Staff Paper

ANALYZING FIRM RESPONSE TO RISK USING MEAN-VARIANCE MODELS

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Analyzing Firm Response to Risk Using Mean-Variance Models

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Summary

This paper suggests that future research will emphasize at least three areas. These include: more general relationships between indirect and direct outcome variables; more complex models with multiple risk responses and source of risk; and dominance analysis, the effect on risk efficiency of including additional risk instruments in the firm's portfolio. The paper then discusses the usefulness of expected utility (EU) models and mean-variance (EV) models in future risk research. Both models will continue to dominate risk analysis, but more complicated risk models will increasingly rely on EV models.

1. Introduction

The expected utility (EU) model is the most widely accepted indexing rule for ordering choices and doing comparative static analysis under risk. Schoemaker (1982) claims it has been the major decision making paradigm since World War II. Although several concerns and empirical contradictions have emerged to challenge the EU model, its dominant role as a theoretical tool for the analysis of the firm under risk continues. There are good reasons for EU's pre-eminent role in the analysis of risk. First, it can be derived from reasonable assumptions. Second, it has produced theoretical results that in most cases correspond with our intuition. And third, it leads to empirically useful ranking tools including stochastic dominance rules (Hadar & Russell, 1969; Hanoch & Levy, 1969; and Meyer, 1977).

The most durable challenger to the EU model has been mean-variance (EV) or the equivalent mean-standard deviation (MS) model. The EV model is popular for many reasons including the following. First, under some conditions EV models can be shown to be consistent with EU models. Second, EV models have produced theoretical results that in most cases correspond with our intuition; namely, that risk averse decision makers prefer increased expected income and dislike increased variation of income (Hawawini, 1978). Third, EV theory leads to an empirically useful efficient EV set or frontier that can be generated by programming models and other maximization algorithms. Fourth, results derived using EV analysis can be described in two dimensional space that facilitate the communication of results. Fifth, compared to EU models, EV models often produce more tractable analytic results. And finally, EV sets permit us to perform dominance analysis; that is, we can examine under what conditions one set of risk instruments is preferred to another set by all risk averse agents.

Differences between the needs of decision makers and analysts have resulted in differentiated preferences for the EU and EV models. EV models are popular in schools of business and among financial market analysts. It is also popular among many applied researchers. On the other hand, the EU model is still used for most theoretical analysis by economists and many agricultural economists. Thus both EU and EV models have proven to be accepted tools of analysis, although EU appears to be most useful as a theoretical tool while EV appears more suited for applied analyses that focus on the identification of efficient sets.

In what follows, we compare the usefulness of EV and EU models for ranking distributions and for theoretical analysis. Then we suggest three areas we expect future risk research to emphasize. We also suggest how EV and EU models may contribute to progress in these three areas. The three areas of future risk research emphasis include the analysis of: (1) more complicated transformations between direct and indirect outcome variables; (2) more complicated decision environments with multiple sources of risk and choice variables that better match the real world conditions facing firms; and (3) dominance conditions analyzed from a theoretical perspective.

2. Ranking empirical distributions using EV and EU models

Consider first EV analysis as a decision theory tool that orders probability distributions into efficient and inefficient sets. Efficient sets are defined with reference to a particular class of decision makers and sometimes restricted to well defined probability distributions. If the well defined class of decision makers unanimously prefer distribution A to distribution B, then distribution A is an efficient choice and distribution B is inefficient.

Risk averse EU decision makers facing normal distributions or distributions that differ by location and scale but otherwise are identically distributed will find their preferred choice in the efficient EV set. Risk averse EU decision makers will also find their preferred choice in the efficient EV set when there is only one risky investment to select that increases in variance and expected return. Under less restrictive conditions, risk averse EU decision makers will find their preferred choice in the second degree stochastic dominance (SSD) efficient sets.

Inconsistencies between EV and SSD sets are not likely to occur in most empirically derived efficient sets. The reason is that Porter (1973) and others who derived SSD choices that were not EV efficient assumed that the distributions being chosen were the true distributions measured without error.[2] Meyer & Rasche (1992) demonstrated that if estimation error is included, large amounts of data are required to find any statistically significant difference between EV and EU efficient sets. Thus, as long as we recognize that we rarely know distributions with certainty and lack large amounts of data for estimating the empirical distributions, there is little basis for distinguishing between EV and SSD efficient sets in empirical work.

Stochastic dominance criteria derived from the EU model do have one important advantage over EV analysis in applied work. It is that efficient sets can be identified for more narrowly defined sets of decision makers using stochastic dominance. EV sets contain the efficient choices for all risk averters while EU efficient sets can be specified for any particular subset of decision makers, usually identified by their absolute risk aversion functions. While an efficient subset can be selected from an EV set, the class of decision makers for whom the subset is efficient is less clear unless the decision makers are all constant absolute risk averters. On the other hand, there is nothing to prevent one from applying stochastic dominance ranking techniques to EV sets. This approach would combine the efficient set selection techniques of EV analysis and the improved decision class identification techniques associated with stochastic dominance with respect to a function.

