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## A SOFTWARE PACKAGE FOR ANALYSIS OF INVESTMENT AND DISINVESTMENT UNDER UNCERTAINTY

Technical Documentation and Programmer's Guide

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## TABLE OF CONTENTS

Page
List of Tables ..... iv
List of Figures ..... v
I. Technical Description ..... 1
A. Introduction ..... 1
B. The Optimal Control Framework ..... 2

1. Variable Definitions ..... 3
2. Model Equations ..... 4
3. Approaches to Finding the Optimal Solution ..... 9
C. GREMP for the Stochastic Case ..... 10
4. Designation of Strategies ..... 10
5. Simulation and Distribution of Outputs ..... 12
6. Stochastic Dominance with Respect to a Function ..... 12
D. A General Test Case ..... 13
7. Assumptions ..... 13
8. Results ..... 21
a. Preferred Strategies ..... 21
b. Model Dynamics ..... 23
E. Summary and Conclusions ..... 26
9. The General Test Case ..... 27
10. The Basic Model ..... 27
F. References ..... 28
II. Program Information ..... 29
A. Program Description ..... 29
11. Operating Environment ..... 29
12. Program Structure ..... 29
a. Investment/Disinvestment Analysis Routines ..... 31
b. Stochastic Dominance Analysis Routines ..... 32
B. Program Implementation ..... 32
13. Job Control Parameters: Data Card Formats and Job Descriptions ..... 33
14. Internal Data ..... 33
a. General Data ..... 34
b. Durable-Specific Data ..... 35
15. Outputs Produced ..... 36
a. Standard Outputs ..... 36
b. Optional Outputs ..... 37

## Table of Contents, Continued

Page
Appendix 1. Variable Definitions ..... 38
Appendix 2. Program List ..... 45
Appendix 3. Sample Print-Outs ..... 68
Table 1. Characteristics of the General Test Case Electric Power Generating Plants ..... 14
Table 2. Parameters of the User Cost, Forced Outage and Variable Maintenance Functions ..... 16
Table 3. Frequency Distributions for Random Exogenous Variables ..... 19
Table 4. Plant Combinations in General Test Case Experimental Design ..... 20
Table 5. General Test Case Results ..... 22

## LIST OF FIGURES

Page
Figure 1. A Flow Chart of the GREMP Model ..... 11
Figure 2. The User Cost Function ..... 15
Figure 3. Triangle Distributions for Two Strategies ..... 24
Figure 4. Plant Utilization Rates ..... 25
Figure 5. AIDU Flowchart ..... 30

## TECHNICAL DESCRIPTION

## Introduction

Investment/disinvestment theory describes the economic decisions associated with production processes using as inputs the services of durables as well as nondurables. Specificially, it considers simultaneously questions of l) when to acquire (invest in) or 2) salvage (disinvest in) durable assets, 3) what size of a durable to acquire, 4) how intensively to extract services from a durable, and 5) how much to maintain a durable. Decision theory, meanwhile, attempts to describe the process whereby decision makers choose among alternative courses of action (or "strategies") whose outcomes are uncertain because they depend on conditions in the environment (or "states of nature") which are themselves uncertain. These two bodies of theory have been integrated and extended in this research project, and it is the objective of this documentation to present a model and computer software package which implements this integrated theory for practical application in decisions of investment and disinvestment in the energy supply industry.

The terms of our EPRI contract refer to analysis of the impacts of asset fixity and uncertainty on energy supply in general. However, in order to have a well-defined problem, we found it necessary to focus on firm-level decisions involving increases and decreases in electric power generating capacity. Nevertheless, the theory developed is general to any production process involving durable factors, and it would be straightforward to generalize the model presented here to address other classes of energy supply decisions, such as oil or gas drilling, coal mining, pipeline distribution, electric power transmission, etc. Indeed, the software package developed for this model is suitable for any such application, reserving the model specifications particular to that application to user-supplied subroutines. For illustrative purposes, however, the remainder of this discussion and the general test case presented as an example refer to decisions with respect to electric power generating capacity.

The next section casts the decision problem in an optimal control framework and suggests the feasibility of numerically obtaining a global solution in the deterministic case. Because of the uncertainty, however, this problem is decidedly not deterministic. Therefore, succeeding sections l) describe the generalized risk-efficient, Monte Carlo programming (GREMP) approach for handling the stochastic case, and 2) discuss the assumptions and results of a general test case designed to illustrate the application of the GREMP approach to this problem. The concluding section of the technical description summarizes recommendations for further developments and refinements of the model.

## The Optimal Control Framework

The general investment/disinvestment problem is a dynamic one involving optimal decisions with respect to when durable factors of production are to be acquired and in what sizes, when they are to be disposed of, at what rate services are to be extracted from them during their lifetime, and when and how much they are to be maintained to extend that lifetimee. An electric utility is a single firm which typically operates several generating plants, each contributing to meeting the firm's load demand. These plants, or individual generators within them, are the durables for which the utility must make the above decisions (as well as other durables, of course, such as transmission lines, etc.). Since the utility generally dispatches plants according to their relative efficiency and is required to meet its load and maintain a reserve capacity, these investment/disinvestment/utilization/maintenance decisions cannot be made for any one plant independently of the others. Therefore, the general problem is stated in an optimal control framework in order to maintain the dynamic and simultaneous character of the decisions to be made in the context of the integrated system.

Two caveats, as obvious as they are, must nevertheless be noted with respect to the optimal control problem stated below. First, there is no implication that a one-time solution to the problem will dictate the pattern and schedule of investment/disinvestment/utilization/maintenance activities over the next T years of the planning horizon. In practice, and even in theory, such an implication would be patently ridiculous. Rather, if the model were to be used in practice, it would typically be solved repeatedly at intervals -- most likely frequent intervals -- dictated by the acquisition of new information on exogenous variables (e.g., prices, regulations, and load forecasts) and control options (e.g., generation technologies).

Secondly, in any case, this model is primarily an economic model, and practical decisions typically require a great deal of additional, noneconomic information -technical, political, social, legal, ecological, etc. Therefore, the model presented here can never claim or be expected to "dictate" decisions; it can at best be only one, hopefully credible source of information for decision making.

The general optimal control problem is stated as follows:
given a system represented by the state equations

$$
x(t)=a[x(t), u(t), z(t)]
$$

where $x=$ vector of state variables
$u=$ vector of control variables
$z=$ vector of noncontrollable exogenous variables
and where $x(t)$ is required to meet certain constraints at every point in time, i.e., $\mathrm{x}(\mathrm{t}) \varepsilon \mathrm{X}$ for all $\mathrm{t} \varepsilon[0, \mathrm{~T}]$, where X is the set of admissible states and T is the planning horizon;
find a control history $u(t)$, also subject to constraints $u(t) \varepsilon U$ for all $t \varepsilon[0, T]$, where $U$ is the set of admissible controls, in order to
maximize the objective function $J=h[x(T)]+\int_{0}^{T} g[x(t), u(t), z(t)] d t$
where h is the contribution to the objective of the state of the system at the end of the planning period and $g$ is the accumulation of contributions over time.

## Variable Definitions

a) Dimensions, $n$ and $k$

For an electric utility operating a number of generating plants, assume k is the number of plants in existence at time zero in the analysis, and $n$ is the maximum number of plants to be considered in the analysis. Therefore, $n-\mathrm{k}$ is the maximum number of additional plants to be considered as options for replacing or augmenting existing capacity over the planning horizon T. Each of the n plants, existing as well as potential, is given a set of characteristics at the beginning of the analysis, e.g., size, operating efficiency and fuel type (coal, oil, nuclear, etc.). Therefore, in order to consider a number of options, n - k will typically be larger than the total number of plants that (or represent greater capacity than) is likely to be needed $T$ years in the future. Note that each of the $n$ "plants" may be interpreted, depending on the needs of the analyst, as a plant, as a generator, or as an aggregation of plants of similar vintage and characteristics.
b) State variables: $x_{i}(t), i=1,2, \ldots, 3 n$

There are $3 n$ state variables. That is, for each plant $j, j=1,2, \ldots, n$ :
$\operatorname{GCAP}_{j}(\mathrm{t})=$ plant capacity (megawatts)
$\operatorname{VALDUR}_{\mathrm{j}}(\mathrm{t}) \quad=$ unit market value of the durable (\$/megawatt)
$\operatorname{VALSER}_{j}(\mathrm{t}) \quad=$ value of services to date (\$).
c) Control variables: $\mathrm{u}_{\mathrm{i}}(\mathrm{t}), \mathrm{i}=1,2, \ldots, 6 \mathrm{n}-2 \mathrm{k}+1$

There are $6 \mathrm{n}-2 \mathrm{k}+1$ control variables, some applicable to all n plants and others applicable only to the $\mathrm{n}-\mathrm{k}$ plants not in existence at time zero.

