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MAXIMUM (MINIMUM) BID (SELL) PRICE MODELS FOR LAND WHEN DEPRECIABLE ASSETS ARE INCLUDED IN THE TRANSACTION

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ABSTRACT

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This paper constructs maximum bid and minimum sell models to be used for the analysis of purchases and sales of real estate. Real estate includes land and depreciable assets sold or purchased together. The models in this paper extend those developed for land by Robison and Burghardt. Including depreciable assets together with the purchase and sale of land allows a more complete analysis of the role of taxes including the effects of the 1986 tax law.

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Introduction

Several studies have examined the break-even price for land, including Lee and Rask's, Baker's, and Robison and Burghardt's. Capital budgeting books, including Bierman and Schmid's, Aplin et al., and Canada and White's, have been written about methods which can be used to value depreciable assets. But frequently land and depreciable assets are sold together as real estate. The purpose of this paper is to develop general methods for finding minimum sell and maximum bid prices for real estate.

The development of the models follows capital budgeting principles suggested by Robison and Burghardt and which are generally accepted by others.[1/] The land models developed by Robison and Burghardt also form the foundations for the land portion of the real estate models developed here. And like their models, the one developed in this paper includes such features as real estate, income and capital gains taxes, transactions costs, holding period length, and financial arrangements. Moreover, the models are developed from both the break-even perspective of the buyer and the seller.

[1/] The five principles suggested by Robison and Burghardt are: (1) the homogeneity of measurements principle; (2) consistency in timing principle; (3) opportunity cost principle; (4) the life of the asset principle; and (5) the total cost and returns principle.

The need for a paper such as this and the models which it develops is because few transactions involve bare land. Most generally, a depreciable asset is included in the sale of the land. Drainage tiles, buildings, machinery, wells, sump pumps and lift stations, fences, mineral rights, and storage facilities are all examples of depreciable assets which may be included in the sale of land.

Including a depreciable asset with the purchase of a nondepreciable asset land complicates maximum bid and minimum sell price calculations in two fundamental ways. First, tax provisions require that the purchase price of real estate be divided between the depreciable and nondepreciable assets. The depreciable and nondepreciable assets are then taxed at different rates. Second, variables such as inflation may have differing effects on depreciable and nondepreciable assets leaving inflation's overall impact ambiguous whereas before it could be unambiguously signed. Consequently, capital gains may accumulate at a rate different than the rate of increase in income.

In what follows, extensions of the Robison and Burghardt (RB) models are developed to calculate maximum bid and minimum sell price models for real estate. The models developed in this paper are consistent with the five principles they suggest for building present value models and consistent with the 1986 tax laws. This paper does not, however, generate formulas for all tax depreciation schemes for depreciable assets which may arise. Instead, we present results consistent with the straight-line depreciation method.[2/]

Finally, the discount rate is generalized to alternatively reflect returns consistent with tax-free bonds, financial instruments whose earnings are taxed at the income tax rate, or investments whose effectivce tax rate

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[2/] A depreciable asset is defined in this paper as any asset whose value creates a tax shield resulting from its book value depreciation.

falls in between zero and the income tax rate including land and depreciable asset purchases. In an effort to avoid confusion, notation used by RB is continued here with minor exceptions. The next section begins by restating those definitions.

Definitions and Notations

Maximum (minimum) bid (sell) price models are one type of model included in the general class of present value models. Present value models consist of three parts: (1) the asset's bid (sell) price valued in current period dollars; (2) a stream of future costs and returns attributable to the acquisition use and maintenance of the asset; and (3) the discount rate which discounts future cash flows to their current period equivalent. If two of the three elements of the present value model are known, the third can be found. Or if all three are known, the difference between the asset's bid (sell) price and the discounted costs and returns can be calculated. This model, a net present value model, indicates whether an investment earns a return greater than, equal to, or less than the discount rate. The sign of the difference, a net present value, is often used as an investment criterion.

An asset's internal rate of return (IRR) can be used as another investment criterion when the asset's price and future cash flows are known. An investment's IRR is the discount rate or rate of return associated with the investment which equates the asset's price and future cash flow. One may compare the IRRs on similar investment opportunities and use the difference as an acceptance criterion. It, of course, goes without saying that multiple IRRs or no IRR at all may exist.[3/]

[3/] Conditions under which the IRR is unique is summarized in Brealey and Myer.

If the discount rate and the future cash flows are known, one can solve for the current period sum of the discounted cash flows and refer to the result as maximum (minimum) bid (sell) model. The term maximum (minimum) bid (sell) model is appropriately named because it represents an equality of attractiveness between the asset being purchased (or sold) and an alternative investment whose internal rate of return is described by the discount rate. Thus, the discount rate in the maximum (minimum) bid (sell) model is also an internal rate of return.

Maximum bid (minimum sell) price models provide information which can be used as a decision criterion for investment problems. If the maximum (minimum) bid (sell) price is greater (less) than the price for which for the asset can be purchased (sold), the investment (disinvestment) should be undertaken since the investment would then earn a rate in excess of the discount rate. But, all three elements of the present value problem must be known before a decision can be made. Thus, the maximum bid (minimum sell) models are designed to find indifference conditions; they do not by themselves provide decision criteria.

Maximum bid and minimum sell price models have many of the same cost and return considerations except they appear in different ways. For example, the maximum bid price for a potential buyer depends on the cost of acquiring the asset, expected returns and costs, tax considerations, and expected future sales prices (which, in turn, depends on expected income and costs of the next buyer). Thus, in the long run, every real estate purchaser eventually becomes a real estate seller. And every seller weighs the sell opportunities with the benefits from continued ownership.

Some of the variables influencing buyers and sellers of bare land are:

r = a real rate of return available to the firm on its investments,

T = a constant proportional income tax rate paid by the firm,

α = the capital gains tax rate where α is a parameter previously valued at .4, but whose value under 1986 tax laws is 1,

T_p = the real estate tax rate paid on the nominal real estate value,

i = the general rate of inflation included in the discount rate,

g = the rate of change in the returns from the real estate within an ownership period which must include the effect of changes in the productivity of land and any attached depreciable assets,

R = net cash return from the land and attached depreciable assets in the base year excluding land costs,

n = the length of time the real estate will be owned and held by each of the m future real estate buyers,

s = the percentage of the real estate's sale price paid as realtor's fee,

c = the percent of the real estate's acquisition price paid as a loan closing fee which includes fees for title searches and points charged to close a loan,

q = the term on the loan used to finance the control of the real estate for the first buyer,

q_s = the term on the loan held by the real estate seller at the time of the sale,

D = the percentage of the real estate's price the buyer pays as a downpayment,

r* = the nominal interest rate paid on funds borrowed by the buyer to acquire the real estate's asset,

r_s* = the nominal interest rate on a loan held by the seller at the time of the real estate sale,

L = the seller's outstanding loan balance at the time of the real estate sale,

V_0 = real estate price paid by the seller when originally acquired.

Using the variables defined above, we will solve for the following maximum bid and minimum sell prices for real estate:

V = the maximum bid price,

V_s = the minimum sell price,

V^* = the maximum bid price with the buyer's purchase being financed at rate r^*,

V_s^* = the minimum sell price with the seller financing the buyer's purchase. In this category, how capital gains are taxed depends on the purchase date and the sale date. Thus, for case 1, V_{s1}^*, $T^* = \alpha T$. For case 2, V_{s2}^*, $T^* = T$. For sales occurring after the effective date of the 1986 tax law, $V_{s1}^* = V_{s2}^*$,

V_s^c = the minimum sell price with a ''due on sale'' clause.

In addition to the variables described above and used in the RB land models, additional variables are required to model real estate sales and purchases. The additional variables include:

D_a = market value in the current period of the depreciable asset attached to the land,

n_d = the tax life of the depreciable asset,

n_o = the age of the seller's depreciable asset or the number of years it has been in service,

D_{ao} = original price of the depreciable asset when put in service n_o periods ago,

$$T^* = \begin{cases} T \text{ if current year (e.g., 1986) less } n_0 > 1980^{4/} \\ \alpha T \text{ if current year (e.g., 1986) less } n_0 < 1981 \end{cases}$$

d = a parameter determining the percentage decline in the remaining useful life of the depreciable asset,

δ = a tax adjusted coefficient applied to the discount rate.

The variable ''d,'' the last variable defined above, determines the useful life of the depreciable asset. The useful life and the rate of inflation in turn determine the value of the depreciable asset exchanged between buyer and seller in future periods. Thus, some discussion of its empirical significance is in order. The value $(1+d)^{-t}$ where t is the number of elapsed time periods approaches zero asymptotically as t gets large so that it gets close but never reaches zero. One could ask, however, how many periods are required for the depreciable asset to be reduced to 5 percent of its original real value?

Such an expression would satisfy the equation:

$$(1) \quad .05 \leq (1+d)^{-t}$$

Alternative values of d and t which satisfy equation (1) are described in Table 1.

Tax Implications of Owning Depreciable Assets

One of the major differences between maximum (minimum) bid (sell) models for land and maximum (minimum) bid (sell) models for real estate is taxes. Land and depreciable assets are taxed differently so that including them both in a present value model requires their tax implications be treated differently. Moreover, consistency requires the discount rate also be adjusted for taxes. But the

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4/ For example, suppose $n_0 = 3$, i.e., the durable is 3 years old at sale time. Then $1986 - n_0 = 1983 > 1980$ and $T^* = T$.

Table 1

Number of Periods Required to Reduce a Depreciable Asset to 5 Percent of Its Original Capacity for Alternative Values of ''d''

d	Periods Required to Reduce a Durable to 5 Percent of Its Original Capacity [t=-ln(.05)/ln(1+d)]
.01	301
.02	151
.03	101
.04	76
.05	61
.06	51
.07	44
.08	39
.09	35
.10	31
.11	29
.12	26
.13	25
.14	23
.15	21
.16	20
.17	19
.18	18
.19	17
.20	16
.21	16
.22	15
.23	14
.24	14
.25	13
.26	13
.30	12
.40	9
.60	6
.80	5
1.00	4

question is at what tax rate should the discount rate be adjusted? The answer is it all depends on what is the next best investment opportunity available to the firm?

