Staff Paper

QUASI-OPTION VALUE

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I. Introduction

In 1964 Burton Weisbrod introduced the concept of option value (OV). OV is an additional value, over and above the expected value of a good's consumption, that is attached to maintaining a good's future availability when faced with uncertainty about its future demand or supply. Research on OV has shown that it derives from risk aversion (see e.g. Graham; Bishop; Smith; Graham-Tomasi and Myers). Generally speaking, the OV literature addresses the correct measurement of welfare change under uncertainty, with alternative institutional structures for managing risk.

In 1974, Arrow and Fisher forwarded a different approach to option value, derived under risk neutrality, and based on a dynamic formulation. Arrow and Fisher examined the effects of learning more over time about the uncertain benefits of preserving an area of wilderness land when its development would be irreversible. These authors demonstrated that, relative to a situation in which the decision-maker ignores opportunities for learning, an extra value is attached to preservation when it is realized that one may learn the true benefits of preservation. This extra value they called quasi-option value (QOV).

In this Chapter, QOV will be investigated, and some of the literature in this area reviewed. Naturally, the basic insight that recognizing opportunities for learning may change decision criteria is applicable beyond the wildland development/preservation scenario investigated by Arrow and Fisher. Hence, levels of general capital investment may be altered in response to learning opportunities (Demers; Epstein, Bernanke; Cukierman). Here, the natural resource connection will be stressed, but the literature is broader than the natural resource examples discussed.

The basic results will establish conditions under which the prospect of receiving "better information" in the future, leads one to adopt "more flexible" decisions today. The intuitive reasoning is clear: if one is in an inflexible situation, so that any alterations of it are costly (or impossible), then one is less willing to respond to changes in beliefs induced by receipt of information. Hence, being in an inflexible position undermines the value of the information to be received. As the information to be received improves, the incentive to remain flexible and take advantage of it increases.

Clearly, the situation studied by Arrow and Fisher is a special case of this more general concern. Undertaking a completely irreversible action, such as the development of a wildland area, results in a more inflexible position than does leaving the area undeveloped today and having a choice of development or preservation tomorrow. And receipt of perfect information about the true value of wilderness is an extreme version of obtaining "more information."

A resource problem that illustrates most of the issues is the depletion of moist tropical rainforests. The resource stock has value both for its timber and the agricultural land (or other uses) it may be converted to, as well as for the ecosystem services it provides and the biodiversity it contains. The values of the goods and services provided by a tropical forest in its natural state are not well-known in comparison to our understanding of the value of harvested timber and agricultural products. The harvest of trees from such forests may result in destruction of the forests, or harvest may be done in a manner (e.g. using helicopters) which preserves much of the standing forest and its ability to provide ecological services. Through scientific research, we can come to learn more about the value of standing forest ecosystems. More informative research is based on larger sample sizes and better procedures. The basic idea of QOV, then, is that the mere prospect of improved research programs on the value of moist tropical forest ecosystems, even allowing for the possibility that they may find that such forests are less valuable than we now believe, should lead to greater conservation of such forests.

II. A General Resource Decision Model

As discussed above, the basic idea of QOV regards the relationship between information and choices that are costly to reverse. In this section we define these ideas in terms of a simple natural resource model with uncertainty.

The Resource System

Let the state of a resource system at time t be given by x_t. This could be the amount of land area in an undeveloped state, or the amount of an exhaustible or renewable resource on hand. Let q_t be a control that is applied to the system by a decision-maker (DM). Examples are new development of land for roads, harvest of timber, or extraction of fossil fuels. The control is constrained to lie in a set Q_t, which may depend on the state of the system. For example, you cannot extract more resource, or develop more wilderness, than you have.

Uncertainty is represented by a random variable s, taking values in the (time invariant) set $S = \{s_1,...,s_n\}$. A realization of s at time t is s_t. Important special cases are where: (i) there is just one true state of nature s^o in S, rather than a sequence of realizations, and (ii) the sequence of realizations s_t forms a Markov process. This uncertainty could arise in a number of ways, affecting the resource itself, or payoffs from its use, or both. Examples would be uncertainty about the demand for wilderness experiences, where s is a parameter of the demand system, or about the value of species in a forest for medicinal purposes, or about a threshold point in the growth function for a renewable resource below which the population is bound for extinction. We shall study a model where either there is just one s_1 or the sequence is i.i.d. Extension to a Markov process is straightforward.

If the system is in state x_t, control q_t is applied, and s_t arises, then the system moves to a new state x_{t+1} according to the transition equation

(1) $$x_{t+1} = g(x_t,q_t,s_t).$$

Payoffs to the DM depend on where the system is, what is done to it, and the realization of s. Letting $z_t = (x_t,q_t,s_t)$, payoffs take the additively separable, discounted form

(2) $$U = \Sigma_t \alpha^t u(z_t),$$

where $\alpha = 1/(1+r)$ is a discount factor. Here, we will assume that this general payoff function takes the particular form

(3) $$u(z_t) = w(x_t,s_t) + v(x_0-x_t, s_t) + [p_t-c_t]q_t.$$

Thus, the benefits are given by the uncertain value of the resource stock, the uncertain value of the cumulative extraction, and the value of current extraction. In the case of forests, w(.) is the value of the standing trees, v(.) is the value of the land freed-up from forests for alternative uses such as pasture, and [p-c] is the net value of todays harvested material. In the case of fossil fuels, w(.) may be zero, while v(.) represents uncertain environmental effects, such as global warming, stemming from burning such fuels. In the case of a renewable resource, such as harvest of whales, v(.) may be zero, while w(.) represents non-use values attached to these creatures.

The decision-maker's problem is to maximize the expected present value of payoffs, based on ones information about s.