3. Using EU and EV models to derive theoretical results

Now consider EU and EV models as theoretical tools. Early efforts to demonstrate compatibility between EU and EV models relied on normality and constant absolute risk aversion (CARA) assumptions that led to

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[2]See also Levy & Markowitz (1979) & Tsiang (1972) for empirical comparisons between EU and EV models.

the linear EV objective function (Freund, 1956).[3] Additional support for the linear EV model was provided by Pratt's EV approximation of an EU model. Another EV model was obtained by demonstrating that a second order Taylor series approximates a von Neumann Morgenstern utility function as a quadratic function whose expected value contains the variance of the random variable as arguments. Neither the assumption of quadratic utility nor the assumption that distributions are normal have proven adequate to justify the use of EV models for theoretical use. Quadratic utility implies increasing absolute risk aversion (IARA) which is unrealistic under most conditions. Worse, quadratic utility functions allows for negative marginal utility. Finally, normality and CARA are also severely limiting assumptions if the goal is to derive theoretical results under more general conditions such as decreasing absolute risk aversion (DARA).

To allow for DARA and IARA income effects using the EV model, Robison & Barry (1987) developed a nonlinear EV model. Choice variable solutions were obtained by maximizing a linear mean variance function at a slope where tangency occurred between the expected utility function and the EV set. They argued that maximizing a linear EV function would provide a solution identical to that obtained using EU analysis as long as all choices were contained in the EV set, a condition always satisfied for models containing a single choice variable. Having found the initial solutions using a linear EV model, Robison & Barry (1987) performed comparative static analysis. Substitution effects were measured as changes in the choice variables in response to parameter changes holding the slope coefficient constant. Income effects for DARA and IARA decision makers were measured as the changes in the choice variables in response to slope changes. In other words, parameter changes that changed expected income or variance were allowed to change the equilibrium slope on the EV frontier.

More recent support for EV models was provided independently by Meyer (1987) and Sinn (1983). They showed that if risky choices differ only by location and scale then the EV set of choices or the mean standard deviation (MS) sets contained the expected utility choices for von Neumann Morgenstern utility maximizers. Meyer went on to show that comparative static results could be obtained for DARA and IARA decision makers that were consistent with the slope conditions earlier derived by Robison & Barry (1987) and Cass & Stiglitz (1972). Since these results are generally well known and available elsewhere we will not dwell further on how EV models can be used for comparative static analysis.

4. Conflicts in theoretical results derived using EU and EV models

When the choice variable(s) analyzed theoretically are not location scale, inconsistencies between EU and EV models results are possible. Two classes of inconsistencies are possible. The first class of inconsistency is that identified by Rothschild & Stiglitz (1971). They pointed out that in a multi variable portfolio model the EV model produced definitive results when none such could be generated using EU analysis. This is because the relationship between probability density functions is more narrowly constrained in an EV model. The second class of conflicts between EV and EU models are those in which definitive results are derived using both models that are inconsistent.

Consider the importance of the two classes of possible conflicts produced using EV and EU models. The first class of conflicts should not be considered a shortcoming of EV analysis. This is because the EV model provides information about conditions in which EV and EU consistency cannot be rejected empirically. For example, portfolios of stocks that Meyer & Rasche (1992) showed were EV efficient can be analyzed

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[3]This linear function also solves for solutions on the EV efficient set linking the theoretical model to its empirical counterpart.

theoretically using EV analysis and produce theoretical results that have proven to be useful. But no such results for portfolio models of multiple random variables have been derived using EU models.

The second class of possible conflicts between EV and EU models is more serious than the first. This class of conflicts, however, is likely limited because definitive results from EU models in more complicated risk settings are possible only in narrowly defined risk settings. To illustrate, to find definitive results using an EU model, Lapan & Moschini (1994) specified random variables whose means were independent of their variances. The second class conflict is removed when the independent assumption is removed. Thus, class two conflicts are likely limited because of the difficulty of obtaining definitive results for increases in risk in more complicated models using EU analysis (Meyer & Ormiston, 1985). In contrast, definitive results for increases in risk in EV models are common.

It is not correct to assert that conflicts between EV and EU models all result from the absence of higher moments of the distributions in EV models. EV and EU models will rank even significantly skewed distributions consistently as long as location and scale conditions are satisfied. And even when location and conditions are not satisfied, consistency between EV and EU models may still be obtained because changes in skewness most often changes the means and variances as well in ways that leave ranking between the two models consistent.

