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RELATIONSHIPS AMONG LOAN MATURITY, TOTAL INTEREST PAID, AND PERIODIC PAYMENT FOR CONSTANT PAYMENT LOANS

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ABSTRACT

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Small changes in loan payments may produce significant changes in loan maturity and total interest paid. This sensitivity, however, depends on the original level of the interest rate and loan maturity. In this article, loan maturity, interest payment, and interest paid/loan maturity point elasticities are derived and then tabulated for various interest rates and loan maturities. The tables provide relevant values for the point elasticities which indicate under what conditions small changes in loan payments may produce significant changes in loan maturity and interest paid.

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Introduction

A recent Wall Street Journal article illustrated the sensitivity of loan maturity to changes in loan payment size. A sensitive relationship can also be demonstrated between loan maturity and total interest paid. To illustrate the sensitivity just described, consider: a $30,000 loan at 15 percent APR to be repaid in 30 years requires monthly payments of $379.33; meanwhile total interest paid equals $106,560. Increasing the payment amount by 10 percent to $417.27 reduces the term of the loan to just over 15 years and total interest paid is reduced to $46,778.

Results are not always so significant. For example, if the loan above had an APR of 8 percent, the monthly payment would equal $220.13 instead of $379.33. Increasing the payment 10 percent would only decrease the term of the loan from 30 years to about 22 years, and total interest paid would be decreased only from $49,247 to $33,718.

It would be useful to know under what conditions changing the loan payment size changes significantly loan maturity and total interest paid. Since there is no generally available source of such information, it is the intent of this paper to provide tables describing the sensitivities between loan payment size, loan maturity, and total interest paid.

Three relationships for a constant payment loan are examined in this paper. They are the relationships between (1) loan maturity and periodic payment amount, (2) total interest paid and periodic payment amount, and (3) total interest paid and loan maturity. We first derive analytic expressions for the three relationships described above. Then we present tables of numerical values for the relationships described above as interest rates and loan maturities change for monthly and annual loan payments.

Elasticity Measures and Notation

When examining the relationships between terms of the loan, the question should be asked: What measure should be used? Absolute measures are not useful since they would be subject to scaling factors such as the amount of the loan. We could, of course, standardize either the periodic payment or the loan amount to 1, but we could not eliminate scaling effects from interest rates or loan maturities. So unless we are interested in results for a particular loan, absolute measures are not useful--nor could we generate enough tables to cover all possible loan arrangements.

Elasticities, a unit-free measure, can be used to solve the scaling dilemma. Elasticities are the ratio of percentages: a percentage change is related to a percentage change. For two variables, x and y, the relationship at a point (x,y) is:

$$\varepsilon_{y,x} = \frac{\Delta y/y}{\Delta x/x}$$

or in the limit as Δx approaches zero:

$$\lim_{\Delta x \to 0} \varepsilon_{y,x} = \frac{dy}{dx} \cdot \frac{x}{y}$$

which, in other words, makes the following comparison between x and y. At some point, (x,y), increasing x by $\frac{\Delta x}{x}$ percent increases y by $\frac{\Delta y}{y}$ percent. Moreover, since the percentage change in y is divided by the corresponding percentage change in x--we could standardize the result and say: a one percent change in x at the point (x,y) results in a $\varepsilon_{y,x}$ percent change in y. In the following paragraphs, we use elasticity measures to examine constant payment loans for the loan relationships described earlier. For consistency, we use the notations of Robison, Koenig, and Brake. The notations are:

V_0 = the present value of the loan;

r = the annual percentage rate (APR) expressed as a decimal;

m = number of conversion periods within a year;

n = loan maturity; i.e., years to complete repayment;

A = the amount of periodic (constant) payment or annuity; and

TI = total interest costs paid over the life of the loan (i.e., mnA - V).

We could, of course, derive elasticity measures for the case where Δx is not close to zero but equal to $x_1 - x_0$. Such an elasticity measure, an arc elasticity, is defined as:

$$\varepsilon_{y,x} = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)\left(\frac{x_1 + x_0}{y_1 + y_0}\right)$$

Arc elasticities, however useful in their own right, also present scaling problems, such as determining the size of $x_1 - x_0$. As a result, arc elasticities are not calculated in this paper. Besides, point elasticities involving the derivative dy/dx are an easier expression to calculate and tabulate. As a result, we derive point elasticity measures in terms of dy/dx which implies the elasticity is measured at a point where Δx is close to zero. The use of point rather than arc elasticities has an important implication. Extrapolations of the tabled point elasticity values to large Δx values may not provide accurate results.

Constant Payment Loans

A constant payment loan is a loan repaid by a uniform series of payments made at equal time intervals. The present value of the payments is obtained by discounting at the interest rate per period and summing the uniform series of discounted payments. Future payments on the loan are equal to the present value of the loan through the amortization formula. The fundamental relationship

between the number of payments nm, the actuarial interest rate r/m, the periodic payment A, and the loan amount is:

$$(1) \quad V_o = \frac{A}{r/m}\left[1 - \left(1 + \frac{r}{m}\right)^{-mn}\right]$$

If the loan amount, interest rate, and number of payments are known, the periodic payment, A, is:

$$(2) \quad A = (r/m)\, V_o \,/\, \left[1 - \left(1 + \frac{r}{m}\right)^{-mn}\right]$$

Knowing A, V_o, and $\frac{r}{m}$, the loan maturity (n or number of years for repayment) can be determined to be:

$$(3) \quad n = \frac{-\ln\left(1 - \frac{r}{m}\frac{V_o}{A}\right)}{m \ln\left(1 + \frac{r}{m}\right)}$$

Moreover, once mn, $\frac{r}{m}$, V_o, and A are known, total interest paid, TI, is simply:

$$(4) \quad TI = (mnA - V_o)$$

The Loan Maturity Elasticity

Now consider the question: If A is increased by some small amount, say 1 percent, holding m, r, and V_o constant, by what percent will the loan maturity change? Before proceeding, however, we must address how loan maturity is to be measured. Two possible measures are the total number of loan payment periods, mn, or the number of years n. We choose to derive the elasticity in terms of n, since m is a nonstandard measure, and for any given loan plan, m will be a constant anyway.

The loan term point elasticity is defined as:

$$(5) \quad \varepsilon_{n,A} = \frac{dn}{dA}\frac{A}{n}$$

An expression for $\frac{dn}{dA}$ is obtained by differentiating (3). The result is:

$$(6)\quad \frac{dn}{dA} = \frac{-r\,V_o}{[\ln\,(1 + \frac{r}{m})]\,(1 - \frac{r}{m}\,\frac{V_o}{A})\,(mA)^2} < 0$$

The derivative $\frac{dn}{dA} < 0$ since $\frac{r}{m}\,\frac{V_o}{A} < 1$ and $\ln\,(1 + \frac{r}{m}) > 0$. Moreover, it has no unique maximum since increasing A monotonically reduces n when constrained by equation (1). Intuitively, if one holds interest rate constant, then increasing the amount of each payment should decrease the time taken to repay the loan, i.e., shorten maturity.

To find $\varepsilon_{n,A}$, $\frac{dn}{dA}$ is multiplied by $\frac{A}{n}$; then A is replaced by the right-hand side of (2) to obtain the result:

$$(7)\quad \varepsilon_{n,A} = \frac{[1 - (1 + \frac{r}{m})^{mn}]}{nm\ln\,(1 + \frac{r}{m})} < 0 \qquad \text{for } r > 0$$

The derivative of $\varepsilon_{n,A}$ with respect to r cannot be signed unambiguously. But numerically, as the values of $\varepsilon_{n,A}$ in Tables 1a and 1b illustrate, $\varepsilon_{n,A}$ becomes increasingly negative as r increases. Moreover, it can be shown mathematically that $\varepsilon_{n,A}$ decreases with r in all cases where $n > 1$. Also, the elasticity measure $\varepsilon_{n,A}$ becomes increasingly negative as n increases. The longer one is scheduled to repay a loan, the more is the effect from an increase in amount of periodic payment.

A simple explanation of these relationships is in order. The periodic loan payment A is made up of principal and interest. At high interest rates and longer loan maturities, a large part of A is interest and a small part is principal payment. Any increase in the amount of payment applied to principal at high interest rates and long loan maturity, the more sensitive will be the changes in n to changes in the amount of A paid each period.

To illustrate, suppose a $100 periodic payment includes $90 of principal and $10 of interest. A $10 increase in the payment will increase the payment on the principal by $10/ 90 or 11 percent. But, if in the same situation, $90 were interest and $10 were principal, the same $10 increase in the amount paid would increase the principal payment in that period by $10/$10 or 100 percent. For this reason, $\varepsilon_{n,A}$ becomes increasingly negative with increases in n and r.