If the next best investment opportunity is a financial instrument whose returns are taxed at the firm or individual's income tax rate T, then the appropriate tax adjustment is to multiply the rate of return by (1-T). If the next best investment is a tax-free bond, then the appropriate discount rate is the rate of return on the bond unadjusted for taxes. Finally, if the next best investment opportunity is an asset whose returns are partially shielded from income tax, the appropriate tax rate by which the discount rate is adjusted is $(1-\delta T)$ where $0 < \delta < 1$. Of course, setting $\delta = 1$ or $\delta = 0$, models the tax adjustment required for financial instruments and tax-free bonds, respectively.

Having introduced taxes into the discount rate, attention is now directed at capturing the tax implications of an investment when a depreciable asset is attached to land.

The first step in calculating the tax implication resulting from including depreciable assets in the purchase of real estate is establishing the depreciable asset's beginning book value. This value, D_a, should correspond to the market value of the depreciable asset and legally (although not always done in practice) must be agreed on by the seller and the buyer. The buyer is then allowed to prorate closing fees of c percent of the purchase price to the depreciable asset establishing a cost basis or beginning book value of $(1+c)D_a$. The periodic tax shield when the depreciable life of the durable is n_d and straight-line depreciation is used then becomes $(1+c)D_a/n_d$.[5/]

[5/] It is recognized that for some depreciable assets, ACRS or other depreciation methods would be used. Model limitations preclude us from generalizing the method of tax depreciation. Instead, n_d can be altered to approximate the actual tax depreciation schedule followed.

Further considerations of the tax implication of real estate purchases which include depreciable assets require the consideration of two separate cases. The first case is the more usual, where the depreciable life is less than or equal to the expected holding period n, ($n_d \leq n$).

In the second case, $n_d > n$, the expected holding period is less than the depreciable life. In this case, the buyer does not take all the tax savings possible. To model these two separate cases, a new variable n_d^* is defined as:

$$n_d^* = \begin{cases} n_d & n_d \leq n \\ n & n_d > n \end{cases}$$

This definition allows us to write the present value of the periodic tax shield as:

$$\text{(1a)} \quad D_1 = \frac{D_a T(1+c)}{n_d[1+(r+i+ir)(1-\delta T)]} + \ldots + \frac{D_a T(1+c)}{n_d[1+(r+i+ir)(1-\delta T)]} n_d^*$$

$$= (1+c) D_a D_1^*$$

where:

$$D_1^* = \frac{T}{n_d(r+i+ir)(1-\delta T)} \left\{ 1-[1+(i+r+ir)(1-\delta T)]^{-n_d^*} \right\}$$

The above formulation assumes that the general rate of inflation included in the discount rate is i percent. Inflation under current tax law, however, does not alter the book value of the asset D_a which was established at the time of purchase. Thus, the periodic tax saving is independent of inflation and equals TD_a/n_d. On the other hand, increasing inflation reduces the present value of those savings because inflation increases the discount rate.

In addition to the inflationary effect on the tax shield, inflation complicates matters at the end of n periods when the land and the attached depreciable assets are sold. If the sale price of the depreciable asset exceeds the depreciated book value, the difference is subject to depreciation recapture provisions. An ordinary income tax obligation on the difference is imposed up to the original purchase price. If the depreciable asset is sold at an amount exceeding the original purchase price, previous to 1987, that difference between the current and original price is taxed at the capital gains rate.

Tax laws passed in 1986 increase the capital gains tax rate to T. This change will affect tax write-offs when the sale price of the depreciable asset exceeds its original book value. This condition is most likely to be satisfied during periods of high inflation rates; that is, when the inflation rate exceeds the productivity decay of the depreciable asset ($i > d$).

At the time of the sale, the buyer and seller must agree to the current value of the depreciable asset. From a tax standpoint, it is in the seller's best interest to allocate a small portion of the sale to the depreciable asset. This approach allows the seller to mitigate the effects of recapture provisions which tax the difference between the depreciable asset's adjusted book value at the time of the sale and the asset's sale price in the nth period. On the other hand, it is in the buyer's best interest to allocate a larger portion of the purchase price to the depreciable asset since establishing a large book value increases the value of future tax shields.

To model real estate transactions requires that the buyer and seller agree to the sale price of the depreciable asset. This is done in our model by discounting the original purchase price of the depreciable asset, D_a, at the rate at which its useful life declines, d, and inflating it at the rate of inflation i. Thus, a depreciable asset whose original purchase price was D_a, is calculated

to have a current market value n periods after purchased of $D_a(\frac{1+i}{1+d})^n$. The current book value of the depreciable asset n_d* periods after purchase is $(1+c)D_a(1-\frac{n_d*}{n_d})$. If the difference between the current market value less the current book value is positive, the seller faces a tax obligation. If the difference is negative, the sale of the depreciable asset creates a tax savings for the seller at the income tax rate.

To calculate the tax obligation assuming the depreciable asset's current market value exceeds its current (depreciated) book value, two cases are considered. First, if i is less than or equal to d, then $D_a(\frac{1+i}{1+d})^n$ is less than or equal to D_a. This implies the current market value of the depreciable asset is less than its original purchase price and the difference between the depreciable asset's current market value and its current book value will create a tax obligation at the income tax rate. This tax obligation is equal to $D_aD_2{}^a$ where $D_2{}^a$ equals:

$$(2) \qquad D_2{}^a = \frac{T[(\frac{1+i}{1+d})^n(1-s) - (1-\frac{n_d^*}{n_d})(1+c)]}{[1+(r+i+ir)(1-\delta T)]^n} \qquad \text{for } (\frac{1+i}{1+d})^n\ (1-s) \leq (1+c)$$

where s is the percentage of the sale price paid by the seller as realtors' fees which are prorated over the market value of the depreciable asset.

On the other hand, if i is greater than d, then $D_a(\frac{1+i}{1+d})^n$ is greater than D_a and the seller will face a future tax obligation taxed at both the income and capital gains tax rates. The tax rate on the market value less the initial purchase price will be taxed at the capital gains tax rate of αT (which after 1986 is T). The difference between the original purchase price and the current book value after n_d^* periods is taxed at the income tax rate. The present value of the future tax obligation where i is greater than d is written as $D_aD_2^b$ where D_2^b

equals:[6/]

$$(3) \quad D_2^b = \frac{T[\alpha(1-s)(\frac{1+i}{1+d})^n + (1+c)\frac{n_d^*}{n} - \alpha(1+c)]}{[1+(r+i+ir)(1-\delta T)]^n} \qquad \text{for } (\frac{1+i}{1+d})^n (1-s) > (1+c)$$

Finally define D_2^* as:

$$(4) \quad D_2^* = \begin{cases} D_2^a & \text{for} \quad (\frac{1+i}{1+d})^n (1-s) \leq (1+c) \\ D_2^b & \text{for} \quad (\frac{1+i}{1+d})^n (1-s) > (1+c) \end{cases}$$

Then the present value of the future tax obligation is simply $D_a D_2^*$. The effect of the new tax law can easily be determined by comparing the results when $\alpha = .4$, as was the case previous to 1987, and $\alpha = 1$ under the new 1986 tax law.

An Example

An example may clarify the formulas developed so far. Suppose drainage tile associated with a land purchase made in 1986 is valued at 300 dollars an acre. The purchaser depreciates the tile using the straight line method over 10 years.[7/] Suppose the discount rate (r+i+ir) is 12 percent of which 7 percent is attributed to inflation and the buyer is in the 25 percent tax bracket. Moreover, assume the buyer intends to hold the land for 8 years. Finally, assume the tile's service capacity depreciates at roughly 4 percent per year, and the

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[6/] Note that the book value of the asset after n periods, $D_a(1-\frac{n_d^*}{n_d})$, may exceed the sale price, $D_a(\frac{1+i}{1+d})^n$, for large d and small i. In such a case, a tax credit is earned at the time of sale in the form of a capital loss.

[7/] The straight-line depreciation is recognized as only an approximation of the actual tax depreciation schedule used--in this case ACRS. Straight-line is an available option and is used when income streams are expected to be greater in later periods.

buyer's closing fees are charged at the rate of 5 percent while the seller's closing fees equal 3 percent. Finally, assume the next best investment has returns taxed at the income tax rate so that δ=1. Summarizing the assumptions on a per acre basis:

s	= 3 percent
c	= 5 percent
T	= 25 percent
n	= 8
n_d	= 10
δ	= 1
α	= 40 percent
n_d^*	= 8
i	= 7 percent
D_a	= \$300
d	= 4 percent
$(r+i+ir)$	= 12 percent

Substituting into (1a), we find $(1+c)D_aD_1^*$ equal to:

$$(5) \quad (1+c)D_aD_1^* = \frac{(1.05)(300)(.25)}{(10)(.12)(.75)} \left\{1-[1+(.12)(.75)]^{-8}\right\} = \$43.59$$

To calculate the present value of the future tax obligation at the time of sale, we note that i = 7 percent is greater than d = 4 percent. Therefore, $D_2^*=D_2^b$ is the appropriate formula and using equation (3) $D_aD_2^*$ equals:

$$\$300\ D_2^* = (\$300)(.25)\ \frac{[\frac{8}{10}(1.05) + .4(\frac{1.07}{1.04})^8.97 - .4(1.05)]}{[1+(.12)(.75)]^8}$$

$$= \$34.14$$

The net tax effect resulting from the purchase of the depreciable asset is then:

$$D_a[(1+c)D_1^* - D_2^*]$$

$$\$43.59 - \$34.14 = \$9.45$$

In addition to the example just constructed, it is useful to examine the comparative static results of the tax effects of a depreciable asset. To do so, let n become large so that:

$$(6) \quad \lim_{n\to\infty} D_a[(1+c)D_1^* - D_2^*] = (1+c)D_a D_1^* = y$$

Then it follows that:

$$(7a) \quad \frac{dy}{dT} \geq 0,$$

$$(7b) \quad \frac{dy}{dD_a} > 0,$$

$$(7c) \quad \frac{dy}{di} < 0,$$

$$(7d) \quad \frac{dy}{dn_d} < 0,$$

$$(7e) \quad \frac{dy}{dr} < 0, \text{ and}$$

$$(7f) \quad \frac{dy}{d\delta} > 0.$$

In other words, the tax savings from depreciable assets relative to tax saving of other investments depend on the tax rate applied to alternative investments. The value of the tax shield increases with increases in the tax rate and in the value of the depreciable asset. The value of the tax shield decreases with increases in inflation, the depreciable life of the asset, and the real interest rate. As the effective tax rate on alternative investments increases, the present value of the tax shield increases.

