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# A Sample Selection Approach to Censored Demand Systems 

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#### Abstract

A censored demand system estimator is proposed by extending the sampleselection model of Heckman. Censoring is governed by a selection mechanism which avoids the restrictive Tobit parameterization. Results of application to household consumption of beverages suggest the estimator produces slightly different elasticity estimates from the Tobit estimator. Demands for beverages are nearly unitary elastic, and net substitution is an obvious pattern.


Key Words: Beverages, censoring, sample selection, Translog demand system.

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Micro survey data present obvious advantages over aggregate time series in modeling consumer demand and other microeconomic relationships. Important features of microdata include censored dependent variables, often prevalent in modeling economic relationships at the disaggregate level. A number of censored system estimators have existed in the literature. Amemiya pioneered a procedure for the linear Tobit system. Wales and Woodland constructed the likelihood function from the Kuhn-Tucker conditions of the constrained maximization of a stochastic utility function. Lee and Pitt, taking the dual approach, used virtual prices to define regime switching. Golan, Perloff and Shen estimated a demand system using the generalized maximum entropy approach. More recently, Yen, Lin and Smallwood applied quasi- and simulated maximumlikelihood approaches to the Tobit system (Amemiya). Other procedures include the less efficient two-step estimators of Heien and Wessell, Perali and Chavas, and Shonkwiler and Yen. This note presents another approach to censored demand system estimation by extending the bivariate sample selection model of Heckman. The model can be viewed as a sample selection generalization to Amemiya's Tobit system, and as a maximumlikelihood (ML) alternative to the two-step procedures of Heien and Wessells and Shonkwiler and Yen for essentially the same model. In the brief empirical section the procedure is demonstrated with a sample used elsewhere, dealing with household consumption of beverages in the United States, and results are compared to those of the Tobit system.

## A Multivariate Sample Selection Model

Let $\mathbf{x}$ be a vector of all explanatory variables and $\theta$ a vector of parameters, and consider a system of $n$ equations in which each dependent variable $w_{i}$ is generated by a deterministic function $f_{i}(\mathbf{x} ; \theta)$, an unobservable error term $v_{i}$, and an indicator variable $d_{i}$ such that

$$
\begin{equation*}
w_{i}=d_{i}\left[f_{i}(\mathbf{x} ; \theta)+v_{i}\right], \quad i=1, \ldots, n . \tag{1}
\end{equation*}
$$

The deterministic components $f_{i}(\mathbf{x} ; \boldsymbol{\theta})$ can be linear or nonlinear functions. In the demand system application below the dependent variables $w_{i}$ are expenditure shares and $f_{i}(\mathbf{x} ; \theta)$ are nonlinear in parameters with cross-equation restrictions. Each indicator $d_{i}$ depends on a vector of conditioning variables $\mathbf{z}$ through a binary mechanism:

$$
\begin{equation*}
d_{i}=1\left(\mathbf{z}^{\prime} \gamma_{i}+u_{i}>0\right), \quad i=1, \ldots, n \tag{2}
\end{equation*}
$$

where $1(\cdot)$ is a binary indicator function, $\gamma_{i}$ is a parameter vector, and $u_{i}$ is a random error. The demand system (1) does not add up to unity as in the uncensored case. While adding-up can be accommodated in other ways, such as the re-mapping procedure in Wales and Woodland, to limit the scope of the current paper we take a simple approach of estimating the first $n-1$ equations and treating the $n$th good as a residual good. This "plausible" and "simple" approach to adding-up, suggested by Pudney (p. 155), was used by Yen, Lin and Smallwood for the Tobit system. The proposed model is an extension of the multivariate Tobit model in that censoring of each dependent variable $y_{i}$ is governed by the sample-selection mechanism (2) and not by $d_{i}=1\left[f_{i}(\mathbf{x} ; \theta)+v_{i}>0\right]$ as in the latter
(Amemiya; Yen, Lin and Smallwood).
To facilitate presentation of the likelihood function, denote $m=n-1$ and define a diagonal matrix $\mathbf{S}=\operatorname{diag}\left(\sigma_{1}, \ldots, \sigma_{m}\right)$ where $\sigma_{1}, \ldots, \sigma_{m}$ are standard deviations of $\mathbf{v}$. Also, let $\mathbf{R}_{\mathrm{uu}}=\left[\rho_{i j}^{\mathrm{uu}}\right], \mathbf{R}_{\mathrm{vu}}=\left[\rho_{i j}^{\mathrm{vu}}\right]$ and $\mathbf{R}_{\mathrm{vv}}=\left[\rho_{i j}^{\mathrm{vv}}\right]$ be $m \times m$ correlation matrices among elements of $\mathbf{u}$ and $\mathbf{u}, \mathbf{v}$ and $\mathbf{u}$, and $\mathbf{v}$ and $\mathbf{v}$, respectively, where $\rho_{i j}^{\text {vu }}$ is the correlation between $v_{i}$ and $u_{j}$ and likewise for $\rho_{i j}^{\mathrm{uu}}$ and $\rho_{i j}^{\mathrm{vv}}$. Assume the concatenated error vector $\left[\mathbf{u}^{\prime}, \mathbf{v}^{\prime}\right]^{\prime} \equiv\left[u_{1}, \ldots, u_{m}, v_{1}, \ldots, v_{m}\right]^{\prime}$ is distributed as (2m)-variate normal with zero mean and covariance matrix