It may, of course, occur that our concern with the loan maturity elasticity is on an after-tax basis. If so, the relevant interest rate is not r but r(1-T) where T is the relevant marginal income tax bracket. This adjustment, however, requires no special calculations. It simply requires that one use the interest rates in Tables 1a and 1b on an after-tax rather than a before-tax basis. For example, the loan term elasticity on a 12 percent loan for a borrower in the 25 percent tax bracket would be examined in the 9 percent, i.e., 12(1-.25) percent, interest rate column.

An Example

Don and Debbie Debtor are considering a 16 percent APR car loan of $7,600 with 48 monthly payments of $215.39 each. The Debtors wonder how much benefit a larger monthly payment would provide. From Table 1b, the loan maturity elasticity for their 16 percent APR loan with loan maturity of 4 years is 1.397. So if they increase their payment by 1 percent, to $217.54, their loan maturity would decrease by 1.397 percent, less than one month.

To decrease their payment by one month on a 48-month planned repayment, a 1/48 or 2.08 percent reduction, would require an increased monthly payment of 2.08/1.397 = 1.489 percent, to about $ 220.75. The table value is only an approximation since elasticity is measured at a point rather than for a segment (arc) of the function; i.e., Δx is close to zero in our elasticity formula,

rather than a 5 or 10 percent change. Hence, Table 1b has limited applicability and applies only to very small changes.

Since one may be interested in actual changes in loan maturity rather than in elasticity, per se, Tables 2a and 2b are provided. They show the decrease in loan maturity in years corresponding to a one percent increase in A. It is constructed by multiplying the elasticity values in Tables 1a and 1b by n. Thus, using Table 1b, for the Debtors' loan, a 1 percent and 5 percent increase in their loan payment reduces their loan maturity by approximately .056 and .280 years (.67 and 3.36 months), respectively. Again, keep in mind that the table value for large changes, such as 5 percent, is only a rough approximation.

Interest Payment Elasticity Measure

Now consider the question: How will total interest paid be affected by a one percent increase in the amount of the loan payment for given values of r, m, and V_o? The answer is given in terms of the elasticity of total interest paid defined as:

$$\varepsilon_{TI,A} = \frac{\Delta TI/TI}{\Delta A/A}$$

or in the limit:

$$\lim_{\Delta A \to 0} \varepsilon_{TI,A} = \frac{dTI}{dA} \frac{A}{TI}$$

To derive $\varepsilon_{TI,A}$ at a point, we must first calculate dTI/dA. Total interest paid was defined in equation (4). For any given payment plan, V is a constant so the derivative of TI with respect to A equals:

$$(8) \quad \frac{dTI}{dA} = A\, m \frac{\partial n}{\partial A} + mn$$

which after restricting n by equation (3) results in the expression:

$$(9)\quad \frac{dTI}{dA} = \frac{1 - (1 + \frac{r}{m})^{mn} - \ln\left[(1 + \frac{r}{m})^{-mn}\right]}{\ln\,(1 + \frac{r}{m})} < 0$$

which is unambiguously negative. Multiplying by A/TI where n in the expression for TI is restricted by equation (3), produces the elasticity measure, $\varepsilon_{TI,A}$, which is equal to:

$$(10)\quad \varepsilon_{TI,A} = \frac{(1 + \frac{r}{m})^{mn} + \ln\left[(1 + \frac{r}{m})^{-mn}\right] - 1}{\ln\,(1 + \frac{r}{m})^{-mn} + \frac{m}{r}\ln\,(1 + \frac{r}{m})\left[1 - (1 + \frac{r}{m})^{-mn}\right]} < 0$$

We know $\varepsilon_{TI,A} < 0$ since it equals a negative value (dTI/dA) multiplied by a positive ratio A/TI. Or intuitively, if the total amount repaid is reduced as a result of an increase in A and a corresponding reduction in n, then total interest paid must be reduced.

Values for $\varepsilon_{TI,A}$ are presented in Tables 3a and 3b. Notice here again that the elasticity $\varepsilon_{TI,A}$ becomes more negative (larger absolute values) with increases in r and n. The reason $\varepsilon_{TI,n}$ becomes more negative with increases in n and r is the reason given earlier for changes in $\varepsilon_{n,A}$. As r and n increase, any change in A has relatively more effect on principal reduction.

An Example

Returning to the Debtor's loan analysis, suppose they want to evaluate how much their total interest paid changes in response to a one percent change in their loan payment. The answer from Table 3b is -1.501 percent. Or if they increase their loan payment by 5 percent, the total interest paid would be reduced by roughly 7.51 percent.

Interest Paid/Loan Maturity Elasticity

For completeness, consider the question: If the loan maturity is increased by one percent, holding r constant, by what percent will interest paid increase? The answer is given by the interest paid/loan term elasticity defined as:

$$\varepsilon_{TI,n} = \frac{\frac{\Delta TI}{TI}}{\frac{\Delta n}{n}}$$

or in the limit as:

$$\lim_{n \to 0} \varepsilon_{TI,n} = \frac{dTI}{dn} \frac{n}{TI}$$

Increasing maturity at a given r must decrease A and increase TI, so $dTI/dn > 0$. Multiplying $\frac{dTI}{dn}$ by $\frac{n}{TI}$, a positive ratio leaves $\varepsilon_{TI,n} > 0$.

To derive $\varepsilon_{TI,n}$ explicitly, recall that $\varepsilon_{TI,n}$ is related to both $\varepsilon_{n,A}$ and $\varepsilon_{TI,A}$. Dividing $\varepsilon_{TI,A}$ by $\varepsilon_{n,A}$ produces the new elasticity measure $\varepsilon_{TI,n}$:

$$\varepsilon_{TI,n} = \frac{\frac{dTI}{dA} \frac{A}{TI}}{\frac{dn}{dA} \frac{A}{n}} = \frac{dTI}{dn} \frac{n}{TI}$$

Hence, any value for $\varepsilon_{TI,n}$ can be found by dividing values from Tables 3a and 3b by the corresponding values in Tables 1a and 1b. For small values of r and n, a one percent increase in n is a small increase and results in a small increase in total interest paid. At large values of n, a one percent increase is a larger value, but so is the corresponding increase in total interest paid.

Some Implications

While anyone conversant with loan term relationships recognizes trade-offs between amount paid per period and loan maturity, several relationships are now more evident. For example, Figure 1 shows combinations of interest rates and

loan maturities with the same elasticity. As loan terms and interest rates increase, a given percentage increase in periodic payment has a larger effect in reducing loan maturity. For a 20-year loan with an APR of 14 percent, a one percent increase in monthly payment would decrease loan maturity by 5 percent (i.e., elasticity = -5). Note, too, that interest rate, as one would expect, has more effect for long loan maturities (say, 25 years) than for short maturities (say, 5 years).

A similar result is obtained for total interest as related to amount of periodic payment as shown in Figure 2. The large (negative) elasticities for long maturities and high interest rates underline the large savings in total interest available from a small percentage increase in periodic payment. Of interest also is the line with elasticity of -1. For example, on a loan of four years at 8 percent, a one percent increase in periodic payment decreases total interest paid by only one percent--a trade-off that seems small compared to the elasticity of -3 obtainable on a 15-year, 12 percent loan.

Concluding Comments

It would be easy to infer more from this paper's presentation than intended. For instance, from a borrower's perspective, it would be incorrect to infer one should agree to a shorter term loan simply because loan term elasticities tend to increase with interest rates and time taken to repay. Whether or not this is the correct response depends on the liquidity of the borrower and the borrower's investment opportunities. Facing favorable returns exceeding the interest rate on borrowed funds could possibly lead the borrower to the opposite strategy--opting for longer terms.

Care should also be exercised concerning inferences from elasticities of interest paid. Minimizing interest paid may not be of overriding concern to the borrower. It is only one of many, often conflicting, goals to consider.

The point is, the loan maturity and interest paid elasticities are information about the loan, much like the interest rate. Because their magnitudes vary significantly, they may be of use when arranging a loan. With better and more complete information, the borrower may make a more appropriate credit decision.