The formulas for D_1^* and D_2^* are now used to calculate the effect of depreciable assets on the maximum bid prices and the minimum sell prices for land. The methods developed in RB are followed closely, assuming that the income pattern described by the net cash returns R and the rate of change in returns g includes the influence of the depreciable asset on the earnings from land. Thus, the RB formulas need only to be modified for tax effects associated with the depreciable asset.[8/]

The Maximum Bid Price Model (V)

A maximum bid price model solves for the highest price which can be offered by the buyer (in the current period) and still earn an internal rate of return equal to the discount rate. To facilitate the calculations of the maximum bid price V in the current period, our calculations are divided into two components: costs and returns.

The cost component, from the perspective of the buyer, can be expressed with two equations. The first cost equation is the purchase price V plus the closing costs cV of the real estate purchase. This can be written as:

(8) $B_1 = (1+c)V$

The second cost equation calculates the present value of real estate taxes. The present value of real estate taxes to be paid over n periods by the current period buyer, hereafter called the first buyer, can be written as B_2 equal to:

[8/] An additional change to the RB formulas is the inclusion of δ which allows us to generalize the tax implications associated with the next best investment.

$$(9)\quad B_2 = \frac{VT_p(1-T)}{[1+(r+i+ir)](1-\delta T)} + \ldots + \frac{V(1+g)^{n-1}T_p(1-T)}{[1+(r+i+ir)(1-\delta T)]^n}$$

$$= \frac{VT_p(1-T)}{[(r+i+ir)(1-\delta T)-g]}\left[1 - \frac{(1+g)^n}{[1+(r+i+ir)(1-\delta T)]^n}\right] = \frac{VT_p(1-T)(..)}{(.)}$$

where:

$$(.) \quad = [((r+i+ir)(1-\delta T))-g]$$

$$(..) \quad = [1 - \frac{(1+g)^n}{(\ldots)}]$$

It should be noted in equation (9) that real estate taxes are tax deductible and, therefore, are reduced by the tax rate T. The value of the real estate to which the tax rate T_p is applied is assumed to inflate at an annual rate of g percent, equal to the rate of real growth in cash returns.[9/]

To complete the model, three return components must be calculated and added and set equal to the present value of costs. The first returns component is cash returns. If returns are inflating at g percent, then the cash return series B_3 for the n periods controlled by the first buyer can be written as:

$$(10)\quad B_3 = \frac{R(1-T)}{[1+(r+i+ir)(1-\delta T)]} + \ldots + \frac{R(1+g)^{n-1}(1-T)}{[1+(r+i+ir)(1-\delta T)]^n}$$

$$= \frac{R(1-T)}{[(r+i+ir)(1-\delta T)-g]}\left(1 - \frac{(1+g)^n}{[1+(r+i+ir)(1-\delta T)]^n}\right) = \frac{R(1-T)(..)}{(.)}$$

[9/] RB showed that the maximum (minimum) bid (sell) price for land would increase at rate g percent. We continue to assume that here even though, as we show later, the RB result does not always hold in the case of real estate.

The second returns component is the tax savings created by the depreciable asset which was calculated to be $D_a[D_1^*(1+c)-D_2^*]$. To this tax savings, however, is added the sale price of the depreciable asset adjusted for realtors' fees. The sum, a return to the buyer, is expressed as B_4^a:

$$(11) \quad B_4^a = D_a[D_1^*(1+c)-D_2^*] + \frac{(1-s)\ D_a\ (\frac{1+i}{1+d})^n}{(\ldots)}$$

where:

$$(\ldots) = [1 + (r+i+ir)(1-\delta T)]^n$$

The third returns component is the value of the land after n periods, V_n, reduced by the sale price of the depreciable asset adjusted for capital gains tax and discounted to the present value. This sum is expressed as B_4^b and equals:

$$(12) \quad B_4^b = \frac{\left\{[V_n-D_a(\frac{1+i}{1+d})^n]\ (1-s) - (V-D_a)(1+c)\right\}\ (1-\alpha T)}{(\ldots)}$$

$$+ \frac{(V-D_a)(1+c)}{(\ldots)}$$

Adding B_4^a to B_4^b, we obtain B_4 equal to:

$$(13) \quad B_4 = \frac{V_n(1-s)(1-\alpha T)}{(\ldots)} + \frac{\alpha TV(1+c)}{(\ldots)} + \frac{\alpha TD_a(\frac{1+i}{1+d})^n(1-s)}{(\ldots)}$$

$$- \frac{\alpha TD_a(1+c)}{(\ldots)} + D_a[D_1^*(1+c)-D_2^*]$$

The maximum bid price model can be summarized by equating returns B_3+B_4 to costs B_1+B_2. Substituting for B_1, B_2, B_3, and B_4 and solving for V, results in:

$$(14)^{10/} \quad V = \frac{R(1-T)(..)}{(.)k_1} + \frac{D_a k_1^*}{k_1} + \frac{V_n(1-s)(1-\alpha T)}{(...)k_1}$$

where:

$$k_1 = (1+c) + \frac{T_p(1-T)(..)}{(.)} - \frac{\alpha T(1+c)}{(...)} \quad \text{and}$$

$$k_1^* = \frac{\alpha T(\frac{1+i}{1+d})^n(1-s)}{(...)} - \frac{\alpha T(1+c)}{(...)} + (1+c)D_1^* - D_2^*$$

Solving equation (14) is a problem since V_n is unknown. One cannot merely assume it is $V(1+g)^n$ because of transactions cost and the tax consequences of the depreciable asset. As a result, V_n is written as the maximum bid price for the buyer purchasing the asset in the nth period (referred to hereafter as the second buyer, etc.):

$$(15) \quad V_n = \frac{R(1-T)(1+g)^n(..)}{(.)k_1} + \frac{V_{2n}(1-s)(1-\alpha T)}{(...)k_1} + \frac{D_a k_1^*}{k_1}\left(\frac{1+i}{1+d}\right)^n$$

Similarly, V_{2n}, the third buyer's maximum bid price in period 2n, can be expressed as:

$$(16) \quad V_{2n} = \frac{R(1-T)(1+g)^{2n}(..)}{(.)k_1} + \frac{V_{3n}(1-s)(1-\alpha T)}{(...)k_1} + \frac{D_a k_1^*}{k_1}\left(\frac{1+i}{1+d}\right)^{2n}$$

One could continue expressing such maximum bid prices up to the mth buyer in the mn^{th} period, which would depend on a terminal value for real estate equal to $V_{n(m+1)}$.

We can avoid assuming a terminal value for real estate for a particular buyer by making successive substitutions and taking the limit of m. To do so,

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10/ Compare with equation (25) in RB.

equation (15) is substituted for V_n, then (16) for V_{2n}, etc. Then finding geometric sums from our equation and taking the limit of m, V can be solved for explicitly in terms of known parameters.

The geometric series obtained after successive substitutions and factorings is:

$$(17) \quad V = \frac{R(1-T)(..)B_5}{(.)k_1} + \frac{D_a k_1^* B_6}{k_1}$$

where:

$$B_5 = 1 + \frac{(1+g)^n(1-s)(1-\alpha T)}{(\ldots)k_1} + \ldots + \frac{(1+g)^{mn}(1-s)^m(1-\alpha T)^m}{k_1^m(\ldots)^m}, \text{ and}$$

$$B_6 = 1 + \frac{(\frac{1+i}{1+d})^n(1-s)(1-\alpha T)}{(\ldots)k_1} + \ldots + \frac{(\frac{1+i}{1+d})^{mn}(1-s)^m(1-\alpha T)^m}{k_1^m(\ldots)^m}$$

Let the geometric factors in the braced expression be represented by k_2 and k_2^* equal to:

$$(18a) \quad k_2 = \frac{(1+g)^n(1-s)(1-\alpha T)}{k_1(\ldots)}$$

and

$$(18b) \quad k_2^* = \frac{(\frac{1+i}{1+d})^n(1-s)(1-\alpha T)}{k_1(\ldots)}$$

It is known that k_2 and k_2^* are less than one because (...) is greater than $(1+g)^n$ or $(\frac{1+i}{1+d})^n$. Thus, the series B_5 and B_6 converge to:

$$(19a) \quad \lim_{m\to\infty} B_5 = 1/(1-k_2)$$

and

$$(19b) \quad \lim_{m\to\infty} B_6 = 1/(1-k_2^*)$$

The maximum bid price V in equation (17) can now be expressed as:

$$(20)\quad V = \frac{R(1-T)(..)}{(.)k_1(1-k_2)} + \frac{D_a k_1^*}{k_1(1-k_2^*)}$$

$$= \frac{R(1-T)(..)}{(.)k_1(1-k_2)} + \frac{D_a k_1^*}{[k_1 - (\frac{1+i}{1+d})^n \frac{(1-s)(1-\alpha T)}{(...)}]}$$

A simplified approach for analyzing this model is to assume the asset is traded only once; that is, assume the length of time between transactions n is infinitely long. Under such an assumption, V can be expressed as:[11/]

$$(21)\quad \lim_{n\to\infty} V = \frac{R(1-T)}{[(r+i+ir)(1-\delta T)-g](1+c)+T_p(1-T)}$$

$$+ \frac{D_a(1+c)T \left\{1-[1+(r+i+ir)(1-\delta T)]^{-n_d}\right\}}{[n_d(r+i+ir)(1-\delta T)] \left\{(1+c) + \frac{T_p(1-T)}{[(r+i+ir)(1-\delta T)-g]}\right\}}$$

In the above formulation, s does not enter since a sales commission is never paid by the first buyer. Otherwise, those variables which can be signed unambiguously are:

$$(22a)\quad \frac{dV}{dR} > 0$$

$$(22b)\quad \frac{dV}{dg} > 0$$

[11/] It is helpful when finding the limit of V to recognize that:

$$\lim_{n\to\infty} k_1 = (1+c) + \frac{T_p(1-T)}{(.)}, \quad \lim_{n\to\infty} (..) = 1, \quad \lim_{n\to\infty} k_2 = 0,$$

$$\lim_{n\to\infty} k_1^* = \frac{(1+c)T}{n_d(r+i+ir)(1-\delta T)} [1 - \frac{1}{[1+(r+i+ir)(1-\delta T)]^{n_d}}], \text{ and } \lim_{n\to\infty} k_2^* = 0$$

(22c) $\frac{dV}{dD_a} > 0$

(22d) $\frac{dV}{dn_d} < 0$

(22e) $\frac{dV}{dT_p} < 0$

(22f) $\frac{dV}{dr} < 0$

(22g) $\frac{dV}{di} < 0$

The remaining variables, T and c, have ambiguous influence on the depreciable asset and the income stream.

