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ECONOMETRIC MODELS OF DISCRETE/CONTINUOUS AGRICULTURAL SUPPLY DECISIONS

by

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1. Introduction

Until recently econometricians have paid very little attention to the discreteness of choices made by ecomic agents. Yet in reality discrete choices, and discrete choices interrelated with continuous choices, are a pervasive phenomenon. For example, a producer is growing a certain crop and faces a binary choice between two alternative production techniques- e.g. whether to use conventional chemical pest control or integrated pest management techniques. Another example is where the farmer must decide whether to participate in a federal commodity program. The discrete choice is which technology to use, or whether to participate in the program; the continuous choice is how much of the crop to produce. In both cases the discrete and continuous choices are fundamentally interrelated: the optimal continuous decision depends on the outcome of the discrete choice, and vice versa. Therefore, both choices should be modelled simultaneously.

Interrelated discrete and continuous choices can readily be accomodated in mathematical programming models simply by defining production under each discrete alternative as a separate activity. However, their incorporation into econometric models has proved more difficult. Over the last decade there have been numerous logit or probit models dealing with the choice of production technique or commodity program participation, but these focus exclusively on the discrete choice and ignore the continuous supply decision. The subsequent introduction of switching and limited dependent variable regression models by Heckman, Lee and Trost, and others, has provided an effective statistical tool for analyzing simultaneous discrete and continuous dependent variables. However, in these models the discrete and continuous choices are not fully integrated, in the sense that they do not both flow from the same underlying economic optimization.

This has now been rectified in work by Duncan, Hanemann, and McFadden. This recent work involves switching regression models generated by an explicit theoretical model of simultaneous discrete and continuous optimization by individual micro-agents, and it explicitly allows for the presence of unobserved variables and variations in tastes or abilities among the micro-agents. This approach has been applied to discrete/continuous models of consumer demand by Hanemann and Dubin and McFadden, and to discrete/continuous models of producer supply under profit maximization by Duncan and McFadden. In order to analyze many agricultural supply issues, including those mentioned above, it would be more appropriate to have a model of discrete/continuous choice under the criterion of expected utility-of-profit maximization, which is the subject of this paper.

We will be concerned here mainly with model formulation rather than estimation; further details on estimation can be found in Hanemann and Tsur. Our object is to explain the general procedure for constructing discrete/continuous models of supply under uncertainty, as well as to present a specific model which is suitable for empirical application. The key concept is the notion of a "random supply" model, in which some components of the supplier's production or utility-of-profit functions are treated as being random from the viewpoint of the econometric investigator. It is this random component which generates the stochastic structure employed in the estimation of the discrete and continuous supply functions. Before describing this random supply model, however, it is convenient to begin by summarizing a "deterministic" model of supply under uncertainty, where this random component is absent-- this is done in section 2. The corresponding random supply model is presented in section 3. Some remarks on estimation follow in section 4.

2. Deterministic Discrete/Continuous Supply Models

We shall first summarize the standard deterministic model of purely continuous supply under uncertainty and then generalize this to the case of discrete/continuous supply choices under uncertainty. We focus on the special case of a supplier of a single product, who faces no explicit constraints on his production decision (such as a limit on the availability of land or credit). His profit, π, is given by

(1) $$\pi = pq - c(w, q) - b$$

where p is the product price, q is the amount of product supplied, $c(\cdot)$ is a variable total cost function generated by some production function, and b is fixed costs. We assume that the producer faces uncertainty with respect to the product price. His subjective density will be denoted f_p, with mean μ and variance σ_p^2.[1] The producer has a utility-of-profit function, $u(\pi)$, with $u' > 0$, and $u'' \gtreqless 0$ depending whether he is risk-prone, risk-neutral, or risk-averse. To allow for the possibility that his risk preferences depend in a parametric manner on his individual characteristics, s, we shall write $u = u(\pi; s)$.

The producer chooses an output level, q, so as to maximize his expected utility

$$\max_q U(q) \equiv \max_q \int u(\pi(q, p); s)\, f_p dp\,. \tag{2}$$

The solution to the producer's maximization problem will be denoted $q(\mu, w, b; s)$. Substituting this into the maximand in (2) yields the ~~indirect expected utility-of-profit function~~, $v(\mu, w, b; s) = U(q(\mu, w, b; s)$.