TABLE 1a

Percentage Change in Time Required to Repay a Loan
Given a One Percent Increase in the Amount of
Periodic Payment, for Selected APR Interest Rates and Loan Maturities

(m = 1 payment per year)

LOAN TERM	INTEREST RATE															
n	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%
	PERCENT															
1	-1.005	-1.010	-1.015	-1.020	-1.025	-1.030	-1.035	-1.039	-1.044	-1.049	-1.054	-1.059	-1.064	-1.068	-1.073	-1.078
2	-1.010	-1.020	-1.030	-1.040	-1.050	-1.061	-1.071	-1.081	-1.091	-1.102	-1.112	-1.122	-1.133	-1.143	-1.154	-1.164
3	-1.015	-1.030	-1.046	-1.061	-1.077	-1.093	-1.109	-1.125	-1.141	-1.158	-1.174	-1.191	-1.208	-1.225	-1.242	-1.260
4	-1.020	-1.041	-1.062	-1.083	-1.104	-1.126	-1.148	-1.171	-1.194	-1.217	-1.241	-1.265	-1.290	-1.315	-1.340	-1.365
5	-1.025	-1.051	-1.078	-1.105	-1.133	-1.161	-1.190	-1.220	-1.250	-1.281	-1.313	-1.345	-1.379	-1.413	-1.447	-1.483
6	-1.030	-1.062	-1.094	-1.127	-1.162	-1.197	-1.233	-1.271	-1.310	-1.349	-1.390	-1.432	-1.475	-1.520	-1.566	-1.613
7	-1.036	-1.073	-1.111	-1.151	-1.192	-1.235	-1.279	-1.325	-1.373	-1.422	-1.473	-1.526	-1.581	-1.638	-1.697	-1.758
8	-1.041	-1.084	-1.128	-1.175	-1.223	-1.274	-1.327	-1.382	-1.440	-1.500	-1.563	-1.628	-1.696	-1.767	-1.842	-1.919
9	-1.046	-1.095	-1.146	-1.199	-1.256	-1.315	-1.377	-1.442	-1.511	-1.583	-1.659	-1.738	-1.822	-1.910	-2.002	-2.098
10	-1.051	-1.106	-1.163	-1.224	-1.289	-1.357	-1.429	-1.506	-1.587	-1.672	-1.763	-1.858	-1.959	-2.066	-2.179	-2.299
11	-1.057	-1.117	-1.182	-1.250	-1.324	-1.401	-1.485	-1.573	-1.667	-1.768	-1.874	-1.988	-2.109	-2.238	-2.376	-2.522
12	-1.062	-1.129	-1.200	-1.277	-1.359	-1.448	-1.542	-1.644	-1.753	-1.870	-1.995	-2.129	-2.274	-2.428	-2.594	-2.771
13	-1.068	-1.141	-1.219	-1.304	-1.396	-1.496	-1.603	-1.719	-1.844	-1.979	-2.125	-2.283	-2.453	-2.637	-2.836	-3.050
14	-1.073	-1.152	-1.239	-1.333	-1.435	-1.546	-1.666	-1.798	-1.941	-2.097	-2.266	-2.450	-2.650	-2.868	-3.105	-3.363
15	-1.078	-1.164	-1.258	-1.361	-1.474	-1.598	-1.733	-1.882	-2.044	-2.222	-2.418	-2.632	-2.866	-3.123	-3.404	-3.713
16	-1.084	-1.177	-1.279	-1.391	-1.515	-1.652	-1.803	-1.970	-2.154	-2.357	-2.582	-2.829	-3.103	-3.404	-3.737	-4.105
17	-1.090	-1.189	-1.299	-1.422	-1.558	-1.709	-1.877	-2.064	-2.271	-2.502	-2.759	-3.045	-3.362	-3.716	-4.108	-4.545
18	-1.095	-1.201	-1.320	-1.453	-1.602	-1.768	-1.954	-2.163	-2.396	-2.658	-2.951	-3.280	-3.648	-4.060	-4.522	-5.039
19	-1.101	-1.214	-1.342	-1.485	-1.647	-1.830	-2.035	-2.268	-2.529	-2.825	-3.159	-3.535	-3.961	-4.441	-4.983	-5.595
20	-1.106	-1.227	-1.364	-1.518	-1.694	-1.894	-2.121	-2.378	-2.671	-3.005	-3.384	-3.815	-4.305	-4.863	-5.497	-6.219
21	-1.112	-1.240	-1.386	-1.553	-1.743	-1.961	-2.210	-2.496	-2.823	-3.198	-3.627	-4.119	-4.684	-5.331	-6.072	-6.922
22	-1.118	-1.253	-1.409	-1.588	-1.794	-2.031	-2.305	-2.620	-2.985	-3.405	-3.891	-4.452	-5.100	-5.849	-6.714	-7.713
23	-1.124	-1.267	-1.432	-1.624	-1.846	-2.104	-2.404	-2.752	-3.157	-3.629	-4.177	-4.816	-5.559	-6.425	-7.432	-8.605
24	-1.130	-1.280	-1.456	-1.661	-1.900	-2.180	-2.508	-2.892	-3.341	-3.869	-4.487	-5.213	-6.064	-7.063	-8.236	-9.611
25	-1.135	-1.294	-1.480	-1.699	-1.956	-2.260	-2.618	-3.040	-3.538	-4.127	-4.824	-5.647	-6.621	-7.773	-9.135	-10.746
26	-1.141	-1.308	-1.505	-1.738	-2.015	-2.343	-2.733	-3.197	-3.749	-4.406	-5.189	-6.122	-7.235	-8.561	-10.143	-12.028
27	-1.147	-1.322	-1.530	-1.779	-2.075	-2.430	-2.854	-3.363	-3.973	-4.706	-5.586	-6.642	-7.912	-9.438	-11.272	-13.475
28	-1.153	-1.336	-1.556	-1.820	-2.138	-2.520	-2.982	-3.539	-4.214	-5.029	-6.016	-7.212	-8.659	-10.413	-12.538	-15.112
29	-1.159	-1.351	-1.583	-1.863	-2.202	-2.615	-3.116	-3.727	-4.470	-5.377	-6.484	-7.835	-9.484	-11.499	-13.959	-16.962
30	-1.165	-1.366	-1.610	-1.907	-2.270	-2.714	-3.258	-3.925	-4.745	-5.753	-6.993	-8.518	-10.396	-12.707	-15.553	-19.056
31	-1.171	-1.381	-1.637	-1.952	-2.339	-2.817	-3.407	-4.136	-5.039	-6.158	-7.545	-9.267	-11.402	-14.053	-17.344	-21.427
32	-1.178	-1.396	-1.665	-1.998	-2.411	-2.925	-3.564	-4.360	-5.354	-6.595	-8.147	-10.087	-12.515	-15.554	-19.355	-24.112
33	-1.184	-1.411	-1.694	-2.046	-2.486	-3.037	-3.729	-4.597	-5.690	-7.066	-8.801	-10.987	-13.746	-17.226	-21.617	-27.155
34	-1.190	-1.427	-1.723	-2.095	-2.564	-3.155	-3.903	-4.850	-6.051	-7.575	-9.512	-11.975	-15.107	-19.092	-24.160	-30.605
35	-1.196	-1.443	-1.753	-2.146	-2.645	-3.278	-4.086	-5.118	-6.437	-8.125	-10.287	-13.059	-16.614	-21.173	-27.021	-34.519
36	-1.203	-1.459	-1.784	-2.198	-2.728	-3.407	-4.280	-5.403	-6.850	-8.718	-11.131	-14.249	-18.282	-23.497	-30.240	-38.959
37	-1.209	-1.475	-1.815	-2.252	-2.815	-3.542	-4.483	-5.705	-7.293	-9.359	-12.050	-15.557	-20.129	-26.091	-33.865	-44.001
38	-1.215	-1.491	-1.847	-2.307	-2.905	-3.683	-4.698	-6.027	-7.768	-10.051	-13.051	-16.993	-22.175	-28.989	-37.949	-49.726
39	-1.222	-1.508	-1.880	-2.364	-2.998	-3.830	-4.925	-6.369	-8.276	-10.800	-14.142	-18.571	-24.443	-32.228	-42.549	-56.231
40	-1.228	-1.525	-1.913	-2.423	-3.095	-3.984	-5.164	-6.732	-8.822	-11.609	-15.332	-20.306	-26.956	-35.848	-47.735	-63.624

TABLE 1b

Percentage Change in Time Required to Repay a Loan
Given a One Percent Increase in the Amount of
Periodic Payment, for Selected APR Interest Rates and Loan Maturities

(m = 12 payments per year)