To understand the signs of the derivatives described above recall the perspective of the first buyer. The first buyer asks: Should a variable, call it x, increase, how must the maximum bid price V change so that I can earn the internal rate of return reflected by the discount rate?

Consider first the variables net cash returns R and the rate of change in the returns g. If R or g become larger, the value of the future income stream increases making ownership more attractive. As a consequence, the first buyer's bid price can increase and still leave the first buyer indifferent about purchasing the real estate or investing elsewhere at the rate of return reflected by the discount rate.

If the market value of the asset D_a, the initial purchase price of the depreciable asset, increases, the future tax shield increases making ownership more attractive. On the other hand, increasing the time period n_d over which the depreciation is taken reduces the present value of the tax shield and makes ownership less attractive. Consequently, V increases in response to an increase in D_a and decreases in response to an increase to n_d in order to remain in equilibrium.

Meanwhile, increasing the real estate tax rate unambiguously reduces the attractiveness of ownership and reduces V.

Increasing the real rate of return r and the general inflation rate i reduces the attractiveness from ownership in two ways. Increasing r and i both increases the attractiveness of investing elsewhere at the discount rate. Or explained another way the present value of future increases in income and the present value of future tax shields are both reduced with increases in i and r requiring V to decrease to maintain equilibrium.[12/]

An interesting question, having solved for V, is: what is V_n? The procedure for finding V_n is the same as for finding V; except initial income is $(1+g)^n R$ and the beginning value of the depreciable asset is $D_a(1+i)^n/(1+d)^n$. Thus, V_n can be written as:

$$(23) \quad V_n = \frac{R(1+g)^n(1-T)(..)}{(.)k_1(1-k_2)} + \frac{D_a k_1^*}{k_1(1-k_2^*)} \left(\frac{1+i}{1+d}\right)^n$$

Then by setting:

$$(24) \quad V - \frac{D_a k_1^*}{k_1(1-k_2^*)} = \frac{R(1-T)(..)}{(.)k_1(1-k_2)}$$

and substituting, we find:

$$(25) \quad V_n = V(1+g)^n + \frac{D_a k_1^*}{k_1(1-k_2^*)} \left[\left(\frac{1+i}{1+d}\right)^n - (1+g)^n\right]$$

[12/] To unambiguously sign r and i, it is important to recognize that:

$$\frac{d\left[\frac{(r+i+ir)(1-T)}{(r+i+ir)(1-T)-g}\right]}{dr} \quad \text{and} \quad \frac{d\left[\frac{(r+i+ir)(1-T)}{(r+i+ir)(1-T)-g}\right]}{di}$$

are both less than zero.

Without consideration of a depreciable asset, RB showed capital gains increased at g percent per year. With depreciable assets, the capital gains earning rate is not just g but also depends on i and d. If $(d+g) > i$, it can be easily shown that V changes by a rate less than g.

The formulas derived in the calculation of V will be useful in the calculation of our second model, the minimum sell price model V_s.

The Minimum Sell Price Model (V_s)

The minimum sell price model calculates the smallest price the seller can accept and still earn on the real estate an internal rate of return equal to the discount rate. The minimum sell models differ in significant ways from the maximum bid models. First, the buying and selling transaction occurs at different times for buyers and sellers. A buyer acquires the asset in the current period and sells it n periods in the future. A seller acquired the asset n_o periods ago and sells it in the current period. This timing difference requires that taxation associated with the depreciable assets be treated differently between the maximum bid and minimum sell price models.

For the most part, for depreciable assets placed in service before 1981, depreciation recapture is ignored if straight-line depreciation has been used. Under these circumstances, all capital gains are taxed at the capital gains tax rate αT. In other words, the difference between the depreciable asset's current market value less its current book value is taxed at the capital gains tax rate.

On the other hand, for depreciable assets placed in service after 1980 and before 1987, the difference between the asset's current market value and its current book value is taxed at the income tax rate. If the depreciable asset is sold for a price exceeding its original purchase price, the difference between its current market value and the original purchase price adjusted for closing

costs is taxed at the capital gains tax rate αT. But, as a result of the new tax law, beginning in 1987 $\alpha = 1$.

This difference in the taxation of depreciable assets requires the definition of a new variable T* equal to:

$$(26) \quad T^* = \begin{cases} T & \text{if depreciable asset was purchased} \geq 1981 \\ \alpha T & \text{if depreciable asset was purchased} < 1981 \end{cases}$$

Maximum bid and minimum sell price models differ significantly in the treatment of taxes. But in other aspects, they are modeled in similar ways. In the maximum bid price model, the comparison was between investing at the after-tax equity at a rate of $(r+i+ir)(1-\delta T)$ versus the opportunity cost of investing in land. The present values of the two investments were then set equal to each other.

The minimum sell price model makes a similar but an alternative comparison. It compares the present value of after-tax proceeds from the sale of land with the alternative of keeping the land for an additional n periods. The proceeds from the land sold are assumed to be invested at the after-tax rate of $(r+i+ir)(1-\delta T)$. Equating the present value of the two investments, the minimum sell price model solves for the selling price which equates the present value of the two alternatives. We now proceed to construct the minimum sell price model.

Assume the seller has held the land and depreciable asset for n_0 periods. Moreover, let the depreciable asset's current market value be D_a, and identify its original market value n_0 periods earlier as D_{ao}. Moreover, the depreciable asset's current book value value is:

$D_{ao}(1-\frac{n_o^*}{n_d})$ where:

$$(27) \quad n_o^* = \begin{cases} n_d & n_d \leq n_o \\ n_o & n_d > n_o \end{cases}$$

The tax obligation created by the sale of the depreciable asset depends on its purchase date and the size of D_a relative to D_{ao}. If D_{ao} is assumed to be less than D_a, then the difference between the depreciable asset's current market value D_a and its current book value $D_{ao}(1-\frac{n_o^*}{n_d})$ will create a tax obligation (or credit) at the tax rate T* which depends on the purchase date. This tax obligation is calculated as D_3^a where:

$$(28) \quad D_3^a = T^*[(1-s)\ D_a - D_{ao}\ (1 - \frac{n_o^*}{n_d})(1+c)]$$

where: $D_a(1-s) < D_{ao}(1+c)$

Since the tax obligation is owed in the current period, the time of sale, D_3^a is not discounted. The tax obligation is reduced by closing fees paid to both purchase and sell the asset.

On the other hand, if $D_a(1-s)$ is greater than or equal to $D_{ao}(1+c)$, then the seller faces a tax obligation at both the income and capital gains tax rates depending on the date of purchase. The tax rate on the current market value less the original purchase price adjusted for closing costs will be taxed at the capital gains tax rate of .4T if the asset was placed into service before 1987. The difference between the original purchase price and the current book value n_o periods later adjusted for closing costs is taxed at rate T* which depends on the purchase date of D_{ao}. This tax obligation where $D_a(1-s)$ is greater than $D_{ao}(1+c)$ is written as D_3^b:

$$(29)\quad D_3^b = [\alpha T\ D_a(1-s) + (1+c)\ D_{ao}\ (\frac{n_o^*}{n_d}\ T^* - \alpha T)]$$

where: $D_a(1-s) \geq D_{ao}(1+c)$

Finally, define D_3^* as:

$$(30)\quad D_3^* = \begin{cases} D_3^a & \text{for} \quad D_a(1-s) < D_{ao}(1+c) \\ D_3^b & \text{for} \quad D_a(1-s) \geq D_{ao}(1+c) \end{cases}$$

Thus, if the land and attached depreciable asset are sold, a current tax obligation on the depreciable asset will be D_3^*.

On the other hand, the sale price of the depreciable asset, adjusted for closing costs, is $(1-s)D_a$. The sale price of the depreciable asset less tax obligations can then be defined as:

$$(31)\quad S_1^a = (1-s)D_a - D_3^*$$

In addition, the sale value of the land, a return to the seller, is calculated as follows. Let V_s be the sale price of the land plus the depreciable asset. Then define S^b as the sale price of the land less taxes which equals:

$$(32)\quad S_1^b = \left\{(V_s-D_a)(1-s)-(1+c)[V_o-D_{ao}]\right\}(1-\alpha T) + [V_o-D_{ao}](1+c)$$

Finally, adding S_1^a to S_1^b, we calculate the net proceeds of the sale as S_1^* equal to:

$$(33)\quad S_1^* = S_1^a + S_1^b$$

$$= V_s(1-s)(1-\alpha T) + \alpha T\ (V_o-D_{ao})(1+c) + \alpha T\ D_a(1-s) - D_3^*$$

The alternative to selling the land and depreciable asset in the current period is to hold it n periods longer. The value of this alternative is calculated in much the same way as it was for the maximum bid model. It includes B_3, the inflating cash returns less the present value of property taxes equal to B_2. It also includes the present value of the tax shield over the remaining depreciable periods.

Define n^*_{do} as:

$$(34) \quad n^*_{do} = \begin{cases} n_d - n_o & n_d - n_o \leq n \\ n & n_d - n_o > n \end{cases}$$

Then the present value of the tax shield is D^*_{10} which equals:

$$(35) \quad D^*_{10} = \frac{T\ D_{ao}}{n_d(r+i+ir)(1-\delta T)} \left\{ 1-[1+(i+r+ir)(1-\delta T)]^{-n^*_{do}} \right\}$$

The tax obligation associated with holding the land and depreciable asset another n periods is calculated next. To do so, consider two cases. First, if the depreciable's initial purchase price adjusted for closing costs, $D_{ao}(1+c)$, is less than its market value net of closing costs n periods in the future, $D_a(\frac{1+i}{1+d})^n(1-s)$, then the difference will be taxed at the tax rate T* which depends on the depreciable asset's purchase date. This future tax obligation is written as D^a_{20} which equals:

$$(36) \quad D^a_{20} = T^*[(\frac{1+i}{1+d})^n D_a(1-s) - D_{ao}\ (1-\frac{n^*_{do}+n_o}{n_d})(1+c)]$$

$$\text{for} \qquad D_a(\frac{1+i}{1+d})^n\ (1-s) \leq D_{ao}(1+c)$$

On the other hand, if $D_a(\frac{1+i}{1+d})^n(1-s)$ is greater than $D_{ao}(1+c)$, the seller may face a future tax obligation at the income and/or capital gains tax rates depending on when the depreciable(s) are placed in service. This future tax obligation is written as D^b_{20} which equals:

$$(37) \quad D^b_{20} = \alpha T[(\frac{1+i}{1+d})^n D_a(1-s) - D_{ao}(1+c)] + T^*[D_{ao}(\frac{n^*_{do} + n_o}{n_d})(1+c)]$$

$$\text{for} \quad D_a(\frac{1+i}{1+d})^n(1-s) > D_{ao}(1+c)$$

Finally, define D^*_{20} as:

$$(38) \quad D^*_{20} = \begin{cases} D^a_{20} & \text{for} \quad D_a(\frac{1+i}{1+d})^n(1-s) \leq D_{ao}(1+c) \\ D^b_{20} & \text{for} \quad D_a(\frac{1+i}{1+d})^n(1-s) > D_{ao}(1+c) \end{cases}$$

Then the future tax obligation is simply $D^*_{20}/[1+(r+i+ir)(1-\delta T)]^n$.

