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UNDER PRICE RISK AND CAPITAL CONSTRAINTS

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RISK AND CAPITAL CONSTRAINTS

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ABSTRACT

This paper investigates whether commodity-linked bonds can offer capital constrained producers an effective means of raising capital and hedging against output price risk. Producers are expected utility maximizers and optimal production and bond issues are characterized graphically in mean-standard deviation space. Results indicate that commodity-linked bonds have considerable potential for hedging price risks.

HEDGING WITH COMMODITY-LINKED BONDS UNDER PRICE RISK AND CAPITAL CONSTRAINTS

Risky commodity prices and lack of investment capital are two problems that plague developing country producers of primary products. A potential instrument for mitigating both of these problems is the commodity-linked bond. The issuer of a commodity bond receives a cash payment upon initial sale of the bond and, at maturity, pays the value of a specified quantity of a commodity to the bond holder. Dividend payments, if any, may also be linked to commodity values. The principal feature distinguishing commodity-linked bonds from conventional bonds is their commodity denominated return structure (Lessard and Williamson). Historical examples include silver bonds issued by The Sunshine Mining Company in 1980, gold bonds issued by the French government, and petroleum bonds issued by the government of Mexico.

This paper investigates whether commodity-linked bonds can offer capital constrained producers an effective means of raising capital and hedging against output price risk. Two issues are examined. First, the optimal levels of commodity production and bond issue are determined for a risk averse producer who has no initial wealth, no access to futures markets, and no conventional source of investment funds. Second, the assumption of no initial wealth and no futures markets are maintained but producers are provided with the opportunity to take out conventional loans, as well as issue commodity-linked bonds. In this case, interest centers not only on the output and bond issue decisions but also on conditions under which issuing bonds or taking out conventional loans will be the dominant strategy for raising capital.

These issues are studied graphically in mean-standard deviation space. In a recent paper, Meyer has shown that expected utility maximization is equivalent to ranking alternatives based on their mean and standard deviation, provided a location and scale (LS) condition is satisfied. The LS condition is shown to hold in the case of the capital constrained commodity producer studied here. Thus, the graphical mean-standard deviation analysis is fully consistent with an expected utility model. The advantages of graphical analysis are that proofs are simplified and results are more intuitive. Meyer and Robison recently used the LS condition in a graphical analysis of futures market hedging under output price randomness.

Hedging With Commodity-Linked Bonds When Producers are Capital Constrained

Suppose a commodity producer faces a stochastic output price at the time resource allocation decisions are made. The producer has no initial wealth, no access to futures markets, and cannot obtain conventional loans. This might represent the situation of a developing country that is experiencing debt servicing problems and has therefore been cut off from new loans. The only means of raising revenue for purchasing inputs is for the producer to issue bonds linked to the price of the commodity being produced. The bonds mature at the time output is realized and require the producer to pay the bond holder an amount equal to the realized spot price of the commodity at maturity. The equilibrium bond price is determined in a competitive market and depends on the probability distribution of the future commodity price. For simplicity, it is assumed that the bonds have no coupon payments.

In this situation, the random profit of the producer can be expressed

$$(1) \qquad \tilde{\pi} = (1+r)[wb - c(q)] + \tilde{p}(q-b)$$

where: π = profit;

r = the interest rate;

w = the price of the bond when issued;

b = quantity of bonds issued;

q = quantity of the commodity produced;

$c(q)$ = a strictly convex cost of production function; and

p = the price of the commodity when output is realized.

The tilde denotes a random variable at the time bonds are issued and resources are allocated.

The producer's production and bond issue decision must also satisfy the constraints

(2) $wb - c(q) \geq 0$; and

(3) $b \geq 0$.

These constraints indicate that production costs cannot exceed the amount of revenue raised by issuing bonds, and that the producer cannot purchase (as opposed to issue) bonds. If the amount of revenue raised by issuing bonds exceeds production costs then the excess is invested by producers at a known interest rate r.