The trade-off between EV and EU models for theoretical analysis should be clear. The preference for EU models is because of its rigorous deductive underpinnings. However, definitive results using EU models will likely be limited unless the generality of the random variables considered are significantly restricted.

The justification for nonlinear EV analysis is that it produces results consistent with EU models when location scale conditions are met and approximates EU results when location scale are not met. The accuracy of the approximation of the non linear EV model to EU model results when location scale are not satisfied has not been carefully examined. The advantage of EV models is that they can often derive definitive results in complex risk models that cannot be produced using EU models. Since definitive EU results are extremely difficult to obtain in more complicated risk models, researchers have often turned to the EV framework to obtain results. Examples include Chavas & Pope (1982), Sarassoro & Leuthold (1991), Coyle (1992), and Robison & Barry (1987). The EV framework has proved useful for more complicated risk problems precisely because it ignores higher moments and specifies relationships between random variables with their covariances.

Thus the decision to use EU or EV analysis is not a right or wrong choice. It is a trade-off between the stronger axiomatic foundations of the EU model versus improved ability of EV models to analyze theoretically more complex risk problems. The use of these two models will depend on are the requirements of risk research in the future.

5. Future areas of emphasis for firm level risk analysis

In our view, there appear three areas in which risk analysis for the firm can be extended theoretically.[4] We introduce and illustrate these three areas in the remainder of this paper. The first area will consider more realistic relationships between indirect and direct outcome variables. The second area will build models with

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[4]We recognize, of course, that theoretical work must needs be followed by appropriate empirical testing. But this later subject must await later discussions.

more than one choice and stochastic variables. And the third area will focus on what we call dominance analysis. All three area have one common element; significant progress will likely require the increased use of EV models. Results could, of course, be obtained using numerical analysis in an EU framework, but the generalization of these results would be limited.

6. Indirect and direct outcome relationships

Von Neumann Morgenstern utility functions are defined over direct outcome variables. Most often the direct outcome variable is final or end of period wealth. Indirect outcome variables such as income contribute to the value of the direct outcome variable but are not themselves the argument of expected utility functions. Most of what we know theoretically about the firm's response to risk has been deduced under a very restrictive assumption about the relationship between direct and indirect outcome variables (Robison & Lev, 1986). The most common assumption is that stochastic wealth, $\tilde{w} \sim (\bar{w}, \sigma_w)$, is related to nonstochastic initial wealth w_0 plus stochastic income $\tilde{y} \sim (\bar{y}, \sigma_y)$:

$$\tilde{w} = w_0 + \tilde{y}(\alpha) \tag{1}$$

where the distribution of $\tilde{y}$ depends on the choice variable α.

The solution for the choice variables in (1) can be found by maximizing the linear mean variance function equal to the sum w_{CE} of expected wealth, $w_0 + \bar{y}(\alpha)$, less the variance of income, σ_y^2 weighted by the EV slope coefficient $\lambda/2$:

$$\underset{\alpha}{Max}\ w_{CE} = w_0 + \bar{y}(\alpha) - \frac{\lambda}{2}\sigma_y^2(\alpha) \tag{2}$$

This simplified relationship between income and wealth suggests that if the choice variables only effect $\tilde{y}$, then the first order conditions contain only derivatives associated with $\bar{y}$ and σ_y^2. Thus from an analytic perspective, little is gained by including explicitly the initial wealth variable in the model unless it somehow constrains the choice variables affecting α. However, in many analyses, even this weak link between $\tilde{y}$ and w_0 described in (1) is ignored.

In the early development of risk models, ignoring the link between initial wealth w_0 and $\tilde{y}$ may be excused. After all, the attention needed to be focused on the effects of particular risk instruments. Further progress in understanding the effects of risk instruments now requires that we test our risk instruments under more real world conditions. That is, we need to test the robustness of our models when more realistic assumptions are made about the relationships between direct and indirect outcome variables. So we next describe models that increase in realism and complexity in their relationships between indirect and direct outcome variables beginning with Sandmo's model of the firm facing output price risk.

6.1. The Sandmo model: $\tilde{w} = \tilde{y}$

Consider Sandmo's production model in which the stochastic element is output price $\tilde{p} \sim (p, \sigma_p)$. The relationship between end-of-period wealth $\tilde{w}$ and income earned during the period $\tilde{y}$ is simply $\tilde{w} = \tilde{y}$. The EV or in this case the MS set is easily found. From the stochastic expression for income $\tilde{y}$, we write the equation:

$$\tilde{y} = \tilde{p}q - C(q) - B \tag{3}$$

In Sandmo's model described in (3), $C(q)$ is the cost function for producing output q such that $C'(q), C''(q) > 0$, $C(0) = 0$, and B is fixed or time costs. For a particular choice of q, $\tilde{y}$ is a linear function of the stochastic variable $\tilde{p}$, satisfying location scale conditions.