$$
\Sigma=\left[\begin{array}{ll}
\Sigma_{11} & \Sigma_{12}  \tag{3}\\
\Sigma_{21} & \Sigma_{22}
\end{array}\right]
$$

where $\Sigma_{11}=E\left(\mathbf{u u}^{\prime}\right)=\mathbf{R}_{\mathrm{uu}}, \Sigma_{21}=\Sigma_{12}^{\prime}=E\left(\mathbf{v u}^{\prime}\right)=\mathbf{S}^{\prime} \mathbf{R}_{\mathrm{vu}}$ and $\Sigma_{22}=E\left(\mathbf{v v}^{\prime}\right)=\mathbf{S}^{\prime} \mathbf{R}_{\mathrm{vv}} \mathbf{S}$.
To construct the likelihood function, consider first a sample regime in which the outcomes of all dependent variables are positive, characterized by

$$
\begin{align*}
& \mathbf{z}^{\prime} \gamma_{i}+u_{i}>0, \\
& w_{i}=f_{i}(\mathbf{x} ; \theta)+v_{i}, i=1, \ldots, m . \tag{4}
\end{align*}
$$

Define $m$-vectors $\mathbf{r} \equiv\left[r_{1}, \ldots, r_{m}\right]^{\prime} \equiv\left[\mathbf{z}^{\prime} \boldsymbol{\gamma}_{1}, \ldots, \mathbf{z}^{\prime} \boldsymbol{\gamma}_{m}\right]^{\prime}$ and $\mathbf{v} \equiv\left[w_{i}-f_{i}(\mathbf{x} ; \theta)\right]$. In addition, let $g(\mathbf{v})$ be the marginal probability density function(pdf) of $\mathbf{v} \sim N\left(0, \boldsymbol{\Sigma}_{22}\right)$ and $h(\mathbf{u} \mid \mathbf{v})$ be the conditional pdf of $\mathbf{u} \mid \mathbf{v} \sim N\left(\mu_{\mathbf{u} \mid \mathbf{v}}, \Sigma_{\mathbf{u} \mid \mathbf{v}}\right)$, where $\mu_{\mathrm{u} \mid \mathbf{v}}=\Sigma_{12} \Sigma_{22}^{-1} \mathbf{v}$ and $\Sigma_{\mathrm{u} \mid \mathrm{v}}=\Sigma_{11}-\Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$. Then, the likelihood contribution for this regime is

$$
\begin{equation*}
L_{1}=g(\mathbf{v}) \int_{\mathbf{u}>-\mathbf{r}} h(\mathbf{u} \mid \mathbf{v}) d \mathbf{u}=g(\mathbf{v}) \Phi_{n}\left(\mathbf{r}+\mu_{\mathbf{u} \mid \mathbf{v}} ; \Sigma_{\mathbf{u} \mid \mathbf{v}}\right) \tag{5}
\end{equation*}
$$

where $\Phi_{n}\left(\mathbf{r}+\mu_{\mathrm{u} \mid \mathrm{v}} ; \Sigma_{\mathrm{u} \mid \mathrm{v}}\right)$ is the $m$-variate normal cumulative distribution function (cdf) with zero mean, covariance matrix $\Sigma_{u \mid v}$ and finite upper integration limits $\mathbf{r}+\mu_{u \mid \mathrm{v}}$.