Beliefs and Information

The DM's beliefs at time t about the random event are summarized by a probability mass function $\pi_t = (\pi_{1,t},...\pi_{n,t})$, where π_{it} is the probability that the realized event at t is s_i. The DM knows that she will receive information over time which can be used to revise her beliefs. The most common representation of this idea is that the DM can observe a "signal" in the form of a random variable y_t, sometimes called the outcome of an "experiment," which is correlated with s_{t+1}. Let Y_t be a set of signals or messages that the DM could receive, where $Y_t = \{y_{1,t}, ...,y_{m,t}\}$.

If the DM's current beliefs about the random event are π_t, and the message received is $y_{j,t}$, then the DM's new beliefs are given by the transformation

(4) $$\pi_{t+1} = B(\pi_t, y_{j,t}, t).$$

The DM faces a sequence of experimental outcomes. Let $\{Y\}_t$ denote the sequence of experiments and associated probabilities the DM faces from date t onward. Note that in much of the literature on sequential experimentation (see e.g. DeGroot) it is supposed that there is one control decision to be made after observing the outcomes of a sequence of experiments, where the number of observations to be made, as well as the experiments to be conducted, are choices. Here, a control decision is made each period, and the sequence of experiments is exogenous.

An important case is when B(.) is the map implied by Bayes Rule. To develop this little further, let β^t, be the matrix of joint probabilities, with typical element β^t_{ij}, i.e. this is the probability that $s_{t+1} = s_{i,t+1}$ and $y_t = y_{j,t}$. The likelihood matrix of conditional probabilities is $\Delta^t = [\delta^t_{ji}] = \text{Prob}(y_t = y_{j,t} | s_{t+1} = s_{i,t+1}]$. Thus, δ gives the probability of observing a particular signal y, given that the true state to arise is s. Let the Θ^t be the posterior probabilities, the matrix of probabilities on s, conditional on having observed a particular y. A typical element of Θ^t is $\theta^t_{ij} = \text{Prob}(s_{t+1} = s_{i,t+1} | y_t = y_{j,t})$. Having observed the signal $y_{j,t}$, the conditional distribution on next period's random event is given by the jth column of Θ^t. Finally, let λ^t be the predictive distribution of y_t, i.e. the marginal distribution regarding which message will be received, given current beliefs.

Then the linkage between the experiment and the random event is summarized by the following relationships (dropping the time notation):

(5) $$\lambda_j = \Sigma_i \beta_{ij} = \Sigma_i \pi_i \delta_{ji}$$
$$\beta_{ij} = \theta_{ij} \lambda_j.$$

Thus, the map B(.) when the DM is using Bayes Rule is

(6) $$\pi_{i,t+1} = \delta^t_{ji} \pi_{i,t} / \Sigma_k \delta^t_{jk} \pi_{k,t}.$$

If there is just one true state of nature s (rather than a sequence of realizations), or if the draws s_t are independent and identically distributed, then the transformation B(.) does not itself depend on time when Bayes Rule is being used. In these situations the sequence of beliefs $\{\pi_t\}_t$ has the Markov property. That is, all that matters to the revision of beliefs is the current state of beliefs π_t, and not the whole history. In the case of a sequence of i.i.d. realizations s_t, the sequence of beliefs also is i.i.d.

The Decision Problem

The DM maximizes the expected present value of payoffs, subject to the transition equation on the resource state, as well as the transition equation on beliefs. Let the state of the system be $(x_t, \pi_t) \equiv Z_t$. The system evolves according to (1) and (5). The DM solves

(7) *P:* $\text{Max}_q \; E \, \Sigma_t \, \alpha^t \, u(x_t, q_t, s_t)$

s.t. $x_{t+1} = g(x_t, q_t, s_t)$

$\pi_{t+1} = B(\pi_t, y_{j,t}, t)$

$q_t \in Q_t(x_t)$

$x_0 = x^o \, (=1); \; \pi_0, \; \{Y\}_0$ given.

It is important to note that the experiment here is exogenous to the DM. There is no choice among experiments to be made, and, in particular, the information to be received does not depend on the control chosen. We will comment on this below.

At each date, the DM chooses the control q as a function of the current state (x_t, π_t); this function is called a plan. Given a plan q(Z), the expected discounted payoffs are

(8) $J(q(Z)) = E\{u(x_0, q(Z_0), s_0) + \Sigma_{t=1} \, \alpha^t \, u(x_t, \, q(Z_t), s_t)\},$

where Z_t evolves according to (1) and (6) and the expectation is relative to the information the DM expects to receive via the experiments.

The problem is to find the best plan from the set of all feasible plans. A plan is feasible if it specifies a control that lies in Q_t for all t. Thus, letting F be the set of all feasible plans, the problem is to find

(9) $V(x_0, \pi_0; \{Y\}_0) = \sup_{q(Z) \in F} J(q(Z)).$

Under some technical conditions, the optimal plan can be characterized using dynamic programming methods. A full treatment of these issues are beyond the scope of this chapter (see, e.g. Blume et al., Blackwell (1965), and Maitra). We assume here that the problem is stationary, so that B(.) does not depend on time. The constraint set Q_t is given by a fixed function of the state, i.e. $Q_t = Q(x_t)$. It further is assumed that Q(x) satisfies the following condition:

(10) $q^1 \epsilon Q(x^1), \; q^2 \epsilon Q(x^2) \Rightarrow kq^1 + (1\text{-}k)q^2 \epsilon Q(kx^1 + (1\text{-}k)x^2) \;\; k\epsilon[0,1].$

In the resource context, such a condition would be satisfied, if Q(x) = [0,x] (e.g. you can't extract more of a resource than you have). Finally, we assume that, for each s, the transition equation is twice differentiable and concave in (x,q), and the reward function is twice differentiable and strictly concave in (x,q).