By a standard application of the envelope theorem it can be shown that

$$\frac{\partial v(\mu, w, b; s)}{\partial \mu} = q(\mu, w, b; s)E\{u'\} \tag{3}$$

$$\frac{\partial v(\mu, w, b; s)}{\partial b} = -E\{u'\} \tag{4}$$

Hence, we have the equivalent of Hotelling's lemma for production decisions under uncertainty

$$q(\mu, w, b; s) = -\frac{\partial v(\mu, w, b; s)/\partial \mu}{\partial v(\mu, w, b; s)/\partial b}\,. \tag{5}$$

It follows that, as with the theory of supply under certainty, there are two methods for generating a particular parametric supply model. The direct (primal) approach is to specify a particular utility function and density, f_p, and then solve the resulting maximization problem (2) for $q(\cdot)$ and $v(\cdot)$. The indirect (dual) approach is to start by specifying an indirect expected utility-of-profit function, $v(\cdot)$, which satisfies the appropriate requirements for such a function, and then to derive the output supply function from (5).

Example.

We assume constant returns to scale with respect to the variable inputs, so that the total variable cost curve can be written

$$(6) \qquad c(w, q) = c(w)q$$

where $c(w)$ is a unit variable cost function. We also assume constant absolute risk aversion:

$$(7) \qquad u(\pi; s) = 1 - e^{-\alpha(s)\pi}$$

where $\alpha(s)$, the absolute risk aversion coefficient, is allowed to vary with the characteristics of the producer. In particular, if the producer's wealth is one of these characteristics, this formulation allows for the possibility of, say, absolute risk aversion declining with wealth <u>across individuals</u> while being constant for a given producer making a given risky decision. It follows from (7) that expected utility is

$$(8) \qquad \upsilon(q) = 1 - M_p(-\alpha(s)q)e^{\alpha(s)c(w)q+\alpha(s)b}$$

where $M_p(\cdot)$ is the moment generating function associated with f_p. If $f_p \sim N(\mu, \sigma_p^2)$, then

$$M_p[-\alpha(s)q] = \exp[-\alpha(s)\mu q + \frac{\alpha(s)^2 q^2 \sigma_p^2}{2}$$

and expected utility becomes

$$(9) \qquad \upsilon(q) = 1 - \exp[-\alpha(s)(\mu q - c(w)q - b) + (\alpha(s)^2 q^2 \sigma_p^2)/2].$$

For the normal case, the maximization of (9) yields

$$(10) \qquad q(\mu, w, b; s) = \frac{\mu - c(w)}{\sigma_p^2} \cdot \frac{1}{\alpha(s)}.$$

Substituting (10) into (9) yields the indirect expected utility-of-profit function

$$(11) \qquad v(\mu, w, b; s) = 1 - \exp[\alpha(s)b - \frac{(\mu - c(w))^2}{2\sigma_p^2}].$$

We now introduce the possibility of a discrete choice by the producer in addition to the continuous supply decision discussed above. Specifically, we assume that the producer faces N mutually exclusive discrete choices. Examples of such discrete choices might be: which of N mutually exclusive production technologies to employ; in which of N mutually exclusive locations to produce; which of N mutually exclusive types of fixed equipment to use alongside of the variable inputs; or whether or not to participate in a federal government commodity program. In general, we can assume that each discrete alternative j presents the producer with a particular vector of variable input prices, w_j; a particular variable cost function, $c_j(w_j, q)$; a particular fixed cost, b_j; and a particular distribution of output prices, $f_{p_j}(p)$, with mean μ_j and variance σ^2_{jp}. We also allow for the possibility that the producer's individual characteristics, s, may vary with the discrete choice, and hence that his utility function $u(\cdot)$, may vary with j.

Suppose, for the moment, that the producer has decided to select the j^{th} discrete alternative. Conditional on this decision, his profit is

$$\pi_j = p_j q_j - c_j(w_j, q_j) - b_j ,$$

where q_j is his output under the j^{th} discrete alternative, and his expected utility is

$$\bar{U}_j(q_j) = \int u_j(\pi_j(q_j, p_j); s_j) f_{p_j} dp_j . \tag{12}$$

His continuous supply decision conditional on this discrete choice is $\bar{q}_j(\mu_j, w_j, b_j; s_j)$ which is obtained by maximizing (12) . His expected utility on making this supply decision is $\bar{v}_j(\mu_j, w_j, b_j; s_j) = \bar{U}_j(\bar{q}_j(\mu_j, w_j, b_j; s_j))$.

