Staff Paper

CROP INSURANCE UNDER CATASTROPHIC RISK

John Duncan and Robert J. Myers

January 1997 No. 97-10

Department of Agricultural Economics
MICHIGAN STATE UNIVERSITY
East Lansing, Michigan

CROP INSURANCE UNDER CATASTROPHIC RISK

John Duncan and Robert J. Myers[1]

Center for Business Research
Department of Applied Economics
University of Cambridge, UK

and

Department of Agricultural Economics
Michigan State University
East Lansing, MI, USA

Department of Agricultural Economics
Staff Paper No. 97-10

[1]Comments and suggestions by Roy Black, Steve Hanson, and Jack Meyer are much appreciated. Acknowledgment is made to the Michigan Agricultural Experiment Station for its support of this research.

CROP INSURANCE UNDER CATASTROPHIC RISK

Some risks cannot be fully insured because of incomplete or costly coverage while others cannot be insured at all because of missing insurance markets. Incomplete insurance is usually explained by asymmetric information problems resulting in moral hazard and adverse selection (e.g., Chambers; Holmstrom; Raviv; Rothschild and Stiglitz; Rubinstein and Yaari). However, catastrophic risks (risks that are highly correlated across insureds) also play a role in reducing or eliminating insurance coverage in some markets. Indeed, the collective damage from natural disasters such as hurricanes, earthquakes, and widespread drought, often lead to large aggregate outlays by insurance firms, causing them to reduce participation or withdraw altogether from these markets (Borden and Sarkar). Thus, catastrophic risk can break down insurance markets by reducing or eliminating gains from trading risks, inducing insurance firms to seek large risk premiums, and/or by requiring insurance firms to hold especially large reserves in order to participate.

One market in which catastrophic risk plays an important role is crop insurance where risks of crop failure are often highly correlated across commodities and regions. Because risks of crop failure are correlated, crop insurance firms cannot eliminate average risk by pooling a large number of farmers. This tends to leave crop insurance firms with considerable residual risk which must be dealt with either by establishing large reserves and/or by charging a risk premium to compensate for the probability of catastrophic loss (Cummins). While this issue of catastrophic risk has been discussed in the crop insurance literature, we are not aware of any published research which has analyzed the effects of catastrophic risk on the existence and nature of equilibrium in crop insurance markets. This is an important issue because the performance of crop insurance markets, and the success of Federal Crop Insurance Corporation (FCIC) policies designed to facilitate crop insurance coverage, depend critically on how insurance firms are responding to the presence of catastrophic risk.

This paper focuses on catastrophic risk as a cause of incomplete crop insurance markets. We show that when the risks faced by different farmers are correlated then insurance firms cannot eliminate average risks through pooling. Furthermore, risk-averse insurers will generally require a risk premium to offer insurance, even in the long run. This risk premium has important implications for the performance of crop insurance markets. We characterize long-run crop insurance equilibrium under catastrophic risk using a reservation preference level rather than the usual zero long-run profit condition. The idea of reservation preference levels was first put forward by Appelbaum and Katz who hypothesized that, in long-run industry equilibrium under uncertainty, firm entry and exit would ensure that the expected utility of participating firms was maintained at a reservation level given by the next best alternative activity. Here we develop the reservation preference level approach in the context of risk averse insurance firms facing catastrophic risk. We also examine the effects of (possibly subsidized) FCIC reinsurance, where insurance firms are able to transfer part of their indemnification risk to the FCIC, possibly at subsidized cost. This is exactly what occurs under current U.S. crop insurance programs.

Results of the analysis show that catastrophic risk increases crop insurance premiums, reduces farmer coverage levels and, under certain conditions, leads to a complete breakdown of the insurance market. Reinsurance, whereby the FCIC shares some of the risks faced by primary insurers, can help reduce these problems and facilitate an equilibrium, particularly if reinsurance is subsidized. The analysis has important implications for the design and management of crop insurance and reinsurance schemes, such as the U.S. federal crop insurance program.

The rest of the paper is organized as follows. First, we characterize a long-run competitive crop insurance equilibrium in a market where insurers face catastrophic risks and may have access to (possibly subsidized) reinsurance. Reinsurance amounts and provisions are assumed to be imposed on primary insurers via government policy, much as in the case of FCIC reinsurance in the current U.S. crop insurance program. The properties of the equilibrium are analyzed, including the role of catastrophic risk in explaining missing insurance markets and low coverage. Reinsurance schemes are then investigated and it is shown that reinsurance can increase coverage levels and reduce premiums, but does not necessarily facilitate equilibrium unless it is subsidized. Implications of these results are then discussed in the context of the current U.S. government crop insurance program. The final section provides concluding comments.

Competitive Insurance Equilibrium Under Catastrophic Risk

The model developed here is similar to that of Rothschild and Stiglitz, with the exceptions that risks are correlated across insureds, insurance firms are risk averse, and we allow subsidized reinsurance. The model makes several simplifying assumptions which help to focus attention on the role of catastrophic risk. The insurance market is characterized by a large number N of individual farmers, each with known potential income M. Each farmer faces a stochastic loss l which takes a value L with probability P and 0 with probability $(1 - P)$. The end-of-period income of each farmer is therefore $M - l$, which is stochastic. All of the farmers face the same marginal probability distribution for l but the losses of each pair may be correlated. Insurance firms are identical and offer contracts to farmers to insure their loss. If an insurance market exists, it is described by a triple (w, ϕ, n), where ϕ is the coverage level, w is the insurance premium per unit of coverage level, and n is the number of contracts held by each insurance firm. The coverage level lies in the range [0, 1] and is quoted as a proportion of the loss. For example, $\phi = 0.5$ indicates 50% of any loss would be reimbursed by the insurance firm. Choosing a higher coverage level entails paying a higher premium because the total premium paid is $w\phi$. The duration of the insurance market is fixed with a definite beginning and definite end for all participants. This fits well for crop insurance where insurance is sold before planting and losses are assessed after harvest.