By well-known results (e.g. Blume et al.) we know that the DM's maximization problem has a solution q(Z), and that the solution is unique. Also, $V(Z_0; \{Y\}_0)$, the optimized objective function, is differentiable, concave in x, and satisfies the recursive relationship

(11) $V(Z_t;\{Y\}_t) = \max_q\{E_t[u(x_t,q(Z_t),s_t) +$

$$\alpha V(g(x_t,q(Z_t),s_t),B(\pi_t,y_t);\{Y\}_{t+1})]|q\epsilon Q(x_t)\},$$

where the expectation is with respect to the DM's current information.

The timing of the observation of the experiment relative to the choice of q and realization of s is important. It is assumed that the DM enters each period in state Z_t, i.e. with x_t on hand and with current beliefs about s of π_t. Then, she must choose current action q. Based on the current resource state and the action q, the resource moves to a new state x_{t+1}, perhaps stochastically. The DM also observes the outcome of the experiment Y_t, and revises her beliefs about next period's random events, according to the map $B(\pi_t,y_t)$.

Being more explicit about the expectations operator, (11) can be written as

(12) $V(Z_t;\{Y\}_t) = \max_{q\epsilon Q}\Sigma_i\ \pi_{i,t}\ u(x_t,q,s_{i,t}) + \alpha\ \Sigma_j\ \Sigma_k\ \pi_{k,t}\delta^t_{jk}\ V(g(x_t,q,s_{i,t}),B(\pi_t,y_{j,t});\{Y\}_{t+1}).$

Of course, $\Sigma_k\ \pi_{k,t}\delta_{jk}$ is just $\lambda_{j,t}$, the probability that one observes signal y_j and time t.

Our task now is to make explicit what is meant by an "irreversibility effect" such that receiving better information induces one to take more flexible positions.

<u>Information</u>

We wish to compare the resource decisions that get made when the DM is to receive information over time from one set of signals to those decisions that are made when improved information is available. There are a number of ways that the idea of improved information has been represented in the literature. Not necessarily referring to the above decision problem, let $W(x,q,s_i)$ be the payoffs from taking action q when the true state is s_i and the resource is in state x. If the DM has just received the message y_j, which arrives with (predictive) probability λ_j, she uses the posterior probabilities θ_{ij} in assessing the chance that s_i will arise. Let Y and Y′ denote two different message systems, with corresponding probabilities (λ,Θ) and (λ',Θ'). We have the following

Df: The message system Y is more valuable than the message system Y′ (written $Y \succeq_v Y'$) if

(13) $\Sigma_j\ \lambda_j\ \max_{q\epsilon Q}\ \Sigma_i\ \theta_{ij}\ W(x,q,s_i) \geq \Sigma_j\ \lambda'_j\ \max_{q\epsilon Q}\ \Sigma_i\ \theta'_{ij}\ W(x,q,s_i)$

Thus, we have that $Y \succeq_v Y'$ if a maximizing decision-maker attains higher expected payoffs observing signals from Y than she does observing signals from Y′. This holds regardless of the risk preferences embodied in W(.).

This definition is equivalent to another, which proves to be extremely useful analytically. Let $\Delta^m \equiv \{(\xi_1,...,\xi_m)|\xi_i \geq 0, \Sigma_i \xi_i = 1\}$, i.e. Δ_m is the set of m-dimensional probability vectors.Let $\Phi(\xi)$ be any convex function defined on Δ_m. We have

Df: Y is more informative than Y′ (written $Y \succeq_I Y'$) if

$$\Sigma_j \lambda_j \Phi(\theta_{ij}) \geq \Sigma_j \lambda'_j \Phi(\theta'_{ij}) \; \forall \text{ convex } \Phi:\Delta_n \rightarrow \mathbb{R}. \tag{14}$$

The following Lemma shows the usefulness of the definition of "more informative."

Lemma 1: $Y \succeq_I Y' \rightarrow Y \succeq_v Y'$.

Thus, more informative message systems are more valuable. Lemma 1 was proved by Bohenblust et al. (see also Marschak and Miyasawa, Theorem 12.1).[1]

These approaches say that, when one has access to better information, then one's initial beliefs are subject to greater revision. Similarly, one can show that if one's initial beliefs are more uncertain then the same information will lead to greater revision of the initial beliefs. Thus, Jones and Ostroy prefer to use the terminology "greater variability of beliefs" rather than "improvement in information."

The above definitions are stated as if there is only a single observation on the experiment. But our general decision problem above involves multiple time periods. The first definition of "more valuable" experiments in (12) is obtained by replacing W by V appropriately. Hence, we have

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[1] Another definition of informativeness of experiments was provided by Blackwell (1951). Let M be a matrix M with non-negative elements, and columns that sum to 1. Then we have that Y is sufficient for the experiment Y′if there is a matrix M such that $\Theta' = M\Theta$ and $\lambda = M\lambda'$. It was proven by Blackwell that sufficient experiments are more informative in the above sense. The result says that, if we can get to the experiment Y′ by taking the experiment Y and subject it to the noise induced by M, then Y is more informative than Y′, and also more valuable.

A final approach to comparing information is to use partitions of the event space S. This is employed by Freixas and Laffont. Recall that a partition $\mathcal{F}$ of a set S is a collection of subsets $\{S_k\}$ such that $S_k \cap S_z = \emptyset$ for all k and z, and $\cup S_k = S$. Suppose that the information to be received is that the true state lies in one element of a partition of S. Clearly, if one information structure is represented by one partition $\mathcal{F}'$, and a second by another $\mathcal{F}$ which is finer than $\mathcal{F}$ (in the sense that any element of $\mathcal{F}$ is contained in one element of $\mathcal{F}'$), then the finer partition provides better information. It was shown by Green and Stokey that finer partitions represent sufficient experiments.

Df: $\{Y\}_t \succsim_v \{Y'\}_t$ if

(15) $V(Z_t; \{Y\}_t) \geq V(Z_t; \{Y'\}_t)$.