It is evident from this derivation that the conditional output supply function, $\bar{q}_j(\cdot)$, and the conditional indirect expected utility-of-profit function, $\bar{v}_j(\cdot)$, have all the standard properties of an output supply function and an indirect expected utility-of-profit function as outlined above.

All of the foregoing is conditional on the producer's selecting discrete alternative j. His discrete choice can be represented by a set of binary valued indices, $d_1, \ldots, d_N$, where $d_j = 1$ if alternative j is selected, and $d_j = 0$ otherwise. His <u>overall</u> continuous and discrete maximization problem is to select $q_1, \ldots, q_N$ and $d_1, \ldots, d_N$ so as to maximize

$$(13) \qquad \sum_1^N d_j \bar{U}_j(q_j) \qquad \text{subject to} \quad d_j = 0 \text{ or } 1, \ \Sigma d_j = 1 .$$

The solution for the discrete choices, denoted $d_j = d_j(\mu_1, \ldots, \mu_N, w_1, \ldots, w_N, b_1, \ldots, b_N; s_1, \ldots, s_N)$ or, more compactly, $d_j = d_j(\mu, w, b; s)$, $j = 1, \ldots, N$, are functions of the full <u>set</u> of input costs and output prices. Similarly, the solution for the continuous choices — the unconditional supply functions — will be denoted $q_j = q_j(\mu, w, b; s)$. Finally, the unconditional indirect expected utility-of-profit function is $v(\mu, w, b; s)$, defined as

$$(14) \qquad v(\mu, w, b; s) = \sum_1^N d_j(\mu, w, b; s)\bar{U}_j(q_j(\mu, w, b; s)).$$

These unconditional functions are related to the conditional functions derived above in the following manner:

$$(15) \qquad d_j(\mu, w, b; s) = \begin{cases} 1 \text{ if } \bar{v}_j(\mu_j, w_j, b_j; s_j) \geq \bar{v}_i(\mu_i, w_i, b_i; s_i), \text{ all i} \\ 0 \text{ otherwise} \end{cases}$$

$$(16) \qquad q_j(\mu, w, b; s) = d_j(\mu, w, b; s)\ \bar{q}_j(\mu_j, w_j, b_j; s_j)$$

$$(17) \qquad v(\mu, w, b; s) = \max\{\bar{v}_1(\mu_1, w_1, b_1; s_1), \ldots, \bar{v}_N(\mu_N, w_N, b_N; s)\}.$$

For the model (6) - (8), under the assumption that the p_j's are independently distributed with $f_{p_j} \sim N(\mu_j, \sigma^2_{jp})$, the conditional output supply and indirect expected utility-of-profit functions are

$$(18) \qquad \bar{q}_j(\mu_j, w_j, b_j; s_j) = \frac{\mu_j - c_j(w_j)}{\alpha_j(s_j)\,\sigma^2_{jp}}$$

$$(19) \qquad \bar{v}_j(\mu_j, w_j, b_j; s_j) = 1 - \exp\{\alpha_j(s_j)b_j - \frac{[\mu_j - c_j(w_j)]^2}{2\sigma^2_{jp}}\}.$$

B. Random Supply Models

A random supply model arises when one assumes that, although all the elements of the producer's decision — his cost function, his subjective probability density for the output price, and his own utility-of-profit function — are known for sure to him, they contain some components which are unobservable to the econometric investigator, and are treated by the investigator as random variables. This formulation embodies two notions which, for practical purposes, are indistinguishable: the idea of a variation in technology, information or preferences among a population of individual economic agents, and the concept of unobserved variables in econometric models. These components will be denoted by ε^c, ε^p and ε^u, which may be scalars or vectors. In each case, they are fixed constants (or functions) for the individual producer, but for the investigator they are random variables. For example, because of unobservables or inter-agent variation in the production technology, the individual producer's cost function appears to the investigator to be of the form $c_j(w_j, q_j; \varepsilon_j^c)$; or, because of differences in perceptions among producers, the individual producer's subjective probability density for output prices appears to the investigator to be of the form $f_{p_j}(p_j; \varepsilon_j^p)$; or, finally, because of variations in risk preferences or unobservable components in profits (including fixed costs), the individual producer's utility-of-profit function appears to the investigator to be of the form $u_j(\pi_j; s_j, \varepsilon_j^u)$.[2]