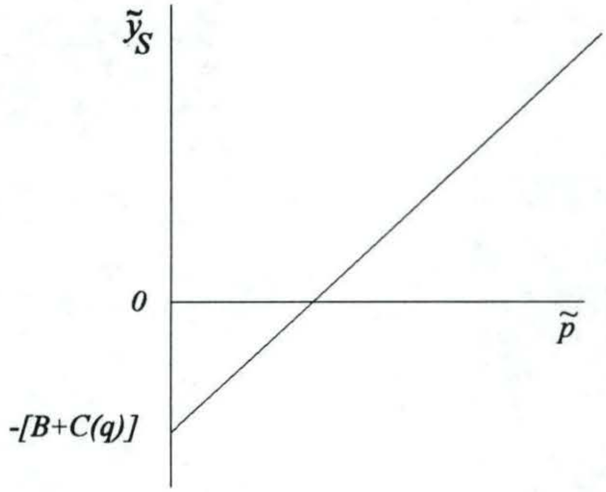

Figure 1. The Relationship Between $\tilde{y}$ and $\tilde{p}$ in the Sandmo Model

Expressions for $\bar{y}$ and σ_y in the Sandmo model are:

$$\bar{y} = \bar{p}q - C(q) - B$$

and:

$$\sigma_y = q\,\sigma_p$$

Solving for q in the expression for σ_y and substituting the result into the expression for $\bar{y}$ produces the MS frontier equal to:

$$\bar{y} = \bar{p}\,\frac{\sigma_y}{\sigma_p} - C\left(\frac{\sigma_y}{\sigma_p}\right) - B \tag{4}$$

The MS graph of the basic Sandmo model is in Figure 2.

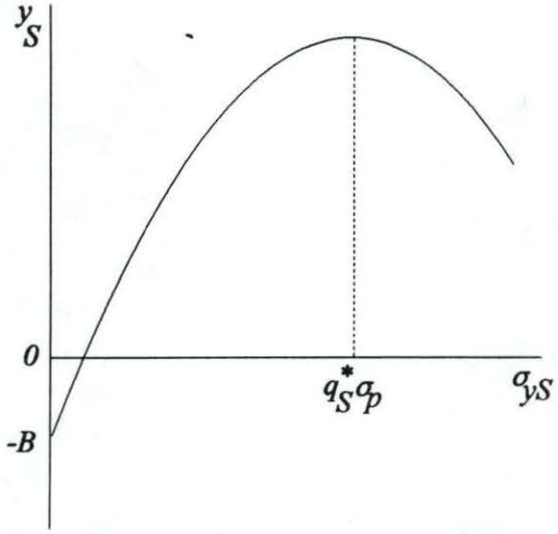

Figure 2. The MS Frontier for the Sandmo Model

6.2. Sandmo with transactions costs

Consider the following relationship between $\tilde{y}$ and initial wealth w_0 in which a transactions cost δ is incurred if the realized stochastic price is less than $p(w_0)$. Stochastic income $\tilde{y}$ can be written as:

$$\tilde{y} = \begin{cases} \tilde{p}q - C(q) - B - \delta & \tilde{p} < p_0(w_0) \\ \tilde{p}q - C(q) - B & \tilde{p} \geq p_0(w_0) \end{cases} \tag{5}$$

In this model, in contrast to the earlier models, income $\tilde{y}$ is not linear in $\tilde{p}$. The kinked linear relationship is described in Figure 3.

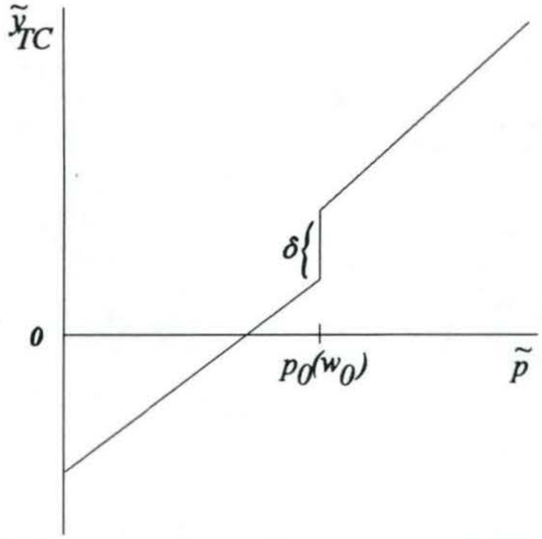

Figure 3. A Jump Relationship Between $\tilde{y}$ and Stochastic Prices $\tilde{p}$

Let the probability distribution function and cumulative density function for $\tilde{p}$ be $dF(\tilde{p})$ and $F(\tilde{p})$ respectively. Then, $\bar{y}$ and σ_y are equal to:

$$\bar{y} = \bar{p}q - C(q) - B - \delta F(p_0)$$

$$\sigma_y^2 = q^2 \sigma_p^2 + \delta^2 F(p_0)[1 - F(p_0)] + 2q\delta[\bar{p}F(p_0) - p_1]$$

where:

$$p_1 = \int_{-\infty}^{p_0} \tilde{p}\, dF(\tilde{p})$$

The first term on the right of σ_y^2 equals that portion of the variance contributed by $\tilde{p}$. The second term to the right of σ_y^2 is the variance associated with the transactions cost δ and the third term to the right of σ_y^2 is the covariance between $\tilde{p}$ and δ. This problem is a good example how changes in skewness caused by transactions costs change the variance as well.