This definition can be written more suggestively by noting that

$$V(Z_t;\{Y\}_t) = \max_{q\epsilon Q(x)} \Big\{\Sigma_i \pi_{i,t}\, u(x_t,q,s_{i,t}) +$$

(16) $$\alpha \{\Sigma_j \lambda_{j,t} \max_{q'\epsilon Q(g(x,q,s))} [\Sigma_k \theta_{kj}\, u(g(x_t,q,s_{i,t}),q',s_{k,t+1}) +$$
$$\alpha\, \Sigma_j \Sigma_k\, \delta^{t+1}{}_j\, \pi_{k,t+1} V(g(g(x_t,q,s_{i,t}),q',s_{k,t+1}),B(B(\pi_t,y_{j,t}),y_{m,t+1});\{Y\}_{t+2})]\}\Big\}.$$

The definition of "more informative" in (13) does not need to be altered at all, if it is understood that Φ is defined on a vector given by sequences $\{\theta_{ij,t}\}_t$.

More informative experiments lead to increases in the value function for the optimization program. The difference between the value function for the improved information structure and the value function for the base information structure equals the expected value of information (VOI) in the improved information. Thus, we offer

Df: The VOI for structure $\{Y\}_t$ relative to that of structure $\{Y'\}_t$ is
$VOI(Y,Y') = V(Z_t;\{Y\}_t) - V(Z_t;\{Y'\}_t)$.

The result on the VOI is recorded in

Theorem 1: If $Y \succsim_I Y'$, the $VOI(Y,Y') \geq 0$.
Proof: This follows immediately from Lemma 1 applied to (16). ■

Examples of "more informative" experiments readily can be provided in particular resource situations. Thus, if we have a wilderness area, then travel cost or contingent valuation studies of the area's value could be based on larger sample sizes. Or, the value of biodiversity could be assessed using a clinical trial of a plant's usefulness as a drug, rather than using laboratory rats, etc.

Flexibility and Irreversibility

The previous section set forth relationships among information structures. In this section we set forth relationships between choices regarding the extent to which they are flexible, and preserving of future options.

One way to do this (Freixas and Laffont) is to consider the size of the constraint set one faces in the next period as it depends on current choices. Consider two decisions, q and q'. In terms of our above model we have

Df: q is more options-preserving than q′ ($q \succeq_o q'$) if
$Q(g(x,q,s)) \supseteq Q(g(x,q',s))$ for every x and s.

Jones and Ostroy focus on the costs of getting from one position to another. They decompose u(x,q,s) into a payoff from being in situation x and a switching cost to get from x_t to x_{t+1}, $C(x_t,x_{t+1},s_t)$.[2] Jones and Ostroy say that position x is more flexible than position x′ ($x \succeq_f x'$) if the set of new positions reachable at a given cost from x is bigger than the set reachable from x′ at that cost. Formally, define

$$G(x_t,s_t,k) \equiv \{x_{t+1} \mid C(x_t,x_{t+1},s_t) \leq k\}.$$

Then we have

Df: $(x \succeq_f x')$ if $G(x_t,s_t,k) \supseteq G(x'_t,s_t,k)$, for all s,k.

The definition here is based on positions of the resource state x, rather than on choices q. This can be translated into a definition for q by applying the ordering $\succeq_f$ to positions reached from q, i.e. by a new ordering $\succeq_F$ defined by

Df: $q \succeq_F q' \leftrightarrow g(x_t,q,s_t) \succeq_f g(x_t,q',s_t)$ for all x,s.

It obviously is the case that, as long as the utility function is decomposable as specified above, then the two definitions of flexibility of choices, $\succeq_o$ and $\succeq_F$ are equivalent.

A special case of the idea of flexibility is perfect irreversibility, a case with which most of the literature is concerned. A perfectly irreversible position is one from which nothing can happen except movement to a new time period. That is, no control can be applied which would move the resource system except to where it would move by itself. Thus, we have

Df. A position x^i is irreversible if, $x^i_{t+1} = g(x^i, q, s)$ for all q in $Q(x^i)$.

Note that this does not necessarily mean that next period's state must bear any particular relationship to this period's state, but it does mean that there is nothing that the DM can do to alter the evolution of the resource state.

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[2]In terms of our model, this is defined as follows. For any given state and realization of the random event, by our previous assumptions there is a unique control required to move the system to a given new state x_{t+1}. This control is $q(x_{t+1};x_t,s_t)$, defined implicitly by $g(x_t,q(x_{t+1};x_t,s_t),s_t) = x_{t+1}$. Suppose that the utility u(x,q,s) takes the separable form

$$u(x,q,s) = u(1-x,s) + w(x,s) + [p-c]q.$$

Then

$$C(x_t,x_{t+1},s_t) \equiv cq(x_{t+1};x_t,s_t),s_t).$$

Note that there is some reversibility in the case of renewable resources, so we do not require strictly irreversible processes in the above definition. Naturally, in the case of exhaustible resources, where g(x,q,s) = x-q and Q(x) = [0,x], any extraction is absolutely irreversible. In the case of forests or wilderness, we may allow some "reversion to the wild" in G(.), or we allow the growth of renewable resources. In these cases, irreversibility is captured by a constraint $q \geq 0$, so that the resource stock cannot be augmented faster than its natural rate of regeneration.

The above definitions regarding flexibility were stated as if they pertained to a two-period model; i.e. they applied to a single control decision at one point in time. However, our basic decision model outlined earlier is a multi-time model. Thus, the above definitions need to be extended to include whole sequences of choices. The most obvious extension is:

Df: The sequence $\{q_t\}_t \succeq_o \{q_t'\}$ if $q_t \succeq_o q_t'$ for all t.

Thus, one sequence is more options preserving than another if each if its elements is more options preserving.

III. Irreversibility Effects and Quasi-Option Values

We now are in a position to state the central results of this literature. These are obtained by applying the above definitions of orderings on information and flexibility of positions (or special cases of them) to the general decision problem stated earlier (or special cases of it).