LOAN TERM	INTEREST RATE															
n	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%
	PERCENT															
1	-1.005	-1.010	-1.015	-1.020	-1.025	-1.031	-1.036	-1.041	-1.046	-1.051	-1.057	-1.062	-1.068	-1.073	-1.078	-1.084
2	-1.010	-1.020	-1.031	-1.041	-1.052	-1.062	-1.073	-1.084	-1.095	-1.107	-1.118	-1.130	-1.141	-1.153	-1.165	-1.177
3	-1.015	-1.031	-1.046	-1.062	-1.079	-1.095	-1.112	-1.130	-1.147	-1.165	-1.184	-1.203	-1.222	-1.241	-1.261	-1.281
4	-1.020	-1.041	-1.062	-1.084	-1.107	-1.130	-1.154	-1.178	-1.203	-1.228	-1.255	-1.282	-1.310	-1.338	-1.367	-1.397
5	-1.025	-1.052	-1.079	-1.107	-1.136	-1.166	-1.197	-1.229	-1.262	-1.296	-1.331	-1.368	-1.406	-1.445	-1.485	-1.527
6	-1.031	-1.062	-1.096	-1.130	-1.166	-1.203	-1.242	-1.282	-1.324	-1.368	-1.414	-1.462	-1.511	-1.563	-1.617	-1.673
7	-1.036	-1.073	-1.113	-1.154	-1.197	-1.242	-1.289	-1.339	-1.391	-1.446	-1.503	-1.563	-1.627	-1.693	-1.762	-1.836
8	-1.041	-1.084	-1.130	-1.178	-1.229	-1.283	-1.339	-1.399	-1.462	-1.529	-1.600	-1.674	-1.753	-1.837	-1.925	-2.018
9	-1.046	-1.096	-1.148	-1.203	-1.262	-1.325	-1.392	-1.463	-1.538	-1.618	-1.704	-1.795	-1.892	-1.996	-2.106	-2.224
10	-1.052	-1.107	-1.166	-1.229	-1.297	-1.369	-1.447	-1.530	-1.619	-1.714	-1.817	-1.927	-2.045	-2.171	-2.308	-2.454
11	-1.057	-1.118	-1.184	-1.256	-1.332	-1.415	-1.504	-1.601	-1.705	-1.817	-1.939	-2.070	-2.212	-2.366	-2.533	-2.714
12	-1.062	-1.130	-1.203	-1.283	-1.369	-1.463	-1.565	-1.676	-1.796	-1.928	-2.071	-2.227	-2.397	-2.583	-2.785	-3.007
13	-1.068	-1.142	-1.223	-1.311	-1.407	-1.513	-1.629	-1.755	-1.894	-2.047	-2.214	-2.398	-2.600	-2.822	-3.067	-3.337
14	-1.073	-1.154	-1.242	-1.340	-1.447	-1.565	-1.696	-1.840	-1.999	-2.175	-2.369	-2.585	-2.824	-3.089	-3.383	-3.710
15	-1.079	-1.166	-1.263	-1.369	-1.488	-1.620	-1.766	-1.929	-2.110	-2.312	-2.538	-2.789	-3.071	-3.385	-3.737	-4.131
16	-1.084	-1.178	-1.283	-1.400	-1.530	-1.677	-1.840	-2.023	-2.229	-2.460	-2.720	-3.013	-3.343	-3.715	-4.134	-4.608
17	-1.090	-1.191	-1.304	-1.431	-1.574	-1.736	-1.918	-2.124	-2.356	-2.620	-2.919	-3.258	-3.643	-4.081	-4.580	-5.148
18	-1.096	-1.204	-1.325	-1.464	-1.620	-1.798	-2.000	-2.230	-2.492	-2.792	-3.134	-3.526	-3.975	-4.490	-5.081	-5.760
19	-1.101	-1.216	-1.347	-1.497	-1.667	-1.862	-2.086	-2.343	-2.638	-2.977	-3.369	-3.820	-4.342	-4.945	-5.644	-6.454
20	-1.107	-1.229	-1.370	-1.531	-1.716	-1.930	-2.177	-2.462	-2.793	-3.177	-3.623	-4.142	-4.747	-5.453	-6.277	-7.241
21	-1.113	-1.243	-1.392	-1.566	-1.767	-2.001	-2.272	-2.589	-2.960	-3.393	-3.900	-4.496	-5.196	-6.020	-6.991	-8.136
22	-1.118	-1.256	-1.416	-1.602	-1.820	-2.074	-2.373	-2.724	-3.138	-3.626	-4.202	-4.884	-5.693	-6.653	-7.795	-9.154
23	-1.124	-1.269	-1.439	-1.639	-1.874	-2.151	-2.479	-2.867	-3.328	-3.877	-4.530	-5.311	-6.244	-7.361	-8.701	-10.311
24	-1.130	-1.283	-1.464	-1.677	-1.931	-2.232	-2.591	-3.019	-3.532	-4.148	-4.888	-5.779	-6.854	-8.153	-9.724	-11.629
25	-1.136	-1.297	-1.489	-1.717	-1.989	-2.316	-2.708	-3.181	-3.751	-4.441	-5.278	-6.294	-7.531	-9.038	-10.879	-13.130
26	-1.142	-1.311	-1.514	-1.757	-2.050	-2.404	-2.832	-3.352	-3.985	-4.758	-5.703	-6.860	-8.282	-10.030	-12.184	-14.842
27	-1.148	-1.326	-1.540	-1.799	-2.113	-2.496	-2.963	-3.535	-4.237	-5.100	-6.166	-7.483	-9.115	-11.141	-13.659	-16.794
28	-1.154	-1.340	-1.566	-1.842	-2.178	-2.592	-3.100	-3.728	-4.506	-5.471	-6.672	-8.169	-10.041	-12.385	-15.327	-19.023
29	-1.160	-1.355	-1.593	-1.886	-2.246	-2.692	-3.246	-3.934	-4.795	-5.872	-7.224	-8.925	-11.070	-13.781	-17.214	-21.569
30	-1.166	-1.370	-1.621	-1.931	-2.317	-2.797	-3.399	-4.154	-5.104	-6.305	-7.826	-9.757	-12.214	-15.347	-19.351	-24.478
31	-1.172	-1.385	-1.649	-1.978	-2.390	-2.907	-3.560	-4.387	-5.437	-6.775	-8.484	-10.674	-13.486	-17.105	-21.772	-27.804
32	-1.178	-1.400	-1.678	-2.026	-2.465	-3.022	-3.731	-4.635	-5.794	-7.283	-9.203	-11.685	-14.901	-19.078	-24.516	-31.609
33	-1.185	-1.416	-1.707	-2.076	-2.544	-3.143	-3.911	-4.899	-6.177	-7.834	-9.989	-12.800	-16.476	-21.296	-27.628	-35.964
34	-1.191	-1.432	-1.737	-2.127	-2.626	-3.269	-4.100	-5.180	-6.588	-8.430	-10.848	-14.030	-18.230	-23.788	-31.157	-40.952
35	-1.197	-1.448	-1.768	-2.179	-2.711	-3.401	-4.301	-5.480	-7.030	-9.077	-11.788	-15.388	-20.184	-26.590	-35.164	-46.666
36	-1.204	-1.464	-1.799	-2.233	-2.799	-3.539	-4.512	-5.799	-7.505	-9.778	-12.816	-16.888	-22.362	-29.741	-39.713	-53.216
37	-1.210	-1.480	-1.831	-2.289	-2.890	-3.683	-4.736	-6.138	-8.016	-10.539	-13.941	-18.544	-24.790	-33.288	-44.880	-60.726
38	-1.216	-1.497	-1.864	-2.346	-2.985	-3.835	-4.972	-6.500	-8.565	-11.364	-15.173	-20.374	-27.497	-37.281	-50.752	-69.343
39	-1.223	-1.514	-1.897	-2.406	-3.083	-3.993	-5.221	-6.886	-9.155	-12.259	-16.521	-22.396	-30.518	-41.777	-57.428	-79.232
40	-1.229	-1.531	-1.932	-2.467	-3.186	-4.159	-5.484	-7.297	-9.789	-13.230	-17.999	-24.632	-33.889	-46.843	-65.020	-90.587

TABLE 2a

Change in Years for Loan Repayment Given a One Percent Increase
in the Amount of Periodic Payment,
for Selected APR Interest Rates and Loan Maturities

(m = 1 payment per year)

