the Texas Panhandle grouped into areas based on relative availability of irrigation water from the Ogallala aquifer.



Area A is in red. Area B is in blue. The number in parentheses is the number of observations taken from producers in the county.

Table 2. Optimum water availability for meeting crop requirements under different sorghum and natural gas prices in Area A.

	Price of Sorghum ($)												
P_{NG}	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
2.00	14.3	15.9	17.3	18.5	19.6	20.5	21.3	22.1	22.7	23.3	23.9	24.4	24.8
2.50	12.7	14.4	15.9	17.2	18.4	19.4	20.2	21.0	21.8	22.4	23.0	23.5	24.0
3.00	11.1	13.0	14.6	15.9	17.2	18.2	19.2	20.0	20.8	21.5	22.1	22.7	23.2
3.50	9.5	11.5	13.2	14.7	15.9	17.1	18.1	19.0	19.8	20.6	21.2	21.9	22.4
4.00	7.9	10.0	11.8	13.4	14.7	16.0	17.0	18.0	18.9	19.7	20.4	21.0	21.6
4.50	6.3	8.5	10.4	12.1	13.5	14.8	16.0	17.0	17.9	18.7	19.5	20.2	20.8
5.00	4.7	7.0	9.0	10.8	12.3	13.7	14.9	16.0	16.9	17.8	18.6	19.3	20.0
5.50	3.0	5.5	7.7	9.5	11.1	12.5	13.8	15.0	16.0	16.9	17.7	18.5	19.2
6.00	1.4	4.0	6.3	8.2	9.9	11.4	12.7	13.9	15.0	16.0	16.9	17.7	18.4
6.50	0.0	2.6	4.9	6.9	8.7	10.3	11.7	12.9	14.0	15.1	16.0	16.8	17.6
7.00	0.0	1.1	3.5	5.6	7.5	9.1	10.6	11.9	13.1	14.1	15.1	16.0	16.8
7.50	0.0	0.0	2.1	4.4	6.3	8.0	9.5	10.9	12.1	13.2	14.2	15.1	16.0
8.00	0.0	0.0	0.8	3.1	5.1	6.9	8.5	9.9	11.1	12.3	13.3	14.3	15.2
8.50	0.0	0.0	0.0	1.8	3.9	5.7	7.4	8.9	10.2	11.4	12.5	13.5	14.4
9.00	0.0	0.0	0.0	0.5	2.7	4.6	6.3	7.8	9.2	10.5	11.6	12.6	13.6
9.50	0.0	0.0	0.0	0.0	1.5	3.5	5.2	6.8	8.3	9.5	10.7	11.8	12.8
10.00	0.0	0.0	0.0	0.0	0.3	2.3	4.2	5.8	7.3	8.6	9.8	10.9	12.0
10.50	0.0	0.0	0.0	0.0	0.0	1.2	3.1	4.8	6.3	7.7	9.0	10.1	11.2
11.00	0.0	0.0	0.0	0.0	0.0	0.1	2.0	3.8	5.4	6.8	8.1	9.3	10.4

Table 3. Optimum water availability for meeting crop requirements under different sorghum and natural gas prices in Area B.

	Price of Sorghum ($)												
P_{NG}	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
2.00	NI	NI	NI	NI	NI	NI	NI	I	I	I	I	I	I
2.50	NI	NI	NI	NI	NI	NI	NI	NI	I	I	I	I	I
3.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	I	I	I
3.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	I	I
4.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	I
4.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
5.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
5.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
6.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI

Table 4. Optimum irrigation applications for meeting crop requirements under different sorghum and natural gas prices in Area A.

	Price of Sorghum ($)												
P_{NG}	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
2.00	7.0	7.8	8.5	9.0	9.5	10.0	10.4	10.7	11.0	11.3	11.6	11.8	12.0
2.50	6.3	7.1	7.8	8.4	9.0	9.4	9.9	10.2	10.6	10.9	11.1	11.4	11.6
3.00	5.5	6.4	7.2	7.8	8.4	8.9	9.3	9.7	10.1	10.4	10.7	11.0	11.3
3.50	4.8	5.7	6.5	7.2	7.8	8.4	8.8	9.3	9.7	10.0	10.3	10.6	10.9
4.00	4.0	5.0	5.9	6.6	7.3	7.8	8.3	8.8	9.2	9.6	9.9	10.2	10.5
4.50	3.3	4.3	5.2	6.0	6.7	7.3	7.8	8.3	8.7	9.1	9.5	9.8	10.1
5.00	2.5	3.6	4.6	5.4	6.1	6.8	7.3	7.8	8.3	8.7	9.1	9.4	9.7
5.50	1.7	2.9	3.9	4.8	5.5	6.2	6.8	7.4	7.8	8.3	8.7	9.0	9.4
6.00	1.0	2.2	3.3	4.2	5.0	5.7	6.3	6.9	7.4	7.8	8.3	8.6	9.0
6.50	0.2	1.5	2.6	3.6	4.4	5.1	5.8	6.4	6.9	7.4	7.8	8.2	8.6
7.00	0.0	0.8	2.0	3.0	3.8	4.6	5.3	5.9	6.5	7.0	7.4	7.8	8.2
7.50	0.0	0.1	1.3	2.4	3.3	4.1	4.8	5.4	6.0	6.5	7.0	7.4	7.8
8.00	0.0	0.0	0.7	1.7	2.7	3.5	4.3	5.0	5.6	6.1	6.6	7.0	7.5
8.50	0.0	0.0	0.0	1.1	2.1	3.0	3.8	4.5	5.1	5.7	6.2	6.7	7.1
9.00	0.0	0.0	0.0	0.5	1.6	2.5	3.3	4.0	4.6	5.2	5.8	6.3	6.7
9.50	0.0	0.0	0.0	0.0	1.0	1.9	2.8	3.5	4.2	4.8	5.4	5.9	6.3
10.00	0.0	0.0	0.0	0.0	0.4	1.4	2.3	3.0	3.7	4.4	4.9	5.5	5.9
10.50	0.0	0.0	0.0	0.0	0.0	0.9	1.8	2.6	3.3	3.9	4.5	5.1	5.6
11.00	0.0	0.0	0.0	0.0	0.0	0.3	1.3	2.1	2.8	3.5	4.1	4.7	5.2