One can generate different random production models depending on which of these sources of randomness one chooses to emphasize and on how one incorporates them. In order to avoid committing ourselves at this point to a specific random production model, we will refer to these random components collectively as ε_j; ε_j could be ε_j^c, ε_j^p, ε_j^u, or some combination of them. Accordingly, we write the direct expected utility-of-profit function associated with the j^{th} discrete alternative in general terms as $\bar{U}_j(q_j; \varepsilon_j)$. A similar set of random terms exists for each discrete alternative. We denote the overall set of random terms by $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_N)$. For the econometric investigator this is a multivariate random variable with some joint density function, denoted $f_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N)$; for the individual producer, however, it is a set of fixed constants.

The individual producer's decision problem is to select $q_1, \ldots, q_N$ and $d_1, \ldots, d_N$ so as to maximize $\sum_1^N d_j \bar{U}_j(q_j;\ \varepsilon_j)$ subject to $d_j = 0$ or 1, $\Sigma d_j = 1$. The supply functions generated by this maximization problem parallel those developed in the previous section, except that they now involve a random component from the point of view of the econometric investigator. Suppose the producer has decided to select the j^{th} discrete alternative. If he maximizes $\bar{U}_j(q_j;\ \varepsilon_j)$ this yields the conditional supply function, $\bar{q}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j)$ and the conditional indirect expected utility-of-profit function, $\bar{v}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j) \equiv \bar{U}_j[\bar{q}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j), \varepsilon_j]$. These still have the properties mentioned in the previous section; in particular,

(20) $$\frac{\partial \bar{v}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j)}{\partial \mu_j} = \bar{q}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j) \cdot E\{u_j'\}$$

(21) $$\frac{\partial \bar{v}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j)}{\partial b_j} = -E\{u_j'\}$$

(22) $$\bar{q}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j) = -\frac{\partial \bar{v}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j)/\partial \mu}{\partial \bar{v}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j)/\partial b_j}\ .$$

The quantities $\bar{q}_j$ and $\bar{v}_j$ are known numbers to the producer but, because his decision is incompletely observed, they are random variables for the investigator.

Similarly, the unconditional discrete choice indices generated by the solution of (13), $d_j(\mu, w, b;\ s, \varepsilon)$, $j = 1, \ldots, N$, are random variables. Let $\bar{z}_i = \bar{v}_i - \bar{v}_j$, $i \neq j$ and let $F_{\bar{z}}(z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_N)$ be the joint c.d.f. of the $\bar{z}_i$'s. Then the mean of the expected value of the discrete choice indices, $E\{d_j\} \equiv P^j$, is

(23) $$P^j(\mu, w, b;\ s) = \Pr\{\bar{v}_j(\mu_j, w_j, b_j;\ s_j, \varepsilon_j) \geq \bar{v}_i(\mu_i, w_i, b_i;\ s_i, \varepsilon_i), \text{ all } i\}$$
$$= F_{\bar{z}}(0, \ldots, 0).$$

The unconditional supply functions generated by (13) denoted $q_j(\mu, w, b; s, \varepsilon)$, $j = 1, \ldots, N$, are also random variables, as is the unconditional indirect expected utility-of-profit function obtained by substituting these unconditional supply functions and the discrete choice functions into the maximand in (13); this will be denoted $v(\mu, w, b; s, \varepsilon)$. These unconditional functions are related to the conditional functions by formulas similar to those for the deterministic production model:

$$(24) \qquad q_j(\mu, w, b; s, \varepsilon) = d_j(\mu, w, b; s, \varepsilon)\bar{q}_j(\mu_j, w_j, b_j; s_j, \varepsilon_j)$$

$$(25) \qquad v(\mu, w, b; s, \varepsilon) = \max\{ \bar{v}_1(\mu_1, w_1, b_1; s_1, \varepsilon_1), \ldots, \bar{v}_N(\mu_N, w_N, b_N; s_N, \varepsilon_N)\}.$$