The Demand for Insurance

The decision problem of a farmer purchasing insurance is to choose a coverage level to maximize his or her preference function defined on end-of-period income. The end-of-period income of a farmer who buys insurance is

$$\pi_d = M - w\phi_d - (1 - \phi_d)l \tag{1}$$

where the subscript d refers to the demand for insurance. The premium $w\phi_d$ is paid irrespective of the state of nature and, if there is a disaster, then $\phi_d L$ of the loss is reimbursed by insurance. The farmer chooses ϕ_d but w is set by competition in the insurance market.

To characterize the demand for insurance we find it convenient to use a linear mean-variance (MV) preference function given by[1]

$$U = M - w\phi_d - (1-\phi_d)\bar{l} - 0.5\lambda\ (1-\phi_d)^2\sigma_l^2 \tag{2}$$

where $\bar{l} = PL$ is the expected loss and σ_l^2 is the variance of the loss. Preferences are increasing in expected income and decreasing in the variance of income. The parameter 0.5λ represents the equilibrium slope at the tangency between an iso-expected utility line and the MV set (Robison and Barry). This slope reflects the degree of risk aversion (higher values indicating higher degrees of risk aversion).

The first-order condition for choosing the optimal coverage level is given by[2]

$$-w + \bar{l} + \lambda\sigma_l^2(1-\phi_d) = 0 \tag{3}$$

which represents the demand for insurance coverage at premium rate w. Of course, the demand for insurance coverage decreases with increases in the premium rate w and increases with increases in the expected loss $\bar{l}$. It can also be shown that the demand for insurance coverage increases with increases in farmer risk aversion λ and with the variance of the loss σ_l^2.

The Supply of Insurance

Insurance is provided by competitive firms which may have access to subsidized reinsurance. In most of the insurance literature, firms providing insurance are assumed to be risk neutral on grounds that: (1) insurance companies are owned by a large number of small shareholders, each holding diversified portfolios, who would therefore instruct managers to maximize expected profits; and/or (2) insurance companies pool a variety of uncorrelated risks thereby effectively eliminating average indemnification risk through diversification (e.g. Rothschild and Stiglitz). However, neither of these arguments is very convincing in the case of crop insurance. Firms providing crop insurance tend to be relatively small, privately owned providers that are not well diversified (U.S. General Accounting Office, 1992). Furthermore, crop losses tend to be correlated across crops and geographic regions, leading to catastrophic risk. In these circumstances, we might expect insurance companies to act as if they are risk averse.

We examine the case of risk averse insurance firms that offer insurance contracts to farmers. The end-of-period profit for a firm selling insurance to n farmers and reinsuring some proportion α of its policies is given by

$$\pi_s = n\phi_s(1-\alpha)(w-c) - \phi_s(1-\alpha-\delta)\sum_{i=1}^{n} l_i \tag{4}$$

where the subscript s refers to the supply of insurance, w is the premium charged per unit of coverage (as before), and c is insurance costs per unit of coverage. The insurance firm gives up some proportion $0 \leq \alpha < 1$ of its premiums (and costs) to a reinsurer and, in return, the reinsurer accepts responsibility for paying some proportion $(\alpha + \delta)$ of the indemnities. The value of δ satisfies $0 \leq \delta < 1 - \alpha$ and represents a subsidy paid by the reinsurer to the insurer. If $\delta > 0$ then the reinsurer covers a higher proportion of indemnities than it receives of premiums, which is an implicit subsidy.

If $\delta = 0$ there is no subsidy. It is assumed that reinsurance is undertaken by the FCIC and that the values of α and δ are set exogenously by government policy. It is important to introduce the possibility of (subsidized) reinsurance because much current U.S. crop insurance policy is designed to encourage participation of private insurance firms via reinsurance arrangements.[3] If the firm chooses to participate then it offers a coverage level ϕ_s to farmers, assuming a given market determined level of w and given government determined levels of α and δ. We investigate the effects of changes in α and δ further below.

As in the case of farmers, the risk averse insurance firms are assumed to have linear MV preferences. Thus, using (4) to compute the mean and variance of firm profits (see appendix 1) then the MV preference function for an individual insurer is given by

$$V = n\phi_s[(1-\alpha)(w-\bar{l}-c) + \delta\bar{l}] - 0.5\psi n\phi_s^2\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho] \tag{5}$$

where ρ is the correlation coefficient between losses of any two farmers, defined as

$$\rho = \frac{Cov(l_i, l_j)}{\sqrt{Var(l_i)Var(l_j)}} . \tag{6}$$

For simplicity, we assume that ρ is the same for every pair of farmers and consider only positive values in order to focus on risks that are catastrophic in nature. The value of ρ measures the degree of catastrophic risk in the system because as ρ increases, the loss correlation between all of the individuals in the market increases, leading to higher average portfolio risk for the insurer.

The insurance firm's short-run problem is to choose a coverage offer ϕ_s to maximize (5) assuming a given premium w and number of policies n. The equilibrium premium w, and number of policies per firm n

(and, hence, the number of firms) is then determined by competition in the insurance market. Assuming a fixed n and w the first-order-condition of the firm is[4]

$$(7) \qquad (1-\alpha)(w-\bar{l}-c) + \delta\bar{l} - \psi\phi_s\sigma_l^2(1-\alpha-\delta)^2\,[1 + (n-1)\rho] = 0$$

which represents the short-run supply of insurance coverage. The short-run supply of insurance coverage increases with increases in the premium w and decreases with an increase in cost (or expected loss $\bar{l}$). It can also be shown that the supply of insurance coverage decreases with an increase in risk aversion of insurance firms or the variance of the loss. If $\rho = 0$ (no catastrophic risk) then an increase in n, the number of farmers insured per insurance firm, does not alter the risk return trade-off of the firm, and therefore does not alter the supply of insurance. In this case, equilibrium ϕ and w do not depend on n and can be found by equating the demand and supply of insurance coverage. If $\rho > 0$ (catastrophic risk), however, then an increase in n increases the riskiness of the portfolio relative to the expected return, thereby shifting the short-run supply of insurance to the left. In this case, equilibrium ϕ and w depend on the number of farmers insured per insurance firm.

