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IMPLEMENTING STOCHASTIC FLEXIBLE FORM ESTIMATION OF PRODUCTION RELATIONSHIPS

by

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1. Introduction

This paper is a user's guide to empirical estimation of stochastic production functions using the flexible, moment-based approach. The purpose is to provide the applied researcher with a working knowledge of moment-based estimation. This knowledge includes a discussion of possible problems and some available solution techniques.

The paper is organized as follows. The second section specifies the reduced form equations that will be used in estimation. The third section describes the econometric techniques used to estimate these equations. Section 4 gives an example of how to implement the technique on actual data. There is a discussion of problems that might arise and possible solutions. Conclusions are drawn in section 5.

2. Specifying the Reduced Form Equations

The econometrician wishes to estimate the distribution of crop yields (or any other random variable), which is stochastic but dependent upon input choices. When the range of possible yields is finite the yield distribution can be estimated arbitrarily closely from knowledge of the moments of the distribution function (Feller). Since the distribution depends on input choice it follows that the moments depend on input choice. The reduced form equations provide empirically estimable relationships among moments and inputs.

Since the observations contain different input combinations, the FMB approach implies that each of the n yield observations is drawn from a different random distribution. The n distribution functions and hence their moments are systematically related to the inputs. The reduced form equations will be econometric specifications of the relationships among moments and inputs.

The i^{th} central moment of distribution j is defined to be $E[Y_j]$ for $i = 1$ and $E[(Y_j - E[Y_j])^i]$ for $i \geq 2$. These moments are denoted by μ_{ij} $i=1, \ldots$. For the j^{th} observation the expected value of output is equal to the first moment of the distribution, that is $E[Y_j] = \mu_{1j}$. Hence $E[Y] = \mu_1$, where μ_1 is the n x 1 column vector of distribution means. Since each moment will depend on the input choices $E[Y_j] = f(X_j)$ and hence $E[Y] = f(X)$. Assuming that f is linear and using Y_j as an observation on μ_{1j} (since $E[Y_j] = \mu_{1j}$) results in

$$(1) \qquad Y = X\beta_1 + \varepsilon_1,$$

where ε_1 is an n x 1 vector of errors with $E(\varepsilon_1) = 0$. Equation (1) so far is identical with the usual econometric estimation of yields as a function of inputs. However, the interpretation of (1) as a relationship explaining only one aspect of the influence of inputs or yields has important consequences that will be explored later.

Taking expectations of (1) shows that $E(Y) = XB_1$ and $Y-E[Y] = \varepsilon_1$. Hence an empirically observable second moment variable is the n x 1 vector $\varepsilon_1^2 \equiv [\varepsilon_{1j}^2]_{j=1}^n$. Assuming that μ_{2j} is a linear function of X_j and using ε_{1j}^2 as an observation on μ_{2j} results in

$$(2) \qquad \varepsilon_1^2 = X\beta_2 + \varepsilon_2.$$

Similarly, the equation for the i^{th} central moment can be written as

$$(3) \qquad \varepsilon_1^i = X\beta_i + \varepsilon_i$$

for $i > 1$, where the n x 1 vector $\varepsilon_1^i \equiv [\varepsilon_{1j}^i]_{j=1}^n$.

The equations (1) and (3) are the set of reduced form equations relating moments of the yield distribution function to inputs.

Throughout this paper the theoretical discussion will be applied to data on Senegalese peanut yields. The data set contains n observations on output and k inputs denoted by the n x 1 column vector Y and the n x k matrix X. Table 1 shows the data on peanut field trials from 1961 to 1982 under different soil preparation and fertilizer applications. Three types of soil preparations used: i) no preparation, ii) hand tilling, and iii) tilling with animal traction. Fertilizer was applied at three different rates: 0 kg/ha, 150 kg/ha, and 200 kg/ha. For each soil preparation and fertilizer application the cell entry shows the yield in kg/ha achieved that year. Yields will be the dependent variable in this analysis; fertilizer application, soil preparation, and dummy variables for years will be the independent variables. Soil preparation is represented by 0-1 dummy variables for hand tilling and tilling with animal traction.