The second return component resulting from holding the asset an additional n periods rather than selling it equals $D_a(D^*_{10}-D^*_{20})$; to that amount is added the sale price of the depreciable adjusted for realtors' fees. Call the result B^a_{40}:

$$(39) \quad B^a_{40} = (D^*_{10}-D^*_{20}) + D_a(1-s)(\frac{1+i}{1+d})^n$$

The third component of returns resulting from holding land an additional n periods is the value of the land after n periods, V_n, reduced by the sale of the depreciable asset adjusted for capital gains recapture tax. This sum is expressed as B^b_{40} and is equal to:

$$(40) \quad B^b_{40} = \left\{[V_n-D_a(\frac{1+i}{1+d})^n]\ (1-s) - [V_o-D_{ao}](1+c)\right\}\ (1-\alpha T)$$

$$+ [V_o-D_{ao}]\ (1+c)$$

Adding B_{40}^{a} to B_{40}^{b} and discounting to the present value, we obtain B_{40} equal to:

$$(41)\quad B_{40} = \Big\{V_n(1-s)(1-\alpha T) + \alpha T\, V_o(1+c) - \alpha T\, D_{ao}(1+c) + \alpha T\, D_a(\tfrac{1+i}{1+d})^n(1-s) + (D_{10}^{*}-D_{20}^{*})\Big\} \;/(\ldots)$$

The minimum sell price is now found by equating returns B_3+B_{40} less costs of B_2 for holding the land plus the depreciable asset to S_1^{*}, the net returns from selling the depreciable asset plus land in the current period:

$$(42)\quad S_1^{*} = B_3 - B_2 + B_{40}$$

Substituting for S_1^{*}, B_3, B_2, B_{40}, V, and V_n and solving for V_s results in the expression:

$$(43)\quad V_s = \frac{R(1-T)(..)S_5}{(.)k_1(1-k_2)(1-s)(1-\alpha T)} - \frac{D_a\, S_5^{*}}{(1-s)(1-\alpha T)} + \frac{\alpha T\, D_a(\frac{1+i}{1+d})^n}{(\ldots)(1-\alpha T)} + \frac{D_{10}^{*}-D_{20}^{*}}{(\ldots)(1-s)(1-\alpha T)} - \frac{\alpha T\, D_{ao}(1+c)}{(\ldots)(1-s)(1-\alpha T)} + \frac{\alpha T\, V_o(1+c)}{(\ldots)(1-s)(1-\alpha T)} + \frac{D_3^{*}}{(1-s)(1-\alpha T)} - \frac{\alpha T\, D_a}{(1-\alpha T)} - \frac{\alpha T(1+c)(V_o-D_{ao})}{(1-s)(1-\alpha T)}$$

where:

$$(44)\quad S_5 = k_1(1-k_2) - \frac{T_p(1-T)(..)}{(.)} + \frac{(1+g)^n(1-s)(1-\alpha T)}{(\ldots)}$$

and

$$(45)\quad S_5^* = \frac{k_1^*}{k_1(1-k_2^*)}\left[\frac{(1-T)T_p(..)}{(.)} - \frac{(\frac{1+i}{1+d})^n(1-s)(1-\alpha T)}{(...)}\right]$$

It is helpful at this point to see what comparative static results can be obtained from the minimum sell price model derived above. Before doing so, however, some simplification is required. The simplication is achieved by allowing n to approach a very large number, suggesting the land is to be traded only once. Under this assumption, V_s can be expressed as:[13/]

$$(46)\quad \lim_{n\to\infty} V_s = \frac{R(1-T)(1+c)}{[((r+i+ir)(1-\delta T)-g)(1+c)+T_p(1-T)](1-s)(1-\alpha T)}$$

$$\frac{-D_aT}{(1-s)(1-\alpha T)n_d(r+i+ir)(1-T)[1+\frac{T_p(1-T)}{(r+i+ir)(1-\delta T)-g}]} - \frac{\alpha T(1+c)V_o}{(1-s)(1-\alpha T)} + \frac{\alpha T(1+c)D_{ao}\left\{\frac{n_o^*}{n_d}\right\}}{(1-s)(1-\alpha T)}$$

By using our simplified assumptions and approximations, comparative static results produce the following unambiguous results:

(47a) $\frac{dV_s}{dR} > 0$	(47e) $\frac{dV_s}{dD_{ao}} > 0$	(47i) $\frac{dV_s}{di} < 0$
(47b) $\frac{dV_s}{dg} > 0$	(47f) $\frac{dV_s}{dn_d} < 0$	(47j) $\frac{dV_s}{d\alpha} > 0$
(47c) $\frac{dV_s}{dV_o} < 0$	(47g) $\frac{dV_s}{ds} > 0$	
(47d) $\frac{dV_s}{dT_p} < 0$	(47h) $\frac{dV_s}{dr} < 0$	

[13/] It is helpful when finding the limit of V_s to recognize that:

$$\lim_{n\to\infty} S_5 = (1+c) \qquad (d > i),$$

$$\lim_{n\to\infty} S_5^* = \frac{T}{n_d(r+i+ir)(1-\delta T)[1+\frac{T_p(1-T)}{(r+i+ir)(1-\delta T)-g}]} \qquad (d > i),$$

and, to simplify matters even more, let:

$$D_3^* = D_3^a \text{ and } T^* = \alpha T$$

Interpretations of the derivatives just obtained are now given. Suppose a variable or parameter other than the minimum sell price V_s increases. Call this parameter or variable x. Then the derivative answers the question: how will the minimum sell price adjust so that the same rate of return would be earned from holding the land another n periods as would be earned from selling it and investing the proceeds at the rate of return reflected by the discount rate?

To illustrate, suppose expected future returns on real estate increase as a result of an increase in R or g. Then the minimum sell price would also have to increase so that returns from the sale of the land would equal the present value of the returns from n more periods of ownership with increased expected earnings.

Consider now the implication of an increase in the original purchase price V_o. The first response is to label V_o as a sunk cost and therefore not economically important. But this would be wrong since increasing V_o reduces capital gains, thereby increasing the attractiveness of the sell option. Thus, an increase in V_o does affect the minimum sell price.

Since increasing real estate taxes makes ownership in the future less attractive, to be indifferent, between ownership in the future or selling now, the sale price must also decline.

Increasing the original depreciable asset's purchase price, D_{ao}, means that there is a larger tax shield for future income. This makes the ownership option more valuable; so the indifference condition requires that the sale option be made more attractive--hence the minimum sell price increases. On the other hand, increasing the tax life of depreciable assets, n_d, means that the tax shield will be available in more distant periods (smaller present value of the tax shield) so that indifference condition requires lowering the minimum sale price.

Increasing realtor fees, increasing s, reduces the actual proceeds received by the seller. There is, of course, some reduction in the capital gains tax as a

result of an increase in s, but this is not nearly as important as the loss in the proceeds from the sale. Thus, for reasons described earlier, indifference requires V_s to increase to offset the increase in s.

An increase in the discount rate through an increase in either r, the real rate of return on an alternative investment, or i, the general inflation rate, would increase the attractiveness of investing the proceeds from the sale of the real estate. Thus, equilibrium requires that both an increase in i and r reduces V_s.

Increasing the capital gains tax α, particularly from .4 to 1, reduces the amount of the purchase price returned to the seller. Thus, the minimum sell price must increase with increases in capital gains tax because the seller can avoid the higher taxes by simply not selling.

Finally, there is one other result not captured by our comparative static results taken in the limit. What is the optimal value of n? Obviously, as inflation i or the real interest rate r increases, so does the present value of the tax shield created by the depreciable asset. As a consequence, more frequent sales of the asset may be advisable so that the differences between book value and market value of the depreciable asset do not get too far out of line.

Consider now the current market value of the depreciable asset D_a. The higher D_a becomes, the greater would be the capital gains tax or income tax paid by the seller. Moreover, since the remaining tax shield is determined by $D_{ao} \frac{n_o^*}{n_o}$ and not by D_a, increasing D_a does nothing for the seller's future tax shield. As a consequence:

$$(48) \quad \frac{dV_s}{dD_a} < 0$$

Building Financial Considerations Into Maximum Bid and Minimum Sell Price Models

The financial terms associated with the sale of real estate alter both the maximum bid and minimum sell prices offered by buyers and accepted by sellers. In the next section of this paper, we examine the effect on the maximum bid price when a buyer is financing his real estate purchase with a loan whose interest rate may differ from that available in the market place or reflected by the discount rate. Then a similar analysis is completed from the seller's perspective by asking: What are the effects on the minimum sell price when a seller provides financing to a buyer at an interest rate which may differ from that available in the market place or reflected by the discount rate? Consider first, however, the analysis from the buyer's perspective.

Before proceeding, however, consider the question: Should financial arrangements be included in the analysis?

In this paper, the financial arrangements are included because their effects cannot be identified independently of other factors operating in the model. Consider the dilemma. If the seller offers or the buyer receives a concessionary interest rate financing, it will require that the seller and buyer be compensated to determine the new maximum bid or minimum sell price. But the amount of the compensation will depend on the seller's or buyer's tax position which depends on what part of the asset is depreciable versus nondepreciable; the rate of inflation which determines the significance of capital gains income tax on the sale; and the depreciable asset's book value and the original price of the depreciable asset which determines what part of the sale price will be paid as income versus capital gains tax.