Competitive Equilibrium

Long-run competitive equilibrium is generally characterized by zero profits brought about by entry and exit of firms. When risk is involved, long-run competitive equilibrium is described by zero expected profits if the producers are risk neutral (Rothschild and Stiglitz). For risk-averse producers Appelbaum and Katz have extended Sandmo's model of the competitive firm under uncertainty to explain the behavior of expected utility maximizing firms in a long-run industry equilibrium. This is achieved by letting stochastic output price be a function of industry output, and making the expected utility of the firm equal to a reservation utility level of some benchmark activity. Entry and exit of firms then ensures that the expected utility of profits equals this benchmark utility level in the long run, and the number of firms is endogenous. In this model, firm output adjusts with free entry and exit of firms, which in turn affects output price and keeps the expected utility of the firm at the reservation level.

For the purposes of this study, a long-run equilibrium concept similar to that suggested by Appelbaum and Katz is adopted. We assume that, in the long run, insurance firms maintain a reservation preference level b, so that $V = b$ in long-run equilibrium. We can now define a long-run insurance equilibrium assuming identical firms and farmers facing identical marginal loss distributions.

Definition: *A competitive equilibrium in the model with catastrophic risk and subsidized reinsurance is a premium level w^0, coverage level Ψ^0, and number of policies n^0, for each firm that satisfy:*

$$-w^0 + \bar{l} + \lambda\sigma_l^2(1-\phi^0) = 0 \tag{8}$$

$$(1-\alpha)(w^0-\bar{l}-c) + \delta\bar{l} - \psi\phi^0\sigma_l^2(1-\alpha-\delta)^2[1 + (n^0-1)\rho] = 0 \tag{9}$$

$$n^0\phi^0[(1-\alpha)(w^0-\bar{l}-c) + \delta\bar{l}] - 0.5\psi n^0(\phi^0)^2\sigma_l^2(1-\alpha-\delta)^2[1 + (n^0-1)\rho] - b = 0 \tag{10}$$

The first expression is the demand for insurance by an individual farmer, the second is the short-run supply of insurance by a competitive firm, and the third is the long-run equilibrium condition that the preference level of each firm equals the reservation level b. The simultaneous solution of the three equations determines the long-run equilibrium premium w^0, coverage level ϕ^0, and the number of policies n^0 sold by each firm. The number of firms is then determined by dividing the total number of farmers N by the number of contracts per firm, n^0.

Notice that we can solve (8) for w and substitute the result into (9) to get a short-run equilibrium relationship between ϕ and n given by

$$(11) \qquad \phi = \frac{(1-\alpha)(\lambda\sigma_l^2 - c) + \delta\bar{l}}{(1-\alpha)\lambda\sigma_l^2 + \psi\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]}$$

For any given n, (11) gives the equilibrium coverage level that equates the demand and short-run supply of insurance. This short-run equilibrium relationship between ϕ and n will be useful in the following analysis.

Existence of Equilibrium

To characterize the nature of an equilibrium, and investigate its existence, we examine the equilibrium equations graphically in figure 1. For now, we assume no reinsurance ($\alpha = \delta = 0$) in order to focus attention on equilibrium in insurance markets where there is catastrophic risk but no reinsurance possibilities. The case of FCIC reinsurance is examined later.

The demand for insurance (8) is a negatively sloped linear relationship between w and ϕ which is graphed in the northeast quadrant of figure 1, and the short-run equilibrium relationship (11) between ϕ and n is graphed in the southeast quadrant of the figure. It is obvious that as n goes to infinity in (11) then ϕ goes to zero asymptotically, and it can be shown that as n goes to zero the associated ϕ intercept lies to the left of the ϕ intercept in the demand equation.

Finally, use (8) and (11) to substitute w and ϕ out of the firm preference function (5). This gives the equilibrium firm preference level as a function of the number of contracts per firm n. We denote this equilibrium preference function $V(n;\, \theta)$ where $\theta = (c,\, \bar{l},\, \psi,\, \lambda,\, \sigma_l^2,\, \rho,\, \alpha,\, \delta)$ is a parameter vector. The equilibrium preference level is graphed as a function of n in the southwest quadrant of figure 1. It can be shown (see appendix 2) that $V(0;\ \theta) = 0$, that $V(n;\ \theta)$ is monotonically increasing in n, and that when $\alpha = \delta = 0$ (no reinsurance) then $V(n;\ \theta)$ converges to a limiting value,

(12) $$\lim_{n \to \infty} V(n;\ \theta) = \frac{(\lambda \sigma_I^2 - c)^2}{2 \psi \rho \sigma_I^2} = \tilde{b}$$

as n goes to infinity.

It is now easy to characterize long-run competitive equilibrium in the crop insurance market with catastrophic risk and no reinsurance. Beginning in the southwest quadrant of figure 1, the long-run reservation preference level b determines the firm preference level that must be attained in long-run equilibrium. Through $V(n;\ \theta)$, this determines the equilibrium number of contracts per firm n^0 which, in turn, determines the equilibrium coverage level ϕ^0 through (11), graphed in the southeast quadrant of figure 1. The equilibrium premium w^0 can then be read off the demand curve (8) in the northeast quadrant of the figure.

It is obvious from figure 1 that long-run competitive equilibrium in this insurance market can fail to exist under catastrophic risk. If the reservation preference level b is high enough, firms will never be able to reach it no matter how many contracts are sold by each firm. Thus, even as the total number of farmers goes to infinity, firms will still not be able to generate enough utility to induce participation in the market. The reason is that increases in n lead to an increase in expected profit but also alter the risk-return trade-off by increasing the variance of profits. Eventually, increases in expected profit no longer compensate for the increase in risk and the $V(n; \theta)$ function becomes bounded. Thus, if the reservation preference level b is above the limiting preference level $\tilde{b}$ then insurance firms will not participate at any n and the equilibrium breaks down. Thus we have the following result.