Some writers in this area study what we call the "irreversibility effect." This is a relationship between better information and the flexibility of initial positions. Establishing the existence of an irreversibility effect requires establishing that an ordering on information induces an ordering on flexibility. This is the approach taken by Freixas and Laffont, and by Jones and Ostroy, among others. In this sense, there is no "quasi-option value" derived explicitly. Formally stated, we have

Df: The irreversibility effect (IE) holds if $\{Y\}_t \succeq_I \{Y'\}_t \Rightarrow \{q_t\}_t \succeq_F \{q_t'\}$.

Another approach is to derive values, the QOVs, as a sequence of taxes which would induce a DM who "ignores the improved information" to choose the same control as that used by a DM who builds the receipt of better information into the decision problem (Arrow and Fisher). But what does it mean to ignore information? And what kinds of taxes should one consider?

Naturally, there is an intimate tie between the existence of an irreversibility effect and the sign of the appropriately defined tax on controls. For example, suppose that the

IE implies that less of the resource is extracted at some date. Then, a tax could be placed on extraction such that a DM facing worse information would use the same control as a DM obtaining the improved information. Thus, if the IE exists, the associated QOV, conceived of as a tax on the control, is positive.

Still another approach identifies the QOV as an expected value of information (VOI), rather than as a tax on development (Conrad; Hanemann). This VOI approach looks at the benefits realized from incorporating information instead of ignoring it. In some circumstances the VOI equals the QOV given by a tax on the control; for example, if $Q(x) = \{0,x\}$. However, in other cases of relevance (e.g. $Q = [0,x]$), this equivalence does not hold (see Hanemann). Here, because of its close relationship to the IE, we submit the appropriate concept of QOV is the tax on the control.

Finally, some authors (e.g. Cukierman) have restricted their attention to a situation where a given decision will be implemented , and the issue is how much information to obtain before doing this, where information is accumulated through time. This is more in line with the literature on sequential experimentation (e.g. DeGroot), and will not be discussed further here.

In most of the literature on the irreversibility effect and QOV, it is supposed that there are just two time periods, and that either one receives perfect information, or no information at all. It further is supposed that the decision space is $Q = \{0,x\}$ or, equivalently, a linear benefit function is imposed with $Q = [0,x]$, in which case either q is set equal to zero or all of the available resource is extracted. This allows sharper results (Hanemann), but we take the more general approach of allowing continuous choices with non-linear payoffs.

In this Chapter, we will examine two resource problems. The first, which we shall call Case E, represents exhaustible resource extraction. The extraction of fossil fuels, and irreversible land development examples are in this class. The transition equation in Case E is $g(x,q,s) = x - q$. The second case is called Case R, for renewable resource extraction. The transition equation in this instance is $g(x,q,s) = F(x) - q$, where $F(x) = x + G(x)$, and G is the growth function for the resource. Of course, Case R becomes Case E when $G(x) = 0$ for all x. In both of these cases we suppose that $Q(x_t) = [0,x_t]$. Note that uncertainty has been expunged from the transition equation for the resource stock, thereby limiting somewhat the scope of our analysis here.

It is straightforward to show that for either of these cases, smaller extractions are more options preserving. We state this as

<u>Lemma 3: In either Case E or Case R, $\{q_t\}_t \succeq_o \{q_t'\}$ if $q_t \leq q_t'$ for all t.</u>

Of course, in the case of renewable resources, this cannot be an "if and only if" statement. This lemma does not characterize all the interesting issues. For example let

G be concave, first rising and then falling. Let x^m be the maximum sustained yield (MSY) biomass level, and take $\{q\}_t$ to be a constant equal to the maximum sustainable harvest, $q^m = G(x^m)$. Consider some alternative $\{q'\}_t$ set equal to a constant level of harvest $q' < q^m$ and associated steady-state stock x' on the upward-sloping portion of the growth function. Relative to q', a larger stock can be obtained, and a more flexible position reached, via the control sequence composed of $q_t = 0$ until $x_t = x^m$, and $q_t = q^m$ thereafter. So the reverse implication of Lemma 3 does not hold. Moreover, this example shows that with multiple time periods and renewable resources, determining the more flexible positions will require some work (see Fisher and Hanemann (1985) for an investigation along these lines).

Unless the general decision model is restricted further, the irreversibility effect does not hold and the QOV can be positive or negative. This is, in some ways, rather surprising, since it makes intuitive sense that the prospect of learning more should lead one to adopt more flexible positions. After all, information is valuable; if one is in an inflexible position, one cannot make use of the information to revise one's actions, and the benefit of learning is foregone. What is required to demonstrate the irreversibility effect is that the gain in the value of information from taking a more flexible position outweighs the opportunity costs of this position.

We assume that the utility function takes the form in (3), i.e. that

$$u(x,q,s) = w(x,s) + v(1\text{-}x,s) + [p\text{-}c]q,$$

where u(.) and v (.) are concave in their first argument for each s. Holding the resource stock may involve maintenance costs implicit in w. In some papers (e.g. Epstein, Graham-Tomasi, Freixas and Laffont) it is supposed that utility is a function of stocks of developed and undeveloped resource alone, with no benefits and/or costs of current development.

The first order necessary condition for a maximum of (16) is

$$(17)\quad [p - c] + \alpha \Sigma_j \Sigma_k \pi_{k,t}\delta^t_{jk}[\partial V(Z_{t+1})/\partial x_{t+1}][\partial g(x_t,q(Z),s_{i,t})/\partial q] \leq 0 \quad \text{if } q(Z) \leq 0$$
$$= 0 \quad \text{if } q(Z) > 0$$

Condition (17) generalizes, by incorporating uncertainty and receipt of information, the usual discrete-time version of the renewable resource problem.