For each plant $j, j=1,2, \ldots, n$ :
$\mathrm{UTILT}_{\mathrm{j}}(\mathrm{t})=$ time utilization rate (proportion of total hours per year)
$\mathrm{UTILC}_{\mathrm{j}}(\mathrm{t})=$ capacity utilization rate (proportion of megawatt capacity)
$\operatorname{VMAINT}_{j}(\mathrm{t})=$ variable maintenance rate to replace lost or used capacity (megawatts/year)
STIME $_{j}=$ time to salvage or disinvestment (year).

For each plant j not existing initially, $\mathrm{j}=\mathrm{k}+\mathrm{l}, \mathrm{k}+2, \ldots, \mathrm{n}$ :
$\mathrm{CAPO}_{j}=$ initial plant capacity (megawatts)
$\mathrm{ACQT}_{j}=$ time of acquisition or investment (year).
Finally, there is one control variable which is not plant specific:
PINRG ( t ) = energy purchased or (if negative) sold (megawatt-hours/year).
d) Exogenous variables: $\mathrm{z}_{\mathrm{i}}(\mathrm{t}), \mathrm{i}=1,2, \ldots, 8 \mathrm{n}+2$

There are $8 \mathrm{n}+2$ exogenous variables which affect the performance of the system but which are assumed to be either beyond the control of the firm, or at least assumed given for purposes of this analysis. These are known functions of time. Included are prices, costs, discount and depreciation rates and load forecasts.
Specifically, for each plant $j, j=1,2, \ldots, n$ :
$\mathrm{FOMC}_{\mathrm{j}}(\mathrm{t})=$ fixed operating and maintenance costs (\$/megawatt-year)
$\mathrm{SCHED}_{\mathrm{j}}=$ scheduled down time for regular maintenance (proportion of hours/year)
VMNTC $_{j}(\mathrm{t})=$ variable maintenance cost (\$/megawatt)
$\operatorname{VICST}_{j}(\mathrm{t})=$ cost of variable input (fuel) (\$ per unit of fuel, e.g., barrel, ton, etc.) $\operatorname{ACCST}_{j}(\mathrm{t})=$ plant acquisition cost (\$/megawatt); this is also a function of the size of the plant, $\mathrm{CAPO}_{j}$
$\operatorname{DCOST}_{j}(\mathrm{t})=$ relicensing cost (\$/megawatt); depends on time from acquisition $D_{j}\left(t-A C Q T T_{j}\right)=$ depreciation schedule (called "time cost" in Chapter III) (proportion per year); depends on time from acquisition.
In addition there are two variables common to the firm as a whole:

$$
\begin{array}{ll}
\mathrm{XLOAD}(\mathrm{t}) & =\text { consumer demand (megawatt-hours per year) } \\
\mathrm{R}(\mathrm{t}) & =\text { opportunity cost of capital (proportion/year). }
\end{array}
$$

Note the simplifying assumption that load is neither price responsive nor controllable by the firm. This assumption could be relaxed, if desired, to include load management policies in the analysis and to capture the secondary impacts of the firm's investment and operating decisions on its load through the primary impacts on price.

## Model Equations

This section describes the state equations representing the dynamics of the system's behavior, the calculation of associated performance variables, the constraints imposed on state and control variables, and the objective function to be optimized. First, for each plant $\mathrm{j}, \mathrm{j}=1,2, \ldots, \mathrm{n}$, the power output capacity, GCAP, changes over time according to the following differential equation:
(1) $\quad \frac{d}{d t} \operatorname{GCAP}_{j}(t)= \begin{cases}\operatorname{VMAINT}_{j}-\operatorname{USCST}_{\mathrm{j}}(\mathrm{t}) & \text { for } \operatorname{ACQT}_{\mathrm{j}} \leq \mathrm{t} \leq \text { STIME }_{J} \\ 0 & \text { else }\end{cases}$
with boundary conditions

$$
\operatorname{GCAP}_{j}(t)=\left\{\begin{array}{cl}
\mathrm{CAPO}_{j} & \text { for } t=\operatorname{ACQT}_{j} \\
0 & \text { for } t<\operatorname{ACQT}_{j} \text { and } t>\text { STIME }_{j}
\end{array}\right.
$$

where for plants existing at time zero $(\mathrm{j}=1,2, \ldots, k)$ we can assume $\operatorname{ACQT}_{\mathrm{j}}=0$, and where USCST is the rate at which power output capacity is lost, in megawatts/year, due to intensity of utilization.

In general, this capacity user cost can be defined as a function of plant capacity, of time utilitization, and of capacity utilization:
(2) $\operatorname{USCST}_{\mathrm{j}}(\mathrm{t})=\mathrm{c}_{\mathrm{j}}\left[\operatorname{GCAP}_{\mathrm{j}}(\mathrm{t}), \operatorname{UTILT}_{\mathrm{j}}(\mathrm{t}), \operatorname{UTILC}_{\mathrm{j}}(\mathrm{t})\right]$.

Such loss in capacity is an important concept in this model, for it influences forced outages and the associated variable maintenance necessary to replace it.

Variable maintenance VMAINT, remember, is a control variable determined in the optimum solution. The analyst may want to specify VMAINT in a control law, such as
(3a) VMAINT $_{j}(t)=m_{l j}$ [CAPO $_{j}-$ GCAP $\left._{j}(t)\right]$
or
(3b) VMAINT $_{j}(\mathrm{t})=\mathrm{m}_{2 \mathrm{j}}$ USCST $\left._{j}\right)$
or perhaps some combination of these. In such a case, optimization would be over the parameters of the control law or over alternative control laws.

The unit market value of the durable (the generating plant) changes over time due to such market conditions as the introduction of improved technologies, changes in the relative prices of fuels, labor and other inputs, and changes in market institutions -- i.e., the pure time cost. This is captured in the model by an exogenously specified depreciation schedule, as follows:
(4) $\frac{d}{d t} \operatorname{VALDUR}_{j}(t)= \begin{cases}-D_{j}\left(t-\operatorname{ACQT}_{j}\right) \operatorname{VALDUR}_{j}(t) \text { for } \operatorname{ACQT}_{j} \leq t \leq \text { STIME }_{j} \\ 0 & \text { else }\end{cases}$
with boundary conditions

$$
\operatorname{VALDUR}_{j}(t)= \begin{cases}\operatorname{ACCST}_{j}(t) & \text { for } t=\operatorname{ACQT}_{j} \\ 0 & \text { for } t<\operatorname{ACQT}_{j} \text { and } t>\operatorname{STIME}_{j}\end{cases}
$$

where $D_{j}$ depends, in general, on the time from acquisition of plant $j$.
Salvage value, in dollars, then is the market value of the plant less a "decommissioning" cost representing disposal costs in general:
(5)

$$
\operatorname{SALVAL}_{j}(\mathrm{t})=\operatorname{VALDUR}_{\mathrm{j}}(\mathrm{t}) \operatorname{GCAP}_{\mathrm{j}}(\mathrm{t})-\operatorname{DCOST}_{j}(\mathrm{t}) \operatorname{CAPO}_{\mathrm{j}}
$$

Notice that the disposal costs are based on the initial size of the plant rather than its current effective capacity. Thus, salvage value is reduced by time cost through VALDUR (equation 4), and increased by maintenance and reduced by user cost through GCAP (equation 1).

The value of services, VALSER, generated by each plant is an important state variable, because it determines the economic life of the plant and whether the plant should be acquired in the first place, as discussed in Chapter III. Since VALSER is derived from the objective function $J$ of the optimal control problem, we turn our attention now to the latter before looking at the former.