Result 1: An insurance market equilibrium will fail to exist under catastrophic risk if the reservation preference level b of insurance firms is greater than the maximum reservation preference level $\tilde{b}$ that will support an equilibrium.

Also notice that if an equilibrium does exist then the higher is n, the number of contracts per firm, the lower is the equilibrium coverage level ϕ and the higher the equilibrium premium w. This results from the higher average risk borne by firms when n increases under catastrophic risk.

It is interesting that an equilibrium always exists in the case of uncorrelated risks $(\rho = 0)$, provided there are a sufficiently large number of farmers N participating in the market. The reason is that if $\rho = 0$ then increases in n increase expected profits but reduce average risk, so that insurance firms are always better off as n rises. The end result it that the supply of insurance does not depend on n, and neither does the equilibrium premium or coverage level [set $\rho = 0$ in (11)]. Furthermore, $V(n;\ \theta)$ is then clearly linear in n. This means that an equilibrium will always exist, no matter how big the reservation preference level b, provided that the number of contracts per firm can be made sufficiently large (i.e. provided N is not too small). The next result follows immediately.

***Result 2*:** An increase in catastrophic risk ρ reduces the set of reservation preference levels that will support an equilibrium by lowering the maximum reservation preference level $\tilde{b}$.

In the limiting case of $\rho = 1$ then $\tilde{b} = (\lambda\sigma_l^2 - c)^2 \ / \ 2\psi\sigma_l^2$ and any reservation preference level greater than this value will not support an equilibrium. Notice that $\tilde{b}$ gets larger (more equilibria supported) as farmers become more risk averse (larger λ) but smaller (fewer equilibria supported) as insurance firms become more risk averse (larger ψ).

Comparative Statics

We have seen that if catastrophic risk is high relative to the reservation preference level of insurance firms then the firms may choose not to participate and the market breaks down. Another interesting question is: what happens to the equilibrium insurance premium, coverage level, and number of contracts per firm, when an equilibrium exists and there is a marginal increase in catastrophic risk?

Assume that, despite the presence of catastrophic risk, the reservation preference level lies below $\tilde{b}$ and an equilibrium exists. Then the marginal effect of an increase in ρ can be calculated using standard comparative statics techniques. We relegate these calculations to appendix 3 where it is shown that $dw/d\rho > 0$, $d\phi/d\rho < 0$, and $dn/d\rho$ is of indeterminate sign. Thus we have the following result.

Result 3. If an equilibrium exists under catastrophic risk without reinsurance then a marginal increase in catastrophic risk increases the equilibrium premium, reduces the equilibrium coverage level, and may increase or decrease the number of policies offered per firm.

Some intuition for these comparative static results can be gleaned from re-examining figure 1. A marginal increase in ρ has no effect on farmer demand for insurance so the demand curve in the northeast quadrant of figure 1 does not shift. On the other hand, the short-run supply of insurance shifts to the left as ρ increases because insurance firms face additional risk without a compensating increase in expected return. Thus, the short-run equilibrium relationship between ϕ and n depicted in the southeast quadrant of figure 1 shifts inward towards the origin. This shift puts downward pressure on the equilibrium coverage level ϕ, and upward pressure on the equilibrium premium w.

The increase in ρ also shifts the $V(n;\ \theta)$ function depicted in the southwest quadrant of figure 1, but the direction of this shift is ambiguous. Other things being equal, the increase in ρ has a direct negative effect on $V(n;\ \theta)$ because it increases risk without a compensating increase in expected return. But the increase in

ρ also decreases ϕ and increases w, which leads to an increase in $V(n;\ \theta)$ for given n, as expected profits of the insurance firms rise. The direction of the net shift in $V(n;\ \theta)$ depends on which of these two opposing effects dominates.

The end result of these shifts is that an increase in ρ always increases w and decreases ϕ. However, the number of contracts offered per firm may have to increase or decrease to maintain the reservation preference level b, depending on whether the direct negative effect of an increase in ρ on the insurance firm's preference level is offset by the indirect positive effect from increasing the premium and lowering the coverage level.

The Role of Reinsurance

We have seen that catastrophic risk increases crop insurance premiums, reduces coverage levels, and may lead to a complete breakdown of the insurance market if the level of catastrophic risk is high relative to the reservation preference level of insurance firms. According to Arrow and Lind, these outcomes may reflect economic inefficiency because they result from the market's inability to spread the risks of crop failure over a large enough number of individuals, thereby eliminating average risk via diversification. In these circumstances there may be a potential role for government policy to bring about a more efficient allocation of the risks of crop failure.

In this section we examine the role of FCIC reinsurance as a means of spreading some of the risks of crop failure across taxpayers, thereby facilitating a market equilibrium and bringing about a more efficient allocation of risk. FCIC reinsurance comes in two main forms, proportional and stop-loss. With proportional reinsurance, insurance firms share a specified proportion of both policy premiums (and costs) and indemnities with the FCIC. The insurance firms themselves continue to collect premiums and service and settle claims. With stop-loss reinsurance the FCIC agrees to reimburse insurance firms if their total indemnity payments exceed a predetermined level. In exchange, the FCIC may receive a premium from the insurance firms. Both types of reinsurance may be subsidized by the government. In 1990 over 90% of U.S. crop insurance policies were marketed by private firms engaging in proportional and stop-loss reinsurance through the FCIC.

In the remainder of this section we focus on proportional reinsurance because it offers a simple way to conceptualize and study the effects of crop reinsurance. It can also be shown that proportional and stop-loss reinsurance have essentially identical effects on the market equilibrium (see Duncan).

