Staff Paper

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Much of the spectacular growth in external borrowing by developing countries that occurred during the 1970s was in the form of general obligation loans denominated in U.S. dollars at floating interest rates. It is now well understood that this strategy involved substantial risks. An unexpected deterioration in a country's terms of trade can quickly erode its ability to service large debts. In turn, this may induce restricted access to new external credit and a period of forced adjustment in domestic consumption and investment. These risks have become all too clear following the developing country debt crisis that began in 1982. A disturbing number of heavily indebted countries have not yet emerged from the resulting difficulties.

This paper investigates how commodity-linked bonds could be used by developing countries to hedge the risks associated with their external debt position. Commodity-linked bonds are bonds that have principal, and possibly coupon payments, linked to future realizations of a specified set of commodity prices. By issuing bonds linked to the prices of commodities that they export, developing countries would be hedging against an unexpected deterioration in export earnings. A major factor contributing to the debt crisis was a decline in earnings from commodity exports, together with a simultaneous increase in debt service obligations. If debt had been issued in the form of comodity-linked bonds, debt service obligations would have fallen along with commodity prices, thus easing the adjustment burden. Of course, other commodity-linked financial instruments, such as futures and options contracts could be used for similar hedging purposes. However, futures and options do not exist for all commodities and those that do exist typically have only short maturities. Thus, for many developing countries, commodity-linked bonds show considerable potential as a financial risk management instrument.

The characteristics of alternative international financial instruments, including commodity-linked bonds, have been discussed extensively elsewhere (e.g. Lessard, Lessard and Williamson, O'Hara). The specific purpose here is to provide an operational rule for choosing an optimal external debt portfolio consisting of commodity-linked bonds and conventional debt. To begin, a simple dynamic model is used to derive optimal rules for issuing commodity-linked bonds and conventional debt in a small open economy. Next, estimation methods which allow these rules to be operationalized are presented. The approach is then illustrated with an application to Costa Rica, where it is found that the optimal external debt portfolio contains a significant proportion of commodity-linked bonds.

A Model of Optimal External Debt Allocation

Consider a small open economy in which all external debt is issued by the government. The government has a utility function, $u(m_t)$, defined over real imports of goods and services per capita in period t. This utility function is meant to capture the contribution that imports make to domestic consumption and growth. It satisfies the Von Neumann-Morgenstern axioms, as well as the conditions $u'(m_t) > 0$ and $u''(m_t) < 0$. Commodity exports by the country are assumed to follow an exogenous stochastic process that is not influenced by the government's external debt decisions. Real exports per capita in period t are denoted x_t.

Without external finance, the value of imports must equal the value of exports so that the current account is in balance each period. However, we assume that the government has access to two sources of external finance.

First, it can take out conventional loans at the constant real interest rate r. Real total debt per capita held in the form of conventional loans at the end of period t is denoted d_t. Second, the government can issue bonds linked to each of n commodities. When issued, these bonds have real prices $w_t = (w_{1t}, w_{2t}, \ldots, w_{nt})$ and the real prices of the underlying commodities are denoted $p_t = (p_{1t}, p_{2t}, \ldots, p_{nt})$. Future commodity prices are stochastic when the government issues the bonds. The bonds mature in one period and require a financial payment at maturity that is equal to the price of the underlying commodity[1]. To simplify the analysis, we assume no coupon payments. The per capita quantity of bonds issued by the government at time t is denoted $b_t = (b_{1t}, b_{2t}, \ldots, b_{nt})'$.

With these assumptions, the constraint facing the government when it chooses an external debt portfolio is

$$(1) \qquad m_t + rd_{t-1} + p_t b_{t-1} \le x_t + (d_t - d_{t-1}) + w_t b_t.$$

The government is also restricted in that it cannot borrow indefinitely to finance ever increasing current account deficits. This constraint is imposed by requiring

$$(2) \qquad \lim_{T\to\infty} (1+r)^{-T} d_T = \lim_{T\to\infty} (1+r)^{-T} w_T b_T = 0.$$

The government's problem is now to choose a portfolio of conventional debt and commodity-linked bonds that maximize the discounted time-additive expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(m_t)$$

subject to the sequence of constraints (1) and the transversality conditions (2).