For investment/disinvestment decisions, the appropriate objective is to maximize the discounted present value of economic gains (in \$) accumulated over a suitable planning horizon [0, T]. Therefore, for the objective function J given above, we define $\mathrm{h}=0$ and g as the discounted gain function (in \$/year). A gain function is used rather than a profit function in order to account for the additional time, control and user costs associated with durable factors of production.

Therefore, for discount rate $\rho$

$$
\begin{align*}
\mathrm{g}(\mathrm{x}, \mathrm{u}, \mathrm{z})= & \mathrm{e}^{-\rho \mathrm{t}_{\overline{\mathrm{g}}}(\mathrm{t})=\mathrm{e}^{-\rho t}\{[\operatorname{CPR}(\mathrm{t})-\operatorname{PPR}(\mathrm{t})] \operatorname{PINRG}(\mathrm{t})}  \tag{6}\\
& +\sum_{j=1}^{n}\left[\operatorname{HPY} \cdot \operatorname{CPR}(\mathrm{t}) \mathrm{U}_{\mathrm{j}}(\mathrm{t}) \mathrm{GCAP}_{\mathrm{j}}(\mathrm{t})-\operatorname{VICST}_{j}(\mathrm{t}) \operatorname{VIUSE}_{\mathrm{j}}(\mathrm{t})\right. \\
& +\left[\operatorname{VALDUR}_{j}(\mathrm{t})-\operatorname{VMNTC}_{j}(\mathrm{t})\right] \operatorname{VMAINT}_{j}(\mathrm{t})-\mathrm{R}(\mathrm{t}) \operatorname{SALVAL}_{\mathrm{j}}(\mathrm{t}) \\
& -\mathrm{D}_{\mathrm{j}}(\mathrm{t}) \operatorname{VALDUR}_{\mathrm{j}}(\mathrm{t}) \operatorname{GCAP}_{\mathrm{j}}(\mathrm{t})-\operatorname{VALDUR}_{j}(\mathrm{t}) \operatorname{USCST}_{j}(\mathrm{t}) \\
& \left.\left.-\operatorname{FOMC}_{\mathrm{j}}(\mathrm{t}) \operatorname{CAPO}_{j}-\operatorname{RCOST}_{\mathrm{j}}(\mathrm{t})-\operatorname{TAX}_{\mathrm{j}}(\mathrm{t})\right]\right\}
\end{align*}
$$

where HPY $=8760$ hours/year, VIUSE = variable input (e.g., fuel) used by the plant (units of fuel/year), CPR and PPR are the consumer price of energy and the price to the firm of purchased energy, respectively (both in \$/megawatt-hour), and where the total plant utilization, $U_{j}$, is
(7) $\quad U_{j}(t)=$ UTILC $_{j}(\mathrm{t}) \mathrm{UTILT}_{\mathrm{j}}(\mathrm{t})$.

Note that the control cost, i.e., the capital cost of the plant, $R(t) \operatorname{SALVAL}_{j}(t)$, is included in the objective function.

The value of services for each plant, then, is defined as that part of the gain which is attributable to the services extracted from that plant. It is determined by the following differential equation, discounting to the plant's acquisition time:

with the boundary condition

$$
\operatorname{VALSER}_{j}(\mathrm{t})=0 \text { for } \mathrm{t}<\operatorname{ACQT}_{\mathrm{j}}
$$

This must be done holding the use of other fixed and variable inputs in a fixed relationship to the durable's use. In the case of one durable $j$, such as an electric power plant, and one variable input, such as fuel, such a relationship could be, for example,
(9) $\quad \operatorname{VIUSE}_{j}(\mathrm{t})=\operatorname{HPY}_{\mathrm{HEATR}_{j}}^{\operatorname{FUELC}_{\mathrm{j}}} \mathrm{U}_{\mathrm{j}}(\mathrm{t}) \operatorname{GCAP}_{\mathrm{j}}(\mathrm{t})$
where $\quad$ HEATR $=$ the heat rate of the plant (BTU/megawatt-hour)
FUELC $=$ the heat content of the plant's fuel (BTU/unit of fuel).
Defining the net price

$$
\mathrm{NP}(\mathrm{t})=\operatorname{CPR}(\mathrm{t})-\operatorname{VICST}_{\mathrm{j}}(\mathrm{t}) \frac{\operatorname{HEATR}_{\mathrm{j}}}{\operatorname{FUELC}_{\mathrm{j}}}
$$

and referring to equations 2 and 3 , we can derive

$$
\text { (10) } \begin{aligned}
\frac{\partial \bar{a}(t)}{\partial U_{j}}= & H P Y \cdot N P_{j}(t) G C A P_{j}(t)-V M N T C_{j}(t) \frac{\partial m_{j}}{\partial U_{j}} \\
& +\left[H P Y \cdot N P_{j}(t) U_{j}(t)-\left(D_{j}(t)+R(t)\right) \text { VALDUR }_{j}(t)\right] \cdot\left[\int_{0}^{t} \frac{\partial m_{j}}{\partial U_{j}} d \tau-\int_{0}^{t} \frac{\partial c_{j}}{\partial U_{j}} d \tau\right]
\end{aligned}
$$

which, given explicit functions for user cost and maintenance ( $c_{j}$ and $m_{j}$, respectively), enables us to find VALSER ${ }_{j}$ by equation 8. If a control law is not specified for maintenance, then $\partial \mathrm{m}_{\mathrm{j}} / \partial \mathrm{U}_{\mathrm{j}}=0$.

As discussed below, further theoretical development is necessary to handle the case of multiple, mutually dependent durables. Therefore, the empirical model implemented
does not attempt to isolate the portion of gain attributable to each durable itself from that of other durable and variable inputs. Thus, for the time being, the total gain generated by the operation of a durable (i.e., for each plant $j$ under the summation sign of equation 6) is used in place of the integral in equation 8 in computing VALSER $_{j}$.

Knowing the value of services for each plant enables us to use a control law for salvage (or replacement) time. Assuming upon salvage a plant is replaced with one identical to it, STIME is determined by comparing the current (not discounted) rate of change of VALSER with the annuity value (annualized average) of VALSER. The annuity value of the existing plant is used as a surrogate for that of the identical replacement, implicitly assuming the replacement experiences the same history as the original plant. Thus,
(ll) $\quad$ STIME $_{j}=t: e^{\rho\left(t-\operatorname{ACQT}_{j}\right)} \frac{d}{d t} \operatorname{VALSER}_{j}(\mathrm{t}) \leq \frac{\rho}{\left.1-e^{-\rho(t-A C Q T}{ }_{j}\right)} \operatorname{VALSER}_{j}(\mathrm{t})$.
Other optimal life criteria could be derived based on other assumptions made regarding the nature of the replacement, e.g., salvage without replacement, technological change, etc. In any case, as in the identification of value of services, the theory upon which equation 11 and any such alternative derivations are based does not yet satisfactorily capture the situation of multiple, mutually dependent durables.

The consumer price, CPR, of the firm's product (e.g., electric power) may be specified as a function of other variables in the model, or it may be projected independently and input to the model. In the latter case, CPR would be included in the list of exogenous variables, $z_{i}$, given above.

In the case of a regulated electric utility, the price to the customer includes a passthrough of costs (fuel, maintenance, purchased power, depreciation, etc.) plus an allowed rate of return on the capital investment. Referring to the objective function (equation 6), since costs are offset on the revenue side by CPR, and insofar as the allowed rate of return included in CPR compensates for the control costs (ignoring time lags in regulatory adjustments), we can see that the objective function will tend to be approximately zero for whatever control strategies are implemented. That is, the regulated price will automatically adjust to maintain a zero economic gain.

In such a situation the objective function specified in equation 6 is unable to distinguish among control strategies. Therefore, the empirical model implemented here uses, for electric utilities regulated in this way, an objective function which omits the revenue side and accounts only for the costs -- essentially a cost minimization problem. That is, the implied objective is to minimize the discounted costs to the consumer.

Another implication of this type of price regulation is that the value of services generated over the lives of the firm's plants will average out to zero. Plants of above average efficiency will have positive value, and those of below average efficiency will have a negative value of services.