The FCIC is providing reinsurance to insurance firms with the goal of facilitating a crop insurance equilibrium, and increasing the coverage level provided to farmers (U.S. General Accounting Office, 1987, 1988). Thus, we examine whether these goals can be achieved using proportional reinsurance in the insurance market model presented here. We initially assume proportional reinsurance without subsidization ($\alpha > 0$ and $\delta = 0$). The case of subsidized reinsurance is examined later.

To investigate whether reinsurance can help facilitate an equilibrium we examine its effect on $\tilde{b}$, the maximum reservation preference that will support a crop insurance equilibrium. If reinsurance increases $\tilde{b}$ then it will have expanded the set of reservation preference levels that can be supported in equilibrium, thereby expanding the opportunity set of available equilibria. From the results in appendix 2, but this time allowing for reinsurance ($\alpha > 0$), but not subsidized reinsurance ($\delta = 0$), we see that the limiting attainable preference level $\tilde{b}$ as n goes to infinity remains as in (12), and is not affected by the availability of reinsurance. Thus, we have the following result.

Result 4: Proportional reinsurance does not expand the opportunity set of available equilibria because it has no effect on the maximum allowable insurance firm preference level that can be supported in equilibrium.

The reason for this result is that reinsurance alters the scale of the insurance firms' operation by transferring some risk (and return) to the FCIC. But as n goes to infinity so does the scale of the firms operation, irrespective of the extent of reinsurance. Thus, reinsurance without subsidization has no effect on the limiting behavior of $\tilde{b}$.

Assuming an equilibrium exists under reinsurance and catastrophic risk, it is of interest to determine the effects of a marginal increase in the reinsurance proportion α when there is no subsidization $(\delta = 0)$. This comparative static analysis is conducted in appendix 3 and leads to the following result.

Result 5: If an equilibrium exists under catastrophic risk and proportional reinsurance, then an increase in proportional reinsurance reduces the equilibrium premium, increases the equilibrium coverage level, and increases the number of policies offered per firm (decreasing the number of firms).

These results occur because proportional reinsurance reduces the risks faced by individual firms, thereby shifting the supply of insurance outward. This increases coverage and reduces the equilibrium premium. Nevertheless, each insurance firm must now share part of their profits with the reinsurer so they have to offer more policies in order to attain the same level of reservation preference as before. This increases the number of policies per firm and reduces the number of firms in equilibrium.

The results of this section show that while unsubsidized reinsurance may be ineffective in facilitating equilibrium in a market that has failed because of catastrophic risk, it will increase coverage levels and reduce premiums if conditions are such that an equilibrium already exists.

Subsidized Reinsurance

We have seen that unsubsidized reinsurance does not expand the opportunity set of available equilibria, and so may be ineffective in facilitating equilibrium where none existed previously because of catastrophic risk. But what about subsidized reinsurance? Can the introduction of a subsidy facilitate equilibrium and expand coverage levels further?

Suppose the reinsurer collects a proportion α of total premiums but pays out a higher proportion $\alpha + \delta$ of the insurance firm indemnities. Then δ represents the size of the subsidy. In appendix 2 it is shown that, under subsidized reinsurance, the equilibrium firm preference function $V(n;\ \theta)$ converges to a limiting value,

$$\lim_{n \to \infty} V(n;\ \theta) = \frac{[(1-\alpha)(\lambda\sigma_l^2 - c) + \delta\bar{l}]^2}{2\psi\rho\sigma_l^2(1-\alpha-\delta)^2} = \tilde{b} \tag{13}$$

as n goes to infinity. As long as $\lambda\sigma_l^2 > c$, which is a condition required for equilibrium with positive coverage levels [see (11)], then a marginal increase in δ increases the size of the numerator in (13) and decreases the size of the denominator. This leads to the following result.

Result 6: An increase in the reinsurance subsidy helps facilitate equilibrium by increasing the maximum allowable reservation preference level that will support an equilibrium, thereby expanding the opportunity set of available equilibria.

Reinsurance itself does not change the opportunity set of available equilibria, but subsidized reinsurance provides addition returns to insurance firms and these additional returns persist even as the scale of the firm's operation goes to infinity. Thus, higher preference levels can be attained with the subsidy than without and subsidization can facilitate equilibrium at reservation preference levels where no equilibrium existed previously.

Assuming an equilibrium exists under subsidized reinsurance, then an increase in the size of the subsidy will have comparative static effects on the equilibrium premium, coverage level, and number of contracts per firm. These comparative static effects are derived in appendix 3 and lead to the following result.

Result 7: If an equilibrium exists with catastrophic risks and subsidized reinsurance then an increase in the reinsurance subsidy reduces the equilibrium premium, increases the equilibrium coverage level and may increase or decrease the number of policies offered per firm.

This result occurs because the subsidy increases the short-run supply of insurance, which puts downward pressure on the premium and upward pressure on the coverage level. The equilibrium firm preference function $V(n;\ \theta)$ also shifts but the direction is ambiguous. The subsidy increase has a direct positive effect on the firm preference level but the lower premium and higher coverage level also lead to an indirect negative effect. The end result is that the equilibrium number of contracts per firm may have to increase or decrease to keep firms at their reservation preference level.

The results of this section show that not only can subsidized reinsurance facilitate an equilibrium which has failed because of catastrophic risk, but the higher the subsidy the higher the coverage levels and lower the equilibrium premium.

Implications for the U.S. Crop Insurance Program

The model outlined in this paper is highly stylized but the results do suggest some interesting implications for crop insurance policy in the presence of catastrophic risk. First, in the absence of government policy, catastrophic risk can lead to higher premiums and lower coverage levels than might occur otherwise. Furthermore, if catastrophic risk becomes high enough it may lead to a complete breakdown of the insurance market. This suggests that catastrophic risk must be seriously considered, along with moral hazard and adverse selection, as a source of inadequate coverage and incomplete markets for crop insurance.

Second, proportional reinsurance without subsidization may increase coverage levels and reduce premiums if an insurance market already exists, but is unlikely to induce market participation if conditions are such that the market has failed previously because of catastrophic risk. Thus, reinsurance by itself may be inadequate for inducing participation and bringing about a market solution to crop insurance provision.