The solution to the government's problem must satisfy (1) and (2), plus the Euler equations

$$(3) \quad u'(m_t) - \beta(1+r)E_t u'(m_{t+1}) = 0$$

$$(4) \quad u'(m_t)w_t - \beta E_t[u'(m_{t+1})p_{t+1}] = 0.$$

Finding a closed form solution is generally impossible without placing strong restrictions on the form of the utility function, and on the probability distribution of prices and exports. Here, however, we follow the literature on the permanent income theory of consumption and assume that the optimal import path can be defined[2]

$$(5) \quad m_t = \alpha[\sum_{i=0}^{\infty} (1+r)^{-i}E_t(x_{t+i}) - p_t b_{t-1} - (1+r)d_{t-1}].$$

Notice that this is just a version of the permanent income theory of consumption—imports are set equal to some proportion, α, of "permanent" exports (a discounted sum of expected future export revenues, minus current external debt).

Equation (5) is not yet a decision rule because the terms $E_t(x_{t+i})$ must be eliminated by expressing them as a function of variables known by the government at time t. Suppose that x_t is the first element of a vector, $y_t = (x_t, p_{1t}, p_{2t}, \ldots, p_{nt}, s_t)'$, that also contains the commodity prices and any other state variables, s_t, useful for predicting future exports. The vector y_t is assumed to follow the autoregressive process

(6) $A(L)y_t = \varepsilon_t$

where $A(L) = I-A_1L-A_2L^2-\ldots-A_qL^q$ is a matrix polynomial in the lag operator and ε_t is a zero mean serially uncorrelated error vector with covariance matrix Ω. In the next section, we show that optimal projections of $E_t(x_{t+i})$, given the autoregressive process (6) and information available at time t, satisfy

(7) $$\sum_{i=0}^{\infty}(1+r)^{-i}E_t(x_{t+i}) = \gamma y_t + B(L)y_{t-1}$$

for some parameter vector γ, and some matrix polynomial in the lag operator $B(L)$. Both γ and $B(L)$ are defined explicitly below in the estimation section.

Substituting (7) into (5) gives the operational decision rule

(8) $$m_t = \alpha[\gamma y_t + B(L)y_{t-1} - p_t b_{t-1} - (1+r)d_{t-1}].$$

However, while this rule defines the optimal level of imports, and thus the optimal level of <u>total</u> external debt, it does not provide the optimal portfolio of commodity-linked bonds. To derive the optimal bond portfolio, notice that the Euler equations (3) and (4) can be substituted and rearranged to give

$$E_t\{u'(m_{t+1})[w_t - p_{t+1}/(1+r)]\} = 0.$$

Next, take a linear approximation of $u'(m_{t+1})$ at m_t so that this equation becomes

(9) $E_t\{m_{t+1}[w_t - p_{t+1}/(1+r)]\} = m_t E_t[w_t - p_{t+1}/(1+r)]$.

Assuming that the expected real return on holding bonds is equal to the real interest rate, $E_t[w_t - p_{t+1}/(1+r)] = 0$, then (9) implies that the conditional covariance between m_{t+1} and p_{t+1} must be zero at an optimum.[3] Leading (8) one period and computing the relevant covariances shows that the optimal commodity-linked bond portfolio satisfies

(10) $\Omega_{py}\gamma' - \Omega_{pp}b_t = 0$

where Ω_{py} is a matrix of conditional covariances between p_{t+1} and y_{t+1}, and Ω_{pp} is the conditional covariance matrix for p_{t+1}. Solving for b_t gives

(11) $b_t = \Omega_{pp}^{-1}\,\Omega_{py}\gamma'$.

A method for estimating (11) is provided in the next section. The per capita amount of total finance raised at time t from issuing an optimal portfolio of commodity-linked bonds can be obtained by premultiplying (11) by the bond prices, w_t. It is interesting to note that (11) is precisely the solution that would be obtained from a two-period mean-variance formulation of the optimal portfolio problem, except that γ would have a one in the first column and zeros elsewhere. In the infinite horizon problem, expectations concerning export performance at <u>all</u> future dates are important and the optimal portfolio must be weighted accordingly.

Estimation

Estimation of the optimal commodity-linked bond portfolio revolves around the vector autoregressive process $A(L)y_t = \varepsilon_t$, which was defined in the previous section. Remembering that y_t contains all of the commodity prices that are linked to bonds (as well as x_t and other state variables helpful in predicting future export revenues), then the conditional covariance matrices Ω_{py} and Ω_{pp} in (11) are clearly just components of Ω, the covariance matrix of ε_t. Thus, estimation of the vector autoregressive process for y_t provides an estimate of Ω which, in turn, can be used directly to operationalize (11).