Obviously, transactions costs increase the variance of the original Sandmo model. Transactions costs may occur because unfavorable price outcomes require firm restructuring by liquidating assets. Or, it may require downsizing with severance costs associated with terminated employees. Or, it may require borrowings with its attendant sacrifices of time and application fees. Whatever the source, it seems reasonable to assume that firms may face the possibility of liquidation fees that are related to its financial wealth. Thus: $p_0 = p_0(w_0)$ and $dp_0/dw_0 < 0$.

Even though this model is no longer linear in $\tilde{p}$ and thus violates location-scale assumptions, still all EU maximizing choices are on an EV frontier.[5] Thus any given EU solution for q can be found by specifying the appropriate slope $\lambda/2$ and by maximizing the certainty equivalent expression equal to:

$$y_{CE} = \bar{y} - \frac{\lambda}{2}\sigma_y^2 \tag{6}$$

where optimal q^* must satisfy:

$$\bar{p} - C'(q^*) - \lambda\left\{q^*\sigma_p^2 + \delta[\bar{p}F(p_0) - p_1]\right\} = 0$$

Increased risk costs associated with δ and lower expected returns caused the MS frontier for the transactions cost model to lie below the original Sandmo model. Moreover, the tradeoff between expected return is less because of increased risk costs, thus lowering optimal q for most risk averse decision makers. Expected profit maximizing q, however, occurs for the same q in both the Sandmo and the transactions cost model but for different variances of income. All of these results easily obtained in the EV model would be much more difficult to deduce in the EU model.

6.3. Sandmo with limited liabilities

Consider a model in which losses of the firm are limited according to the specification below:

$$\tilde{w} = \tilde{y} = \begin{cases} p_0(w_0)q - C(q) - B & \tilde{p} \le p_0(w_0) \\ \tilde{p}q - C(q) - B & \tilde{p} \ge p_0(w_0) \end{cases} \tag{7}$$

The motivation for this model is that institutionally imposed safeguards may limit the actual loss incurred by a firm. Bankruptcy laws may protect firms against losses beyond minimum levels as well as do unemployment insurance. Thus realistic examples of the limited liabilities model exist.

As in the transactions cost model, $\tilde{y}$ is no longer linear in the random variable $\tilde{p}$, but all choices remain on the EV frontier. The relationship between $\tilde{y}$ and $\tilde{p}$ is described in Figure 4.

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[5]It can be shown that because the indirect outcome variable does satisfy location-scale, the analysis can be made consistent with EU through the appropriate transformation of the utility function in final outcome space (Robison, 1994).

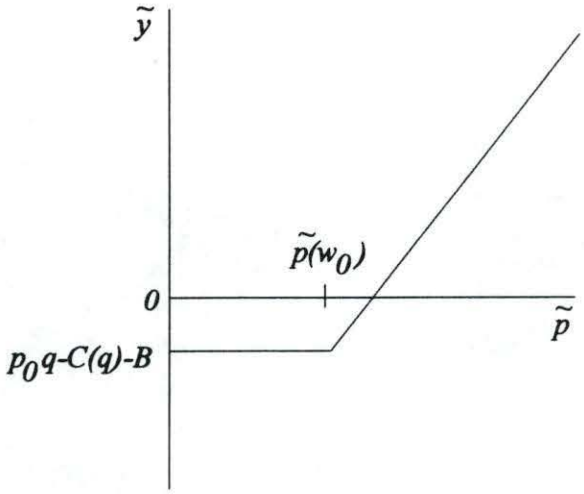

Figure 4. Limited Liabilities Depending on W_0

The expected value and variance of $\tilde{y}$ in the limited liabilities model are equal to:

$$\bar{y} = \left[p_0 F(p_0) + p_2\right] q - C(q) - B$$

where:

$$p_2 = \int_{p_0}^{\infty} \tilde{p}\, dF(\tilde{p})$$

and:

$$\sigma_y^2 = q^2 \{p_0 F(p_0)\, [p_0 (1 - F(p_0))] - 2\, p_2] + \sigma_2^2\}$$

where:

$$\sigma_2^2 = \int_{p_0}^{\infty} (\tilde{p} - p_2)^2\, dF(\tilde{p})$$

Again, all choices of q are on the EV frontier. Thus the linear EV model can be made consistent with any EU model results. And again as in the transaction costs, any changes in the skewness also affects the variance.