The confluence of these factors in the maximum bid and minimum sell models defies simple subtractions or additions of the effects of financial arrangements. Therefore, this paper argues that they logically belong in the models.

The Maximum Bid Price for a Buyer Financed Purchase (V*)

Suppose a buyer is able to finance (1-D) percent of his real estate purchase's acquisition price at a concessionary interest rate r*, which is less than the discount rate of (r+i+ir). Such a favorable rate may result from a governmental sponsored loan program, such as might be available from the Farmer's Home Administration, from the transfer of a previous loan contracted by the seller in periods of lower interest rates, or from a land contract offered by the seller as an inducement to complete the transaction.

Without financial terms provided the buyer, the maximum bid price was found to be V. Let the maximum bid price with financial terms provided the buyer be V*. Moreover, let the amount loaned be (1-D)V* where DV* is the amount of the purchase price V* paid as a down payment. The after-tax present value of the loan (1-D)V* repaid at interest rate r* when the market rate of interest is (r+i+ir) was calculated earlier in RB to be V*f where f equals:[14/]

$$(49) \quad f = D + \frac{(1-D)r^*}{(r+i+ir)} \frac{[1-(\ldots)q^{-1}]}{[1-(\ldots)q^{*-1}]} + \frac{T(1-D)r^*(\ldots.)}{(.)^*}$$

where:

$$(\ldots)q = [1+(r+i+ir)(1-\delta T)]^q,$$

$$(\ldots)q^* = (1+r^*)^q,$$

$$(.)^* = [(r+i+ir)(1-\delta T)-r^*], \text{ and}$$

$$(\ldots.) = \frac{[(\ldots)q^{*-1}-(\ldots)q^{-1}]}{[1-(\ldots)q^{*-1}]}$$

[14/] In the special case where r* equals (r+i+ir)(1- δT), f can be expressed as:

$$f = 1-T + DT + \frac{qTr^*(1-D)}{(1+r^*)^{q+1}(1+r^*)}$$

An investment of $(V^*f + cV^*)$ is now compared to which could be invested at the after-tax rate of return of $(r+i+ir)(1- T)$ the present value of an after-tax stream of earnings from real estate. The maximum bid price under such a comparison can be obtained from the equality below. The opportunity cost is $V^*f + cV^*$ because the closing cost is calculated as a percentage of the contract price V^* while the present value of the contract price repaid over q periods at interest rate r^* is fV^*. One can also show that when $r^* = (r+i+ir)$, $V = V^*$.

$$(50) \quad B_1^* = B_3 - B_2 + B_4^*$$

where:

$B_1^* = fV^* + cV^*$, and

$$B_4^* = \frac{V_n(1-s)(1-\alpha T)}{(\ldots)} + \frac{\alpha TV^*(1+c)}{(\ldots)} + \frac{\alpha T\ D_a(\frac{1+i}{1+d})^n(1-s)}{(\ldots)} - \frac{\alpha T\ D_a(1+c)}{(\ldots)} + D_a[D_1^*(1+c)-D_2^*]$$

Replacing V in B_2 with the right-hand side of (20), and V_n in B_4^* with the right-hand side of (23), we solve for V^* and obtain the expression below:

$$(51) \quad V^* = \frac{R(1-T)(..)S_5}{k_3k_1(1-k_2)(.)} - \frac{D_a\ S_5^*}{k_3}$$

$$+ \frac{D_a}{k_3(\ldots)}[\alpha T(1-s)(\frac{1+i}{1+d})^n - \alpha T(1+c) + (D_1^*(1+c)-D_2^*)(\ldots)]$$

where:

$$k_3 = f+c - \frac{\alpha T(1+c)}{(\ldots)}$$

<u>The Minimum Sell Model with Seller Providing Financing</u> (V_s^*)

Now consider how the minimum sell price for real estate may be altered when the seller provides financing. Assume the seller, as a condition of sale, offers

a land contract to the buyer for (1-D) percent of the purchase price at an interest rate r* which may be different than the discount rate (i+r+ir). The seller is quite willing to offer the loan of amount (1-D) percent of the sale V_s^* at r* less than (greater than) his discount rate of r+i+ir for q periods provided V_s^* is increased (decreased) to offset the loss (gain) in interest income. Of course, the higher the seller's tax bracket, the more important will be the tax savings which occurs from having the firm's returns taxed at the capital gains tax rate (αT) rather than at the income tax rate T.

The 1981 tax law change requires we examine two separate cases. The tax law change treats differently depreciable assets placed in service before 1981 and after 1980. Case 1 finds the minimum sell price for a seller financed land plus depreciable asset sale when the depreciable asset was placed in service before 1981. Its solution is V_{s1}^* where $T^* = \alpha T$. Case 2 finds the minimum sell price when the depreciable asset was placed in service after 1980. Its solution is V_{s2}^* for $T^* = T$.

Case 1: V_{s1}^* for $T^* = \alpha T$

Seller financed sales introduce at least one complicating factor in the analysis. The complication arises because the purchase price is received in installments, only part of which could be considered capital gains. Thus, a part of the principal will be received over time; in addition, part of the capital gains taxes will also be paid over future time periods. Any interest received by the seller is, of course, taxed at the seller's ordinary income tax rate. But the portion of the principal which is taxed depends on when the depreciable asset was originally purchased.

If the depreciable asset plus land were purchased before 1981, then all the difference between the current adjusted book value and the depreciable asset's

sale price is taxed at the capital gains tax rate of αT. Moreover, if the sale price including the depreciable asset is V^*_{s1}, then the percentage of the downpayment and principal payments taxed at the capital gains tax rate is equal to:

$$(52) \quad w = 1 - \frac{(1+c)(V_o - D_{ao}\frac{n^*_o}{n_d})}{V^*_{s1}(1-s)}$$

To calculate the present value of a seller financed sale, the after-tax benefits of the loan which is scheduled to be financed for q periods at a concessionary interest rate r* must be calculated. Define S^a_{10} as the net present value of after-tax payments P received by the seller for the land sold plus the downpayment less the sales commission. It can be expressed as:

$$(53) \quad S^a_{10} = (D-s)\, V^*_{s1}\, (1-w\alpha T)$$

$$+ \frac{P(1-\alpha Tw)}{(1+r^*)^q[1+(r+i+ir)(1-\delta T)]} + \ldots + \frac{P(1-\alpha Tw)}{(1+r^*)[1+(r+i+ir)(1-\delta T)]^q}$$

$$+ \frac{P[1-(1+r^*)^{-q}](1-T)}{[1+(r+i+ir)(1-\delta T)]} + \ldots + \frac{P[1-(1+r^*)^{-1}](1-T)}{[1+(r+i+ir)(1-\delta T)]^q}$$

Since the downpayment and sales commission were paid initially, the loan payment P must be sufficient to retire the amount $V^*_s(1-D)$ at interest rate r* in q periods. This relationship is expressed below:

$$(54) \quad (1-D)V^*_s = \frac{P}{r^*}\,[1 - \frac{1}{(1+r^*)^q}]$$

Then, solving for P, obtains:

$$(55) \quad P = \frac{r^*(1-D)V^*_{s1}}{[1-(1+r^*)^{-q}]}$$

Next we substitute into S^a_{10} for P to obtain an expression for V^*_{s1}. Simplifying, after summing geometrically, produces the result:[15/]

$$(56)\quad S^a_{10} = V^*_{s1}\, f^{**} + [V_o - D_{ao}\frac{n^*_o}{n_d}]\,(1+c)\,\alpha T\,[(D-s) + \frac{r^*(1-D)(\ldots.)}{(.)^*}]/(1-s)$$

where:

$$f^{**} = (1-\alpha T)(D-s) + \frac{r^*(1-D)[1-(\ldots)q^{-1}]}{(r+i+ir)[1-(\ldots)q^{*-1}]} + \frac{r^*(1-D)T(1-\alpha)(\ldots.)}{(.)^*}$$

For reasons already given, $df^{**}/dr^* > 0$.

The opportunity cost is again associated with the alternative of holding real estate another n periods equal to $(B_3 - B_2 + B_{40})$. This equality is then expressed as:

$$(57)\quad S^a_{10} = B_3 - B_2 + B_{40}$$

And after making the appropriate substitutions, the expression above simplifies to:

$$(58)\quad V^*_{s1} = \frac{R(1-T)(..)S_5}{f^{**}(.)k_1(1-k_2)} - \frac{D_a\, S^*_5}{f^{**}}$$

$$+ \frac{\alpha T\, D_a(\frac{1+i}{1+d})^n(1-s)}{(\ldots)} + \frac{D^*_{10}-D^*_{20}}{(\ldots)f^{**}} - \frac{\alpha T\, D_{ao}(1+c)}{(\ldots)f^{**}}$$

$$+ \frac{\alpha T\, V_o(1+c)}{(\ldots)f^{**}} - \frac{[V_o - D_{ao}\frac{n^*_o}{n_d}]\,[(1+c)\alpha T]\,[D-s + \frac{r^*(1-D)(\ldots.)}{(.)^*}]}{(1-s)\, f^{**}}$$

[15/] Note that there is some discrepancy between f** above and f** in RB; even with s and D_{ao} set equal to zero.

Case 2: V^*_{s2} for $T^* = T$

For depreciable assets placed in service after 1980, a significant tax cost is introduced into the minimum sell model for a seller financed sale. The additional cost is: the difference between the depreciable asset's sale price up to the original purchase price, D_{ao}, less the current depreciated book value of the depreciable asset taxed at the income tax rate. Moreover, this tax obligation is due at the time of the sale regardless of the fact that principal payments for the depreciable asset may not be received until sometime in the future.

In order to describe this tax obligation due at the time of sale, a new variable, D^*_a, is defined equal to:

$$(59) \quad D^*_a = \begin{cases} D_{ao} & \text{if } (1-D) \quad D_a \geq D_{ao} \quad (1+c) \\ D_a & \text{if } (1-D) \quad D_a < D_{ao} \quad (1+c) \end{cases}$$

The tax due at the time of sale is:

$$(60) \quad T[D^*_a(1-s) - (1+c)D_{ao}\,(1 - \frac{n^*_o}{n_d})]$$

Having already paid part of the tax obligation that which remains including any created when $D^*_a = D_{ao}$ is repaid over time. It's calculating requires we define w^* equal to:

$$(61) \quad w^* = [(V^*_{s2}-D^*_a)(1-s) - (V^*_o-D_{ao})(1+c)]/V^*_{s2}(1-s)$$

$$= 1 - \frac{V_o(1+c)}{V^*_{s2}(1-s)} + \frac{D_{ao}(1+c) - D^*_a(1-s)}{V^*_{s2}(1-s)}$$

The variable w* calculates the percentage of the purchase price required to pay the tax on the capital gains associated with the land plus any tax due on the sale of the depreciable asset not already paid.