Estimation of the vector autoregressive process for y_t is also useful for another reason. In order to compute optimal bond issues from (11), we need to know the parameter vector γ. From (7), recall that γ represents coefficients on y_t in the optimal prediction of a discounted sum of future realizations of export revenues, given current and past values of y_t. From a formula derived by Hansen and Sargent, this optimal predictor is

$$(12) \quad \sum_{i=0}^{\infty} (1+r)^{-i}E_t(x_{t+i}) = \phi A[1/(1+r)]^{-1}y_t + B(L)y_{t-2}$$

where ϕ is a row vector with a one in the first column and zeros elsewhere, and B(L) satisfies

$$B(L) = \phi A[1/(1+r)]^{-1}\{\sum_{j=1}^{q-1}[\sum_{k=j+1}^{q}(1+r)^{j-k}A_k]L^{j+1}\}.$$

Thus, γ is simply the first row of $A[1/(1+r)]^{-1}$. Evidently, estimation of the autoregressive parameters in A(L) provides a direct estimate of γ (conditional on knowledge of the real interest rate r). Actual estimation of the parameters in A(L) and Ω can be accomplished via vector autoregression[4].

The final piece of the estimation puzzle lies in obtaining an estimate of the real interest rate, r. Once r has been found, and A(L) and Ω have been estimated, then computing the optimal commodity-linked bond issues is straightforward using (11). In many cases, prior information will be available on real interest rates. In the following application to Costa Rica, we present optimal external debt portfolios for a number of different real interest rates and find that the results are not sensitive to the size of this parameter.

The Case of Costa Rica

Costa Rica is a small country that depends on a handful of agricultural commodities for the bulk of its export earnings. In recent years, coffee, beef, and bananas have accounted for over half of total export revenues. Figure 1 shows gross national product, consumption, and investment for Costa Rica between 1966 and 1986, all in real per capita terms. The pronounced slump which began around 1981 and continued into 1983 is indicative of the problems that many developing countries have experienced since the onset of the debt crisis. Figure 2 shows Costa Rica's terms of trade index and total foreign debt in U.S. dollars. The data suggest that the economic slump of 1981-83 was preceeded by a sharp negative terms of trade shock and a dramatic increase in debt servicing requirements. By linking debt service to commodity price realizations, commodity-linked bonds might help facilitate adjustment and avoid future slumps of this severity.

To illustrate the estimation of optimal external debt portfolios, we examined a Costa Rican portfolio of conventional loans and bonds linked to three major export commodities; coffee, beef, and bananas. Real prices of these commodities are denoted p_t^c, p_t^b, and p_t^a respectively. The first task was to estimate the vector autoregression for real exports and the three real commodity prices. To simplify, we did not examine possibilities for including other variables in the model. Nominal commodity prices and nominal per capita export revenues, all in U.S. dollars, were each deflated by an index of (dollar) import prices for Costa Rica. The commodity prices were obtained from World Bank (1986) and all other data are from World Bank (1987). The data are annual and the sample runs from 1966 through 1985.

In view of the small number of observations, an equation by equation approach to model specification was used. Preliminary investigations revealed no strong evidence of nonstationarity or heteroscedastic errors so the models were estimated in the levels of each variable, assuming a constant conditional covariance matrix. We initially specified an overfitted equation with lags of all four variables included. Then F-tests were used to test zero restrictions on sets of coefficients. Estimation results for the final model specification are shown in table 1, where the system was estimated using seemingly unrelated regression. Table 1 also contains the resulting estimated conditional covariance matrix.

The optimal external debt portfolio was computed for 1985, the last year of the sample. The matrices Ω_{py} and Ω_{pp} come directly from $\hat{\Omega}$ in table 1 and the parameter vector γ is computed from the autoregression coefficients in table 1 (as shown above). Each optimal commodity-linked bond issue was multiplied by an estimate of its real price. This estimate was obtained by using the vector autoregression to forecast real commodity prices into 1986, given information available in 1985, and then discounting back using the real interest rate. The final bond revenue figure is then expressed as a proportion of the actual level of total external debt in Costa Rica in 1985.

The estimated optimal external debt portfolio is presented in table 3 under three different real interest rate assumptions. Clearly, the portfolios are not very sensitive to the real interest rate used. The results suggest that over 30% of total debt should be issued in the form of commodity-linked bonds, with the bulk of these issues being split between coffee and bananas. The optimal portfolio of external debt for Costa Rica in 1985 therefore appears to contain a significant proportion of commodity-linked bonds.

Concluding Comments

This paper provides a simple dynamic model that can be used to estimate optimal portfolios of external debt. We focused on the potential role of commodity-linked bonds in hedging against terms of trade shocks. The approach was applied to Costa Rica where it was found that a significant proportion of external debt should be issued in the form of commodity-linked bonds.