In either Case R or E, $\partial g(x_t,q(Z),s_{i,t})/\partial q = -1$. Thus,

$$(18)\quad [p - c] = \alpha \Sigma_j \Sigma_k \pi_{k,t}\delta^t_{ji}[\partial V(x + F(x) - q(x_t,\pi_t),\pi_{t+1})/\partial x_{t+1}] \quad \text{if } q(Z) > 0,$$
$$[p - c] \leq \alpha \Sigma_j \Sigma_k \pi_{k,t}\delta^t_{ji}[\partial V(x + F(x) - q(x_t,\pi_t),\pi_{t+1})/\partial x_{t+1}] \quad \text{if } q(Z) = 0,$$

Hence, if the expected marginal shadow value of the resource stock exceeds the value of a unit of extracted stock, then extraction will be zero. However, if extraction is to be positive, it is carried out to the point which balances the current marginal gains from extraction and the expected shadow value of the resource in situ. Naturally, if the problem were reversible, so that the resource stock could actively be augmented (i.e. q could be negative), then (18) would hold as an equality for all t. It is the second line of (18) where irreversibility is realized.

We see immediately that if the term on the RHS of (18) is convex in π_{t+1}, then an improvement in information leads to increases in flexibility by reducing the current extraction of the resource. This is so by Lemmas 1 and 2, and the assumed concavity of w and v in their arguments. We state this as

> Theorem 2: If payoffs are concave in x for all (x,s) and $\partial V(Z_{t+1})/\partial x_{t+1}$ is convex in π_{t+1}, then the irreversibility effect holds, i.e. an improvement in information leads to an increase in flexibility.
> Proof: If the value function is convex in π for given x, an improvement in information increases the RHS of (18), by Lemma 1. In order to maintain the equality in (18), by the concavity of V(.) in x, x_{t+1} must increase, requiring a decrease in $q(Z_t)$. This corresponds to an increase in flexibility, according to Lemma 2. ■

This is an application of Epstein's Theorem 1 to a multi-time resource extraction problem. It also is a reformulation of the result in Freixas and Laffont. They show that if the derivative with respect to the state variable of the expected value of information (conditional on the choices with inferior information), is positive, then the irreversibility effect holds in their model. But this will hold if the expected shadow value function is convex in beliefs. The result extends both of these papers to include net values for current extraction.

To define the QOV and its relationship to the irreversibility effect, suppose that a DM operates using information system $\{Y'\}_t$ and ignores the availability of an improved information system $\{Y^o\}_t$. In order to induce the same choice of extraction, a unit tax can be placed on extraction. The appropriate tax is the expected shadow value of the resource with the improved information, less its expected value with the inferior information. That is, we have the definition

Df: Suppose that $\{Y^o\}_t \succeq_I \{Y'\}_t$. Then

$$(19)\quad QOV(\{Y^o\},\{Y'\}) = \alpha \Sigma_j \Sigma_k \pi^o_{k,t}\delta^{ot}_{ji}[\partial V(x + F(x) - q(x_t,\pi^o_t),\pi^o_{t+1})/\partial x_{t+1}] - \alpha \Sigma_j \Sigma_k \pi'_{k,t}\delta'^t_{ji}[\partial V(x + F(x) - q(x_t,\pi'_t),\pi'_{t+1})/\partial x_{t+1}] .$$

Thus, the QOV equals the difference between two shadow prices of the stock. It is not given by a value of information, which is the difference between to value functions. An immediate corollary to Theorem 2 is that, under the stipulated conditions, an improvement in information leads to a positive QOV. Formally, we have,

Corollary: Under the conditions of Theorem 2, QOV $\geq$ 0.

The trick, of course, is to determine conditions under which the derivative of the value function with respect to the state variable is convex in beliefs. We show that, under the conditions stipulated so far, the expected shadow value of the stock is in fact a convex function of beliefs in Case E and Case R..

Theorem 3: For Case E or Case R, the expected shadow value of the stock, $E\{\partial V(x,\pi;\{Y\})/\partial x\}$, is convex in π.
Proof: See the Appendix.

In the proof it is revealed that the result depends on information which might provide both "good" and "bad" news. That is, if all the information that one might receive leads to an expected shadow value of the stock which exceeds the current value of extraction, then a positive level of extraction is undertaken under all beliefs. In this case the news is good in the sense that it is not discovered that past extraction was excessive. Then, improved information has no effect, the irreversibility effect does not hold, and the QOV is zero. Similarly, if all the information leads to bad news, so that the constraint always is binding, then the IE does not hold, and QOV is zero. Only if some messages lead to good news and some to bad does the prospect of improved information imply an increase in flexibility and a positive QOV.[3]

Bernanke/Graham-Tomasi Quasi-Option Value

As discussed briefly above, different authors treat the scenario of ignoring the improved information differently. Bernanke assumed that the ignorance case involves maximizing each period's payoffs separately, in a sequence of myopic optimization problems. Thus, each period, the DM who ignores information solves

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[3]A different question is the effect of an increase in the riskiness of the decision environment, holding fixed the information to be received. Rothschild and Stiglitz have shown that if a distribution of a random variable is subjected to a mean-preserving spread, then the expected value of any convex function of the random variable increases.

It is the curvature of the shadow value function in s that is important to determining the impact of an increase in risk. This is not the same as curvature in beliefs, so the above proof in Theorem 3 implies nothing about increases in risk. See Epstein for further results in this vein.

$$\max_q E\{[p-c]q + \alpha[w(g(x,q),s) + v(1-g(x,q),s)]\} \text{ s.t. } q \in Q(x).$$

Imposing the separable utility function as in (3), we see that the solution to this problem is characterized by the condition

$$(20) \quad \alpha\Sigma_i \pi_{i,t} [\partial w(x_t,s_{i,t})/\partial x - \partial v(1-x_t,s_{i,t})/\partial x] = p - c.$$

Thus, the DM in this version sets the discounted expected net marginal benefits of increasing the resource stock (by decreasing q) equal to the net benefits foregone from the decrease in q.