The second regime is one in which the values of all dependent variables are zeros, characterized by

$$
\begin{equation*}
\mathbf{z}^{\prime} \gamma_{i}+u_{i} \leq 0, i=1, \ldots, m \tag{6}
\end{equation*}
$$

The likelihood contribution is

$$
\begin{equation*}
L_{2}=\int_{\mathbf{u} \leq-\mathbf{r}} f\left(\mathbf{u} ; \Sigma_{11}\right) d \mathbf{u}=\Phi_{m}\left(-\mathbf{r} ; \Sigma_{11}\right), \tag{7}
\end{equation*}
$$

where $f\left(\mathbf{u} ; \boldsymbol{\Sigma}_{11}\right)$ is the marginal pdf of $\mathbf{u} \sim N\left(0, \Sigma_{11}\right)$. The last regime is one in which, without loss of generality, the first $\ell$ dependent variables are not censored and the rest are zeros, characterized by

$$
\begin{array}{ll}
\mathbf{z}^{\prime} \gamma_{i}+u_{i}>0, w_{i}=f_{i}(\mathbf{x} ; \theta)+v_{i}, & i=1, \ldots, \ell  \tag{8}\\
\mathbf{z}^{\prime} \gamma_{i}+u_{i} \leq 0, & i=\ell+1, \ldots, m
\end{array}
$$

Define $\ell$-vector $\tilde{\mathbf{v}} \equiv\left[w_{i}-f_{i}(\mathbf{x} ; \theta)\right]$. Then, $\left[\mathbf{u}^{\prime}, \tilde{\mathbf{v}}\right]^{\prime}$ is $(m+\ell)$-variate normal with zero mean and covariance matrix $\tilde{\boldsymbol{\Sigma}}$, where $\tilde{\boldsymbol{\Sigma}}$ is an $(m+\ell) \times(m+\ell)$ sub-matrix containing the first ( $m+\ell$ ) rows and columns of the error covariance matrix $\Sigma$ in (3). Partition $\tilde{\Sigma}$ at the $m$ th row and column such that

$$
\tilde{\Sigma}=\left[\begin{array}{ll}
\Sigma_{11} & \tilde{\Sigma}_{12} \\
\tilde{\Sigma}_{21} & \tilde{\Sigma}_{22}
\end{array}\right]
$$

Let $g(\tilde{\mathbf{v}})$ be the marginal pdf of $\tilde{\mathbf{v}} \sim N\left(0, \tilde{\Sigma}_{22}\right)$ and $h(\mathbf{u} \mid \tilde{\mathbf{v}})$ be the conditional pdf of $\mathbf{u} \mid \tilde{\mathbf{v}} \sim N\left(\mu_{\mathbf{u} \mid \tilde{\mathbf{v}}}, \Sigma_{\mathbf{u} \mid \tilde{\mathbf{v}}}\right)$, where $\mu_{\mathbf{u} \mid \tilde{\mathbf{v}}}=\tilde{\Sigma}_{12} \tilde{\Sigma}_{22}^{-1} \tilde{\mathbf{v}}$ and $\Sigma_{\mathbf{u} \mid \tilde{\mathbf{v}}}=\Sigma_{11}-\tilde{\Sigma}_{12} \tilde{\Sigma}_{22}^{-1} \tilde{\Sigma}_{21}$. Then, the
likelihood contribution for this regime is

$$
\begin{align*}
L_{3} & =g(\tilde{\mathbf{v}}) \int_{u_{1}>-r_{1}} \cdots \int_{u_{\ell}>-r_{\ell}} \int_{u_{\ell+1} \leq-r_{t+1}} \cdots \int_{u_{m} \leq-r_{m}} h\left(u_{1}, \ldots, u_{m} \mid \tilde{\mathbf{v}}\right) d u_{n} \cdots d u_{1}  \tag{9}\\
& =g(\tilde{\mathbf{v}}) \Phi_{n}\left[\mathbf{D}\left(\mathbf{r}+\mu_{\mathbf{u} \mid \tilde{\mathbf{v}}}\right) ; \mathbf{D}^{\prime} \Sigma_{\mathbf{u | |}} \mathbf{D}\right],
\end{align*}
$$

where $\mathbf{D} \equiv \operatorname{diag}\left(2 d_{1}-1, \ldots, 2 d_{m}-1\right)$. The sample likelihood function is the product of the likelihood contributions $L_{1}, L_{2}$ or $L_{3}$ across observations, depending on the regime for each observation.