LOAN TERM	INTEREST RATE															
n	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%
	YEARS															
1	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.011	-.011	-.011	-.011	-.011	-.011
2	-.020	-.020	-.021	-.021	-.021	-.021	-.021	-.022	-.022	-.022	-.022	-.022	-.023	-.023	-.023	-.023
3	-.030	-.031	-.031	-.032	-.032	-.033	-.033	-.034	-.034	-.035	-.035	-.036	-.036	-.037	-.037	-.038
4	-.041	-.042	-.042	-.043	-.044	-.045	-.046	-.047	-.048	-.049	-.050	-.051	-.052	-.053	-.054	-.055
5	-.051	-.053	-.054	-.055	-.057	-.058	-.059	-.061	-.063	-.064	-.066	-.067	-.069	-.071	-.072	-.074
6	-.062	-.064	-.066	-.068	-.070	-.072	-.074	-.076	-.079	-.081	-.083	-.086	-.089	-.091	-.094	-.097
7	-.072	-.075	-.078	-.081	-.083	-.086	-.090	-.093	-.096	-.100	-.103	-.107	-.111	-.115	-.119	-.123
8	-.083	-.087	-.090	-.094	-.098	-.102	-.106	-.111	-.115	-.120	-.125	-.130	-.136	-.141	-.147	-.154
9	-.094	-.099	-.103	-.108	-.113	-.118	-.124	-.130	-.136	-.142	-.149	-.156	-.164	-.172	-.180	-.189
10	-.105	-.111	-.116	-.122	-.129	-.136	-.143	-.151	-.159	-.167	-.176	-.186	-.196	-.207	-.218	-.230
11	-.116	-.123	-.130	-.138	-.146	-.154	-.163	-.173	-.183	-.194	-.206	-.219	-.232	-.246	-.261	-.277
12	-.127	-.135	-.144	-.153	-.163	-.174	-.185	-.197	-.210	-.224	-.239	-.256	-.273	-.291	-.311	-.333
13	-.139	-.148	-.159	-.170	-.182	-.194	-.208	-.223	-.240	-.257	-.276	-.297	-.319	-.343	-.369	-.397
14	-.150	-.161	-.173	-.187	-.201	-.216	-.233	-.252	-.272	-.294	-.317	-.343	-.371	-.402	-.435	-.471
15	-.162	-.175	-.189	-.204	-.221	-.240	-.260	-.282	-.307	-.333	-.363	-.395	-.430	-.468	-.511	-.557
16	-.173	-.188	-.205	-.223	-.242	-.264	-.289	-.315	-.345	-.377	-.413	-.453	-.496	-.545	-.598	-.657
17	-.185	-.202	-.221	-.242	-.265	-.291	-.319	-.351	-.386	-.425	-.469	-.518	-.572	-.632	-.698	-.773
18	-.197	-.216	-.238	-.262	-.288	-.318	-.352	-.389	-.431	-.478	-.531	-.590	-.657	-.731	-.814	-.907
19	-.209	-.231	-.255	-.282	-.313	-.348	-.387	-.431	-.481	-.537	-.600	-.672	-.753	-.844	-.947	-1.063
20	-.221	-.245	-.273	-.304	-.339	-.379	-.424	-.476	-.534	-.601	-.677	-.763	-.861	-.973	-1.099	-1.244
21	-.234	-.260	-.291	-.326	-.366	-.412	-.464	-.524	-.593	-.672	-.762	-.865	-.984	-1.119	-1.275	-1.454
22	-.246	-.276	-.310	-.349	-.395	-.447	-.507	-.576	-.657	-.749	-.856	-.979	-1.122	-1.287	-1.477	-1.697
23	-.258	-.291	-.329	-.373	-.425	-.484	-.553	-.633	-.726	-.835	-.961	-1.108	-1.279	-1.478	-1.709	-1.979
24	-.271	-.307	-.349	-.399	-.456	-.523	-.602	-.694	-.802	-.929	-1.077	-1.251	-1.455	-1.695	-1.977	-2.307
25	-.284	-.323	-.370	-.425	-.489	-.565	-.654	-.760	-.885	-1.032	-1.206	-1.412	-1.655	-1.943	-2.284	-2.687
26	-.297	-.340	-.391	-.452	-.524	-.609	-.711	-.831	-.975	-1.146	-1.349	-1.592	-1.881	-2.226	-2.637	-3.127
27	-.310	-.357	-.413	-.480	-.560	-.656	-.771	-.908	-1.073	-1.271	-1.508	-1.793	-2.136	-2.548	-3.043	-3.638
28	-.323	-.374	-.436	-.510	-.599	-.706	-.835	-.991	-1.180	-1.408	-1.685	-2.019	-2.425	-2.916	-3.511	-4.231
29	-.336	-.392	-.459	-.540	-.639	-.758	-.904	-1.081	-1.296	-1.559	-1.880	-2.272	-2.750	-3.335	-4.048	-4.919
30	-.350	-.410	-.483	-.572	-.681	-.814	-.977	-1.178	-1.424	-1.726	-2.098	-2.555	-3.119	-3.812	-4.666	-5.717
31	-.363	-.428	-.507	-.605	-.725	-.873	-1.056	-1.282	-1.562	-1.909	-2.339	-2.873	-3.535	-4.357	-5.377	-6.642
32	-.377	-.447	-.533	-.639	-.772	-.936	-1.140	-1.395	-1.713	-2.110	-2.607	-3.228	-4.005	-4.977	-6.194	-7.716
33	-.391	-.466	-.559	-.675	-.820	-1.002	-1.230	-1.517	-1.878	-2.332	-2.904	-3.626	-4.536	-5.685	-7.134	-8.961
34	-.405	-.485	-.586	-.712	-.872	-1.073	-1.327	-1.649	-2.057	-2.576	-3.234	-4.072	-5.137	-6.491	-8.214	-10.406
35	-.419	-.505	-.614	-.751	-.926	-1.147	-1.430	-1.791	-2.253	-2.844	-3.601	-4.571	-5.815	-7.411	-9.457	-12.082
36	-.433	-.525	-.642	-.791	-.982	-1.227	-1.541	-1.945	-2.466	-3.138	-4.007	-5.130	-6.581	-8.459	-10.887	-14.025
37	-.447	-.546	-.672	-.833	-1.041	-1.310	-1.659	-2.111	-2.698	-3.463	-4.458	-5.756	-7.448	-9.654	-12.530	-16.280
38	-.462	-.567	-.702	-.877	-1.104	-1.399	-1.785	-2.290	-2.952	-3.820	-4.959	-6.457	-8.427	-11.016	-14.420	-18.896
39	-.476	-.588	-.733	-.922	-1.169	-1.494	-1.921	-2.484	-3.228	-4.212	-5.515	-7.243	-9.533	-12.569	-16.594	-21.930
40	-.491	-.610	-.765	-.969	-1.238	-1.594	-2.065	-2.693	-3.529	-4.644	-6.133	-8.122	-10.783	-14.339	-19.094	-25.449

TABLE 2b

Change in Years for Loan Repayment Given a One Percent Increase
in the Amount of Periodic Payment,
for Selected APR Interest Rates and Loan Maturities

(m = 12 payments per year)