The producer is assumed to choose a bond issue and production level to maximize the expected value of an increasing and strictly concave Von Neumann-Morgenstern utility function defined over profits. Notice, however, that the random profit function (1) is linear in a single random variable, $\tilde{p}$. Thus, different (b,q) combinations lead to profit distributions that have the same shape, differing only by their position along the horizontal axis (location) and/or their tightness around the mean (scale). This means that the producer's decision satisfies Meyer's LS condition, which can be stated formally as follows.

<u>Definition</u> A decision problem satisfies the LS condition if every two cumulative distribution functions, $F_1(\pi)$ and $F_2(\pi)$, describing elements in the choice set satisfy $F_1(\pi) = F_2(\alpha + \beta\pi)$ for all π and for some α and some $\beta > 0$.

Meyer has shown that when the LS condition is satisfied, then maximization of expected utility is equivalent to maximizing a preference function $V(\mu,\sigma)$ of the mean and standard deviation of profits. Furthermore, for a risk averse producer $V(\mu,\sigma)$ is increasing in μ, decreasing in σ, and concave in μ and σ. Thus, the slope of a risk averse producer's indifference curves in mean-standard deviation space is always positive:

$$(4) \qquad S(\mu,\sigma) = -V_\sigma(\mu,\sigma)/V_\mu(\mu,\sigma) > 0.$$

The concavity of $V(\mu,\sigma)$ implies that indifference curves in mean-standard deviation space are convex to the origin.

The following important assumption is made about the equilibrium bond price.

<u>Assumption 1</u> Bond issuers must pay bond holders a risk premium to hold the bond; $\bar{p}/w > (1+r)$.

This says that the expected gross return on holding bonds is greater than the gross rate of return on a riskless asset. Since commodity-linked bonds are risky financial instruments, and the certain interest rate r is available to investors, this assumption simply states that there is the usual risk/return trade-off among assets.

Now consider the opportunity set of a capital constrained producer in mean-standard deviation space. The mean and standard deviation of profits are

(5) $\mu = (1+r)[wb - c(q)] + \bar{p}(q - b)$; and

(6) $\sigma = |q-b|\sigma_p$

where: $\bar{p}$ = expected output price; and

σ_p = standard deviation of the output price.

The shape of the producer's opportunity set in mean-standard deviation space is illustrated graphically in figure 1. The opportunity set can be derived in three steps using (5), (6), and assumption 1.

First, suppose that the quantity produced is fixed at some level and b is set equal to this production level, b=q. Substituting the equality into (5) and (6) gives $\mu = (1+r)[wq - c(q)]$ and $\sigma = 0$. This defines a point on the μ axis which is in the opportunity set (see figure 1).

Second, suppose that the quantity produced remains fixed at q and b is decreased below the point at which b=q. Then differentiating (5) and (6) gives

(7) $d\mu = [(1+r)w - \bar{p}]db$; and

(8) $\quad d\sigma = -\sigma_p db.$

Dividing these equations gives the slope of the opportunity set as b decreases:

(9) $\quad d\mu/d\sigma = [\bar{p} - (1+r)w]/\sigma_p.$

This part of the opportunity set is indicated by the positively sloped ray moving out from the μ axis in figure 1. Assumption 1 ensures that the slope is positive as b decreases. Nevertheless, b cannot fall too far below q because of the capital constraint. That is, the opportunity set becomes truncated at the point where the revenue raised by issuing bonds is just equal to the cost of producing the fixed output q. Any further reductions in b beyond this point are infeasible because then there would not be enough revenue available to purchase the inputs required to produce q. This truncation point is also illustrated on figure 1.

Third, suppose that the quantity produced remains fixed at q and b is increased above the point at which b=q. Then (8) becomes $d\sigma = \sigma_p dp$ and the slope of the opportunity set is now

(10) $\quad d\mu/d\sigma = -[\bar{p} - (1+r)w]/\sigma_p.$

This part of the opportunity set is indicated by the negatively sloped ray moving out from the μ axis in figure 1. Assumption 1 ensures that the slope of the opportunity set is negative as b increases. In this case, however, there is no truncation point because more and more revenue is being raised from bond issues.