To demonstrate the proposed estimator we use the Translog demand system (Christensen, Jorgenson and Lau), with deterministic shares

$$
\begin{equation*}
f_{i}(\mathbf{x} ; \theta)=\frac{\alpha_{i}+\sum_{j=1}^{n} \beta_{i j} \log \left(p_{j} / E\right)}{-1+\sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{k j} \log \left(p_{j} / E\right)}, i=1, \ldots, n \tag{10}
\end{equation*}
$$

where $E$ is total expenditure, $p_{j}$ are prices, and $\alpha_{i}$ and $\beta_{i j}$ are parameters. Demographic variables $d_{k}$ are incorporated in the demand equations (10) by parameterizing $\alpha_{i}$ such that $\alpha_{i}=\alpha_{i 0}+\sum_{k} \alpha_{i k} d_{k}, i=1, \ldots, m$. The symmetry restrictions $\beta_{i j}=\beta_{j i} \forall i, j$ are also imposed.

Because the dependent variables are censored, elasticities are calculated by differentiating the unconditional means of the expenditure shares. Based on the marginal distribution of each $\left[u_{i}, v_{i}\right]^{\prime}$, which is bivariate normal, the unconditional means of $w_{i}$ are

$$
\begin{equation*}
E\left(w_{i}\right)=\Phi\left(\mathbf{z}^{\prime} \gamma_{i}\right) f_{i}(\mathbf{x} ; \theta)+\sigma_{m+i, i} \phi\left(\mathbf{z}^{\prime} \gamma_{i}\right), i=1, \ldots, m \tag{11}
\end{equation*}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are univariate standard normal pdf and cdf, respectively.
Differentiation of the unconditional mean (11) gives demand elasticities for the first $m$ goods, ${ }^{1}$ and elasticities for the $n$th goods can be derived by the adding up restriction (Yen, Lin and Smallwood).

## Data and Application

The data are drawn from the 1996-97 National Food Stamp Program Survey, conducted by Mathematica Policy Research, Inc. for U.S. Department of Agriculture. The beverages considered are milk, fruit juice, soft drink and 'coffee \& tea’. Prices (unit values) were derived from reported expenditures and quantities, and missing prices for non-consuming households were replaced with regional averages. While this zero-order imputation is parsimonious and put the current application within scope, further applications might address this missing-price issue more carefully. Besides prices, demographic variables are also used in the analysis (see table 1). Detailed definitions and sample statistics for all variables are available from the authors. Among the beverages considered, the percentages of consuming households are milk (91\%), juice (75\%), soft drink (82\%) and coffee \& tea (72\%).

ML estimates are presented in tables 1 and 2. About half of the demographic variables are significant at the 0.05 level or lower in the selection equations. Presence of children is significant in the share equation for juice and presence of the elderly is significant in the soft drink equation. All but one of the quadratic price coefficients ( $\beta_{i j}$ ) are significant at the 0.01 level. The error correlation estimates, presented in table 2, suggest that among the selection equations only error correlation between juice and soft drink is significant. All other coefficients with and among errors of the demand equations are significant at the 0.05 level or lower. Thus, apart from the need to impose crossequation restrictions, the significance of these error correlations also supports estimation of the equations in a system.

Table 3 presents the demand elasticities and their asymptotic standard errors, calculated by the delta method. All uncompensated own-price elasticities are significant at the 0.01 level. The own-price elasticities are slightly below unity for milk and juice and are slightly above unity for soft drink and coffee \& tea. Only two of the cross-price elasticities are significant. All expenditure elasticities are very close to unity. All compensated elasticities are significant at the 0.01 level, with the compensated own-price elasticities much smaller than their Marshallian counterparts. The cross-price elasticities are much smaller than the own-price elasticities and suggest net substitutability among all beverages.