6.4. Sandmo with background risk

Background risk reflects an investment environment in which the firm begins the decision making period with some resources committed to a risky investment. For example, assume that the firm has committed resources that return a stochastic return of $\tilde{w}_0$. This model, Sandmo's with background risk, is represented in equation (10).

$$\tilde{w} = \tilde{w}_0 + \tilde{p}q - C(q) - B. \tag{8}$$

The question to be answered is how do these pre-existing and fixed commitments change the investment decision compared to the earlier Sandmo model? To answer this question and others we find the expected value and variance of ending wealth described next:

$$\bar{w} = \bar{w}_0 + \bar{p}q - C(q) - B$$
$$\sigma_w^2 = \sigma_{w_0}^2 + q^2\sigma_p^2 + 2q\rho\sigma_p\sigma_{w_0}$$

where $\tilde{w}_0 \sim (\bar{w}_0, \sigma_{w_0})$ and the covariance between stochastic committed investments and stochastic income from production is $2q\rho\sigma_p\sigma_{w_0}$. The effect of background risk depends on its correlation with other stochastic investments and may or may not increase q. The analysis can be easily completed using the nonlinear *EV* model.

7. Dominance analysis and more complicated risk models

An advantage of the EV model for theoretical analysis is that it permits the analysis to occur in two separate spaces. The analysis can occur in the utility space confined by available choices on the EV set or the discussion can focus on the EV set and how it is changed by altering parameters in the existing model or by adding risk instruments. This later approach considers what we refer to as dominance analysis. Dominance analysis defined here is finding the conditions under which the addition of a new risk instrument produces a new EV set that is preferred by risk averse agents to the EV set previously available. This topic appears to us particularly relevant for the analysis of the firm facing risk because so many practical questions involve choices between risk instruments.

Dominance analysis requires that we identify risk instruments as efficient or inefficient. Efficient risk instruments permit the following. Beginning at an existing level of expected return and variance, the addition of an efficient risk instrument allows to move to a more efficient EV location not on the original EV frontier. The improvement permitted by adding an efficient risk instruments is described in Figure 5.[6]

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[6]Of course, the risk instrument may exhibit both risk efficiency and inefficiency depending on the value of other parameters and variables in the model.

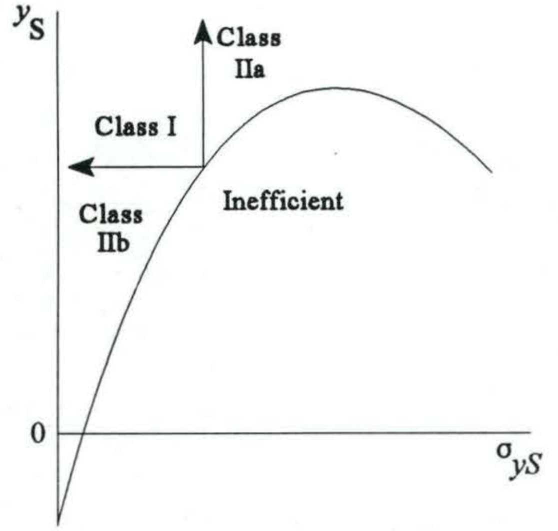

Figure 5. Class I and II Risk Efficient Instruments

Efficient risk instruments are of two classes described in Figure 5. Class one instruments allow us to move to EV locations with increased means and/or reduced variances. Class two efficient risk instruments allow us to move to EV locations in spaces IIa and IIb which increase (decrease) in mean and variance a rate greater than (less than) possible with the existing EV set. Risk instruments that fail to qualify as class one or class two efficient risk instruments are inefficient risk instruments. To describe precisely the conditions required for first class risk efficiency assume the original relationship between final wealth $\tilde{w}$ and initial wealth w_0 to be:

$$\tilde{w} = w_0 + \tilde{y}(w_0,\alpha) \tag{9}$$

where the choice variable α and initial wealth w_0 affect the distribution of stochastic income.

Now suppose there exists another risk instrument that could be added to the model. This second instrument that produces stochastic income is represented as $\tilde{z}(\beta;w_0)\sim(\bar{z},\sigma_z;\beta)$ where β is a vector of choice variables associated with the new risk instrument. Furthermore, let the correlation between $\tilde{y}$ and $\tilde{z}$ be ρ. Then the new risk instrument is efficient if the following two conditions are satisfied where strict inequality holds in one of the two equations:

$$\begin{gathered} E[\tilde{z}(\beta;w_0)] \geq 0 \\ \sigma_y^2 + \sigma_z^2 + 2\rho\sigma_y\sigma_z \leq \sigma_y^2 \end{gathered} \tag{10}$$

If strict inequality holds in the second requirement then it follows that the correlation coefficient must be negative and less than the negative ratio of the variance of the new risk instrument to twice the variance of the old risk instrument. This relationship is described in the equation below. The implication of this second condition is that the variance of the new instrument can be larger than the variance of the old instrument and still qualify as risk efficient. But only negatively correlated instruments can qualify as first class efficient risk instruments.