Third, subsidized reinsurance does expand the opportunity set of available equilibria, and therefore can encourage establishment of a crop insurance market where none existed previously because of catastrophic risk. At the same time, the subsidy can encourage additional farmer participation, in the form of higher coverage levels and lower premium rates. Thus, subsidized reinsurance can achieve some of the stated objectives of U.S. crop insurance policy, such as creating new insurance opportunities for farmers, keeping premiums low, and expanding crop insurance coverage (U.S. General Accounting Office, 1987, 1988).

Conclusions

This paper investigates the role of catastrophic risk in contributing to inadequate or incomplete crop insurance coverage. A long-run insurance market equilibrium is defined assuming risk averse farmers and insurance firms, and using the concept of a reservation preference level that must be obtained for insurance firms to participate in the market in the long run. The reservation preference level plays a critical role in determining the effects of catastrophic risk, and of reinsurance and subsidies which are designed to counteract catastrophic risk.

Results indicate that high levels of catastrophic risk can reduce coverage levels, increase premiums and, if high enough, lead to a complete breakdown of the market. This occurs because insurance firms are risk averse and, with correlated risks, they cannot reduce average risk by simply expanding their portfolio of crop insurance contracts. Assuming that the government should operate as a risk-neutral agent because it can diversify risk over a very large number of taxpayers, then catastrophic risk may therefore lead to an inefficient allocation of risks and a potential role for government policy in reallocating and spreading risks across taxpayers (Arrow and Lind).

Unsubsidized reinsurance arrangements with the FCIC can help increase coverage levels and reduce premiums if an equilibrium already exists, but does not expand the opportunity set of available equilibria. It is therefore of limited use in providing incentives for market creation where none existed previously because of catastrophic risk. On the other hand, subsidized reinsurance can help facilitate an equilibrium by expanding the opportunity set of available equilibria, and can also increase coverage levels and reduce the equilibrium premium. Overall, the results suggest that catastrophic risk can play an important role in crop insurance markets and has serious implications for the effects of government policies designed to increase coverage and facilitate market equilibrium.

References

Appelbaum, E., and E. Katz. "Measures of Risk Aversion and Comparative Statics of Industry Equilibrium." *Amer. Econ. Rev.* 76(June 1986):524-29.

Arrow, K.J., and R.C. Lind. "Uncertainty and the Evaluation of Public Investment Decisions." *Amer. Econ. Rev.* 60(June 1970):364-78.

Borden, S., and A. Sarkar. "Securitizing Property Catastrophic Risk." *Current Issues in Economics and Finance* 2(August 1996).

Chambers, R.G. "Insurability and Moral Hazard in Agricultural Insurance Markets." *Amer. J. Agr. Econ.* 71(August 1989):604-16.

Cummins, J.D. "Statistical and Financial Models of Insurance Pricing and the Insurance Firm." *J. of Risk and Insurance* 63(June 1991):261-302.

Duncan, J. *The Economic Effects of Alternative Institutional Designs for U.S. Crop Insurance*. Unpublished Ph. D. Dissertation, Department of Agricultural Economics, Michigan State University, East Lansing, MI, 1994.

Holstrom, B. "Moral Hazard and Observability." *Bell J. of Econ.* 10(Spring 1979):74-91.

Raviv, A. "The Design of an Optimal Insurance Policy." *Amer. Econ. Rev.* 69(March 1979):84-96.

Robison, L.J., and P.J. Barry. *The Competitive Firm's Response to Risk*, Macmillan Publishing Company, New York:NY, 1987.

Rothschild, M., and J. Stiglitz. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly J. of Econ.* 90(November 1976):629-50.

Rubinstein, A., and M. Yaari. "Repeated Insurance Contracts and Moral Hazard." *J. of Econ. Theory* 30(March 1983):74-97.

Sandmo, A. "On the Theory of the Competitive Firm Under Price Uncertainty." *Amer. Econ. Rev.* 61(March 1971):65-73.

U.S. General Accounting Office. *Crop Insurance: Overpayment of Claims by Private Companies Costs the Government Millions*. Report to Chairman, Subcommittee on Conservation, Credit, and Rural Development, Committee on Agriculture, House of Representatives, Washington, DC, GAO/RCED-88-7, November, 1987.

________. *Crop Insurance: FCIC Needs to Improve Its Oversight of Reinsured Companies*. Report to Congressional Requesters, Washington, DC, GAO/RCED-89-10, October, 1988.

________. *Crop Insurance: Program Has Not Fostered Significant Risk Sharing by Insurance Companies*. Report to the Chairman, Committee on Agriculture, Nutrition, and Forestry, U.S. Senate, Washington, DC, GAO/RCED-92-25, January, 1992.

Appendix 1. The Variance of Insurance Profits

The variance of a sum of random variables is

$$Var\left(\sum_{i=1}^{n} l_i\right) = \sum_{i=1}^{n} Var(l_i) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} Cov(l_i, l_j).$$

Since, in this study, all the random variables have the same marginal distribution, and the covariance between any two random variables is positive and identical, the correlation coefficient between any two of the random variables is

$$\rho = \frac{Cov(l_i, l_j)}{\sigma_l^2}$$

Substituting ρ into the variance expression then gives

$$Var\left(\sum_{i=1}^{n} l_i\right) = \sum_{i=1}^{n} \sigma_l^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \rho \sigma_l^2$$

or

$$Var\left(\sum_{i=1}^{n} l_i\right) = n\sigma_l^2[1 + (n - 1)\rho].$$

which is the result used in (5).