The framework could be extended in a variety of directions. In particular, while optimal portfolios of external debt have been computed we have not yet determined the extent of resulting reductions in the variance of real imports. This information is critical in determining the hedging effectiveness of commodity-linked bonds. Future research might also examine expanded portfolios, perhaps looking at other commodity-linked instruments, such as futures, options, and bonds linked to indexes of commodity prices.

There are also a number of practical difficulties that deserve additional attention. In this paper, we have simply assumed that markets for commodity-linked bonds exist, and that such markets have no risk premia. However, it seems likely that commodity-linked bonds would be priced at a discount, especially if issued by developing countries subject to significant default risk. In fact, the size of risk premia may be an important reason why there is currently such little use made of commodity-linked bonds. Nevertheless, the analysis presented above suggests a potential hedging role for commodity-linked bonds, provided that diversified markets for these contracts can emerge and grow.

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FOOTNOTES

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1. For simplicity, attention is restricted to bonds with a one-period maturity. However, an extension to longer-term maturities would be relatively straightforward.
2. Assumptions sufficient to guarantee this equation is an exact solution are: (a) the expected real return to holding bonds is equal to the real interest rate $E_t[w_{t+i} - p_{t+1+i}/(1+r)] = 0$ for $i=0,1,\ldots$; and <u>either</u> (b) utility is quadratic; <u>or</u> (c) utility features constant absolute risk aversion and m_t is normally distributed with a variance that depends only on i; <u>or</u> (d) utility features constant relative risk aversion and log m_t is normally distributed with a variance that depends only on i. See Evans and the references therein for more details on the latter three conditions. A complete derivation of equation (5) under these conditions is available from the authors on request.
3. This implies that there are no risk premia in commodity-linked bond prices. If investors are risk averse and cannot diversify all of the risks of investing in the bonds, then the bonds may be priced at a discount to conventional debt (Schwartz).
4. No discussion of vector autoregression estimation techniques is included here. Those interested should consult Engle and Bollerslev, Engle and Granger, Sims, and others.

Table 1
Estimation Results[a]

$$x_t = \underset{(3.78)}{219.55} + \underset{(4.14)}{0.63}\, x_{t-1} - \underset{(0.78)}{0.11}\, x_{t-2}; \qquad \bar{R}^2 = 0.36$$

$$p_t^c = \underset{(3.92)}{2.82} + \underset{(2.63)}{0.42}\, p_{t-1}^c - \underset{(0.65)}{0.10}\, p_{t-2}^c; \; ; \qquad \bar{R}^2 = 0.23$$

$$p_t^b = \underset{(1.13)}{0.61} + \underset{(3.87)}{0.84}\, p_{t-1}^b - \underset{(0.22)}{0.05}\, p_{t-2}^b; \qquad \bar{R}^2 = 0.57$$

$$p_t^a = \underset{(1.12)}{0.05} + \underset{(3.11)}{0.48}\, p_{t-1}^a + \underset{(3.75)}{0.65}\, p_{t-2}^a - \underset{(4.14)}{0.04}\, p_{t-1}^b; \qquad \bar{R}^2 = 0.58$$

$$\hat{\Omega} = \begin{bmatrix} 2,238.4 & 39.2 & 9.9 & 0.9 \\ 39.2 & 0.99 & 0.15 & 0.02 \\ 9.9 & 0.15 & 0.44 & -0.081 \\ 0.9 & 0.02 & -0.081 & 0.001 \end{bmatrix}$$

[a]Values in parentheses are t-values.

Table 2
Optimal Portfolios in 1985 as a Proportion of Total External Debt

r	General Obligation Loans	Coffee	Beef	Bananas
0.00	.655	0.144	0.035	0.166
0.05	.652	0.145	0.036	0.167
0.10	.649	0.147	0.036	0.168