3. Estimating the Reduced Form Equations

Although the OLS estimator $B_1 = (X'X)^{-1} X'Y$ is an unbiased estimator of β_1, the OLS estimates of the standard errors of B_1 are biased because the covariance matrix $E[\varepsilon_1 \varepsilon_1']$ is not of form $\sigma^2 I$ (Antle). Equation (2) implies that the diagonal of $E[\varepsilon_1 \varepsilon_1']$ is equal to the n x 1 vector $X\beta_2$. Similarly, $E[\varepsilon_i \varepsilon_i']$ has diagonal equal to the n x 1 vector $X B_{2i}$ - Diagonal $((XB_i)(XB_i)')$.

The above results imply that the simplest, internally consistent, estimation procedure is to use a GLS regression where the covariance matrix for equation i has form

$$\sigma^2 \begin{bmatrix} W^i & & & 0 \\ & \cdot & & \\ & & \cdot & \\ 0 & & & W^i \end{bmatrix}$$

There are two important consequences. First, hypothesis testing using the OLS covariance matrix will be inaccurate. Second, XB_{2i} should be positive for all i and for each observation, for this is a necessary condition that $E[\varepsilon_{ij}^2]$ be positive (here ε_{ij}^2 is the diagonal element in position j, j in the covariance matrix $E[\varepsilon_i \varepsilon_i']$).

A theoretically correct method for estimating equations (1) and (3) is given by Antle. First, use the OLS estimator B_1 to construct $\varepsilon_1^{\,i}$ for all needed i. Second, estimate equation 2i taking the estimation results from equation i as given and imposing constraints on $B_i i$ so that the variance terms from equation i are nonnegative. Third, calculate the GLS regression of equation i using the constrained variance estimates from equation 2i. Fourth, repeat steps 2 and 3 for the desired moment i.

4. Implementing the GLS Procedure.

This section discusses some of the problems that can arise in FMB estimation of probability distributions. It is not a complete listing and the proposed solutions are neither guaranteed nor exhaustive. The purpose of this section is to provide a starting point from which the applied econometrician can proceed quickly to obtain results.

a. OLS Estimation

The results of the OLS regressions of the Senegalese peanut data are shown in Table 2. Each type of soil preparation increases mean yields as seen by the positive coefficients in the first moment regression. Fertilizer also increases mean yields, with the estimated coefficient of 2.89 implying that application at the recommended rate of 150 kg/ha will increase yields by 433 kg/ha. This figure agrees with agronomic intuition on increased yields. Soil preparation has a negative effect on variance, indicated by the negative sign in the second moment regression. Fertilizer also has a negative effect. This is somewhat surprising since it is generally considered that fertilizer has only a small effect in bad years but a large effect in good years, leading to an increased dispersion of yields. The OLS regressions suggest that this is not so, although the sign of this coefficient changes to positive in the GLS regressions. In the third moment regression hand tilling has a positive coefficient while tilling with animal traction and fertilizer have negative signs. Although the third moment is related to the skewness of the distribution there is very little intuition as to the effects of soil preparation and fertilizer on skewness.

The standard errors reported for the three regressions in Table 2 are calculated from the OLS covariance matrix. Recall that these errors are biased: the improved efficiency from the GLS estimation will generally produce smaller errors. This will be especially important in regards to the peanut data since none of the coefficients are significant at the 5% level based on the OLS statistics. Standard errors will be discussed further in subsection C.

b. Choice of Dependent Variables

Using the errors from OLS estimation of equation (1) to construct the dependent variables for the remaining equations is incongruous with the rejection of OLS methods in favor of GLS methods for estimating the parameter vector β_1. An alternative procedure would be to use the GLS errors from equation 1 to reestimate a constrained equation (2), to use the reestimated (2) to construct a second GLS regression for (1), and repeat until convergence is reached.

Three justifications for the simple, OLS estimation of dependent variables are available. First, the OLS estimation B_1 is unbiased and hence the $\varepsilon_1^{\,i}$ are unbiased estimators of μ_i. Second, the iterated estimation procedure may not significantly alter the error. Third, if the iteration procedure does not converge then some artificial stopping point must be imposed, which will lead to the same conceptual problems as stopping after the OLS estimation of dependent variables.

An examination of the estimated errors is provided in Table 3. The first column shows the errors from OLS regression, $\varepsilon_1 = Y - X\,B_1$. The second column shows the errors from GLS estimates based on constrained second moment regressions.

c. Imposing the Constraints

The constrained estimation for moment 2i should be thought of as choosing an estimator B_{2i} which minimizes the sum of squared errors subject to a constraint that $XB_{2i} < C$ where C is an n x 1 vector of constrains. If i=1 then C=0; If $i > 1$ then $C_i = (X\,B_i)^2$, which is a constant in the $2i^{th}$ moment regression. There are n constraints involved, one for each observation.