The opportunity set shown in figure 1 is for changes in bond issues while quantity produced is kept constant. Notice, however, that changes in q simply move this opportunity set up and down the μ axis. Furthermore, since the producer prefers higher profit means to lower ones, and the slope of the opportunity set does not depend on the quantity produced, then the optimal q maximizes the intercept of the opportunity set on the μ axis. The optimal quantity produced therefore satisfies

$$(11) \quad c'(q) - w = 0.$$

Thus, the optimal production level depends only on marginal costs and the bond price. This <u>separation property</u> is a familiar result from the literature on futures market hedging where it has been found that the optimal quantity produced depends only on marginal costs and the futures price (Danthine, Holthausen, Meyer and Robison). Equation (11) shows that a similar result holds in the case of a capital constrained producer issuing commodity-linked bonds except that the bond price, not the futures price, is the action certainty equivalent price for the producer's output decision.

Having determined the optimal quantity produced, the next step is to characterise the optimal bond issue graphically in mean-standard deviation space. Three possible cases are illustrated in figure 1. In each case, the negatively sloped portion of the opportunity set is irrelevant since indifference curves are convex and positively sloped.

In panel (a) of figure 1 the optimum is defined by a tangency between the producer's indifference curve and the opportunity set. In this situation, the producer issues less bonds than the quantity being produced, $b<q$. However, the revenue raised by issuing bonds is greater than the cost of production. Bonds are issued in excess of the level needed to finance production because

they provide an output price hedge. Revenues generated in excess of production costs are invested at the known interest rate, r.

Panel (b) of figure 2 represents an optimum where the slope of the producer's indifference curve is greater than the slope of the opportunity set. This is a corner solution where the optimal bond issue equals the quantity produced, b=q, and the variance of profit is reduced to zero. It will occur when producers are very risk averse and want to completely eliminate all risk. Once again, the optimal hedge requires bond issues that raise revenue in excess of the amount required to finance production costs and the excess is invested at the interest rate r.

Finally, panel (c) of figure 2 illustrates a constrained optimum where the slope of the producer's indifference curve is less than the slope of the opportunity set. In this case the producer is not very risk averse and would like to issue less bonds for hedging purposes (remember that the producer must pay the bond holder a risk premium to invest in the bond). However, bonds sufficient to cover production costs must always be issued and so the optimum occurs at the truncation point on the upward-sloping portion of the opportunity set.

These results illustrate the effect of producer risk preferences on the optimal risk/return trade-off from issuing commodity-linked bonds. The risk premium on the bonds causes mean profit to fall whenever the producer issues more bonds. However, the principal payment on the bond is positively correlated with the commodity output price. Thus, the bonds provide a hedge against output price risk. If the producer is very risk averse, then there will be a complete hedge, b=q. If the producer is not very risk averse then the bond issue will only cover production costs. And if producer risk preferences lie between these two extremes, then the revenue raised by bond issues will be greater than that required to finance production costs, but not great enough to provide a complete hedge and eliminate all risk.

Hedging With Commodity-Linked Bonds and Conventional Loans

Suppose that the producer of the previous section now has access to conventional loans at the known interest rate, r. Everything else remains as before, including the existence of a market for commodity-linked bonds. Given perfect capital markets, the availability of conventional loans does not change the profit function (1). What does change is the capital constraint (2). Since any amount can be borrowed or lent at the interest rate r, the producer is no longer constrained to issue enough bonds to cover production costs--the money can always be borrowed instead.

The effects that conventional loans have on optimal production and bond issue decisions are easy to derive graphically. To begin, consider the shape of the producer's opportunity set when conventional loans are available. Given some fixed level of output, q, the opportunity set for changes in b is almost identical to the previous case without conventional loans. The only difference is that the positively sloped ray is no longer truncated at the point where revenue from bond sales equals production costs. Since production costs can now be financed by conventional loans as well as bonds, bond issues can feasibly be reduced all the way to zero. However, the non-negativity constraint (3) continues to hold so that truncation now occurs at b=0. This opportunity set is illustrated in figure 2.