Suppose that the improved information scenario involves applying condition (19). To deduce the QOV we need an expression for $\partial V/\partial x$. Using the envelope theorem, we find that

$$(21) \quad \partial V(Z_t)/\partial x = \Sigma_i \pi_{i,t} [\partial w(x_t,s_{i,t})/\partial x - \partial v(1-x_t,s_{i,t})/\partial x] + [(1+G'(x))/(1+r)] \; \Sigma_j \Sigma_k \pi_{k,t}\delta^t_{ji}[\partial V(Z_{t+1})/\partial x].$$

Combining this with (20) we can see that the QOV for this comparison is

$$(21) \quad [(1+G'(x))/(1+r)] \; \Sigma_j \Sigma_k \pi_{k,t}\delta^t_{ji}[\partial V(Z_{t+1})/\partial x].$$

This is proportional to the term on the RHS of (19). Bernanke QOV is positive.

Why would a DM ignore the future and act myopically as Bernanke supposes? In an independent derivation of the same result as Bernanke's, Graham-Tomasi considered a model of wilderness development in which new development is costless and confers no direct benefit. Then, if one is not considering the receipt of information, the optimal dynamic policy is to behave myopically. That is, assume no revision of beliefs, so that a dynamic optimization problem is solved, but using $\pi_t = \pi_0$ for all t. We have $x_t = 1 - \Sigma_{s\leq t} q_t$ and $U = u(\Sigma_{s\leq t} q_t, 1 - \Sigma_{s\leq t} q_t, s_t)$. It is easy to show that the optimal policy is $\{q\}_t = (q^m,0,0,...)$, where q^m solves $u_1 - u_2 = 0$, where u_i is the derivative of u with respect to its ith argument. This is the equivalent of the Bernanke approach in this model. Then, introducing information leads to setting $u_1 - u_2 =$ QOV, defined as above. Graham-Tomasi shows that, in this model, the shadow price function is convex in beliefs. Thus, the IE holds and QOV is positive.

IV. Discussion

There are variety of ways that QOV has been discussed. Originally, the motivation was that decision-makers, such as government agencies, appeared to ignore possibilities for learning in their natural resource decisions. Cost-benefit analysts typically employ simplified procedures when uncertainty is present, and use current information to replace random variables by their means or to compute expected future benefits and costs. A concern then arises that such incomplete decision rules lead to decisions biased in a particular direction. The demonstration that QOV is positive implies that typical benefit-cost decision rules are biased in favor of irreversible investments.

This Chapter has applied these ideas to a more general set of concerns. This more general framework reveals the wide applicability of the notion of QOV. It seems that the concept is fundamental to problems of resource use. The difficulty, of course, lies in empirical treatments which might establish the magnitude of the bias in particular situations. In fact, few such empirical applications of the ideas have been attempted (see Fisher and Hanemann (1986) for one effort.

There are three extensions of the above analysis that warrant discussion. First, the approach removed uncertainty from the transition equation. Some of the arguments of environmentalists pertain to the difficulty of undertaking decisions when the laws governing ecosystem function are poorly understood. The model above can handle some of these concerns. For example, we should not be cavalier about substitution of capital for resources, since we know little about which are the necessary resources. This introduces uncertainty into the demand for the resource stock, as above. However, there needs to be uncertainty in the transition equation to handle problems of hysteresis and uncertain thresholds below which extinction occurs.

Second, the basic arguments could be applied to more general-equilibrium, macro-oriented concerns. Adding capital to the model would allow this. It would be reasonable, then, to introduce uncertainty in the production function relating capital to resource stocks of various kinds. The value function would play the role of an appropriate national income equation, and the shadow prices appropriately would consider uncertainty. Thus, national income accounts augmented to include resources should employ shadow values which include recognition of uncertainty as well as opportunities for learning.

Finally, the information structure here was taken as exogenous. But the amount to be learned obviously depends on research expenditures, taken out of current output. Moreover, as Miller and Lad pointed out, the amount learned may depend on extraction decisions themselves. Thus, with oil exploration in wilderness, to learn more about the oil stock requires some development of wilderness. In this case, the QOV results derived above are undermined. However, in many circumstances, learning will increase with conservation, which will only serve to reinforce the irreversibility effect. Perhaps more importantly, the amount to be learned via research effort should be made endogenous.

APPENDIX

Here, we prove Theorem 3, which states that the value function is convex in beliefs. First, we consider a finite-horizon problem and show that each value function is convex using an induction argument. Then, we use a result that the infinite-horizon value function retains the features of the finite horizon value function in the limit as the time-horizon becomes arbitrarily long. The proof is based on one by Demers.

Step 1. Consider a finite time horizon version of the above problem *P*, with end-date T. Let $V^N(Z_{T-N})$ be the value function with N dates remaining. Define an operator $\Gamma(V(x,\pi)) = \max_{q\geq 0} \Sigma_i \pi_i\{w(x,s_i) + v(1-x,s_i) + [p-c]q + \alpha\Sigma_j\Sigma_k \delta_{jk}\pi_k V(x+F(x)-q,B(\pi,y_j))$. If C(S) is the space of bounded, continuous functions $V:S\rightarrow\mathbb{R}$, with S compact, previous assumptions ensure that $\Gamma:C(S)\rightarrow C(S)$. Give C(S) the sup norm, under which C(S) is a complete metric space. Then Γ satisfies the properties of a contraction mapping on a complete metric space, due to arguments in Blackwell (1965). Every contraction map on a complete metric space has a unique fixed point (Rudin). Thus, the equation $\Gamma V = V$ has a unique solution, which is the infinite-horizon value function for *P*. Moreover, letting Γ^m be the application of Γ m times, $\|\Gamma^m V^o - V\|$ goes to zero uniformly (since it is convergence in the sup norm) as m goes to infinity, for any initial V^o. Concavity and differentiability are established by Blume et al.