The econometrics literature contains several methods for calculating constrained estimations; the technique used in this paper is the Lagrangian multiplier (for a survey of techniques see Judge et al., Appendix B). The problem with imposing Lagrangian multipliers when each observation is constrained is that there are as many multipliers as constraints. Each of these multipliers must be estimated together with the parameter vector B, which leads to an underdetermined system when there is only one yield observation associated with each input combination. However, it is likely that not all the constraints will be binding. The Kuhn-Tucker theorem implies that the multiplier on a non-binding constraint is zero (Luenberger, p. 249). Hence the econometrician need estimate the multipliers only for binding constraints.

Algebraically the constrained minimization problem is

$$(4)\quad \text{MIN } (Y-XB)'(Y-XB) \text{ s.t. } XB \geq C.$$

The solution for B is

$$(5)\quad B = (X'X)^{-1} X' (Y-\lambda)_1$$

which shows that the estimated B depends on knowledge of the λ.

A numerical method which uses the Kuhn-Tucker condition is outlined in Table 4. First, note that the unconstrained estimates $B_0 = (X'X)^{-1}X'Y$, can be written as a solution to the equation, $A_0B = D_0$ where $A_0 = I_{nxn}$ and $D_0 = X^+Y$. Second, check to see if any constraints are violated. If there is a violation pick the worst violation, observation j, where worst can be defined in terms of absolute value or percentage deviation. The third step is to find an expression for the constrained, optimal B. The estimate of β_i that minimizes SSE subject to the constraint will be written as a solution to $A_1B = D_1$. A_1 and D_1 can be constructed from A_0 and D_0. Concantenating the column X_j^+Y to the right border of A_0 captures the effect of the matrix λ in equation (5) when entry j is the only nonzero entry. The constraint $X_jB=C_j$ is imposed with equality by adding to A_0 a $k+1^{st}$ row equal to $(X_j,0)$ and constructing on D_0 the $k+1^{st}$ row equal to the constraint constant C_i. Thus $A_1 = \begin{bmatrix} I_{nxn} & X_j^+Y \\ X_j & 0 \end{bmatrix}$ and $D_1 = \begin{bmatrix} X^+Y \\ C_j \end{bmatrix}$. The fourth step is to solve these equations.

The solution $B_1 = A_1^{-1} D_1$ may still violate the covariance constraints. The fifth step will accommodate these constraints. Check again to see if any constraints are violated. If there are still violations, then follow the above procedure for constraining the (new) worst violation by constructing A_2 and D_2 from A_1 and D_1.

The number of repetitions needed is dependent on the quality of the data and the explanatory power of the model. For example, the constraint on the second moment regression determining B_2 is that $XB_2 = 0$. Since the dependent variable in this regression, ε_1^2, is nonnegative by construction, the predicted values XB_2 will be negative only because of large sampling error or specification error. Hence it is not expected that the covariance constraint will be binding for a large number of observations. However, note that each time a multiplier is required, a linear relationship among the elements of B is imposed. Thus the greater the number of multipliers imposed the less flexibility is left for B in minimizing SSE (although $B_i = 0$ for all moments i will always satisfy all constraints). This minimization is generally the economically relevant part of the regression ($B_i = 0$ is uninformative). Hence, an increase in the number of multipliers needed to impose the constraints means a corresponding decrease in the information content of the results.

c. Using the Constrained Results as GLS Weights

Successful completion of the constrained estimation of the $2i^{th}$ moment regression allows the econometrician to compute the i^{th} moment GLS covariance matrix V under the assumptions made above. Specifically, the diagonal of V will be the vector $X B_{2i} - C$, where $C = 0$ if $i=1$ and $C = X B_{2i} - [(XB_{i(j,1)})^i]_{j=1}^n$ if $i \geq 2$; the off-diagonal elements of V are zero.

Since the GLS estimator B and the standard deviations of the estimated coefficient depend on V^{-1} problems will arise whenever a constraint has been imposed. Imposing the covariance constraint on observation i implies that $v_{ii} = 0$, and hence that V is singular.