Now having identified requirements for first risk efficient instruments, we intend to demonstrate how risk instruments can be tested for efficiency. It is, of course, obvious, that dominance analysis requires more complicated risk models be considered. Thus our dominance analysis combines two of the new areas of analysis into one model: including more than one risk instrument in dominance analysis.

7.1. Sandmo plus hedging: $\tilde{w} = \tilde{y}$

$$\rho < -\frac{\sigma_z}{2\sigma_y} \tag{11}$$

Consider again the Sandmo model with a new risk instrument included. The risk instrument considered first is hedging with a futures contract. Hedging allows the firm to exchange an uncertain spot price $\tilde{p}$ for a certain future price p_f. As long as $p_f \geq \bar{p}$, selling futures satisfies the conditions required for first class risk efficiency. If $p_f < \bar{p}$ then negative hedging (speculation) increases return at a faster rate than income earned without hedging and may satisfy the conditions required for second class risk efficiency.

Let the amount of q hedged be h. Then, hedged income can be expressed as:

$$\tilde{y} = \tilde{p}q + (p_f - \tilde{p})\, h - C(q) - B \tag{12}$$

Assume for the moment that the futures market is unbiased so that $p_f = p$ so that the first condition for first class risk efficiency is satisfied with equality. Then consider how hedging has altered the variance. The variance of the hedged income can be expressed as:

$$\sigma_y^2 = q^2\sigma_p^2 + h^2\sigma_p^2 - 2qh\,\sigma_p^2 \tag{13}$$

The first term on the right hand side of the equation is the variance of unhedged production. The second term is the variance associated with the hedge. The third term with its implied correlation coefficient of negative one is the covariance. As long as $|h| < 2q$ the second condition for first class risk efficiency is satisfied and hedging is a first class risk efficient instrument.[7]

[7]The properties of the hedged Sandmo model are well known (Meyer & Robison, 1988; Feder, Just, & Schmitz, 1980; Holthausen, 1979). Output is chosen independent of risk. If the futures price equals the expected value of the spot price the amount hedged equals the amount produced. If the futures price exceeds the expected value of the spot price, then the amount hedged exceeds the amount produced.

The hedging model is an important prototype. It introduces into the risk model a risk reducing input; that is, increasing h reduces σ_y^2. Other examples of risk reducing variables are the purchase of insurance, investing in information, investing in kill functions that reduce a pest or capital investment that moderates the effects of drought, floods, and frost.

7.2. Hedging with limited liabilities

An important contribution of dominance analysis is that it focuses on the unique risk environment of the firm. Recognizing the uniqueness of the risk environment facing each firm leads to the recognition that a particular risk instrument such as hedging added to the firm's portfolio will produce differential effects depending on the firm's unique risk environment. To illustrate, suppose that the firm has its liabilities associated with unfavorable prices limited to $p(w_0)$. This limitation might result from government policies or the firm's adoption of other risk strategies. We want to know if the hedging results are robust; that is, we want to know if hedging is an efficient risk instrument when the firm's liabilities are limited.

Figure 6 illustrates how the effectiveness of a hedge is altered by the changed risk environment. Earlier, hedging could completely eliminate the firm's risk. And if the futures price was unbiased, the firm would desire a hedge equal to its production. Now, as can be seen from Figure 6, no single futures hedge can eliminate risk. Furthermore, the expected output price is no longer the expected spot price because of the limited liability. Therefore, an unbiased futures price is less than the expected value of the firm's returns from production. Thus, limited liabilities has the effect of converting a class one instrument to a class two instrument. As a result, a hedge equal to production levels will no longer eliminate risk. Clearly, the optimal hedge will be quite different than those derived in earlier studies.