For comparison, we also estimated the Tobit system (Amemiya; Yen, Lin and Smallwood). Elasticity estimates from the Tobit system are presented in the appendix (table A1). In general, most elasticities are fairly close between the two sets of estimates. More notable differences are seen in the elasticities of coffee \& tea. Specifically, the Tobit estimates suggest much lower expenditure elasticity for coffee \& tea, and that coffee \& tea is a gross substitute to juice and soft drink, whereas such substitution is nonexistent according to results of the proposed model. In addition, the own-price effects are also slightly different between the two sets of estimates. Specifically, the proposed estimator produces lower compensated and uncompensated own-price elasticities for juice, soft drink and coffee \& tea but slightly higher (compensated and uncompensated) own-price elasticities for milk than those generated by the Tobit system.

## Summary

With the growing popularity of microdata in empirical analysis, interest in the censored data issues has continued to grow. This note contributes to the censored demand system literature by proposing a sample selection approach to censoring. A multivariate generation of the bivariate sample selection model (Heckman) and a sample-selection generalization to the Tobit system (Amemiya), the proposed procedure accommodates censoring in an equation system with a separate selection mechanism for each equation and avoids the Tobit parameterization that is known to be restrictive. The procedure is fairly easy to implement and allows imposition of cross-equation restrictions. We demonstrate the procedure in a consumer demand system but the procedure is equally applicable to other linear or nonlinear systems of equations. The trivariate cdf's in our application of a four-equation consumer demand system are calculated by conventional means. For a larger system, the higher-level probability integrals can be evaluated with existing simulation or Bayesian techniques. The estimator is applied to household consumption of beverages and the findings suggest demands for these beverage products are nearly unitary elastic. Net substitution is the obvious pattern. The estimator also produces slightly different elasticity estimates than the Tobit estimator.

## Footnote

1 Elasticity formulas are available upon request from the author.

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Table 1. ML Estimates of Multivariate Sample Selection Model: Translog Demand System

| Variables | Milk |  | Juice |  | Soft drink |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. | S.E. | Coeff. | S.E. | Coeff. | S.E. |
| Selection equations ( $\gamma_{i}$ ) |  |  |  |  |  |  |
| Constant | $1.190^{\ddagger}$ | 0.315 | 0.128 | 0.147 | $0.787^{\ddagger}$ | 0.159 |
| Income | 0.060 | 0.152 | -0.072 | 0.072 | $0.231{ }^{\dagger}$ | 0.099 |
| Compare prices | 0.256* | 0.151 | 0.041 | 0.068 | -0.051 | 0.072 |
| Use coupons | $0.334^{\dagger}$ | 0.149 | -0.017 | 0.066 | 0.101 | 0.074 |
| Elderly present | 0.014 | 0.164 | 0.051 | 0.095 | $-0.277^{\ddagger}$ | 0.102 |
| Children present | 0.554 | 0.165 | $0.262^{\ddagger}$ | 0.095 | 0.139 | 0.106 |
| Black | -0.400 | 0.247 | $0.388^{\ddagger}$ | 0.120 | -. 030 | 0.131 |
| White | 0.183 | 0.260 | $0.269^{\ddagger}$ | 0.103 | 0.042 | 0.113 |
| Northeast | $-0.440^{*}$ | 0.267 | $0.216^{\dagger}$ | 0.100 | -0.039 | 0.116 |
| Midwest | -0.105 | 0.220 | $0.149^{*}$ | 0.085 | $-0.287^{\ddagger}$ | 0.093 |
| South | $-0.544^{\ddagger}$ | 0.209 | 0.051 | 0.094 | $-0.174^{*}$ | 0.094 |
| High school | -0.080 | 0.146 | 0.030 | 0.063 | 0.041 | 0.070 |
| Rural | $-0.390^{*}$ | 0.205 | $0.162^{\dagger}$ | 0.080 | 0.072 | 0.088 |
| Demand system: demographic variables ( $\alpha_{i j}$ ) |  |  |  |  |  |  |
| Constant | $-0.360^{\ddagger}$ | 0.068 | $-0.182^{\ddagger}$ | 0.067 | $-0.169^{\ddagger}$ | 0.067 |
| Household size | -0.022 | 0.015 | -0.014 | 0.024 | -0.009 | 0.021 |
| Children present | 0.006 | 0.041 | $-0.200^{\dagger}$ | 0.090 | 0.008 | 0.077 |
| Elderly present | $0.062^{*}$ | 0.037 | -0.057 | 0.062 | $0.139^{\dagger}$ | 0.060 |
| Quadratic price terms ( $\beta_{i j}$ ) |  |  |  |  |  |  |
| Milk | $0.120^{\ddagger}$ | 0.039 |  |  |  |  |
| Juice | $0.135^{\ddagger}$ | 0.028 | -0.049 | 0.031 |  |  |
| Soft drink | $0.151^{\ddagger}$ | 0.022 | $0.071{ }^{\ddagger}$ | 0.017 | $0.134^{\ddagger}$ | 0.016 |