In order to construct the probability distributions of these random variables, we introduce the sets $A_j = \{\varepsilon | \bar{v}_j(\mu_j, w_j, b_j; s_j, \varepsilon_j) \geq \bar{v}_i(\mu_i, w_i, b_i, s_i, \varepsilon_i), \text{ all } i\}$, $j = 1, \ldots, N$. Let $f_{\varepsilon|\varepsilon\in A_j}$ be the conditional joint density of $\varepsilon_1, \ldots, \varepsilon_N$ given that $\varepsilon \in A_j$; i.e., given that discrete alternative j is selected. Then the probability density of $\bar{q}_j$, i.e., the ~~conditional probability $\Pr\{q_j = q | q_j > 0\}$~~, denoted $f_{q_j|q_j>0}(q)$, can be obtained by a change of variable from $f_{\varepsilon|\varepsilon\in A_j}$ using (22). The probability density of q_j, i.e., the unconditional probability $\Pr\{q_j = q\}$, denoted $f_{q_j}(q)$, therefore has the form

$$(26) \qquad f_{q_j}(q) = \begin{cases} 1 - P^j & q = 0 \\ f_{q_j|q_j>0}(q)\cdot P^j & q > 0 . \end{cases}$$

Thus, given a sample of T producers, where j^* is the index of the discrete choice selected by the t^{th} producer and q^*_t is his observed supply, the likelihood function of the sample is, from (26)

$$(27) \qquad L = \prod_{t=1}^{T} \{P_t^{j*} \cdot f_{q_{j*t}|q_{j*t}>0}(q_t^*)\}.$$

This completes our account of the general structure of random supply discrete/continuous choice models. The crucial ingredients in these models are the conditional indirect expected utility-of-profit functions, $\bar{v}_j(\mu_j, w_j,$

$b_j; s_j, \varepsilon_j)$, $j = 1, \ldots, N$, and the joint density $f_\varepsilon(\cdot)$. With these one can construct the densities $f_{\tilde{z}}(\cdot)$, $f_{\varepsilon|\varepsilon \in A_j}(\cdot)$, which are used to form the discrete choice probabilities and the conditional and unconditional densities of the q_j's. As noted above, different random supply models can be generated by allowing the ε_j's to enter the conditional indirect expected utility-of-profit functions in different ways or by making different assumptions about their joint distribution, but these models will all conform to the general structure outlined above.

4. Estimation

First we must draw attention to an alternative way of representing the unconditional supply functions, besides (24). Purely for notational convenience we consider the case where $N = 2$. The unconditional supply functions may be written

$$(28) \qquad q = \begin{cases} -\dfrac{\partial \bar{v}_1(\mu_1, w_1, b_1; s_1, \varepsilon_1)/\partial \mu_1}{\partial \bar{v}_1(\mu_1, w_1, b_1; s_1, \varepsilon_1)/\partial b_1}, & \text{if } \bar{v}_1(\mu_1, w_1, b_1; s_1, \varepsilon_1) \geq \bar{v}_2(\mu_2, w_2, b_2; s_2, \varepsilon_2) \\ -\dfrac{\partial \bar{v}_2(\mu_2, w_2, b_2; s_2, \varepsilon_2)/\partial \mu_2}{\partial \bar{v}_2(\mu_2, w_2, b_2; s_2, \varepsilon_2)/\partial b_2} & \text{otherwise}. \end{cases}$$

Since the $\bar{v}_j$'s are functions of several variables — μ_j, w_j, b_j, s_j, etc. — it is convenient at this point to refer explicitly to the coefficients of these variables, which we denote by the vector β. Therefore, we now write the conditional indirect expected utility-of-profit functions as $\bar{v}_j(\mu_j, w_j, b_j; s_j, \varepsilon_j, \beta)$. Then (28) can be written symbolically as

$$(29) \qquad q = \begin{cases} g_1(\mu_1, w_1, b_1; s_1, \varepsilon_1, \beta) & \text{if } h(\mu_1, \mu_2, w_1, w_2, b_1, b_2; s_1, s_2, \varepsilon_1, \varepsilon_2, \beta) \geq 0 \\ g_2(\mu_2, w_2, b_2; s_2, \varepsilon_2, \beta) & \text{otherwise} \end{cases}$$

where $g_1(\cdot)$ and $g_2(\cdot)$ are the ratios of the derivatives of $\bar{v}_1(\cdot)$ and $\bar{v}_2(\cdot)$, and $h(\cdot) \equiv \bar{v}_1(\cdot) - \bar{v}_2(\cdot)$; these functions will be linear or nonlinear in β depending upon the underlying structure of the $\bar{v}_j(\cdot)$ functions.