Optimal output and bond issue decisions are characterized by considering preference maximization subject to remaining in this opportunity set. Three different situations are illustrated in figure 2. In panel (a), the optimum is defined by a tangency between the producer's indifference curve and the opportunity set. This may occur where revenues raised from bond issues are greater than, less than, or equal to production costs. At this solution, bond sales are strictly positive and the optimal output level satisfies (11). The producer is risk averse enough to hedge by issuing bonds but not risk averse enough to eliminate all risk by setting b=q. Panel (b) of figure 4 illustrates the corner solution when the producer is very risk averse and does issue enough bonds to reduce the profit variance to zero.

Panel (c) shows the interesting case in which conventional loans dominate commodity-linked bonds, b=0. This occurs when the slope of the indifference curve is smaller than the slope of the opportunity set at the optimum:

$$(12) \quad S(\mu,\sigma) < [\bar{p} - (1+r)w]/\sigma_p.$$

Equation (12) has an intuitive economic interpretation. The slope of the indifference curve represents the "cost" of producing unhedged output and bearing the full risk of output price uncertainty. The slope of the opportunity set represents the "cost" of the risk premium that producers must pay to bond holders in order to facilitate a transfer of risk. If this "cost" of producing unhedged output is less than this "cost" of paying the risk premium, then all production costs are financed with conventional loans and no bonds are issued. The less risk averse are producers, the more likely that conventional loans will dominate commodity-linked bonds

The final task is to determine the optimal output level when b=0. If no bonds are issued then output is completely unhedged. Thus, at an optimum q must satisfy

$$(13) \quad S(\mu,\sigma) = [\bar{p} - (1+r)c'(q)]/\sigma_p.$$

The slope of the producer's indifference curve in mean-standard deviation space equals the slope of an opportunity set defined by variations in q with bond issues fixed at $b=0$.

Conclusion

This paper investigated the behavior of capital constrained commodity producers managing output price risk with commodity-linked bonds. The study was motivated by the problems of heavily indebted developing countries that have exhausted conventional sources of credit and face commodity price risks. Futures markets are not available because many commodities produced by developing countries do not have futures markets, and those that do exist are typically located in major international financial centers where developing countries may face substantial basis risk.

Results of the investigation indicate that commodity-linked bonds could have an important role to play in hedging commodity price risks. If producers are highly risk averse, and the risk premium in the bond price is "not too high," then the optimal bond issue will equal the quantity produced and the producer will be fully hedged. As producers get less risk averse and the risk premium on the bonds gets bigger, the optimal bond issue declines. If the risk premium is high enough, and producers are "not too risk averse," then no bonds will be issued provided conventional loans are available. If conventional loans are not available, only enough bonds to cover production costs will be issued.

These results were derived using a graphical mean-standard deviation approach that is fully consistent with expected utility maximization. The graphical approach is more intuitive and leads to simple proofs for the various results.