Table 5. Optimum irrigation strategy under different sorghum and natural gas prices in Area B.

	Price of Sorghum ($)												
P_{NG}	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
2.00	NI	NI	NI	NI	NI	NI	I	I	I	I	I	I	I
2.50	NI	NI	NI	NI	NI	NI	NI	I	I	I	I	I	I
3.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	I	I	I	I
3.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	I	I	I
4.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	I	I
4.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
5.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
5.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
6.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI

Table 6. Optimum percent potential evapotranspiration for grain sorghum production in Area A.

	Price of Sorghum ($)												
P_{NG}	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
2.00	66.4	69.7	72.6	75.1	77.2	79.2	80.9	82.4	83.8	85.0	86.2	87.2	88.1
2.50	63.0	66.6	69.7	72.4	74.8	76.8	78.7	80.3	81.8	83.1	84.3	85.5	86.5
3.00	59.7	63.6	66.9	69.7	72.3	74.5	76.4	78.2	79.8	81.2	82.5	83.7	84.8
3.50	56.4	60.5	64.0	67.1	69.8	72.1	74.2	76.1	77.8	79.3	80.7	82.0	83.1
4.00	53.0	57.4	61.2	64.4	67.3	69.8	72.0	74.0	75.8	77.4	78.9	80.2	81.5
4.50	49.7	54.4	58.3	61.8	64.8	67.4	69.8	71.9	73.8	75.5	77.1	78.5	79.8
5.00	46.4	51.3	55.5	59.1	62.3	65.1	67.6	69.8	71.8	73.6	75.3	76.8	78.2
5.50	43.1	48.2	52.6	56.4	59.8	62.7	65.4	67.7	69.8	71.7	73.5	75.0	76.5
6.00	39.7	45.1	49.8	53.8	57.3	60.4	63.1	65.6	67.8	69.8	71.6	73.3	74.8
6.50	36.4	42.1	46.9	51.1	54.8	58.0	60.9	63.5	65.8	67.9	69.8	71.6	73.2
7.00	33.1	39.0	44.1	48.5	52.3	55.7	58.7	61.4	63.8	66.0	68.0	69.8	71.5
7.50	29.8	35.9	41.2	45.8	49.8	53.3	56.5	59.3	61.8	64.1	66.2	68.1	69.8
8.00	26.4	32.9	38.4	43.1	47.3	51.0	54.3	57.2	59.8	62.2	64.4	66.4	68.2
8.50	23.1	29.8	35.5	40.5	44.8	48.6	52.0	55.1	57.8	60.3	62.6	64.6	66.5
9.00	19.8	26.7	32.7	37.8	42.3	46.3	49.8	53.0	55.8	58.4	60.8	62.9	64.9
9.50	16.5	23.6	29.8	35.1	39.8	43.9	47.6	50.9	53.8	56.5	58.9	61.2	63.2
10.00	13.1	20.6	27.0	32.5	37.3	41.6	45.4	48.8	51.8	54.6	57.1	59.4	61.5
10.50	9.8	17.5	24.1	29.8	34.8	39.2	43.2	46.7	49.8	52.7	55.3	57.7	59.9
11.00	6.5	14.4	21.3	27.2	32.3	36.9	41.0	44.6	47.9	50.8	53.5	55.9	58.2

Table 7. Optimum irrigation strategy based on ET requirements under different sorghum and natural gas prices in Area B.

	Price of Sorghum ($)												
P_{NG}	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
2.00	NI	NI	NI	NI	NI	NI	NI	NI	I	I	I	I	I
2.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	I	I	I	I
3.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	I	I
3.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	I
4.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
4.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
5.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
5.50	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI
6.00	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI	NI

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