Having calculated the tax obligation and a new variable w*, we are prepared to find V^*_{s2} using much the same procedure used to find V^*_{s1}.

First, set S^b_{10} equal to:

$$(62)\quad S^b_{10} = (D-s)V^*_{s2}(1-w^*\alpha T) - T[D^*_a(1-s)-(1+c)D_{ao}(1-\frac{n^*_o}{n_d})]$$

$$+ \frac{P(1-\alpha Tw)}{(1+r^*)^q[1+(r+i+ir)(1-\delta T)]} + \ldots + \frac{P(1-\alpha Tw)}{(1+r^*)[1+(r+i+ir)(1-\delta T)]^q}$$

$$+ \frac{P[1-(1+r^*)^{-q}](1-T)}{[1+(r+i+ir)(1-\delta T)]} + \ldots + \frac{P[1-(1+r^*)^{-1}](1-T)}{[1+(r+i+ir)(1-\delta T)]^q}$$

The loan payment P is similar to our earlier expression:

$$(63)\quad P = \frac{r^*(1-D)V^*_{s2}}{[1-(1+r^*)^{-q}]}$$

Substituting into S^b_{10} for P to obtain an expression for V^*_{s2}, after summing geometrically, produces the result:

$$(64)\quad S^b_{10} = V^*_{s2}f^{**} + \frac{V_o(1+c)\alpha T}{(1-s)}[D-s + \frac{r^*(1-D)(\ldots.)}{(.)^*}]$$

$$- \frac{\alpha T[D_{ao}(1+c)-D^*_a(1-s)](\ldots.)}{(1-s)(.)^*}[D-s + r^*(1-D)]$$

where f** is already defined.

The opportunity cost is again associated with the alternative of holding the real estate another n periods equal to $B_3 - B_2 + B_{40}$. Thus, one can write as an equality:

$$(65) \quad S^b_{10} = B_3 - B_2 + B_{40}$$

And after making the appropriate substitutions, the expression above simplifies to:

$$(66) \quad V^*_{s2} = \frac{R(1-T)(..)S_5}{f^{**}(.)k_1(1-k_2)} - \frac{D_a S^*_5}{f^{**}}$$

$$+ \frac{\alpha T D_a \left(\frac{1+i}{1+d}\right)^n (1-s)}{(...)} + \frac{D^*_{10} - D^*_{20}}{(...)f^{**}}$$

$$- \frac{\alpha T D_{ao}(1+c)}{(...)f^{**}} + \frac{\alpha T V_o(1+c)}{(...)f^{**}}$$

$$+ \frac{\alpha T[D_{ao}(1+c) - D^*_a(1-s)](....)}{(1-s)(.)^* f^{**}} [D-s+r^*(1-D)]$$

$$- \frac{V_o(1+c)\alpha T}{(1-s)f^{**}} \left[D-s+\frac{r^*(1-d)(....)}{(.)^*}\right]$$

<u>Minimum Sell Model With A ''Due on Sale''</u>
<u>Clause With No Seller Financing</u> (V^c_s)

Suppose a seller holds a loan with a balance of L to be repaid at interest rate r_s^* over the next q* periods. This loan, unfortunately, must be paid in full at the time of sale of the asset. When the loan balance is repaid at the time of the sale, the seller pays L to the financial intermediary which originally provided the loan. On the other hand, if the seller repays the loan over q* periods, the proceeds from keeping the land another n periods is reduced by the amount Lf* where f* equals f when D=0 and with r* replaced by r_s^*. The equality between these two options is expressed as:

(67) $S^*_1 - L = B_3 - B_2 + B_{40} - Lf^*$

After substituting for S^*_1, B_2, B_3, and B_{40} and replacing V_s with V_s^c to indicate a minimum sell price with a due on sale clause, the equality below is obtained:

(68) $V_s^c = V_s + \frac{(1-f^*)L}{(1-s)(1-\alpha T)}$

It should be obvious that when r_s^* equals (r+i+ir), f* equals one and V_s^c equals V_s[16/]. Moreover, for r_s^* less than (r+i+ir), f* is less than 1 so that V_s^c is greater than V_s. Similarly, V_s^c is less than V_s when r_s^* is greater than (r+i+ir).

While the absolute value or significance of financial arrangements can only be determined by running the models, the direction of the effects of changes in r*, r_s^*, q, q*, and D can be described logically. Or for the interested and industrious reader, the limit of V*, V_s^*, V_{s1}^*, V_{s2}^*, and V_{sc}^* can be easily obtained by letting n, q, and q* approach very large numbers. Under such assumptions:

(69a) $\lim_{n,q,q^*\to\infty} f = D + \frac{(1-D)V^*}{(r+i+ir)}$

(69b) $\lim_{n,q,q^*\to\infty} f^{**} = (1-\alpha T)\ (D-s) + \frac{r^*(1-D)}{(r+i+ir)}$

(69c) $\lim_{n,q,q^*\to\infty} f^* = \frac{r_s^*}{(r+i+ir)}$

16/ This can be shown by replacing (r+i+ir) with r* when D equals zero in the expression for f. This results in the expression:

$$f = \frac{1-[(1+r^*)(1-T)]^{-q}}{1-(1+r^*)^{-q}} - \frac{[(1+r^*)^{-q}-[1+r^*(1-T)]^{-q}]}{[1(1+r^*)^{-q}]}$$

$$= \frac{1-[(1+r^*)^{-q}]}{1-[(1+r^*)^{-q}]}$$

The two variables of interest in the limit are, of course, r^* and r_s^*. Increasing r^* or r_s^* increases f, f^{**}, and f^*, respectively.

From the buyer's perspective, if the interest rate on the loan financing his purchase increases, then his maximum bid must decrease. Otherwise, the internal rate of return would fall below the discount rate. Thus:

$$(70) \quad \frac{dV^*}{dr^*} < 0$$

From the seller's perspective, regardless of whether the purchase was made before 1981 or after 1980, if the interest rate earned by the seller from his loan to the buyer increases, selling becomes more attractive, allowing him to accept a lower minimum sell price and remain in equilibrium. Thus, one can write:

$$(71) \quad \frac{dV_{s1}^*}{dr^*}, \frac{dV_{s2}^*}{dr^*} < 0$$

Finally, a due on sale clause forces the seller to give up his concessionary interest rate loan at the time of sale. This represents a sacrifice for the seller only if $r_s^* < r+i+ir$ (the interest rate on the seller's loan is below the discount rate).

If the seller is to be induced to give up the concessionary interest rate loan, then V_{sc}^* must be greater than V_s^*, the minimum sell price without any financial instrument sacrifice. But as r_s^* increases, less of a sacrifice is made by the seller by giving up his or her loan; V_{sc}^* then approaches from above V_s^*. Thus:

$$(72) \quad \frac{dV_s^c}{dr_s^*} < 0$$

Empirical Results

Before reporting empirical results generated by the models, however, we emphasize the role that maximum bid and minimum sell models play in economic analysis. Without additional constraints, the models do not necessarily reflect market prices for an asset; they only reflect break-even conditions for the buyer or seller. And if the buyer or seller may have opportunities different than market rates of return, the model results will not reflect market prices.

The model results do though provide useful investment criteria information. The break-even price can be compared to the market determined price and the difference between the two prices is useful investment information. If the buyer's (seller's) maximum (minimum) bid (sell) price less the market price is positive (negative), then the net present value association with the purchase (sale) is positive.

Now we proceed with the description of the model's empirical results. This description begins with a base case against which the results from changes in the base model variables are compared.

The base case assumes the market generates the following variables. Let the real rate of interest be 4 percent. Assume the inflation rate is 4.5 percent and that the land in question is 100 acres of farmland which in 1981 was valued at $1,000 per acre. Also assume that attached to the land is a swine finishing structure constructed in 1981 at a cost of $250,000, or $2,500 of building invested per acre of land. The original purchase price of the real estate in 1981 was therefore $3,500 per acre.

The returns from the swine operation on a per acre basis are $250. The land plus the swine operation are expected to generate returns of $400 per acre the first year and returns are expected to increase at the rate of 4 percent.

The swine finishing structures are expected to be reduced to 5 percent of their original capacity after 21 years which implies d=.16. For tax purposes, the structure will be fully depreciated in 5 years. The current market value per acre of the structure is $2,500.

The buyer and seller both pay income taxes at the proportional rate of 15 percent, and the transaction is planned to occur before the effective date of the 1986 tax law so that . equals .4. Other tax considerations are property taxes which are paid at the rate of 2.5 percent and the opportunity tax weight coefficient is δ=1.

Transactions costs include 5 percent realtor fees paid by the seller and 2.5 percent closing fees paid by the buyer. Financial arrangements include a 7.5 percent note held by the seller, with $2,500 per acre outstanding, with 20 years remaining until maturity. The buyer, on the other hand, can negotiate a FmHA loan at 5 percent for 20 years to cover 75 percent of the purchase. Finally, if the purchase is completed, the buyer is expected to hold the land and attached buildings for 20 years.

A summary of these base line data assumptions used in the example is provided in Table 2.

The model results from the base line data are also described in Table 2. Ignoring financial considerations, the maximum bid price is $5,694.22. The minimum sell price is $6,569.87. Without financial considerations, the buyer and seller would never transact with each other since the buyer's maximum bid price is less than the seller's minimum sell price. This, of course, implies the seller could not sell to himself or herself and make money on the transaction.