If the model is correctly specified constraint i will be binding when there is a large sample error associated with observation i. Hence the econometrician may wish to omit the effect of this observation in calculating and the standard errors of the estimated coefficients. This can be done by defining V^{-1} to be the n x n diagonal matrix with diagonal elements v_{ii}^{-1} when $v_{ii} = 0$ and 0 when $v_{ii} = 0$. An alternative procedure is to use the mean of the nonzero v_{ii} as a proxy for the values of the v_{ii} constrained to equal zero (this implies renormalization of V so that tV=n)[1]. Finally, the econometrician could eliminate the observation entirely. The justification for this is similar to the justification for removing outliers when there is an a priori belief that the outlying observation represents measurement error, and of course problems with removal of outliers apply to the methods discussed here.

The second moment regression of peanut yields resulted in eleven of the one hundred thirty-five observations having predicted values (first moment variance) essentially equal to zero. Of the zero values five were directly constrained to be zero; six were indirectly forced to zero by the imposition of the five constraints. The fact that five constraints affected eleven observations may be due to the use of dummy variables as independent variables, since altering the coefficient on soil preparation #1 (for example) affects forty-five observations in exactly the same manner.

The GLS regression are presented in Table 4. In the first moment regression each coefficient has the same sign as in the OLS regression. The coefficients generally have the same order of magnitude in the GLS and OLS regressions. Soil preparation seems to be economically unimportant and is statistically insignificant. Fertilizer is highly

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[1]This suggestion is due to John Hoehn.

significant, and application of the recommended 150 kg/ha dose increases average yields by 420 kg/ha. This is only 3% different from the OLS figure, which is not surprising since this coefficient is highly significant according both the GLS and the OLS standard error estimates. The GLS coefficients on the soil preparation variables are much smaller than their OLS counterparts, although none are statistically significant.

The advantage of the FMB approach over OLS methods for estimating the yield equations is apparent from a comparison of the estimated standard errors. For the year dummy variables the OLS standard errors are clearly nonsense. However, some of the problems with equality of OLS standard errors across coefficients is due to the combination of many dummy variables and the GAUSS procedure for calculating the standard errors. Even so, the greater precision of the GLS method is apparent. For example, the coefficient on Dummy-1969 is insignificant at the 10% level in the OLS regression but significant at the 10% level in the GLS regression.

The second moment GLS estimates are similar to the second moment OLS estimates, although the standard errors are again very different. It is interesting to note that the fertilizer coefficient switched signs, although it is not signficant in either regression.

Problems with estimating the sixth moment constrained estimation prevented GLS estimation of a third moment regression. The problem is that the magnitude of the elements of the dependent variable in the sixth moment regression can be very different. For example, the dependent variables for observation 112 is the first moment error for the observation, -593, raised to the sixth power. This is of order 10^{16}. However for observation 29 the dependent variable is 2.08. The GAUSS inversion procedure cannot easily handle such wide discrepencies in element size, and hence it was not possible to perform the calculations necessary to estimate a constrained sixth moment regression. Solutions are to write a subroutine conformalizing matrices or else to move to a language with this capability built in, such as APL.

While the programming problem can probably be solved, a more severe criticism is that the OLS regression indicates that the inputs have very little explanatory power in the third moment regression. Thus it appears best to limit the analysis to the first two moments.

4. Conclusions

This paper has pointed out some of the problems that occur when using a flexible, moment-based approach to stochastic production analysis. The Senegalese peanut data used in this paper illustrate some of these problems. Proposed solution are implemented on the peanut data in order to test their efficiency and to derive econometric results.

The problems in using the FMB method on Senegalese peanut data worsen as higher moments are estimated. Third and higher moments cause extreme problems, implying that the data may not be amenable to more than mean-variance analysis.

The FMB approach appears to have a significant advantage over the nonstochastic function approach even in estimating only the yield function (1). The FMB approach accounts for heteroskedasticity in (1) and significantly improves the precision of the estimated parameters.