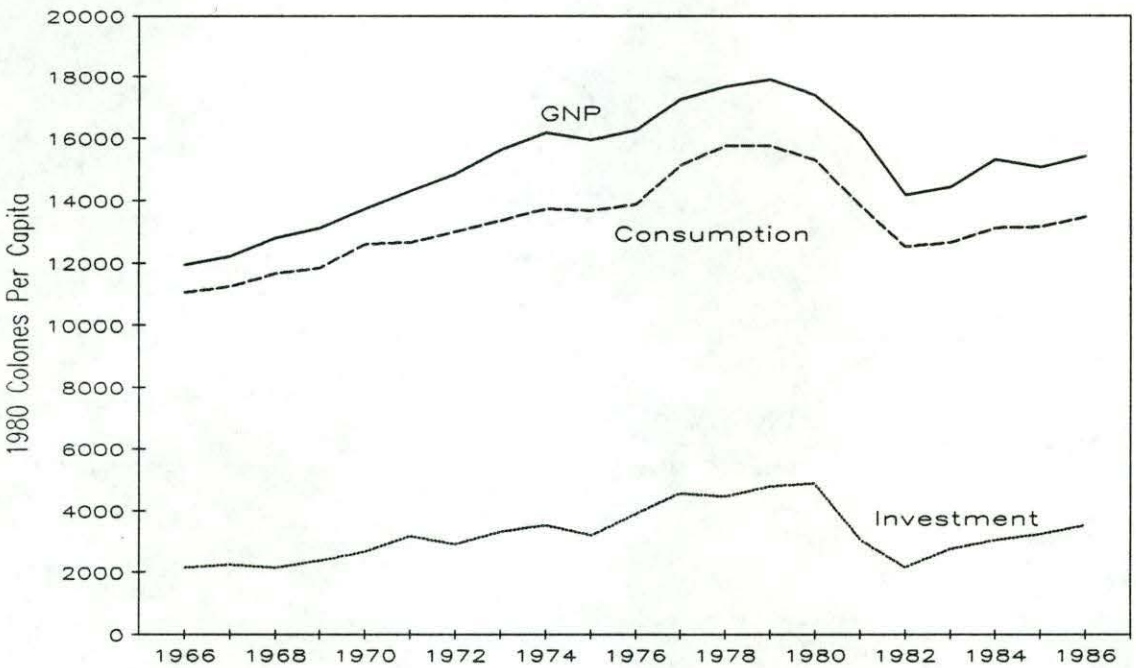

Figure 1. Real Per Capita Gross National Product, Consumption and Investment for Costa Rica

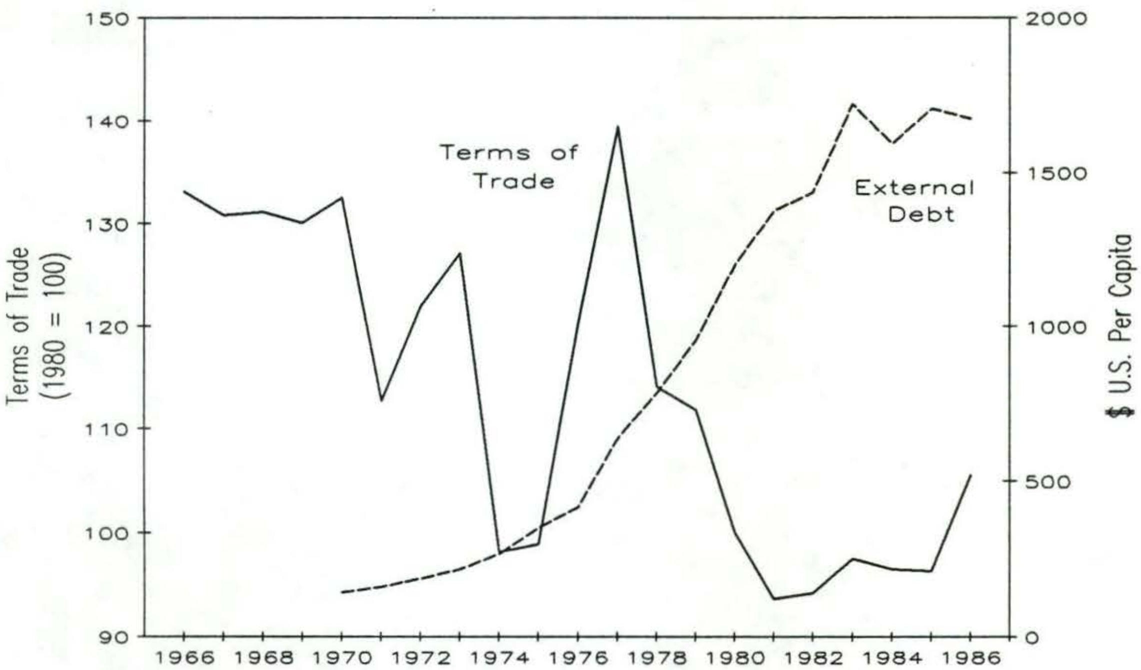

Figure 2. Terms of Trade and Per Capita Total External Debt for Costa Rica