LOAN TERM	INTEREST RATE															
n	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%
	YEARS															
1	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.010	-.011	-.011	-.011	-.011	-.011	-.011	-.011
2	-.020	-.020	-.021	-.021	-.021	-.021	-.021	-.022	-.022	-.022	-.022	-.023	-.023	-.023	-.023	-.024
3	-.030	-.031	-.031	-.032	-.032	-.033	-.033	-.034	-.034	-.035	-.036	-.036	-.037	-.037	-.038	-.038
4	-.041	-.042	-.042	-.043	-.044	-.045	-.046	-.047	-.048	-.049	-.050	-.051	-.052	-.054	-.055	-.056
5	-.051	-.053	-.054	-.055	-.057	-.058	-.060	-.061	-.063	-.065	-.067	-.068	-.070	-.072	-.074	-.076
6	-.062	-.064	-.066	-.068	-.070	-.072	-.075	-.077	-.079	-.082	-.085	-.088	-.091	-.094	-.097	-.100
7	-.073	-.075	-.078	-.081	-.084	-.087	-.090	-.094	-.097	-.101	-.105	-.109	-.114	-.118	-.123	-.128
8	-.083	-.087	-.090	-.094	-.098	-.103	-.107	-.112	-.117	-.122	-.128	-.134	-.140	-.147	-.154	-.161
9	-.094	-.099	-.103	-.108	-.114	-.119	-.125	-.132	-.138	-.146	-.153	-.162	-.170	-.180	-.190	-.200
10	-.105	-.111	-.117	-.123	-.130	-.137	-.145	-.153	-.162	-.171	-.182	-.193	-.204	-.217	-.231	-.245
11	-.116	-.123	-.130	-.138	-.147	-.156	-.165	-.176	-.188	-.200	-.213	-.228	-.243	-.260	-.279	-.299
12	-.127	-.136	-.144	-.154	-.164	-.176	-.188	-.201	-.216	-.231	-.248	-.267	-.288	-.310	-.334	-.361
13	-.139	-.148	-.159	-.170	-.183	-.197	-.212	-.228	-.246	-.266	-.288	-.312	-.338	-.367	-.399	-.434
14	-.150	-.162	-.174	-.188	-.203	-.219	-.237	-.258	-.280	-.304	-.332	-.362	-.395	-.432	-.474	-.519
15	-.162	-.175	-.189	-.205	-.223	-.243	-.265	-.289	-.317	-.347	-.381	-.418	-.461	-.508	-.561	-.620
16	-.174	-.189	-.205	-.224	-.245	-.268	-.294	-.324	-.357	-.394	-.435	-.482	-.535	-.594	-.661	-.737
17	-.185	-.202	-.222	-.243	-.268	-.295	-.326	-.361	-.401	-.445	-.496	-.554	-.619	-.694	-.779	-.875
18	-.197	-.217	-.239	-.263	-.292	-.324	-.360	-.401	-.449	-.503	-.564	-.635	-.715	-.808	-.915	-1.037
19	-.209	-.231	-.256	-.284	-.317	-.354	-.396	-.445	-.501	-.566	-.640	-.726	-.825	-.940	-1.072	-1.226
20	-.221	-.246	-.274	-.306	-.343	-.386	-.435	-.492	-.559	-.635	-.725	-.828	-.949	-1.091	-1.255	-1.448
21	-.234	-.261	-.292	-.329	-.371	-.420	-.477	-.544	-.622	-.712	-.819	-.944	-1.091	-1.264	-1.468	-1.709
22	-.246	-.276	-.311	-.352	-.400	-.456	-.522	-.599	-.690	-.798	-.924	-1.075	-1.253	-1.464	-1.715	-2.014
23	-.259	-.292	-.331	-.377	-.431	-.495	-.570	-.659	-.766	-.892	-1.042	-1.221	-1.436	-1.693	-2.001	-2.372
24	-.271	-.308	-.351	-.403	-.463	-.536	-.622	-.725	-.848	-.996	-1.173	-1.387	-1.645	-1.957	-2.334	-2.791
25	-.284	-.324	-.372	-.429	-.497	-.579	-.677	-.795	-.938	-1.110	-1.319	-1.574	-1.883	-2.260	-2.720	-3.283
26	-.297	-.341	-.394	-.457	-.533	-.625	-.736	-.872	-1.036	-1.237	-1.483	-1.784	-2.153	-2.608	-3.168	-3.859
27	-.310	-.358	-.416	-.486	-.571	-.674	-.800	-.954	-1.144	-1.377	-1.665	-2.021	-2.461	-3.008	-3.688	-4.535
28	-.323	-.375	-.439	-.516	-.610	-.726	-.868	-1.044	-1.262	-1.532	-1.868	-2.287	-2.812	-3.468	-4.291	-5.327
29	-.336	-.393	-.462	-.547	-.651	-.781	-.941	-1.141	-1.390	-1.703	-2.095	-2.588	-3.210	-3.997	-4.992	-6.255
30	-.350	-.411	-.486	-.579	-.695	-.839	-1.020	-1.246	-1.531	-1.892	-2.348	-2.927	-3.664	-4.604	-5.805	-7.343
31	-.363	-.429	-.511	-.613	-.741	-.901	-1.104	-1.360	-1.685	-2.100	-2.630	-3.309	-4.181	-5.302	-6.749	-8.619
32	-.377	-.448	-.537	-.648	-.789	-.967	-1.194	-1.483	-1.854	-2.331	-2.945	-3.739	-4.768	-6.105	-7.845	-10.115
33	-.391	-.467	-.563	-.685	-.840	-1.037	-1.290	-1.617	-2.038	-2.585	-3.296	-4.224	-5.437	-7.028	-9.117	-11.868
34	-.405	-.487	-.591	-.723	-.893	-1.111	-1.394	-1.761	-2.240	-2.866	-3.688	-4.770	-6.198	-8.088	-10.594	-13.924
35	-.419	-.507	-.619	-.763	-.949	-1.190	-1.505	-1.918	-2.461	-3.177	-4.126	-5.386	-7.065	-9.306	-12.307	-16.333
36	-.433	-.527	-.648	-.804	-1.008	-1.274	-1.624	-2.088	-2.702	-3.520	-4.614	-6.080	-8.050	-10.707	-14.297	-19.158
37	-.448	-.548	-.678	-.847	-1.069	-1.363	-1.752	-2.271	-2.966	-3.899	-5.158	-6.861	-9.172	-12.317	-16.606	-22.469
38	-.462	-.569	-.708	-.892	-1.134	-1.457	-1.889	-2.470	-3.255	-4.318	-5.766	-7.742	-10.449	-14.167	-19.286	-26.350
39	-.477	-.591	-.740	-.938	-1.203	-1.557	-2.036	-2.686	-3.570	-4.781	-6.443	-8.735	-11.902	-16.293	-22.397	-30.901
40	-.492	-.613	-.773	-.987	-1.274	-1.664	-2.194	-2.919	-3.916	-5.292	-7.200	-9.853	-13.555	-18.737	-26.008	-36.235

TABLE 3a

Percentage Change in Total Interest Paid Given a
One Percent Increase in the Amount of Periodic Payment,
for Selected APR Interest Rates and Loan Maturities

(m = 1 payment per year)

LOAN TERM	INTEREST RATE															
n	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%
								PERCENT								
1	-.504	-.508	-.512	-.517	-.521	-.525	-.529	-.533	-.537	-.541	-.545	-.549	-.553	-.558	-.562	-.566
2	-.677	-.687	-.697	-.707	-.717	-.727	-.738	-.748	-.758	-.769	-.779	-.790	-.800	-.811	-.822	-.832
3	-.766	-.783	-.800	-.816	-.833	-.851	-.868	-.886	-.904	-.922	-.940	-.958	-.977	-.995	-1.014	-1.033
4	-.823	-.846	-.870	-.894	-.918	-.943	-.969	-.994	-1.021	-1.047	-1.074	-1.102	-1.130	-1.158	-1.187	-1.216
5	-.863	-.893	-.924	-.956	-.988	-1.021	-1.056	-1.090	-1.126	-1.162	-1.200	-1.238	-1.277	-1.316	-1.357	-1.399
6	-.893	-.931	-.969	-1.009	-1.050	-1.092	-1.136	-1.180	-1.227	-1.274	-1.323	-1.373	-1.425	-1.478	-1.532	-1.589
7	-.918	-.963	-1.009	-1.057	-1.107	-1.159	-1.213	-1.268	-1.326	-1.386	-1.448	-1.512	-1.578	-1.647	-1.718	-1.791
8	-.939	-.991	-1.046	-1.103	-1.162	-1.224	-1.289	-1.356	-1.427	-1.500	-1.577	-1.657	-1.740	-1.826	-1.917	-2.010
9	-.957	-1.017	-1.080	-1.146	-1.216	-1.289	-1.365	-1.446	-1.530	-1.619	-1.712	-1.810	-1.912	-2.020	-2.132	-2.250
10	-.973	-1.041	-1.112	-1.188	-1.268	-1.353	-1.443	-1.538	-1.638	-1.743	-1.855	-1.973	-2.097	-2.229	-2.367	-2.513
11	-.988	-1.063	-1.144	-1.230	-1.321	-1.419	-1.522	-1.632	-1.750	-1.874	-2.007	-2.147	-2.297	-2.456	-2.624	-2.803
12	-1.001	-1.085	-1.175	-1.271	-1.375	-1.485	-1.604	-1.731	-1.867	-2.012	-2.168	-2.335	-2.513	-2.703	-2.907	-3.125
13	-1.014	-1.106	-1.205	-1.313	-1.429	-1.554	-1.688	-1.834	-1.990	-2.159	-2.340	-2.536	-2.747	-2.974	-3.218	-3.481
14	-1.026	-1.126	-1.236	-1.355	-1.484	-1.624	-1.776	-1.941	-2.120	-2.314	-2.525	-2.754	-3.001	-3.270	-3.561	-3.877
15	-1.037	-1.146	-1.266	-1.397	-1.540	-1.696	-1.867	-2.053	-2.257	-2.480	-2.723	-2.989	-3.279	-3.595	-3.940	-4.317
16	-1.048	-1.166	-1.296	-1.439	-1.597	-1.770	-1.961	-2.171	-2.402	-2.656	-2.936	-3.243	-3.581	-3.952	-4.360	-4.808
17	-1.059	-1.186	-1.327	-1.483	-1.656	-1.848	-2.060	-2.295	-2.556	-2.845	-3.164	-3.518	-3.910	-4.344	-4.825	-5.356
18	-1.069	-1.205	-1.357	-1.527	-1.716	-1.927	-2.163	-2.426	-2.719	-3.046	-3.410	-3.817	-4.270	-4.776	-5.340	-5.968
19	-1.079	-1.225	-1.388	-1.572	-1.778	-2.010	-2.270	-2.563	-2.892	-3.261	-3.675	-4.141	-4.664	-5.251	-5.911	-6.652
20	-1.089	-1.244	-1.420	-1.618	-1.842	-2.096	-2.383	-2.708	-3.075	-3.490	-3.961	-4.493	-5.094	-5.775	-6.546	-7.418
21	-1.099	-1.264	-1.451	-1.665	-1.908	-2.185	-2.501	-2.860	-3.270	-3.737	-4.269	-4.875	-5.566	-6.354	-7.252	-8.276
22	-1.109	-1.283	-1.483	-1.713	-1.976	-2.278	-2.624	-3.021	-3.477	-4.000	-4.601	-5.291	-6.083	-6.992	-8.037	-9.238
23	-1.118	-1.303	-1.516	-1.762	-2.046	-2.374	-2.753	-3.191	-3.697	-4.283	-4.960	-5.743	-6.649	-7.698	-8.912	-10.317
24	-1.128	-1.322	-1.549	-1.812	-2.118	-2.474	-2.888	-3.370	-3.932	-4.585	-5.347	-6.235	-7.271	-8.478	-9.886	-11.528
25	-1.137	-1.342	-1.582	-1.863	-2.192	-2.578	-3.030	-3.560	-4.181	-4.910	-5.766	-6.772	-7.953	-9.341	-10.972	-12.890
26	-1.146	-1.362	-1.616	-1.916	-2.269	-2.686	-3.179	-3.760	-4.447	-5.259	-6.220	-7.356	-8.702	-10.296	-12.184	-14.421
27	-1.155	-1.382	-1.651	-1.970	-2.349	-2.799	-3.334	-3.971	-4.730	-5.633	-6.710	-7.994	-9.526	-11.355	-13.538	-16.145
28	-1.164	-1.402	-1.686	-2.025	-2.431	-2.917	-3.498	-4.195	-5.031	-6.035	-7.241	-8.689	-10.432	-12.528	-15.050	-18.085
29	-1.174	-1.423	-1.722	-2.082	-2.516	-3.039	-3.670	-4.432	-5.353	-6.467	-7.815	-9.449	-11.428	-13.829	-16.740	-20.272
30	-1.183	-1.443	-1.758	-2.140	-2.604	-3.166	-3.850	-4.682	-5.696	-6.931	-8.438	-10.278	-12.525	-15.272	-18.631	-22.738
31	-1.192	-1.464	-1.795	-2.200	-2.695	-3.299	-4.040	-4.947	-6.061	-7.430	-9.112	-11.183	-13.733	-16.875	-20.747	-25.521
32	-1.201	-1.485	-1.833	-2.262	-2.788	-3.438	-4.239	-5.228	-6.452	-7.966	-9.844	-12.173	-15.064	-18.655	-23.116	-28.662
33	-1.210	-1.506	-1.872	-2.325	-2.886	-3.582	-4.448	-5.525	-6.868	-8.544	-10.637	-13.255	-16.531	-20.633	-25.771	-32.210
34	-1.219	-1.527	-1.911	-2.389	-2.986	-3.733	-4.667	-5.840	-7.313	-9.165	-11.498	-14.439	-18.149	-22.833	-28.748	-36.221
35	-1.228	-1.549	-1.951	-2.456	-3.090	-3.890	-4.898	-6.173	-7.788	-9.834	-12.433	-15.735	-19.934	-25.280	-32.086	-40.756
36	-1.237	-1.571	-1.992	-2.524	-3.198	-4.053	-5.141	-6.527	-8.295	-10.555	-13.447	-17.153	-21.905	-28.003	-35.833	-45.888
37	-1.247	-1.593	-2.033	-2.594	-3.310	-4.224	-5.397	-6.901	-8.837	-11.331	-14.549	-18.706	-24.081	-31.036	-40.039	-51.697
38	-1.256	-1.616	-2.076	-2.666	-3.425	-4.403	-5.665	-7.299	-9.417	-12.168	-15.747	-20.408	-26.485	-34.413	-44.763	-58.277
39	-1.265	-1.638	-2.119	-2.740	-3.545	-4.589	-5.948	-7.720	-10.036	-13.070	-17.048	-22.273	-29.142	-38.178	-50.072	-65.732
40	-1.274	-1.661	-2.163	-2.816	-3.669	-4.783	-6.245	-8.167	-10.699	-14.042	-18.463	-24.318	-32.079	-42.375	-56.041	-74.184