TABLE 1

SENEGALESE PEANUT YIELDS

SOIL PREPARATION #1

Year	Fert = 0kg/ha	Fert = 150kg/ha	Fert = 200kg/ha
1966	2200	1925	2150
1967	1380	1656	1897
1968	1378	2001	2341
1969	1005	1205	1570
1970	1234	1267	1673
1971	1660	2463	2897
1972	1636	1929	2180
1973	1607	2316	2155
1974	1436	2069	2377
1975	1465	1888	2283
1976	1766	2814	2645
1977		745	948
1978		1588	1909
1979		1061	1330
1980		1110	1291
1981		1876	2977
1982		1644	1802

SOIL PREPARATION #2

Year	Fert = 0kg/ha	Fert = 150kg/ha	Fert = 200kg/ha
1966	1837	2075	2062
1967	1226	1532	1599
1968	1344	2040	2134
1969	978	1116	1677
1970	1279	1420	1778
1971	1656	2077	2566
1972	1494	1908	2017
1973	1741	2354	2418
1974	1894	2450	2465
1975	1862	1531	1756
1976	1662	2535	2589
1977		776	883
1978		2056	2398
1979		1328	2177
1980		1020	1147
1981		2182	2873
1982		1747	1858

TABLE 1 (CONTINUED)

SENEGALESE PEANUT YIELDS

SOIL PREPARATION #3

Year	Fert = 0kg/ha	Fert = 150kg/ha	Fert = 200kg/ha
1966	1737	2150	2062
1967	1267	1677	1552
1968	1548	2250	2204
1969	1319	1765	1893
1970	1316	1668	1673
1971	1808	2278	2294
1972	1581	2102	2140
1973	1702	2238	2241
1974	2111	2575	2780
1975	1133	1170	1540
1976	2390	2685	2447
1977		896	969
1978		2072	2497
1979		1477	1961
1980		1005	1260
1981		2309	2886
1982		1575	1863

TABLE 2

OLS REGRESSIONS ANALYSIS OF THE STOCHASTIC YIELD FUNCTION FOR PEANUTS

	First Moment	Second Moment	Third Moment
Constant	1203.6	29,790	2,230,276
	(105.4)	(29,355)	(20,155,000)
Soil Prep #2	31.9	-25,661	6,163,772
	(48.4)	(13,439)	(9,241,000)
Prep #3	82.9	-27,294	-4,423,582
	(49.1)	13,654)	(9,241,000)
Fert	2.89	-41.2	-18,459
	(0.26)	(73.2)	(50,303)
Dummy-1966	442.6	43,947	15,721,540
	(119.6)	(33,258)	(22,835,000)
Dummy-1967	-47.6	13,757	-1,573,500
	(119.6)	(33,258)	(22,835,000)
Dummy-1968	336.1	11,864	-449,040
	(119.6)	(33,258)	(22,835,000)
Dummy-1969	-187.4	31,648	-4,094,149
	(119.6)	(33,258)	(22,835,000)
Dummy-1970	-100.7	10,984	-2,418,870
	(119.6)	(33,258)	(22,835,000)
Dummy-1971	609.4	44,363	12,323,365
	(119.8)	(33,258)	(22,835,000)
Dummy-1972	304.5	411	-1,195,370
	(119.8)	(33,314)	(22,871,000)
Dummy-1973	502.8	10,428	-841,124
	(119.8)	(33,312)	(22,071,000)
Dummy-1974	656.7	38,737	-7,687,941
	(119.8)	38,737)	(22,871,000)
Dummy-1975	42.4	126,861	1,314,848
	(119.8)	(33,312)	(22,871,000)
Dummy-1976	809.6	52,130	-1,849,094
	(119.8)	(33,312)	(22,871,000)
Dummy-1977	-878.7	-3526	435,209
	(130.0)	(36,130)	(24,807,000)
Dummy-1978	338.5	52,446	-4,214,151
	(130.0)	(36,130)	(24,807,000)
Dummy-1979	192.5	100,310	20,031,840
	(130.0)	(36,130)	(24,807,000)
Dummy-1980	-609.3	1,220	543,312
	(130.0)	(36,130)	(24,807,000)
Dummy-1981	769.0	113,180	-8,680,435
	(130.0)	(30,130)	(24,807,000)
	R^2=.81	R^2=.21	R^2=10.113