An important constraint on the system (for an electric utility) requires that the load must be met at every point in time, i.e.,
(12) $\operatorname{PINRG}(\mathrm{t})+\sum_{\mathrm{j}=1}^{n} \operatorname{HPY} \cdot \mathrm{U}_{\mathrm{j}}(\mathrm{t}) \operatorname{GCAP}_{\mathrm{j}}(\mathrm{t})=\operatorname{XLOAD}(\mathrm{t})$.

Other constraints on state and control variables are
(13) $0 \leq$ GCAP $_{j}(\mathrm{t}) \leq \mathrm{CAPO}_{j}$

$$
\begin{align*}
& \text { for } j=1,2, \ldots, n \\
& \text { for } j=k+1, k+2, . . \\
& \text { for } j=k+1, k+2, . . \\
& \text { for } j=1,2, \ldots, n  \tag{18}\\
& \text { for } j=1,2, \ldots, n \\
& \text { for } j=1,2, \ldots, n .
\end{align*}
$$

(14) $\mathrm{CAPO}_{j} \varepsilon \mathrm{~F}_{j}$
(15) $\mathrm{O} \leq \mathrm{ACQT}_{\mathrm{j}}$
(16) $\mathrm{ACQT}_{\mathrm{j}} \leq$ STIME $_{\mathrm{j}}$
(17) UTILC $_{\mathrm{j}}(\mathrm{t}) \varepsilon\left\{0\right.$, UCMIN $\left.\left._{\mathrm{j}}, 1\right]\right\}$ $0 \leq$ UTILT $_{j}(\mathrm{t}) \leq 1-$ SCHED $_{\mathrm{j}}-$ FORC $_{j}(\mathrm{t})$

Note that equation 14 says the initial plant capacity may be constrained to selected sizes or a certain range of sizes F ; and equation 17 says, if a plant is going to be operated at all at time $t$, it must be utilized at least UCMIN percent of capacity. Equation 18 is necessary to allow for scheduled and forced down time.

## Approaches to Finding the Optimal Solution

The size and degree of nonlinearity of this problem make an analytic solution virtually impossible. However, there are a number of techniques available, some in "canned" software packages, for finding numerical solutions (e.g., see Kirk, 1970, and Luenberger, 1973). The gradient projection method and adaptations of it appear particularly suitable for our constrained problem; indeed, this method requires the control variables to be in a bounded region.

We have not gone this route, because of the part of the problem we haven't discussed yet -- the uncertainty part. Uncertainty is introduced by specifying probability density functions for a subset, if not all, of the above exogenous variables, as we have done for the illustrative test case described below. While a numerical solution appears to be feasible (although probably costly) for the deterministic case, a different approach must be found for the stochastic case. This is not to say that finding a numerical,
deterministic solution to the problem described above wouldn't be instructive and useful -- indeed, it certainly would be -- but we have not done so in this project. For the stochastic case, King, 1979, suggests an approach which is used here as described in the next section.

## GREMP for the Stochastic Case

We have elected to solve for the stochastic case with the GREMP package as presented in King, 1979. GREMP is an acronym for "generalized risk efficient Monte Carlo programming". The technique is particularly well suited for problems in which it is difficult or impossible to determine a solution analytically. It does not necessarily identify the optimal strategy but rather a nearly optimal strategy. This is accomplished by examining a large number of alternatives under a variety of states of nature and selecting those strategies which perform "best" according to a given criterion.

Three major processes are included in the model -- strategy generation, simulation and distribution of outputs, and evaluation. These are illustrated by the flow chart in Figure 1. Strategy generation may be accomplished through 1) a process of random selection, 2) specification of a set of strategy choices deemed apt by the decision maker, 3) an experimental design, or some combination of these. Each strategy is simulated repeatedly for a number of states of nature, and a distribution of outputs likely for that strategy is generated. The distributions of outputs for the various strategies are then evaluated by applying the criterion of stochastic dominance with respect to a function. Those strategies which are not dominated by any other comprise the efficient set of strategies.

## Designation of Strategies

King, in his application of the GREMP model, constructed strategies at random. We have chosen instead to furnish a pre-selected set of strategies to the model. Random selection would have involved choices of plant size, plant type, acquisition time, utilization rate and other variables for a number of plants. The combination of all these factors would have yielded a vast magnitude of strategies, many of which would obviously be inappropriate. Computational constraints would limit random evaluation to a very small fraction of the total, resulting in possible omission of favorable strategies.

For the electricity generating capacity decision problem, an experimental design has been constructed, for the illustrative test case described below, for plant capacities and acquisition times, while decision rules have been implemented for utilization rates and


Figure 1. A flow chart of the GREMP Model

Source: Figure taken from King, 1979.
maintenance and replacement policies. The a priori selection of the experimental design enables the analyst to tap the expertise of the decision maker and leads to a more efficient use of computing resources. Similarly, standard decision rules are useful for such variables as the utilization rate of a plant. It is dependent on the load and the relative efficiencies, capacities and utilization rates of itself and other plants and, thus, is better specified in the form of a decisin rule (i.e., a dispatch rule) also drawn from knowledgeable expertise, as opposed to a pre-specified rate.

## Simulation and Distribution of Outputs

We have constructed a computer model of the investment/disinvestment problem presented in the previous section. It simulates each strategy under the same set of random states of nature.

The exogenous variables represented within the states of nature include consumer demand and prices of variable inputs. Probability density functions are furnished for each of these variables and a random number generator is used to construct the states of nature. The seed of the random number generator is reset for each strategy to ensure that all strategies are simulated under the same set of conditions.

Given a strategy, the model generates the gain (actually, cost) function for that strategy for each state of nature. These results are then ordered from smallest to largest gain in order to obtain the cumulative distribution of gains for that strategy.

## Stochastic Dominance with Respect to a Function

The GREMP model evaluates alternative strategies under the criterion of stochastic dominance with respect to a function (King, 1979). Each strategy is evaluated after its cumulative distribution of gains has been generated. It is compared with other alternatives and if dominated by any, it is removed from further consideration. If not dominated by any other strategy, it is retained for further consideration. Any previously retained strategy which is dominated by a new strategy is dropped at this time. Those strategies remaining after all have been evaluated are referred to as the efficient set of strategies. Within the criterion specified, it is not possible to proclaim any one to be superior to the others.

Stochastic dominance with respect to a function does not necessarily yield a unique strategy. Rather, it provides the decision maker with a set of strategies favorable to maximizing the gain function under uncertain conditions. The efficient set may not include a global optimum but a large sample of wisely selected alternative strategies
assures that nearly optimal strategies will be identified and gives a high probability that the global optimum is included.

## A General Test Case

In order to fully test and illustrate the decision features of the model, as an operationalization of the investment/disinvestment/utilization/maintenance decision theory, we have developed a general test case of an electric utility with a number of existing generating plants, a number of options for future plants, and a load forecast. There is much that could be done to make this general test case more realistic; therefore, the assumptions and results presented in this report must be considered tentative and for illustrative purposes only.

## Assumptions

The assumptions defining the general test case (itemized below) include explicit functions and parameter values for the model presented above and in some cases variances from that model.

1. Planning horizon and discount rate
$\mathrm{T}=40$ years, $\rho=11.75 \%$
2. Plants to be considered

Four plants are assumed to exist at time zero. The number of plants to be analyzed for expansion of capacity will vary with the strategy, but four different sizes and types of such additional plants are assumed to be available. Table 1 summarizes the characteristics of the initial and additional plants.
3. User cost, forced outage and variable maintenance

The explicit functions for user cost ( $c_{j}$ in equation 2), variable maintenance $\left(m_{j}\right.$ in equation 3), and forced outage for a plant $j$ are assumed to derive from the degree to which the plant's capacity utilization exceeds a threshold level and the duration of that excess.