Let $V^N(x,\pi) = \Gamma V^{N-1}(x,\pi)$. Since V^N is differentiable and converges to V uniformly. Assuming that $\partial V^N(x,\pi)/\partial x$ converges, it does so uniformly to $\partial V(x,\pi)/\partial x$ (Rudin). Since the limit of a sequence of convex functions is convex, it suffices to show that $\partial V^N(x,\pi)/\partial x$ is convex in π for every N.

Step 2. We need to show that $V^N(x,\pi)/\partial x$ is convex. Pick two beliefs, π^o and π^1, and let $\pi^\mu = \mu\,\pi^o + (1-\mu)\,\pi^1$, for $\mu \in [0,1]$. Thus, we need to show that

$$\text{(A1)} \qquad \partial V^N(x,\pi^\mu)/\partial x \leq \mu\partial V^N(x,\pi^o)/\partial x + (1-\mu)\partial V^N(x,\pi^1)/\partial x\,.$$

From equation (21),

$$\text{(A2)} \quad \partial V^N(x,\pi^\mu)/\partial x = \Sigma_i\, \pi^\mu_{i,t}\, [\partial w(x_t,s_{i,t})/\partial x - \partial v(1-x_t,s_{i,t})/\partial x\,] + [(1+G'(x)/(1+r)]\, \Sigma_j\, \Sigma_k\, \pi^\mu_{k,t}\delta^t_{ji}[\partial V^{N-1}(x+G(x)-q(x,\pi^\mu),\, \pi^\mu_{t+1})/\partial x].$$

Since $V^0 = 0$, $\partial V^1(x,\pi^\mu)/\partial x$ is linear in π and hence convex. Using an induction argument, we know that $\partial V^1/\partial x$ is convex in π, and will assume that $\partial V^2/\partial x, \partial V^3/\partial x, \ldots, \partial V^{N-1}/\partial x$ are all convex in π; we shall then show that $\partial V^N/\partial x$ is convex in π.

Use $\pi^\mu = \mu\,\pi^o + (1-\mu)\,\pi^1$ in the first summation in A(2), and add and subtract

$\mu[(1+G'(x)/(1+r)]\, \Sigma_j\, \Sigma_k\, \pi^o_{k,t}\delta^t_{ji}[\partial V^{N-1}(x+G(x)-q(x,\pi^o),\, \pi^o_{t+1})/\partial x]$ and

$(1-\mu)\, [(1+G'(x)/(1+r)]\, \Sigma_j\, \Sigma_k\, \pi^1_{k,t}\delta^t_{ji}[\partial V^{N-1}(x+G(x)-q(x,\pi^1),\, \pi^1_{t+1})/\partial x]$.

This yields

$$(A3)\qquad \partial V^N(x,\pi^\mu)/\partial x = \mu\partial V^N(x,\pi^o)/\partial x + (1-\mu)\partial V^N(x,\pi^1)/\partial x +$$
$$[(1+G'(x)/(1+r)]\ \Sigma_j\ \Sigma_k\ \pi^\mu_{k,t}\delta^t_{ji}[\partial V^{N-1}(x+G(x)-q(x,\pi^\mu),\ \pi^\mu_{t+1})/\partial x]\ -$$
$$\mu[(1+G'(x)/(1+r)]\ \Sigma_j\ \Sigma_k\ \pi^o_{k,t}\delta^t_{ji}[\partial V^{N-1}(x+G(x)-q(x,\pi^o),\ \pi^o_{t+1})/\partial x]\ -$$
$$\mu[(1+G'(x)/(1+r)]\ \Sigma_j\ \Sigma_k\ \pi^1_{k,t}\delta^t_{ji}[\partial V^{N-1}(x+G(x)-q(x,\pi^1),\ \pi^1_{t+1})/\partial x]\ .$$

Thus, we need to show that the sum of the last three lines is non-positive. Divide through these terms by $(1+G')/(1+r)$. Then the terms are related to the first order conditions for a maximizing choice of q in (18). We must consider whether the alternative beliefs π^μ, π^o, and π^1, lead to corner solutions or interior solutions. We shall consider the alternatives in turn.

A) Suppose first that the three beliefs all lead to interior solutions. By the first order condition for q, the first term is (p-c), while the second and third terms are μ(p-c) and $(1-\mu)$(p-c). Thus, the sum of these is zero. In this case, the expected shadow value of the resource is linear in beliefs and it does not respond to improvements in information. Then too, neither does extraction respond to improvements in information, so there is no irreversibility effect and QOV is zero.

B) Suppose that all three beliefs lead to a corner solution. We cannot say what happens to the expected shadow value of the resource, since it generally will be non-linear in π. But, since all three extractions are zero, there is no irreversibility effect, and QOV again is zero.

C) Suppose now, without loss of generality that π^o leads to an interior solution, while π^1 leads to a corner solution. We must consider two further cases regarding π^μ: C(i) π^μ leads to an interior solution, and C(ii) π^μ leads to a corner solution.

In case C(i), we have that the second line in A3 equals (p-c), and the third equals μ(p-c). Since π^1 leads to a corner solution the fourth line does not fall short of $(1-\mu)$(p-c), as shown in (18). Assume that it exceeds it. Then in case C(i), the sum of these terms is negative, and the expected shadow price is convex.

For Case C(ii), the second line exceeds (p-c), the third line equals μ(p-c), while the fourth line exceeds $(1-\mu)$(p-c). We use the induction assumption that $\partial V^{N-1}/\partial x$ is convex in π. Taking the summation term by term, and dividing through by π^μ, the sum of these three lines would be non-positive if all these derivatives were evaluated at the same stock, namely, with $q=0$. However, setting $q > 0$ in the third line in A3 increases this term above even more, by the concavity of V^{N-1} in x. Hence, by the induction hypothesis, the sum of these three lines is negative and the expected shadow price is convex. ■

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