| Coffee \& tea | $0.088^{\ddagger}$ | 0.026 | $0.048^{\ddagger}$ | 0.017 | $0.060^{\ddagger}$ | 0.015 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Std. dev. $\left(\sigma_{i}\right)$ | $0.217^{\ddagger}$ | 0.007 | $0.242^{\ddagger}$ | 0.008 | $0.255^{\ddagger}$ | 0.008 |
| Log-likelihood | -256.105 |  |  |  |  |  |

Note: Daggers $\ddagger$ and $\dagger$ denote significance at the $1 \%$ and $5 \%$ levels and asterisk (*) at the $10 \%$ levels, respectively. The coefficient of the quadratic log-price term ( $\beta_{44}$ ), not reported due to space consideration, is 0.137 and has a standard error of 0.031 .

Table 2. ML Estimates of Error Correlation Coefficients

|  | Selection equations |  |  |  | Share equations |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Milk $\left(u_{1}\right)$ | Juice $\left(u_{2}\right)$ | Soft drink $\left(u_{3}\right)$ |  | Milk $\left(v_{1}\right)$ |  |
| Juice $\left(u_{2}\right)$ | -0.044 |  | Juice $\left(v_{2}\right)$ |  |  |  |
|  | $(0.079)$ |  |  |  |  |  |
| Soft drink $\left(u_{3}\right)$ | -0.101 | $-0.214^{\ddagger}$ |  |  |  |  |
|  | $(0.071)$ | $(0.050)$ |  |  |  |  |
| Milk $\left(v_{1}\right)$ | $0.411^{\dagger}$ | $-0.411^{\ddagger}$ | $-0.462^{\ddagger}$ |  |  |  |
|  | $(0.201)$ | $(0.042)$ | $(0.041)$ |  |  |  |
| Juice $\left(v_{2}\right)$ | $-0.200^{\ddagger}$ | $0.957^{\ddagger}$ | $-0.328^{\ddagger}$ | $-0.328^{\ddagger}$ |  |  |
|  | $(0.059)$ | $(0.017)$ | $(0.047)$ | $(0.047)$ |  |  |
| Soft drink $\left(v_{3}\right)$ | $-0.206^{\ddagger}$ | $-0.311^{\ddagger}$ | $0.968^{\ddagger}$ | $-0.466^{\ddagger}$ | $-0.415^{\ddagger}$ |  |
|  | $(0.058$ | $(0.044)$ | $(0.014)$ | $(0.039)$ | $(0.041)$ |  |

Note: Daggers $\ddagger$ and $\dagger$ denote significance at the $1 \%$ and $5 \%$ levels, respectively.