Appendix 2. Properties of the Firm Preference Function

The equilibrium firm preference function $V(n;\theta)$ can be written as,

$$V(n;\theta) = n\phi(n)\{(1-\alpha)[w(n)-\bar{l}-c] + \delta\bar{l}\} - 0.5\psi n[\phi(n)]^2\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]$$

We can add and subtract $0.5\psi n[\phi(n)]^2\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]$ to this without changing anything. Collecting terms then gives,

$$V(n;\ \theta) = n\phi(n)\{(1-\alpha)[w(n)-\bar{l}-c] + \delta\bar{l} - \psi\phi(n)\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]\}$$
$$+ 0.5\psi n[\phi(n)]^2\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]$$

In equilibrium, (9) implies that the $\{\cdot\}$ term is zero which leaves,

$$V(n;\ \theta) = 0.5\psi n[\phi(n)]^2\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]$$

where $\phi(n)$ is given by (11). The properties of $V(n;\ \theta)$ are now easy to derive.

1. $V(0;\ \theta) = 0$ is trivial.

2. $$\frac{dV(n;\ \theta)}{dn} = \frac{\partial V}{\partial n} + \frac{\partial V}{\partial \phi}\cdot\frac{\partial \phi}{\partial n}$$
$$= \frac{\partial V}{\partial n}$$

because, in equilibrium, $\partial V/\partial\phi = 0$. Thus

$$\frac{dV(n;\ \theta)}{dn} = 0.5\psi[\phi(n)]^2\sigma_l^2(1-\alpha-\delta)^2(1-\rho+2n\rho) > 0$$

and $V(n;\ \phi)$ is monotonically increasing.

3. $$\lim_{n\to\infty} V(n;\ \theta) = \lim_{n\to\infty} 0.5\psi n[\phi(n)]^2\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]$$

$$= \lim_{n\to\infty} 0.5\psi n[\phi(n)]^2\sigma_l^2(1-\alpha-\delta)^2(1-\rho) + \lim_{n\to\infty} 0.5\psi n^2[\phi(n)]^2\ \sigma_l^2(1-\alpha-\delta)^2\rho$$

From figure 1 it is easy to see that as $n\to\infty$ then $\phi\to 0$. And since $[\phi(n)]^2$ is of second order it will go to zero faster than $n\to\infty$ and the first limit is therefore zero (a formal proof is straightforward). The second limit depends on

$$\lim_{n\to\infty} n^2[\phi(n)]^2 = \lim \frac{n^2[(1-\alpha)(\lambda\alpha\sigma_l^2-c) + \delta\bar{l}]^2}{\{(1-\alpha)\lambda\sigma_l^2 + \psi\sigma_l^2(1-\alpha-\delta)^2[1 + (n-1)\rho]\}^2}$$

$$= \lim_{n\to\infty} [(1-\sigma)(\lambda\sigma_l^2-c) + \delta\bar{l}]^2 \Bigg/ \left[\frac{(1-\alpha)\lambda\sigma_l^2 + \psi\sigma_l^2(1-\alpha-\delta)^2}{n} + \frac{\psi\sigma_l^2(1-\alpha-\delta)^2(n-1)\rho}{n}\right]^2$$

$$= \frac{[(1-\alpha)(\lambda\sigma_l^2-c) + \delta\bar{l}]^2}{[\psi\sigma_l^2(1-\alpha-\delta)^2\rho]^2}$$

Thus, we have

$$\lim_{n \to \infty} V(n;\ \theta) = \frac{0.5\psi\sigma_l^2(1-\alpha-\delta)^2\rho[(1-\alpha)(\lambda\sigma_l^2 - c) + \delta\bar{l}]^2}{[\psi\sigma_l^2(1-\alpha-\delta)^2\rho]^2}$$

$$= \frac{[(1-\alpha)(\lambda\sigma_l^2 - c) + \delta\bar{l}]^2}{2\psi\rho\sigma_l^2(1-\alpha-\delta)^2}$$

Setting $\alpha = \delta = 0$ gives the limiting value (12) assuming no reinsurance. Setting $\alpha > 0$ (reinsurance but no subsidization) again leads to (12). Allowing α and δ to both be positive gives the general result (13).

Appendix 3. Comparative Statics Results

We undertake a standard comparative statics analysis using the equilibrium conditions (8) -(10) and the implicit function theorem. Rewrite (8)-(10) as a vector of equations $F(x, \theta) = 0$ where $x = (w, \phi, n)$ and $\theta = (c, \bar{l}, \psi, \lambda, \sigma_l^2, \rho, \alpha, \delta)$ are the exogenous variables. Assuming the conditions of the implicit function theorem are satisfied then, at an equilibrium, $x = G(\theta)$ is an implicit function defined by $F[G(\theta), \theta] = 0$ and characterized by

$$F_x(x, \theta).G_\theta(\theta) = -F_\theta(x, \theta)$$

where the subscripts indicate matrices of partial derivatives. Particular derivatives, such as $dw/d\rho$, $d\phi/d\rho$, and $dn/d\rho$, can be computed via Cramer's rule.

Differentiating (8)-(10) and rearranging terms we see that

$$F_x(x, \theta) = \begin{bmatrix} -1 & -\lambda\sigma_l^2 & 0 \\ 1-\alpha & -\psi\sigma_l^2(1-\alpha-\delta)^2[1+(n-1)\rho] & -\psi\phi\sigma_l^2(1-\alpha-\delta)^2\rho \\ n\phi(1-\alpha) & 0 & 0.5\psi\phi^2\sigma_l^2(1-\alpha-\delta)^2(1-\rho) \end{bmatrix}$$

Furthermore,

$$-F_\rho(x, \theta) = \begin{bmatrix} 0 \\ \psi\phi\sigma_l^2(1-\alpha-\delta)^2(n-1) \\ 0.5\,\psi n\phi^2\sigma_l^2(1-\alpha-\delta)^2(n-1) \end{bmatrix}$$

Straightforward calculations show that $\det[F_x(x, \theta)] > 0$. Now consider the sequence of matrices $F_i(x, \theta)$, $i = 1, 2, 3$ which represent $F_x(x, \theta)$ but with the ith column replaced by $-F_\rho(x, \theta)$. It is again straightforward to show that for $n > 1$, $\det[F_1\ (x,\ \theta)] > 0$, $\det[F_2(x,\ \theta)] < 0$, and $\det[F_3(x,\ \theta)]$ is of indeterminate sign. Thus, since $x = (w,\ \phi,\ n)$, Cramer's rule implies $dw/d\rho > 0$ $d\phi/d\rho < 0$, and $dn/d\rho$ could be positive, zero, or negative. These are the comparative static derivatives for Result 3.