Table 2
Base Line Input Assumptions and Model Results

Value	Symbol	Description
4.0%	r	Real rate of return available to the firm on investments.
15.0%	T	Constant proportional income tax rate.
40.0%		The capital gains tax rate as a percentage of income tax.
2.5%	T_p	Real estate tax rate paid on the nominal real estate value.
4.5%	i	General inflation rate implicit in the discount rate.
4.0%	g	Inflation rate applied to the returns from the asset within an ownership period, including any productivity changes of the land and its attached depreciable assets.
$400.00	R	Cash return from the real estate in the first period.
20	n	Length of time the real estate will be owned by the current owner and the m subsequent owners.
5.0%	s	Realtor's fee as a percentage of the real estate's sale price.
2.5%	c	Percent of the acquisition price paid as a closing fee, including title searches and points paid on loans.
20	q	Maturity of the loan used to finance the purchase.
20	q_s	Maturity of the loan held by seller, at sale time.
25.0%	D	Percentage of the real estate's price the buyer paid as a downpayment.
5.0%	r^*	Nominal interest rate paid on funds borrowed by the buyer to acquire the real estate.
7.5%	r^*_s	Nominal interest rate on a loan held by the seller at the time of the real estate sale.
$2,500.00	L	Seller's outstanding loan balance at the time of sale.
$2,250.00	D_a	Market value of the depreciable asset attached to the land in the current period.
5	n_d	Tax life of the depreciable asset.
16.0%	d	Parameter determining the decline in the remaining useful life of the depreciable asset.
1981	T^*	Purchase year of depreciable assets; e.g., 1981.
$3,500.00	V_o	Price paid when the seller acquired the real estate.
5	n_o	Current age of depreciable when placed in service by the first buyer.
$2,500.00	D_{ao}	Original purchase price of depreciable asset.
1		A tax adjustment coefficient applied to the discount rate. (=0 implies the firm's next best investment is a tax-free bond while =1 implies the next best investment is a financial instrument.)

Base Value		Model
$5,876	V	Maximum Bid Price
$6,529	V_s	Minimum Sell Price
$7,080	V^*	Maximum Bid Price When Financed
$7,546	V^*_s	Minimum Sell Price With Seller Providing Financing
$6,744	V_{sc}	Minimum Sell With Due on Sale Clause

On the other hand, if the buyer's purchase is partially financed at 5 percent, the maximum bid price increases to $6,889.34 exceeding the minimum sell price without financial consideration and also greater than the minimum sell price with a due on sale clause. As one would expect, however, if the seller must provide the 5 percent financing of the buyer, the minimum sell price increases to $7,568.70 and no sale is possible.

Table 3 reports sensitivity results in dollar values of the base model to changes in model parameters. The top line reflects a 25 percent increase ceteris paribus. The second line reflects a 25 percent decrease, ceteris paribus.

Table 4 reports the same results reported in Table 3, only in percent terms. To obtain Table 4, the numbers in Table 3 are divided by their respective base line values.

It is interesting to compare the direction of the percentage changes in the maximum bid prices and minimum sell prices reported in Table 4 to increases in the model parameters and variables. In all cases, they conform to the deductive results when the limiting versions of the models were differentiated.

One change not reported is the effect of the changed tax laws increasing some taxes from the rate of .4T to T. For the seller, this change reduces his incentive to sell, increasing his or her minimum sell price. For the buyer, it reduces the tax advantage of asset ownership and lowers his or her maximum bid price. The combined effect then will be to reduce the number of sales for which willing sellers and buyers can be matched.

Table 3
Sensitivity Results in Dollar Measures for Maximum Bid and Minimum Sell Models in Response to ±25 Percent Changes in Parameter Values

Input Parameter	+25% -25%	Maximum Bid V	Maximum Bid V*	V_s	Minimum Sell V_s^*	V_s^c
Base Line		**5694**	**6889**	**6570**	**7569**	**6808**
r	5.00%	5018	6357	5764	6983	6190
	3.00%	6588	7586	7626	8332	7654
T	18.150%	5782	6978	6779	7671	7021
	11.250%	5592	6786	6336	7451	6569
	30.00%	5907	7112	6524	7544	6736
	50.00%	5845	7047	6534	7548	6752
T_p	3.125%	5245	6345	6020	6931	6258
	1.875%	6228	7536	7223	8326	7461
i	5.625%	4949	6303	5681	6922	6128
	3.375%	6713	7684	7773	8440	7776
g	5.00%	6653	8050	7743	8929	7980
	3.00%	4980	6024	5696	6555	5934
R	$500.00	7078	8565	8263	9532	8501
	$300.00	4311	5214	4877	5606	5115
n	25	5808	7013	6747	7777	6985
	15	5531	6713	6322	7276	6560
s	6.25%	5658	6846	6613	7643	6854
	3.75%	5731	6933	6529	7496	6763
c	3.125%	5657	6834	6562	7566	6800
	1.857%	5732	6946	6578	7572	6816
q	25	5694	7115	6570	7774	6808
	15	5694	6639	6570	7337	6808
q_s	25	5694	6889	6570	7569	6845
	15	5694	6889	6570	7569	6764
D	31.25%	5694	6770	6570	7471	6808
	18.75%	5694	7012	6570	7669	6808
r*	6.35%	5694	6448	6570	6966	6808
	3.750%	5694	7366	6570	8271	6808
r_s^*	9.375%	5694	6889	6570	7569	6426
	5.625%	5694	6889	6570	7569	7166
L	$3125.00	5694	6889	6570	7569	6867
	$1875.00	5694	6889	6570	7569	6748
D_a	$2812.50	5734	6936	6556	7435	6794
	$1687.50	5654	6842	6583	7850	6821
n_d	10	5661	6849	6389	7634	6626
	15	5634	6817	6324	7652	6562
d	20.00%	5693	6888	6568	7584	6806
	12.00%	5697	6893	6574	7554	6812
T*	1980	5876	7080	6321	7555	6536
V_o	$4375.00	5694	6889	6443	7496	6681
	$2625.00	5694	6889	6696	7641	6934
n_o	7	5694	6889	6523	7515	6761
	3	5694	6889	6420	7615	6658
D_{ao}	$3125.00	5694	6889	6660	7851	6897
	$1875.00	5694	6889	6480	7462	6718

Table 4
Sensitivity Results in Percentage Change for Maximum Bid and Minimum Sell Models in Response to ±25 Percent Changes in Parameter Values

Input Parameter	+25% -25%	Maximum Bid V	Maximum Bid V*	V_s	Minimum Sell V_s^*	V_s^c
r	5.000%	-11.87	-7.72	-12.26	-7.74	-9.07
	3.000%	15.69	10.12	16.07	10.09	12.43
T	18.150%	1.54	1.28	3.18	1.35	3.13
	11.250%	-1.79	-1.50	-3.56	-1.55	-3.50
	30.000%	3.74	3.23	-.70	-.33	-1.06
	50.000%	2.65	1.83	-.55	-.28	-.82
T_p	3.125%	-7.89	-7.90	-8.37	-8.42	-8.08
	1.875%	9.38	9.38	9.94	10.01	9.60
i	5.625%	-13.09	-8.52	-13.52	-8.55	-9.98
	3.375%	17.90	11.53	18.32	11.51	14.22
g	5.000%	16.83	16.85	17.85	17.97	17.23
	3.000%	-12.55	-12.56	-13.31	-13.39	-12.84
R	$500.00	24.30	24.32	25.77	25.94	24.87
	$300.00	-24.30	-24.32	-25.77	-25.94	-24.87
n	25	2.01	1.79	2.70	2.75	2.60
	15	-2.86	-2.56	-3.77	-3.87	-3.64
s	6.250%	-.63	-.63	.66	.98	.68
	3.750%	.64	.64	-.63	-.96	-.65
c	3.125%	-.66	-.80	-.12	-.04	-.12
	1.857%	.67	.82	.12	.04	.12
q	25	.00	3.28	.00	2.71	.00
	15	.00	-3.64	.00	-3.06	.00
q_s	25	.00	.00	.00	.00	.54
	15	.00	.00	.00	.00	-.65
D	31.250%	.00	-1.73	.00	-1.29	.00
	18.750%	.00	1.79	.00	1.32	.00
r*	6.350%	.00	-6.40	.00	-7.97	.00
	3.750%	.00	6.92	.00	9.28	.00
r_s^*	9.375%	.00	.00	.00	.00	-5.61
	5.625%	.00	.00	.00	.00	5.26
L	$3125.00	.00	.00	.00	.00	.87
	$1875.00	.00	.00	.00	.00	-.87
D_a	$2812.50	.70	.68	-.21	-1.77	-.20
	$1687.50	-.70	-.68	.21	3.71	.20
n_d	10	-.59	-.59	-2.76	.87	-2.66
	15	-1.05	-1.06	-3.74	1.10	-3.61
d	20.000%	-.03	-.03	-.03	.20	-.03
	12.000%	.06	.06	.06	-.19	.06
T*	1980	3.19	2.76	-3.78	-.18	-3.99
V_o	$4375.00	.00	.00	-1.93	-.96	-1.86
	$2625.00	.00	.00	1.93	.96	1.86
n_o	7	.00	.00	-.71	-.71	-.68
	3	.00	.00	-2.29	.62	-2.21
D_{ao}	$3125.00	.00	.00	1.36	3.74	1.32
	$1875.00	.00	.00	-1.36	-1.41	-1.32

Epilogue

Real estate buy and sell decisions can be very complicated to calculate. They are complicated by differential tax effects depending on cut-off dates on tax rate changes, property taxes, and tax shields created by depreciation and capital gains. They are further complicated by special financial terms which affect buyers and sellers differently. As more realism is included, maximum bid and minimum sell models become more complex.

This paper's approach has been to build comprehensive bid and sell models piece by piece and then combine the results. Approached in that manner, the process can be understood and analyzed.

Models of the type constructed in this paper are intended to be decision aids--and not the decision making model. They cannot be the only basis for decisions because no one can supply the perfectly accurate data they require for perfect estimates of maximum bid and minimum sell prices. At best they can be estimated under alternative scenarios to find the range of possible outcomes.

The usefulness of present value models as an analytic tool has been frequently overlooked. One possible explanation for their lack of analytic use is that they can quickly become complicated. To be analytically useful present value models must often be simplified. The models in this paper were simplified by increasing the holding period n to a very large number. Then once simplified the models were differentiated to predict directional response to variable and parameter changes. In all cases, the analytic predictions were consistent with the direction of nominal changes obtained using the more complicated models.

We recommend that the use of present value models as an analytic as well as empirical tool be increased. This, of course, will require that present value models be constructed using geometric series rather than simply expressed as numerical sums. There is a saving, however, from using geometric series: future

cash flow streams can be expressed in terms of geometric means--variables more likely to be estimable than mn individual data points. Hopefully, the models and approach followed in this paper have provided a useful illustration of how useful analytic and empirical present value models can be constructed and used.

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