If $\mathrm{UTH}_{\mathrm{j}}$ is a measure of the amount and duration of capacity utilization exceeding a threshold, then user cost, in megawatts/year, is defined as
(19) $\quad \mathrm{USCST}_{\mathrm{j}}(\mathrm{t})=\mathrm{UCMAX}_{\mathrm{j}} \cdot \mathrm{UTH}_{\mathrm{j}}(\mathrm{t})^{\mathrm{UCEXP}}{ }_{\mathrm{j}} \cdot \operatorname{GCAP}_{\mathrm{j}}(\mathrm{t})$
where $\mathrm{UTH}_{\mathrm{j}}$ is defined in such a way that, for threshold capacity $\mathrm{CAPTH}_{\mathrm{j}}, 0 \leq \mathrm{UTH}_{\mathrm{j}} \leq$ (1$\mathrm{CAPTH}_{\mathrm{j}}$ )/CAPTH $\mathrm{Cl}_{\mathrm{j}} \leq 1$. This function behaves as shown in Figure 2, where we have assumed the parameter values shown in Table 2.

Table 1
Characteristics of the General Test Case Electric Power Generating Plants

| Characteristics | Initial Plants |  |  |  | Additional Plants |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | A | B | C | D |
| Fuel Type | Oil | Coal | Oil | Coal | Coal | Coal | Coal | Coal |
| Capacity (megawatts) | 200 | 200 | 400 | 600 | 200 | 400 | 600 | 800 |
| Heat rate (BTU/kwh) | 8,600 | 10,500 | 8,400 | 10,000 | 10,000 | 9,800 | 9,600 | 9,500 |
| Cost (\$/kw)* | 700 | 1,200 | 500 | 950 | 1,150 | 1,050 | 950 | 850 |
| Estimated life (years) | 30 | 34 | 30 | 34 | 34 | 34 | 34 | 34 |
| Scheduled down time (proportion of time) | . 10 | . 13 | . 10 | . 14 | . 13 | . 13 | . 14 | . 15 |
| Construction lead time (years) | 7 | 7 | 7 | 8 | 7 | 7 | 8 | 9 |
| Fixed operating and maintenance cost (\$/kw-yr)* | 15.0 | 16.0 | 15.0 | 12.6 | 16.0 | 14.7 | 12.6 | 10.4 |

*In dollars of time zero.


Figure 2
The User Cost Function

Table 2
Parameters of the User Cost, Forced Outage and Variable Maintenance Functions

|  | Initial Plants |  |  |  | Additional Plants |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | A | B | C | D |
| UCMAX (proportion/year) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| UCEXP (no units) | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| CAPTH (proportion of capacity) | . 80 | . 80 | . 80 | . 80 | . 80 | . 80 | . 80 | . 80 |
| FOR (proportion of time) | . 05 | . 10 | . 05 | . 11 | . 10 | . 10 | . 11 | . 12 |
| VMNT (proportion of mw/year) | . 99 | . 99 | . 99 | . 99 | . 99 | . 99 | . 99 | . 99 |

Forced outage and variable maintenance are assumed related to one another in that it is during periods of forced outage that variable maintenance is required and takes place. Both are related to user cost in the previous time period. Thus,

$$
\begin{align*}
& \text { FORC }_{j}(\mathrm{t})=\mathrm{FOR}_{\mathrm{j}}+\mathrm{UTH}_{\mathrm{j}}(\mathrm{t}-\mathrm{DT}) \mathrm{UCEXP}_{\mathrm{j}}  \tag{20}\\
& \operatorname{VMAINT}_{\mathrm{j}}(\mathrm{t})=\mathrm{VMNT}_{\mathrm{j}} \cdot \mathrm{USCST}_{\mathrm{j}}(\mathrm{t}-\mathrm{DT}) \tag{21}
\end{align*}
$$

where $\quad$ FOR $=$ forced outage due to factors other than utilization rate (proportion of hours per year)
VMNT = maintenance policy (proportion of user cost).
The maintenance policy is, in principle, considered a strategy to be searched over, but for purposes of this test it is constant. The values assumed for VMNT and other parameters of these functions are shown in Table 2.

## 4. Plant dispatching

Within each simulation period ( $\mathrm{DT}=.25$ year), a load duration curve is assumed and plants are dispatched in order of decreasing efficiency (as determined by cost of fuel needed to generate one kilowatt hour of electricity), assuming utilization does not exceed the user-specified parameter UCOPT. Any remaining load will be satisfied by further utilizing plants in increasing order of efficiency until the load is met or all plants are fully utilized. Then, any remaining load is met with purchased power. By setting UCOPT equal to or less than the capacity threshold, CAPTH, user cost on the more efficient plants is avoided. That is, the risk of forced outage on these base plants is minimized. If UCOPT is set to 1.0 , the more efficient plants will be used to capacity before less efficient ones are dispatched.
5. Exogenous variables

Some of the exogenous variables are assumed to be random variables reflecting the uncertainty in the problem, while others are deterministic. The deterministic random variables include the cost of capital, R , which is assumed to be a constant $11.75 \% /$ year for all plants, and the time cost or depreciation rate, D , which is assumed to be a constant $2.5 \% /$ year for all plants after the first year of acquisition and $75 \%$ during the first year. In addition, acquisition cost, ACCST, and fixed operating and maintenance cost, FOMC, are assumed to grow exponentially at 6.5 and $5.0 \% /$ year, respectively. Variable maintenance cost and disposal cost are each assumed to be $10 \%$ of acquisition cost, ACCST. Purchased power is assumed to cost $110 \%$ of the endogenous consumer price, CPR.

The random variables also are assumed to grow exponentially, with the growth rates (except in one instance) being drawn each year from given frequency distributions. The one exception is the relicensing cost, where it is the deviation from a baseline projection rather than the growth rate which is random. Data assumed for these distributions are given in Table 3.
6. Salvage time and replacement

The salvage time criterion used for coal plants (equation ll) is one which assumes a plant is replaced with one identical to it. It is further assumed that, when it is decided to salvage a plant, it is maintained in operation during the construction period of its replacement before actually being taken off line. Oil plants are assumed not to be replaced after retirement, except by preplanned plants as identified by the strategy (see below).

For this version of the model, a plant's value of services, VALSER, which is key to the salvage time criterion, is not computed according to equation 8 . Instead, as discussed above, the total gain from the operation of each plant (as given under the summation sign in the objective function shown in equation 6) is allocated to the services of the durable. Further theoretical development is necessary in order to isolate the portion of the gain attributable to the plant's services from that attributable to other inputs.

A moving average of the rate of change of value of services is computed and compared to the annualized average of the value of services (or, in the case of no replacement, the control cost) to indicate the salvage time. If the moving average falls below the annualized average (or control cost), the plant is salvaged. In addition, no durable being used at more than five percent of its capacity is retired unless it is to be replaced.
7. Control strategies and optimal search

For this test case, and since control laws are assumed for utilization rates, maintenance rates and salvage times, the control strategies subject to the optimizing search are the sizes and acquisition times of the alternative future plants under consideration. In addition, alternative control laws for utilization (i.e., dispatch rules) are tested. In total, ten strategies are tested, combining five sets of plant aquisition schedules and two dispatch rules.

Either four or five additional plants are included in each strategy, in order to add a total of 2400 megawatts of capacity to the system over the 40 -year planning horizon. Table 4 summarizes the combinations of plants tested.