The purpose of the formulation (29) is to demonstrate how our theoretical random supply model generates a statistical switching regression model. The general (binary) single-equation switching regression model can be written in the form

$$(30)\qquad Y = \begin{cases} G_1(X_1;\, \beta_1,\, \xi_1) & \text{if } H(Z;\, \gamma,\, \eta) \geq 0 \\ G_2(X_2;\, \beta_2,\, \xi_2) & \text{otherwise} \end{cases}$$

where Y is the dependent variable, X_1, X_2 and Z are exogenous variables, β_1, β_2, and γ are the coefficients to be esimated, and ξ_1, ξ_2, and η are random error terms. Our supply model (29) is clearly a special case of (30) where, because the discrete and continuous choices both flow from the same underlying expected utility-of-profit maximization problem, the variables X_1 and X_2 are known transformations of the variables in Z, the coefficients β_1 and β_2 are the same as the coefficients γ, and the random terms ξ_1 and ξ_2 are directly related to the random term η. We can therefore estimate the random supply model (29) by any of the techniques developed for the switching regression model (30) while taking advantage of the special structure of our model. This is discussed by Hanemann and Tsur.

Example.

Our starting point is the deterministic discrete/continuous supply (18) and (19). To allow for the unobservable elements which are treated by the econometric investigator as random variables, we might in general write

$$(31a)\qquad c_j(w_j;\, \varepsilon_j^c) = \hat{c}_j(w_j) + \varepsilon_j^c$$

$$(31b)\qquad \sigma_{jp}^2(\varepsilon_j^p) = \hat{\sigma}_{jp}^2 + \varepsilon_j^p$$

$$(31c)\qquad \alpha_j(s_j;\, \varepsilon_j^u) = \hat{\alpha}_j(s_j) + \varepsilon_j^u$$

where "^" signifies the nonstochastic variables or functions observed by the investigator. Substitution of (31) into (18) and (19) yields

$$(32)\qquad \bar{q}_j(\mu_j,\, w_j,\, b_j;\, s_j,\, \varepsilon_j) = \frac{\mu_j - \hat{c}_j(w_j) - \varepsilon_j^c}{\hat{\sigma}_{jp}^2 + \varepsilon_j^p} \cdot \frac{1}{\hat{\alpha}_j(s_j) + \varepsilon_j^u}$$

$$(33)\qquad \bar{v}_j(\mu_j, w_j, b_j;\, s_j,\, \varepsilon_j) = 1 - \exp\{\hat{\alpha}_j(s)b_j + \varepsilon_j^u b_j - \frac{[\mu_j - \hat{c}_j(w_j) - \varepsilon_j^c\,]^2}{2\hat{\sigma}_{jp} + 2\varepsilon_j^p}\}.$$

The model is closed by specifying a joint distribution for $\varepsilon_1^c, \ldots, \varepsilon_N^c, \varepsilon_1^p, \ldots, \varepsilon_N^p, \varepsilon_1^u, \ldots, \varepsilon_N^u$.

We will work through these steps for a simplified version of this model in which the random terms ε_j^c and ε_j^p are omitted, leaving only the random term ε_j^u. That is, we assume that the random supply model arises from unobservable variation in the producer's risk preferences. Dropping the "u" superscript, we rewrite (31c) as $\alpha_j(s_j; \varepsilon_j) = S_j\beta_j + \varepsilon_j$, where S_j is a row vector of K observed exogenous variables representing attributes of the individual producer or the discrete alternative which influence his degree of risk aversion, and β_j is the associated (K×1) vector of coefficients to be estimated — for the sake of generality we allow both S and β to vary with the discrete choice, j. We assume that $\varepsilon_1 \varepsilon_2$ have a bivariate normal distribution with mean zero and some covariance matrix Σ. Our formulation thus allows for the possibility that $\text{cov}(\varepsilon_1, \varepsilon_2) \neq 0$. The correlation of the random terms across the discrete choices could be generated by assuming that the coefficients $\beta_{j1}, \ldots, \beta_{jK}$ are themselves random, in the manner of Hausman and Wise . More generally, it could arise because the same extraneous unobserved factors influence the producer's risk aversion in a similar way across different discrete choices. Finally, to simplify the model further, we assume that the investigator observes the unit costs, c_j, and so does not have to estimate the cost functions, $c_j(w_j)$. Thus, for each given producer, the observed variables are c_1, c_2, μ_1, μ_2, σ_{1p}^2, σ_{2p}^2, b_1, b_2, S_1, and S_2, as well as the producer's actual supply decision — both his discrete choice and his continuous choice. The unknowns are β_1, β_2, and the elements of Σ.