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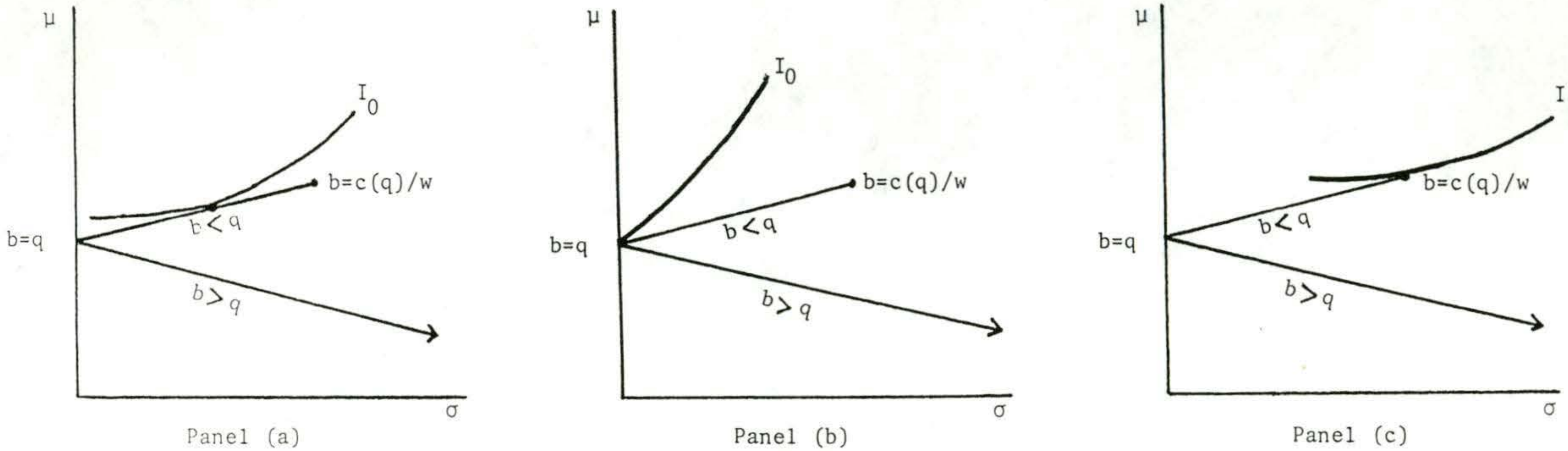

Figure 1. Optimal Bond Issues Under Capital Constraints

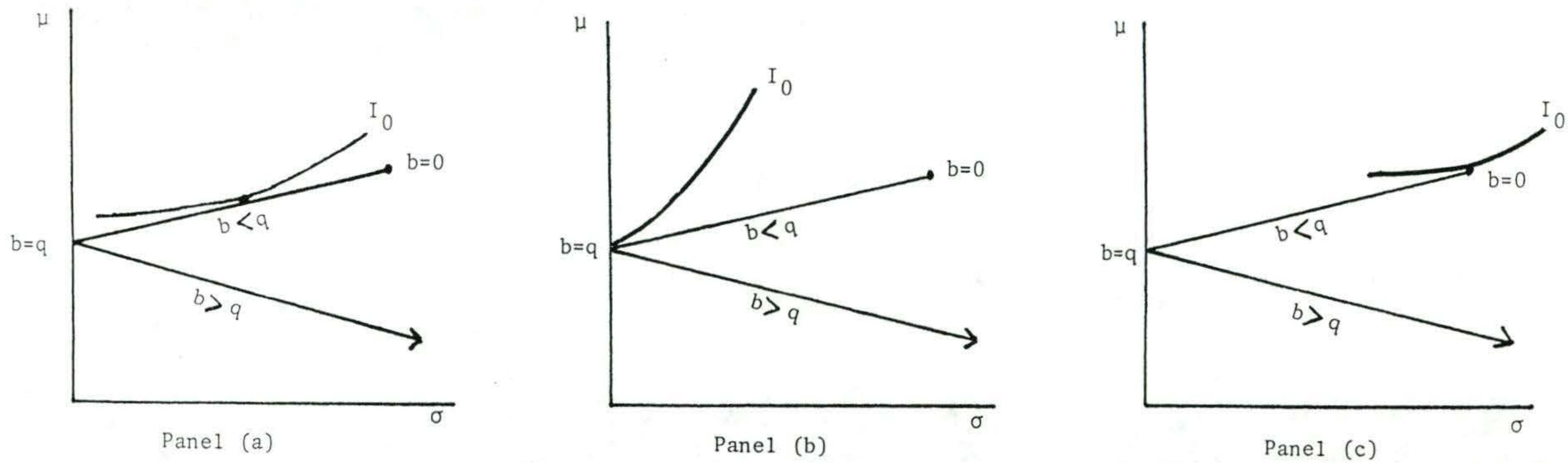

Figure 2. Optimal Bond Issues With Conventional Loans