One dispatch rule tested assumes that plants will be operated at full capacity in decreasing order of efficiency (UCOPT $=1.0$ ). The second rule (UCOPT $=$ CAPTH)

Table 3
Frequency Distributions for Random Exogenous Variables

|  | Variable | Base | Growth Rate (proportion/year) $\mathrm{x} / \mathrm{P}(\mathrm{x})$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Oil price (\$/barrel), PROIL | 25 | $\begin{aligned} & .03 \\ & .0 \% \end{aligned}$ | $\begin{array}{r} .09 \\ 20 \% \end{array}$ | $\begin{array}{r} .12 \\ 40 \% \end{array}$ | $\begin{array}{r} .15 \\ 10 \% \end{array}$ | $\begin{aligned} & .18 \\ & 20 \% \end{aligned}$ |
| 2. | Coal price (\$/ton), PRCOAL | 30 | $\begin{array}{r} .08 \\ 30 \% \end{array}$ | $\begin{array}{r} .10 \\ 40 \% \end{array}$ | $\begin{array}{r} .12 \\ 30 \% \end{array}$ |  |  |
| 3. | Load (mwh/yr), XLOAD | $\begin{aligned} & 6.5 \\ & \text { million } \end{aligned}$ | $\begin{array}{r} .01 \\ 25 \% \end{array}$ | $\begin{array}{r} .02 \\ 30 \% \end{array}$ | $\begin{array}{r} .03 \\ 25 \% \end{array}$ | $\begin{gathered} .04 \\ .0 \% \end{gathered}$ | $\begin{array}{r} .05 \\ 10 \% \end{array}$ |
| 4. | Deviation from baseline relic cost (proportion of baseline), |  | $\begin{aligned} & -.30 \\ & 10 \% \end{aligned}$ | $\begin{aligned} & -.10 \\ & 20 \% \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 40 \% \end{aligned}$ | $\begin{array}{r} .10 \\ 20 \% \end{array}$ | $\begin{aligned} & .30 \\ & 10 \% \end{aligned}$ |

Table 4
Plant Combinations in General Test Case Experimental Design

| Combination No. | Plant No. | $\begin{gathered} \text { Capacity } \\ (\mathrm{mw}) \end{gathered}$ | Acquisition Time | Combination No. | Plant <br> No. | Capacity (mw) | Acquisition Time |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 400 | 0 | 4 | 5 | 600 | 0 |
|  | 6 | 600 | 2 |  | 6 | 600 | 6 |
|  | 7 | 600 | 12 |  | 7 | 400 | 12 |
|  | 8 | 800 | 20 |  | 8 | 800 | 20 |
| 2 | 5 | 400 | 0 | 5 | 5 | 600 | 0 |
|  | 6 | 400 | 2 |  | 6 | 600 | 6 |
|  | 7 | 400 | 6 |  | 7 | 600 | 12 |
|  | 8 | 400 | 10 |  | 8 | 600 | 20 |
|  | 9 | 800 | 20 |  |  |  |  |
| 3 | 5 | 200 | 0 |  |  |  |  |
|  | 6 | 200 | 2 |  |  |  |  |
|  | 7 | 600 | 4 |  |  |  |  |
|  | 8 | 600 | 12 |  |  |  |  |
|  | 9 | 800 | 20 |  |  |  |  |

attempts to minimize user cost by operating the more efficient plants at or below the capacity threshold unless the less efficient plants are not capable of meeting load when operating at full capacity.

For each of the ten strategies, twenty states of nature are run, generating a cumulative distribution function of the objective function value J. The GREMP package reduces the ten distributions to an efficient set using the criterion of stochastic dominance with respect to a function. The results are presented in the next section.

## Results

This section discusses the results of the general test case with respect to l) the selection of preferred strategies and 2) the dynamic behavior of the simulation model.

## Preferred Strategies

Each of the ten strategies was simulated for 20 states of nature, and means and standard deviations of the objective function values were computed. Remember that, because the consumer price is endogenous as discused above, the objective is to minimize the discounted present value of costs paid by the consumer. Costs include time, user and control costs as well as operating, maintenance and variable input (fuel) costs. Table 5 summarizes the strategies and the outcome statistics.

Strategy \#l has the minimum expected (mean) cost and the second smallest standard deviation. If minimizing the expected value of the objective function were the decision criterion, Strategy \#l would be selected. The criterion used in GREMP, however -stochastic dominance with respect to a function -- considers other statistical properties of the outcomes as well in relation to assumed bounds on the decision maker's absolute risk aversion function, and it generally results in an efficient set of strategies rather than a unique optimum. In this general test case, the efficient set contains only one strategy -- Strategy \#l.

It is interesting to note, however, that the most costly strategy, Strategy \#8, has an expected value of its outcome only about $6 \%$ greater than that of Strategy \#l yet exhibits a standard deviation almost twice as large. This suggests that, for normally distributed outcomes, the odds are about one in five (about a $19.5 \%$ probability) that Strategy \#8, the "worst" in expected value, would have a better outcome than the mean of Strategy \#l, a better than one in twenty chance (about a $5.7 \%$ probability) of costing less than the least cost ( 5.788 billion dollars) experienced in the 20 samples of Strategy \#l, and an even chance of bettering the worst of Strategy \#1's 20 outcomes ( 6.451 billion dollars). The smaller standard deviation of Strategy \#l, on the other hand, gives it only about a $5 \%$

Table 5

## General Test Case Results

| Strategy <br> No. | Dispatch <br> Threshold <br> UCOPT* | Plant <br> Combination <br> No.+ | Statistics on Outcomes  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mean++ <br> (bill. $\$$ ) Std. Dev. <br> (bill.\$) Rank |  |  |  |  |  |
| 1 | 1.00 | 1 | 6.090 | .220 | 1 |
| 2 | 1.00 | 2 | 6.382 | .233 | 9 |
| 3 | 1.00 | 3 | 6.161 | .259 | 2 |
| 4 | 1.00 | 4 | 6.167 | .206 | 3 |
| 5 | 1.00 | 5 | 6.231 | .242 | 5 |
| 6 | .80 | 1 | 6.352 | .418 | 8 |
| 7 | .80 | 2 | 6.276 | .243 | 7 |
| 8 | .80 | 3 | 6.450 | .420 | 10 |
| 9 | .80 | 4 | 6.230 | .299 | 4 |
| 10 | .80 | 5 | 6.271 | .295 | 6 |

* UCOPT $=1.0$ means more efficient plants are used to full capacity before less efficient plants are dispatched.
$\mathrm{UCOPT}=0.8$ means more efficient plants are used at $80 \%$ capacity, in order to minimize user cost and forced outage, unless less efficient plants aren't sufficient to meet load.
+ See Table 4 for definitions of the plant combinations tested.
++ Discounted present value of costs, including input, maintenance, time, user and control costs.
probability of doing worse than Strategy \#8's mean, about a $78 \%$ chance of costing more than Strategy \#8's least cost outcome ( 5.924 billion dollars), and virtually no chance of doing worse than Strategy \#8's worst outcome ( 7.531 billion dollars). Pictorially, the two distributions are roughly compared in Figure 3, which constructs triangle distributions from the sample statistics.

It is also noteworthy that the five plant combinations tested rank differently depending on the dispatch threshold. That is, Combination \#l is best and Combination \#2 is worst when UCOPT $=1.0$, while UCOPT $=0.8$ results in Combination $\# 4$ being best and Combination \#3 being worst. On the whole, however, except for Plant Combination \#2, full utilization of the most efficient plants ( $U C O P T=1.0$ ) gives lower cost results than trying to reduce user cost (and hence forced outages) for those plants (UCOPT = 0.8). As indicated above, the differences are small. Nevertheless, such differences as these are due to the dynamic impacts of the dispatch rule on utilization rates and, thence, values of services and salvage times. Furthermore, these results depend on the particular user cost functions assumed. Other, possibly more realistic, functions could have different impacts.

## Model Dynamics

The dispatch rule determines plant capacity utilization by dispatching plants to full capacity from the most efficient to the least efficient until the load is met. Efficiency is calculated according to the cost of fuel, the heat content of the fuel and the heat rate of the plant. Furthermore, plant time utilization depends on an assumed down time for scheduled maintenance and forced outage, which is a function of user cost and maintenance regimes. Therefore, the plant utilization rates (the product of time and capacity utilization) provide a composite picture of the dynamic interations among the various plants of fuel costs, plant efficiencies, user costs, maintenance and load forecasts. Figure 4 shows the time paths of utilization rates for the eight plants of Strategy \#l over the $40-$ year planning horizon.

As new plants come on line following their construction lead times (i.e., at times 7, 10,20 and 29 for plants $5,6,7$ and 8 , respectively), existing plants experience a drop in utilization to the extent they are less efficient than the new capacity. The more efficient, new plants are then utilized to full capacity, allowing for scheduled and forced down time, while the remaining plants steadily increase in utilization to meet the increasing load.

Plants 1 and 3 are oil plants and are assumed to be retired without replacement when the current increment to value of services, averaged over a year, does not cover control


Figure 3
Triangle Distributions for Two Strategies


Figure 4
Plant Uitlization Rates
cost. Under this rule, plant 3 is retired at time 15.75. Plant 1 also meets this criteria at about the same time. However, since it is being used at a utilization rate greater than $5 \%$ and therefore presumably needed to meet load, it is retained. Conditions do not dictate retirement again until after plant 7 comes on line at time 20.00 , and plant $l$ is retired at time 24.25 .