Accordingly, for a given producer the model can be written as:

$$q_1 = \left(\frac{\mu_1 - c_1}{\sigma_{1p}^2}\right)\frac{1}{S_1\beta_1 + \varepsilon_1} \quad \text{if } I' \geq 0$$

$$q_2 = \left(\frac{\mu_2 - c_2}{\sigma_{2p}^2}\right)\frac{1}{S_2\beta_2 + \varepsilon_2} \quad \text{if } I' < 0$$

$$I' = \left[1 - \exp\left\{b_1 S_1\beta_1 + b_1\varepsilon_1 - \frac{(\mu_1 - c_1)^2}{2\sigma_{1p}^2}\right\}\right] - \left[1 - \exp\left\{b_2 S_2\beta_2 + b_2\varepsilon_2 - \frac{(\mu_2 - c_2)^2}{2\sigma_{2p}^2}\right\}\right],$$

Define $Y_j \equiv (\mu_j - c_j)/q_j\sigma_j^2$, $j = 1, 2$. An equivalent way of formulating the model is

(34a) $$Y_1 = S_1\beta_1 + \varepsilon_1 \quad \text{if } I \geq 0$$

(34b) $$Y_2 = S_2\beta_2 + \varepsilon_2 \quad \text{if } I < 0$$

(34c) $$I = b_2S_2\beta_2 - b_1S_1\beta_1 + \frac{(\mu_1 - c_1)^2}{2\sigma_{1p}^2} - \frac{(\mu_2 - c_2)^2}{2\sigma_{2p}^2} + b_2\varepsilon_2 - b_1\varepsilon_1 - b_1\varepsilon_1,$$

which is a standard switching regression model.

FOOTNOTES

1. This assumption of output price uncertainty can also be extended to include the notion of yield uncertainty: interpret q as the ex-ante anticipated output and p as the "effective price"- i.e., actual price times the ratio of actual to anticipated output. It is necessary under this interpretation to assume that variable production costs, c(·), depend on planned output rather than actual output, which is not unreasonable.

2. In all these examples we can actually assume intra-agent as well as inter-agent variability- i.e., although an individual's technology, information and preferences are fixed at the point of each decision, they may vary between decisions in a manner which is partly unobservable to the investigator and is taken by the investigator to be random.

REFERENCES

Duncan, Gregory M. "Formulation and Statistical Analysis of the Mixed, Continuous/Discrete Dependent Variable Model in Classical Production Theory," *Econometrica*, 48 (1980), 839-852.

Dubin, Jeffrey A., and Daniel McFadden, "An Econometric Analysis of Residential Electric Appliance Holdings and Consumption," Cambridge: Massachusetts Institute of Technology Department of Economics Working Paper, 1981.

Hanemann, W. Michael, "Discrete/Continuous Models of Consumer Choice," to appear in *Econometrica*

______, and Yacov Tsur, "Econometric Models of Discrete/Continuous Supply Decisions under Uncertainty," Berkeley: University of California, Berkeley Department of Agricultural & Resource Economics Working Paper No. 195, March 1982.

Hausman, Jerry A., and David A. Wise, "A Conditional Probit Model for Qualitative Choice: Discrete Decisions Recognizing Interdependence and Heterogeneous Preferences," *Econometrica*, 46 (1978), 403-426.

Heckman, James J., "Sample Selection Bias as a Specification Error," *Econometrica*, 47 (1979), 153-161.

Lee, Lung-Fei, and R.P. Trost, "Estimation of Some Limited Dependent Variable Models With Application to Housing Demand," *Journal of Econometrics*, 8 (1978), 357-382.

McFadden, D. "Econometric Net Supply Systems for Firms with Continuous and Discrete Commodities," Cambridge: Massachusetts Institute of Technology Department of Economics Working Paper, 1979.