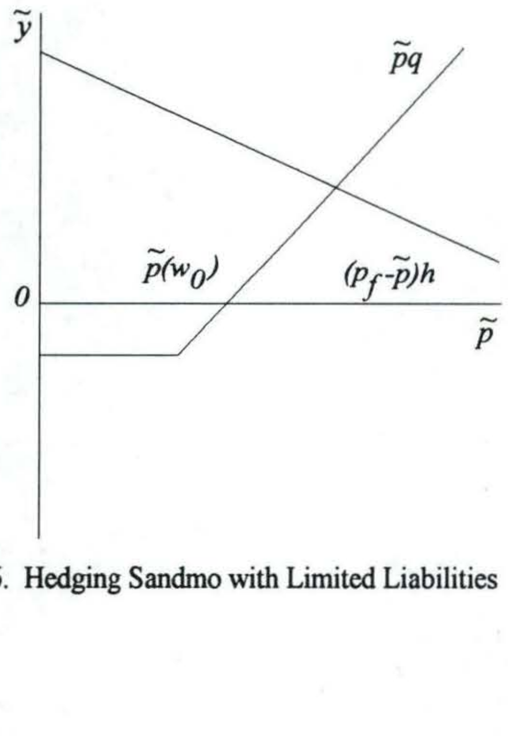

Figure 6. Hedging Sandmo with Limited Liabilities

7.3. Options and limited liabilities

In the case of a truncated output price, futures were no longer able to provide a perfect hedge because they were linear in the cash price (see Figure 6). Now consider the role of options when the producer faces limited liability. For simplicity assume there is no basis risk, basis is zero, and the producer's minimum cash price is $p(w_0)$. The producer can use a call option to manage risk which has a payout of:

$$\tilde{V} = \begin{cases} \tilde{p} - p_0(w_0) & \tilde{p} \geq p_0(w_0) \\ 0 & \tilde{p} \leq p_0(w_0) \end{cases} \tag{14}$$

Assume the cost of the option is r and is assumed to be unbiased in the sense that $r = E(\tilde{V})$. Now the producer's profit function becomes:

$$\tilde{y} = \tilde{p}q - C(q) - B + z(r - \tilde{V}) \tag{15}$$

where z is the number of call options (sold is positive) held by the hedger. Stochastic income in (15) can be rewritten as follows:

$$\tilde{y} = \begin{cases} p(w_0)q - C(q) - B + zr & \tilde{p} \leq p(w_0) \\ \tilde{p}q - C(q) - B + z[r - \tilde{p} + p(w_0)] & \tilde{p} \geq p(w_0) \end{cases} \tag{16}$$

Because the options are priced at their expected value, adding options to the portfolio has no impact on the expected profit level. However, since they reduce variance they are a class one risk instrument. Choosing to sell q call options ($z=q$), riskless profits become:

$$y = [p(w_0) + r]q - C(q) - B \tag{17}$$

The variance of profit has been eliminated while the expected profit is unaffected. The call option has provided a perfect hedge when the producer faces the truncated cash price.[8]

8. Summary and conclusions

This paper has suggested three areas of future research emphasis. First, we expect future research will analyze more general relationships between indirect and direct outcome variables. Relationships between indirect and direct outcome variables examined in the past have often been linear. But institutions that limit liabilities, create transactions costs, provide various kinds of insurance and options, and facilitate shared risk arrangements may all produce nonlinear relationships between direct and indirect outcome variables. Second,

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[8]See Lapan, Moschini, & Hanson (1991) for a discussion of a model with combined production, hedging, and options.

we expect future risk research to explain the results of increasingly complex numerical models and simulations. Thus, risk models will be required to include multiple choice variables and sources of risk. And third, we expect future risk research to consider dominance analysis. This analysis will ask what combinations of risk instruments are efficient. In EU, this analysis would require numerical simulations and the applications of stochastic dominance rules. EV analysis allows us to approach the problem more generally. This is because the EV set is efficient for risk averse decision makers and relationships between risk instruments can be used to evaluate their efficiency.

We expect that progress in all three areas of future research will make increased use of EV models. The reason is that the analysis of increases in risk are not easily analyzed in the EU model unless severe limitations are placed on the variables considered.

Many questions must still be answered about the plausibility of the implicit assumptions about preferences in EV models before EV models are generally accepted as a theory tool. The analogy between results obtained using the nonlinear EV model and those obtained in the EV model when location scale conditions hold is the following. In econometrics certain properties of models are obtained that depend on large samples. Yet there are models that need analyzing even when large samples are not available. The best that can be done in such circumstances is to infer from the large sample properties to the models that are estimated with small samples. We find ourselves in a similar circumstance in our application of the EV model. We have properties that are developed under location scale. We infer these same properties even when location scale conditions do not hold in our use of the nonlinear EV model.

So future risk research will require trade-offs. Some important work remains to be done using models that are EV and EU consistent. This work can proceed using EU or EV models and obtain consistent results. Work in the three areas discussed in this paper, however, will likely require models that are not location scale. We expect only limited progress using EU models to solve more complicated models and even this progress may often require restrictions on choice and random variables that limit the usefulness of the results. On the other hand, progress is much more likely using EV models. But EV models do not yet have the strong axiomatic basis of EU models leaving us less certain about the usefulness of the results. Perhaps this tradeoff between EV and EU models suggests still a fourth area of future research: empirical and theoretical testing of conditions required for consistency between EV and EU models.

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