The remaining plants are coal-fired and assumed to be replaced with identical units when the current increment to value of services, averaged over a year, is less than the annualized average of value services to date, the latter representing the value of the replacement. Under this criterion, plants 2 and 4 are replaced, following lead times for construction of the replacements, at times 17.50 and 37.75 , respectively.

## Summary and Conclusions

We have presented a preliminary model to operationalize theoretical developments integrating the economic theory of investment and disinvestment decision making with the decision analysis theory of decision making under uncertainty. The operational model presented is oriented to decisions regarding electric power generating capacity, although it is generalizable to other durable factors of energy production and distribution.

The decision problem is conceptually cast in an optimal control framework in order to reflect the dynamic and simultaneous features of decisions, over some planning horizon, concerning: 1) when to acquire and dispose of durable productive assets (i.e., generating capacity), 2) what sizes to acquire, 3) what rate services are to be extracted from them, and 4) when and how much they are to be maintained. While it would be virtually impossible to solve such a complex, nonlinear problem analytically, techniques are available for finding numerical solutions in the deterministic case. Our situation is decidedly not deterministic, however, because of the uncertainty part of our problem. Therefore, we have embedded the model in a software package (GREMP) that 1) systematically selects prespecified control strategies, 2) generates a distribution of the objective function value for each such strategy through Monte Carlo simulation, and 3) identifies the efficient set of distributions (i.e., strategies), based on the criterion of stochastic dominance with respect to a function.

Finally, a general test case is constructed to test and demonstrate the range of decision features of the model. The results of this test are discussed with respect to 1 ) the dynamics of the problem for a single strategy and state of nature, and 2) the overall "optimal" solution, which is an efficient set of one strategy.

Experience developing the model and specifying and evaluating the test case has suggested a number of avenues for further theoretical and model development and
experimentation. These are briefly enumerated below, grouped into those relating to the specifics of the test case and those concerning the basic model itself.

## The General Test Case

1. Investigation of and experimentation with the parameters and functional form assumed for the stochastic dominance criterion. The one used here resulted in an efficient set of only one strategy. Was this due to the criterion function or to the peculiarities of the test problem as specified?
2. Reconsideration and, where necessary, revision of particular assumptions in the test case to increase its realism and relevance. This would include, for example:
a) data on plant characteristics and costs,
b) the dispatch rule (e.g., see Booth, 1971),
c) the functional forms relating user cost, forced outage and variable maintenance to one another and to the rate of plant utilization, and
d) whether a control law could or should be specified relating plant acquisition time to load.

## The Basic Model

3. Revision of the equation computing the value of services accrued to each plant. Currently, the total economic gain generated by a plant is attributed to the services of the plant itself. Further theoretical development is necessary for the multiple durable case, however, in order to exclude the gain attributable to other inputs used in conjunction with the plant's services.
4. Reconsideration of the criteria determining the optimal salvage time for a plant, i.e., STIME in equation 11.
a) The derivation of the criterion for replacement with an identical unit assumes the value of services is well-behaved over time; specifically, that the secondorder conditions for maximization are met. This is not the case in general, primarily due to the interdependency of plants through the dispatch rule and other constraints imposed on a plant's operation. That is, the criterion was developed for a single durable. Further theoretical development is necessary to derive a criterion which considers the interactions among the multiple durables of a single firm which is constrained to produce a given level of output over time.
b) The criteria implemented here were derived assuming either no replacement or that the durable would be replaced with an identical unit which faces
identical price, cost and use patterns over time. Alternative criteria need to be derived for other cases, such as replacement with altogether different types of units.
5. Implementation of an algorithm to find a numerical solution to the deterministic optimal control problem. A software package to do so could be appended to the GREMP system. Such a solution would be instructive and useful to check the logic and realism of the problem specification and to compare with the set of efficient strategies resulting from the GREMP algorithm.

## References

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## PROGRAM INFORMATION

The mathematical model described in the previous section is implemented for simulation on the computer in program AIDU--Analysis of Investment and Disinvestment under Uncertainty. This section describes the operating environment and structure of AIDU and how to implement it on the computer.

## Program Description

## Operating Environment

AIDU is programmed in standard FORTRAN and has been implemented on Michigan State University's CDC computer as follows:

Machine: CDC Cyber 750
Operating System: MSU Hustler 2, LSD 49.56
Compiler: CDC FTN V4.8--P498
Core requirements: 12,269 decimal words
CP compile time: 1.724 CP seconds
CP execution time: 659.654 CP seconds for the general test case of 20040 -year simulations ( 10 strategies and 20 states of nature), for an average of 3.30 seconds/ simulation and 0.08 seconds/year.

## Program Structure

Program AIDU is structured in two nested DO-loops, as shown in Figure 5. The outer loop is for strategies, and for each strategy, the inner loop cycles over the states of nature. For each strategy and state of nature, subroutine SIMIDA (Simulation for Investment/Disinvestment Analysis) contains the simulation time loop.

The following sections describe the functions of the subroutines, while the symbolic variable names used in the program are defined in

Figure 5. AIDU Flowchart


Appendix 1. Subprograms are considered standard parts of the AIDU package unless indicated (by an asterik *) as being user-supplied particular to each application. The complete computer program, including user-supplied routines for the general test case described here, is listed in Appendix 2.

Investment/Disinvestment Analysis Routines
AIDU - Executive PROGRAM of model for the analysis of investment and disinvestment under uncertainty.

ACQUIR - SUBROUTINE which accounts for the acquisition and implementation of new durables.
*DATGEN - BLOCKDATA which contains projections of the growth rates of exogenous data.
*EXDAT - BLOCKDATA which initializes constant parameters and includes data relating to existing durables.
*EXOG - SUBROUTINE which computes exogenous variables.
*INDAT - SUBROUTINE which initializes both the variables common to the entire system and variables specific to individual durables.
*LODUR - SUBROUTINE which constructs a load duration curve for the system.

OBJECT - SUBROUTINE which computes the objective function and also depreciation, income taxes and the customer price of energy.
*OUTPUT - SUBROUTINE which prints detailed outputs from the model.
SALVGE - SUBROUTINE which evaluates durables for salvage and indicates replacements or retirement at salvage time.

SIMIDA - Executive SUBROUTINE of the investment and disinvestment analysis model.

STATES - SUBROUTINE which computes the state variables of the model.

| *STRAT | - SUBROUTINE which generates decision strategies for analysis <br> by the model. |
| :--- | :--- |
| *UTLMNT | - SUBROUTINE which determines utilization, maintenance and <br> energy dispatch for each of the durables. |
| STEP | - Utility FUNCTION which implements a step function. |
| TABEL | - Utility FUNCTION which implements a table look-up function. |

Stochastic Dominance Analysis Routines
NSTDO - Executive SUBROUTINE of the stochastic dominance analysis model.

CUMCAL - SUBROUTINE which generates a cumulative distribution from a sample of observations.

RORDER - SUBROUTINE which arranges a sample of observations in ascending order.

SD - SUBROUTINE which compares two functions according to the criterion of stochastic dominance.

UT

- FUNCTION which computes the upper bound of absolute risk aversion.

U2 - FUNCTION which computes the lower bound of absolute risk aversion.

UI1 - FUNCTION which computes the inverse of the upper bound of absolute risk aversion.

UI2 - FUNCTION which computes the inverse of the lower bound of absolute risk aversion.

## Program Implementation

AIDU may be implemented by providing 1) the user-supplied subroutines (indicated above) and 2) data, both of which define a particular application. Job control parameters--i.e., defining the size of the experiment, print
options, etc.--are specificed on data cards read in the main program as input from logical unit 5 at the start of each job. These are specified below, after which all other data requirements, which are defined in DATA statements and assignment statements, are described. Finally, the outputs generated by AIDU are outlined.