TABLE 3b

Percentage Change in Total Interest Paid Given a One Percent Increase in the Amount of Periodic Payment, for Selected APR Interest Rates and Loan Maturities

(m = 12 payments per year)

LOAN TERM	INTEREST RATE															
n	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%
	PERCENT															
1	-.929	-.936	-.942	-.949	-.955	-.961	-.968	-.975	-.981	-.988	-.994	-1.001	-1.008	-1.015	-1.022	-1.028
2	-.973	-.986	-1.000	-1.013	-1.027	-1.041	-1.055	-1.069	-1.083	-1.098	-1.113	-1.128	-1.143	-1.158	-1.173	-1.189
3	-.993	-1.013	-1.033	-1.054	-1.076	-1.098	-1.120	-1.142	-1.166	-1.189	-1.213	-1.237	-1.262	-1.288	-1.314	-1.340
4	-1.006	-1.033	-1.062	-1.090	-1.120	-1.150	-1.181	-1.213	-1.246	-1.279	-1.314	-1.349	-1.386	-1.423	-1.461	-1.501
5	-1.017	-1.052	-1.087	-1.124	-1.162	-1.202	-1.243	-1.285	-1.328	-1.373	-1.419	-1.467	-1.517	-1.568	-1.621	-1.676
6	-1.027	-1.069	-1.112	-1.158	-1.205	-1.254	-1.305	-1.359	-1.414	-1.472	-1.532	-1.594	-1.659	-1.726	-1.797	-1.870
7	-1.036	-1.085	-1.137	-1.191	-1.248	-1.308	-1.370	-1.436	-1.504	-1.576	-1.651	-1.730	-1.813	-1.900	-1.990	-2.084
8	-1.044	-1.101	-1.162	-1.225	-1.292	-1.363	-1.438	-1.517	-1.600	-1.687	-1.780	-1.878	-1.981	-2.090	-2.204	-2.325
9	-1.052	-1.117	-1.186	-1.260	-1.338	-1.421	-1.508	-1.602	-1.701	-1.806	-1.918	-2.037	-2.164	-2.298	-2.442	-2.594
10	-1.060	-1.133	-1.212	-1.295	-1.384	-1.480	-1.582	-1.691	-1.808	-1.933	-2.067	-2.210	-2.364	-2.529	-2.705	-2.894
11	-1.068	-1.149	-1.237	-1.331	-1.432	-1.542	-1.659	-1.786	-1.922	-2.069	-2.228	-2.399	-2.583	-2.782	-2.997	-3.230
12	-1.076	-1.166	-1.263	-1.368	-1.482	-1.606	-1.740	-1.885	-2.043	-2.215	-2.401	-2.603	-2.823	-3.062	-3.323	-3.606
13	-1.084	-1.182	-1.289	-1.406	-1.533	-1.672	-1.824	-1.990	-2.172	-2.370	-2.588	-2.826	-3.086	-3.372	-3.685	-4.029
14	-1.091	-1.198	-1.316	-1.444	-1.586	-1.742	-1.913	-2.101	-2.309	-2.537	-2.790	-3.068	-3.375	-3.714	-4.089	-4.503
15	-1.099	-1.215	-1.343	-1.484	-1.641	-1.814	-2.006	-2.219	-2.455	-2.717	-3.008	-3.331	-3.691	-4.092	-4.539	-5.036
16	-1.107	-1.232	-1.370	-1.525	-1.697	-1.889	-2.103	-2.343	-2.610	-2.909	-3.244	-3.619	-4.039	-4.511	-5.041	-5.637
17	-1.115	-1.249	-1.398	-1.567	-1.755	-1.967	-2.206	-2.474	-2.775	-3.115	-3.499	-3.932	-4.422	-4.976	-5.603	-6.313
18	-1.122	-1.266	-1.427	-1.610	-1.816	-2.049	-2.313	-2.612	-2.952	-3.337	-3.775	-4.274	-4.843	-5.491	-6.231	-7.076
19	-1.130	-1.283	-1.456	-1.654	-1.878	-2.134	-2.426	-2.759	-3.140	-3.575	-4.075	-4.648	-5.306	-6.062	-6.933	-7.937
20	-1.138	-1.300	-1.486	-1.699	-1.943	-2.223	-2.544	-2.914	-3.340	-3.832	-4.399	-5.056	-5.816	-6.697	-7.720	-8.910
21	-1.146	-1.318	-1.517	-1.745	-2.010	-2.315	-2.669	-3.078	-3.554	-4.107	-4.751	-5.501	-6.378	-7.403	-8.603	-10.009
22	-1.154	-1.336	-1.548	-1.793	-2.079	-2.411	-2.799	-3.252	-3.782	-4.403	-5.132	-5.989	-6.998	-8.187	-9.592	-11.254
23	-1.162	-1.354	-1.579	-1.842	-2.151	-2.512	-2.937	-3.436	-4.026	-4.722	-5.546	-6.522	-7.682	-9.061	-10.703	-12.663
24	-1.170	-1.373	-1.612	-1.893	-2.225	-2.617	-3.081	-3.632	-4.286	-5.065	-5.995	-7.106	-8.437	-10.033	-11.951	-14.261
25	-1.178	-1.391	-1.645	-1.945	-2.301	-2.726	-3.233	-3.838	-4.564	-5.435	-6.482	-7.745	-9.271	-11.117	-13.355	-16.073
26	-1.186	-1.410	-1.678	-1.998	-2.381	-2.840	-3.392	-4.058	-4.861	-5.833	-7.012	-8.446	-10.193	-12.325	-14.934	-18.130
27	-1.194	-1.429	-1.712	-2.053	-2.463	-2.959	-3.560	-4.290	-5.178	-6.262	-7.588	-9.214	-11.212	-13.674	-16.711	-20.467
28	-1.202	-1.449	-1.747	-2.109	-2.549	-3.084	-3.737	-4.536	-5.517	-6.725	-8.214	-10.057	-12.341	-15.179	-18.714	-23.125
29	-1.210	-1.468	-1.783	-2.167	-2.637	-3.213	-3.923	-4.798	-5.880	-7.224	-8.896	-10.981	-13.590	-16.861	-20.971	-26.148
30	-1.218	-1.488	-1.820	-2.227	-2.729	-3.349	-4.118	-5.075	-6.269	-7.762	-9.637	-11.996	-14.974	-18.741	-23.518	-29.590
31	-1.226	-1.509	-1.857	-2.288	-2.823	-3.490	-4.324	-5.369	-6.684	-8.343	-10.444	-13.111	-16.507	-20.843	-26.392	-33.511
32	-1.235	-1.529	-1.895	-2.351	-2.922	-3.638	-4.540	-5.681	-7.129	-8.971	-11.323	-14.336	-18.208	-23.196	-29.639	-37.981
33	-1.243	-1.550	-1.934	-2.416	-3.023	-3.792	-4.769	-6.013	-7.605	-9.648	-12.281	-15.683	-20.094	-25.830	-33.307	-43.080
34	-1.252	-1.571	-1.973	-2.482	-3.129	-3.953	-5.009	-6.365	-8.114	-10.380	-13.325	-17.165	-22.188	-28.781	-37.455	-48.900
35	-1.260	-1.592	-2.014	-2.551	-3.238	-4.122	-5.262	-6.739	-8.661	-11.171	-14.463	-18.795	-24.514	-32.088	-42.148	-55.547
36	-1.268	-1.614	-2.055	-2.621	-3.352	-4.298	-5.528	-7.136	-9.246	-12.026	-15.705	-20.589	-27.097	-35.796	-47.460	-63.142
37	-1.277	-1.635	-2.097	-2.694	-3.469	-4.482	-5.809	-7.558	-9.873	-12.951	-17.060	-22.565	-29.968	-39.957	-53.476	-71.825
38	-1.286	-1.658	-2.140	-2.768	-3.591	-4.674	-6.105	-8.007	-10.546	-13.952	-18.539	-24.741	-33.161	-44.627	-60.293	-81.758
39	-1.294	-1.680	-2.184	-2.845	-3.718	-4.875	-6.417	-8.484	-11.268	-15.034	-20.154	-27.140	-36.712	-49.871	-68.020	-93.126
40	-1.303	-1.703	-2.229	-2.924	-3.849	-5.085	-6.746	-8.991	-12.042	-16.206	-21.918	-29.785	-40.664	-55.762	-76.784	-106.143