Table 3. Demand Elasticities

|  | Price of |  |  |  | Total <br> Expend. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Milk | Juice | Soft drink | Coffee \& tea |  |
| Uncompensated elasticities |  |  |  |  |  |
| Milk | $-0.972^{\ddagger}$ | $-0.031{ }^{\dagger}$ | -0.002 | 0.015 | $0.990^{\ddagger}$ |
|  | (0.015) | (0.013) | (0.007) | (0.013) | (0.003) |
| Juice | -0.027* | $-0.944^{\ddagger}$ | 0.005 | 0.009 | $0.956{ }^{\ddagger}$ |
|  | $(0.017)$ | (0.020) | (0.010) | (0.010) | (0.009) |
| Soft drink | -0.008 | -0.008 | $-1.010^{\ddagger}$ | $0.016^{\dagger}$ | $1.010^{\ddagger}$ |
|  | (0.007) | (0.009) | (0.007) | (0.008) | (0.005) |
| Coffee \& tea | -0.010 | 0.006 | 0.017 | $-1.080^{\ddagger}$ | $1.067^{\ddagger}$ |
|  | (0.027) | (0.017) | (0.015) | (0.033) | (0.013) |
| Compensated elasticities |  |  |  |  |  |
| Milk | $-0.632^{\ddagger}$ | $0.182^{\ddagger}$ | $0.284^{\ddagger}$ | $0.165^{\ddagger}$ |  |
|  | (0.016) | (0.015) | (0.011) | (0.014) |  |
| Juice | $0.302{ }^{\ddagger}$ | $-0.738^{\ddagger}$ | $0.281{ }^{\ddagger}$ | $0.154^{\ddagger}$ |  |
|  | (0.019) | (0.020) | (0.013) | (0.013) |  |
| Soft drink | $0.339^{\ddagger}$ | $0.210^{\ddagger}$ | $-0.719^{\ddagger}$ | $0.169^{\ddagger}$ |  |
|  | (0.013) | (0.013) | (0.014) | (0.013) |  |
| Coffee \& tea | $0.357^{\ddagger}$ | $0.236^{\ddagger}$ | $0.325^{\ddagger}$ | $-0.918^{\ddagger}$ |  |
|  | (0.029) | (0.018) | (0.017) | (0.035) |  |

Note: Asymptotic standard errors in parentheses. Daggers $\ddagger$ and $\dagger$ denote significance at the $1 \%$ and $5 \%$ levels and asterisk (*) at the 0.10 level, respectively.

## Appendix

Table A1. Demand Elasticities Based on Tobit System

|  | Price of |  |  |  | Total <br> Expend. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Milk | Juice | Soft drink | Coffee \& tea |  |
| Uncompensated elasticities |  |  |  |  |  |
| Milk | $-0.967^{\ddagger}$ | $-0.062^{\dagger}$ | $0.058{ }^{\ddagger}$ | -0.015 | $0.986^{\ddagger}$ |
|  | (0.036) | (0.030) | (0.017) | (0.021) | (0.017) |
| Juice | $-0.113^{\ddagger}$ | $-1.081^{\ddagger}$ | 0.029 | $0.067{ }^{\ddagger}$ | $1.099^{\ddagger}$ |
|  | (0.038) | (0.044) | (0.019) | (0.026) | (0.025) |
| Soft drink | 0.035 | $0.032^{*}$ | $-1.186^{\ddagger}$ | $0.035^{*}$ | $1.083^{\ddagger}$ |
|  | (0.025) | (0.020) | (0.030) | (0.021) | (0.025) |
| Coffee \& tea | 0.060 | $0.265^{\ddagger}$ | $0.173^{\ddagger}$ | $-1.165^{\ddagger}$ | $0.668^{\ddagger}$ |
|  | (0.063) | (0.065) | (0.039) | (0.060) | (0.093) |
|  | Compensated elasticities |  |  |  |  |
| Milk | $-0.619^{\ddagger}$ | $0.186^{\ddagger}$ | $0.322^{\ddagger}$ | $0.111^{\ddagger}$ |  |
|  | (0.033) | (0.031) | (0.018) | (0.023) |  |
| Juice | $0.274^{\ddagger}$ | $-0.805^{\ddagger}$ | $0.323^{\ddagger}$ | $0.207^{\ddagger}$ |  |
|  | (0.038) | (0.044) | (0.022) | (0.028) |  |
| Soft drink | $0.417^{\ddagger}$ | $0.305^{\ddagger}$ | $-0.895^{\ddagger}$ | $0.173^{\ddagger}$ |  |
|  | (0.032) | (0.028) | (0.041) | (0.028) |  |
| Coffee \& tea | $0.295^{\ddagger}$ | $0.433^{\ddagger}$ | $0.352^{\ddagger}$ | $-1.080^{\ddagger}$ |  |
|  | (0.058) | (0.056) | (0.040) | (0.062) |  |

Note: Asymptotic standard errors in parentheses. Daggers $\ddagger$ and $\dagger$ denote significance at the $1 \%$ and $5 \%$ levels and asterisk (*) at the 0.10 level, respectively.