TABLE 3A
FIRST MOMENT REGRESSION ERRORS

OLS ERRORS

553.76	223.99	-161.78	-11.23	131.10
-153.00	127.86	-99.46	-424.35	218.97
-247.24	-13.92	-388.09	-384.09	81.74
-530.59	6.40	-155.18	66.04	27.26
-245.18	-269.84	216.04	-13.08	175.58
-225.30	208.02	366.80	44.42	-211.74
-259.74	118.09	425.75	19.75	-74.83
162.39	222.61	-24.83	-8.49	505.39
61.39	-161.93	-93.82	426.50	21.28
158.89	38.11	-227.65	-70.10	144.23
188.88	-46.00	2.66	1.77	584.10
-383.11	-14.79	48.03	-148.96	-40.12
-256.46	77.53	-37.05	-89.83	34.39
-366.05	-148.71	-201.83	-65.95	181.71
123.82	-180.84	55.93	-52.44	245.38
555.38	-57.77	289.88	43.88	-194.70
-167.48	-16.25	50.29	64.63	142.51
-101.60	101.06	-5.82	-100.49	-34.71
7.88	28.10	-74.67	219.88	130.21
-87.89	-10.01	-87.35	167.75	-195.90
293.87	54.19	13.02	-50.97	-106.14
-180.47	-145.47	-13.06	4.15	193.37
231.93	48.26	-51.84	77.03	14.69
197.80	-592.85	154.92	-17.45	293.37
288.37	4.20	251.87	-2.12	-245.71
-265.49	2.72	215.28	-91.38	-180.49
-29.61	-126.95	258.15	-367.50	-227.72

TABLE 3B
GLS ERRORS

546.24	250.73	-205.28	-60.45	95.32
-162.74	89.25	-128.34	-492.90	197.09
-277.75	-62.71	-453.30	-433.61	114.70
-560.11	8.10	-152.97	102.51	-6.50
-284.68	-295.89	216.02	-41.97	156.43
-284.13	195.87	346.01	-1.12	-273.71
306.02	154.29	399.47	24.69	-69.38
202.10	192.08	-61.08	-31.30	508.61
65.22	-148.37	119.93	447.06	33.21
180.84	94.33	-241.68	-89.85	137.92
-169.14	-55.14	3.25	-37.30	591.69
-384.15	-34.11	12.29	-169.01	22.30
-256.51	108.70	-5.37	-23.88	30.09
-376.08	-145.29	-172.37	-65.37	192.03
94.46	-163.52	64.61	-68.52	212.88
538.57	7.89	293.07	78.30	-159.78
-98.29	-17.31	43.51	71.29	175.22
-97.77	114.62	-31.93	-79.93	-22.78
33.20	87.69	-85.32	203.50	127.28
-64.79	-15.78	-83.38	132.04	-184.94
296.20	38.24	-19.34	-67.65	-40.33
-177.15	-110.93	21.98	73.47	192.45
225.27	55.06	-19.01	80.98	28.39
171.82	-572.16	166.97	-30.16	264.24
274.93	73.25	258.43	35.65	-207.42
192.93	5.04	211.87	-81.34	144.42
-22.41	-110.01	235.41	-343.57	-212.42

TABLE 4
GLS REGRESSION ANALYSIS OF THE STOCHASTIC YIELD FUNCTION FOR PEANUTS

Independent Variable	First Moment (kg/ha)	Second Moment $(kg/ha)^2$
Constant	1211.7	13,908
	(58.4)	(11,702)
Soil Prep. #2	2.4	-10,273
	(43.2)	(11,021)
Soil Prep. #3	30.0	-20,220
	(43.2)	(10,364)
Fertilizer	2.80	19
	(0.2)	(23)
Dummy-1966	442.1	25,579
	(90.4)	(28,252)
Dummy-1967	-82.4	20,139
	(57.7)	(8309)
Dummy-1968	371.6	1972
	(54.6)	(6934)
Dummy-1969	-146.2	37,056
	(79.2)	(20,401)
Dummy-1970	-73.0	631
	(53.1)	(6322)
Dummy-1971	611.1	28,884
	(90.7)	(28,513)
Dummy-1972	335.1	-3029
	(44.3)	(3506)
Dummy-1973	523.7	6598
	(49.7)	(5508)
Dummy-1974	717.2	22,873
	(83.5)	(23,693)
Dummy-1975	56.2	128,372
	(141.2)	(77,168)
Dummy-1976	832.1	37,528
	(94.8)	(31,989)
Dummy-1977	-828.2	7295
	(46.0)	(6744)
Dummy-1978	405.4	38,674
	(114.4)	(39,438)
Dummy-1979	-141.3	94,748
	(153.8)	(75,191)
Dummy-1980	-640.6	-4429
	(46.0)	(3135)
Dummy-1981	800.2	92,114
	(162.7)	(84,690)
	$R^2 = 0.80$	$R^2 = 0.20$

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