FIGURE 1

Iso Elasticities of Loan Maturity with Respect to Amount Paid Per Period, m = 1

$$\frac{dn}{dA} \cdot \frac{A}{n}$$

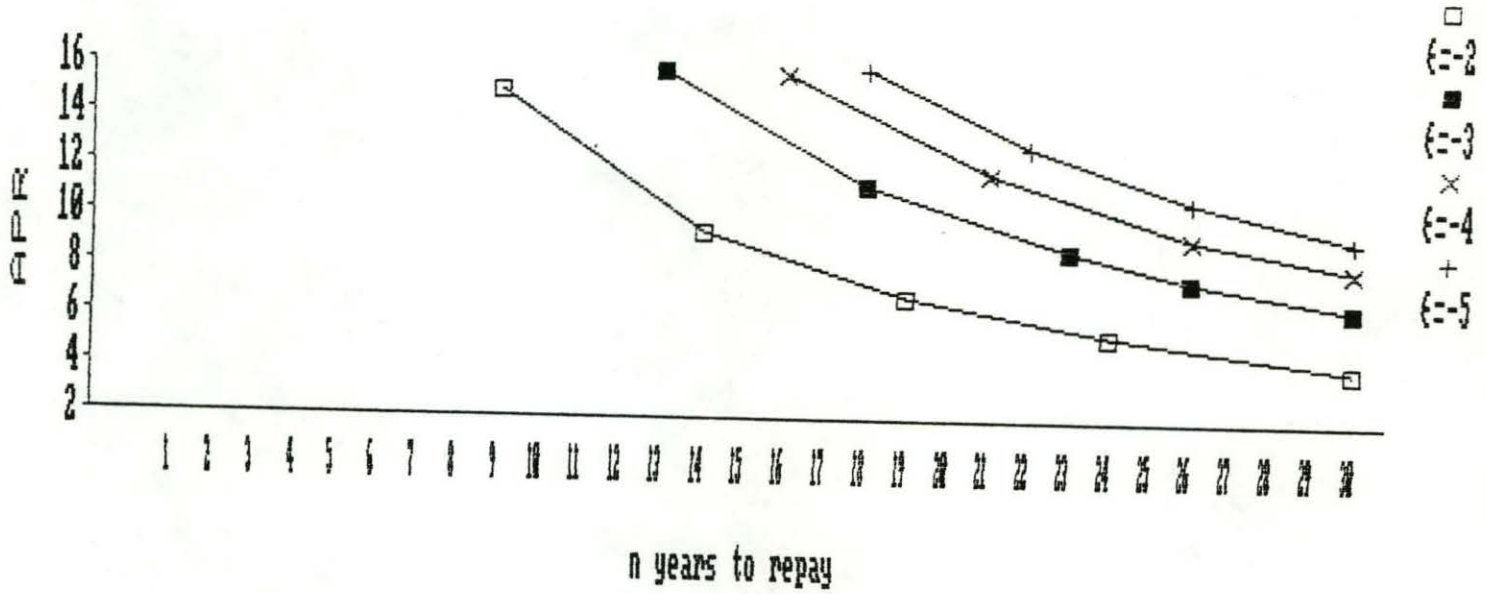
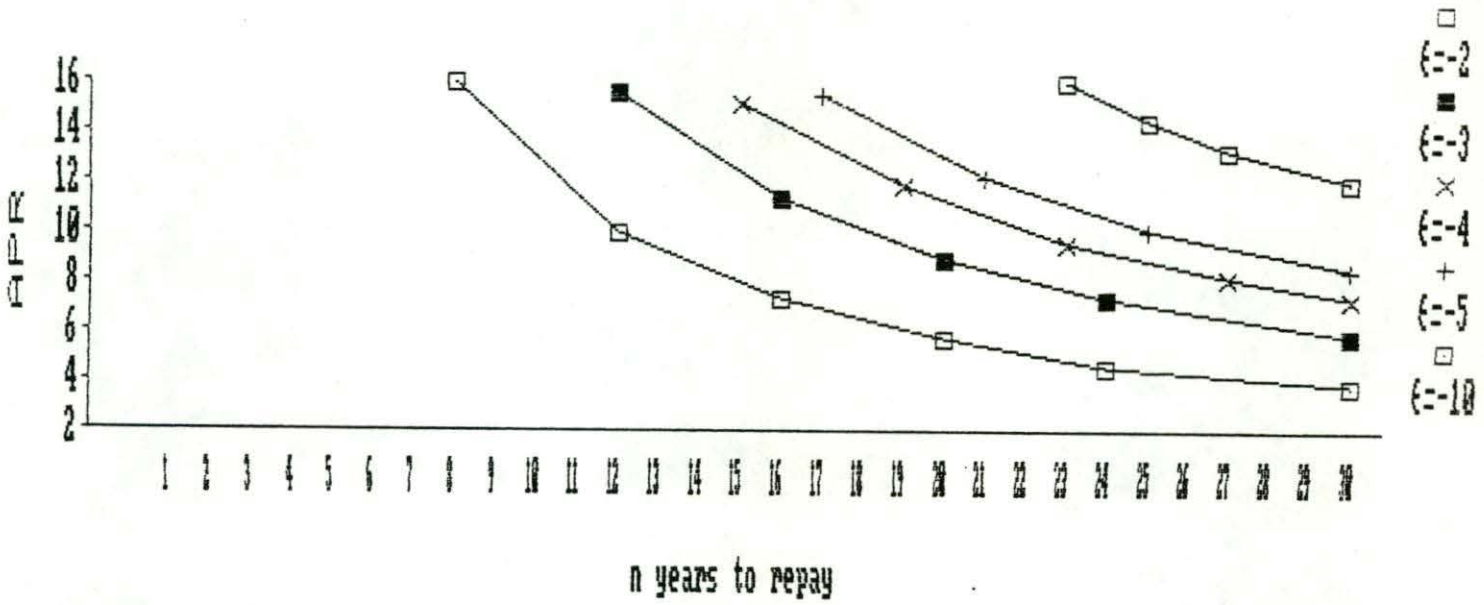


FIGURE 2

Iso Elasticities of Total Interest Paid with Respect to Amount Paid Per Period, m = 1

$$\frac{dTI}{dA} \cdot \frac{A}{TI}$$



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