Next note that if $\delta = 0$ then

$$-F_{\alpha}(x,\ \theta) = \begin{bmatrix} 0 \\ -\psi\phi\sigma_l^2(1-\alpha)[1+(n-1)\rho] \\ 0 \end{bmatrix}$$

and consider the sequence of matrices $F_i(x,\ \theta)$, $i = 1, 2, 3$ which represent $F_x(x,\ \theta)$ but with the ith column replaced by $-F_{\alpha}(x,\ \theta)$. In this case we can show that $\det[F_1\ (x,\ \theta)] < 0$, $\det[F_2(x,\ \theta)] > 0$, and $\det[F_3(x,\ \theta)] > 0$. Thus, since $x = (w,\ \phi,\ n)$, Cramer's rule implies $dw/d\alpha < 0$, $d\phi/d\alpha > 0$, and $dn/d\alpha > 0$. These are the comparative static derivatives for Result 5.

Finally, note that

$$-F_{\delta}(x,\ \theta) = \begin{bmatrix} 0 \\ -\{\bar{l}+2\psi\phi\sigma_l^2(1-\alpha-\delta)[1+(n-1)\rho]\} \\ -n\phi\{\bar{l}+\psi\phi\sigma_l^2(1-\alpha-\delta)[1+(n-1)\rho]\} \end{bmatrix}$$

and consider the sequence of matrices $F_i(x,\ \theta)$, $i = 1, 2, 3$ which represent $F_x(x,\ \theta)$ but with the ith column replaced by $-F_{\delta}(x,\ \theta)$. In this case it is straightforward to show that $\det[F_1\ (x,\ \theta)] < 0$, $\det[F_2(x,\ \theta)] > 0$, and $\det[F_3(x,\ \theta)]$ is of indeterminate sign. Thus, since $x = (w,\ \phi,\ n)$, Cramer's rule implies $dw/d\delta < 0$ $d\phi/d\delta > 0$, and $dn/d\delta$ could be positive, zero, or negative. These are the comparative static derivatives for Result 7.

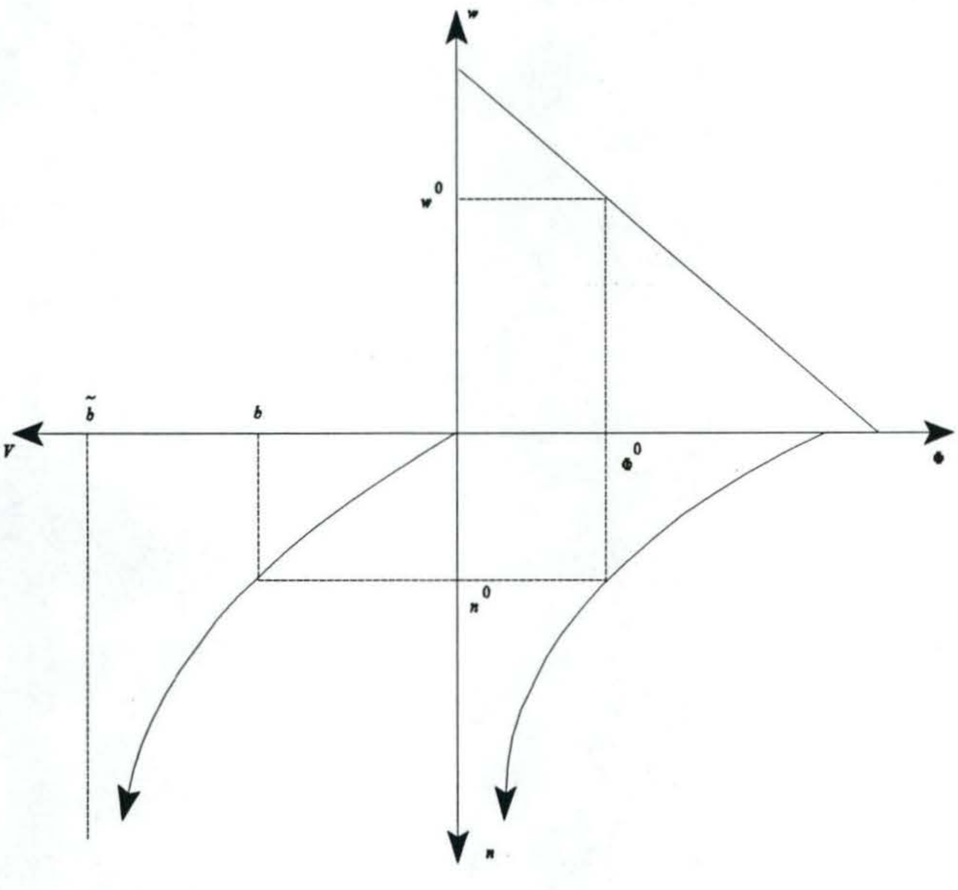

Figure 1. **Competitive insurance market equilibrium under catastrophic risk**

ENDNOTES

1. While recognizing the limitations of the MV framework we find that it is adequate to demonstrate the main results of the study without unduly complicating the presentation.

2. The second-order condition for a maximum is immediately satisfied because $-\lambda\sigma_I^2 < 0$.

3. In 1990 over 90% of U.S. crop insurance policies were marketed by private firms engaging in reinsurance with the FCIC.

4. The second-order condition for a maximum is satisfied because $-\psi\sigma_I^2(1-\alpha-\delta)^2[1 + (n-1)\rho] < 0$.