Job Control Parameters: Data Card Formats and Descriptions.

| Card | Columns | Format | Description |
| :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & 1-5 \\ & 6-10 \end{aligned}$ | $\begin{aligned} & \text { I5 } \\ & \text { I5 } \end{aligned}$ | NSTRAT - Number of strategies to be evaluated <br> NSAMP - Number of sample observations (states of nature) |
| 2 | $\begin{gathered} 1-5 \\ 6-10 \\ 11-15 \end{gathered}$ | $\begin{aligned} & \text { F5. } 2 \\ & \text { F5.0 } \\ & \text { F5.0 } \end{aligned}$ | DT - Simulation time increment in years <br> BTIME - Initial time of simulation <br> ETIME - Ending time of simulation |
| 3 | 1-5 | I5 | IPRT - Switch for detailed model output $\begin{aligned} & (0=0 f f) \\ & (\neq 0=O n) \end{aligned}$ |
| 4 | 1-5 | I5 | NDUR - Number of durables |

For the general test case sample run, these parameters had the following values:

```
NSTRAT = 10
BTIME = 0.0
IPRT = 1
NSAMP = 20
ETIME = 40.0
NDUR = 10
```

$D T=0.25$

## Internal Data

Values for all other parameters, initial conditions, and exogenous variables are defined within the program in DATA statements or assignment statements. The lists below describe each variable requiring such definition
(for the general test case) and the appropriate unit of measure, and give the routines where it is defined. The first list is for data general to the system being modeled, while the second list is for data required for each durable modeled. The values used in the general test case are shown in the program list in Appendix 2.

General Data

| Variable Name | Routine | Description |
| :---: | :---: | :---: |
| ALPK | EXDAT | Minimum load as a proportion of peak load |
| FUELC | EXDAT | Heat content of fuels (Btu/unit) |
| PRCOAL | INDAT | Initial price of coal (\$/ton) |
| PROIL | INDAT | Initial price of oil (\$/barrel) |
| R | EXDAT | Opportunity cost (proportion/year) |
| RA | AIDU | Boundary coefficients for absolute risk aversion analysis |
| RETINV | EXDAT | Authorized return on capital base (proportion/year) |
| RHO | EXDAT | Discount rate (proportion/year) |
| TAXRAT | EXDAT | Income tax rate (proportion/year) |
| XALIC, XVLIC | DATGEN | Probability distribution of random coefficient of relicensing cost |
| XALD, XVLD | DATGEN | Probability distribution of growth rate of load |
| XAPRC, XVPRC | DATGEN | Probability distribution of growth rate of price of coal |
| XAPRO, XVPRO | DATGEN | Probability distribution of growth rate of price of oil |
| XLOAD | INDAT | Initial demand for electric power (MWH/year) |

Durable-Specific Data
In general, initial conditions and characteristics for durables existing at time zero are given in EXDAT, and data characterizing durables which may be acquired later are specified in STRAT, where they may be strategy-dependent.

| Variable | Defined in Routine | Description |
| :---: | :---: | :---: |
| ACCSTO | EXDAT, STRAT | Acquisition cost in present dollars (\$/MW) |
| ACQTO | EXDAT, STRAT | Acquistion time (year) |
| CAPO | EXDAT, STRAT | Rated capacity (MW) |
| CAPTH | EXDAT, STRAT | Capacity threshold as proportion of boilerplate rating |
| DEPLIF | EXDAT, STRAT | Estimated life for purposes of depreciation (Years) |
| FOMCO | EXDAT, STRAT | Fixed operating and maintenance costs (\$/MW/year) |
| FOR | EXDAT, STRAT | Forced outage rate (proportion) |
| HEATR | EXDAT, STRAT | Heat rate (Btu's/MNH) |
| INTP | EXDAT, STRAT | $\begin{array}{lll}\text { Input Type } & \text { 1. } & 0 i 1 \\ & \text { 2. } & \text { Coal }\end{array}$ |
| ISTAT | $\begin{aligned} & \text { EXDAT, } \\ & \text { INDAT, } \\ & \text { STRAT } \end{aligned}$ | Initial Status of Durable <br> -1 Pre-construction <br> 0 Retired <br> 1 Under construction <br> 2 Active <br> 3 Active with replacement under construction |
| PSCRP | EXDAT, STRAT | Scrap value as proportion of acquisition cost |
| PTIRAT | EXDAT, STRAT | Property tax and insurance rate as proportion of acquisition cost |


| RCOSTO | EXDAT,STRAT | Cost of relicensing (\$) |
| :--- | :--- | :--- |
| SCHED | EXDAT, STRAT | Scheduled outage rate (Proportion) |
| TLEAD | EXDAT,STRAT | Lead time for design and construction (Years) <br> TLIC |
| UCEXP | EXDAT, STRAT | Period of time for which operating license <br> is valid (Years) |
| UCMAX | EXDAT,STRAT | Parameter for computation of user cost <br> (No units) |
| UCOPT | EXDAT, STRAT | Parameter for computation of user cost <br> (No units) |
|  |  | Optimal operating rate as proportion of <br> boilerplate rating. |

Outputs Produced
All output is written on logical unit 6 . The standard output described below is always printed, while the optional output is given only for IPRT $\neq 0$. Appendix 3 contains samples of these outputs for the general test case.

Standard Outputs

The model produces a number of standard outputs for each execution. The outputs from the analysis of each strategy is preceded by a short description of the strategy: the capacity, acquisition time and maintenance policy for each durable.

Each strategy is simulated under a number of states of nature and a summary of each simulation is printed. It includes a message issued at the acquisition time, implementation time and salvage time of each durable.

The message gives the capacity of the durable, its accumulated value of services, the rate of change of the value of services and the annualized average of the value of services at that instant of time. At the end of the run, the same information is printed for all active durables along with the value of the objective function.

At the end of the run, a list of the strategies in the efficient set is printed. Each of the efficient strategies is identified and the mean and standard deviation of the sample distribution is printed.

## Optional Outputs

More detailed outputs may be obtained via the output switch IPRT. The status of each durable and selected variables pertaining to the electric power system (for the general test case) as a whole are printed at each time interval. This option is designed for use in debugging and is not recommended in general since it will lead to voluminous outputs.

Currently, detailed outputs include, for each durable, its capacity salvage value, accumulated value of services, instantaneous rate of change of value of services, annualized average of value of services, production, utilization rate, variable input use, and maintenance. System variables include the objective function, load, purchased energy, customer price, purchased energy price, and prices of variable inputs.

## APPENDIX 1

## VARIABLE DEFINITIONS

The FORTRAN variable symbolic names used in AIDU are defined in this appendix, and units of measure are given where appropriate. The variables are grouped by labeled common block and are listed alphabetically within each group. Variables specific to the general test case application are indicated by an asterisk (*).
/CNTRL/, Control variables

| Variable | $\frac{\text { Description }}{\text { BTIME }}$ |
| :--- | :--- |
| DT | Beginning time of model (year) |
| ETIME | Time interval |
| IPRT | Ending time of model (year) |
|  | Print switch <br> 0 <br> 1 Summary output only |
|  | 1 |


| Variable | Description |
| :---: | :---: |
| ACCSTO | Acquisition cost in current dollars, \$/megawatt |
| ACQTO | Scheduled acquisition time |
| CAPO | Boiler plate capacity, megawatts |
| *CAPTH | Capacity threshold, proportion of boilerplate capacity at which user cost will begin to accrue |
| DEPLIF | Lifetime for purposes of depreciation, years |
| FOMCO | Fixed operation and maintenance costs in current dollars, \$/megawatt |
| *FOR | Forced outage rate, proportion |
| *HEATR | Heat rate, Btu's/megawatt-hour |
| IDDUR | Durable indicator |
| IMNT | Maintenance policy, l=Maintain durable to proportion of capacity represented by VMNT |
| *INTP | $\begin{array}{r} \text { Fuel type, } \\ 1 \text { 0il } \\ 2 \text { Coal } \end{array}$ |
| IREPLC | Replacement policy <br> 0 Retire with no replacement <br> 1 Replace with identical unit |

Number of durables

NDUR
PSCRP
*PTIRAT
SCHED
*TLEAD
*TLIC

Scrap value, proportion of acquisition cost
Property tax and insurance rate, proportion of acquisition cost

Relicensing cost in current dollars
Scheduled outage rate, proportion
Lead time for design and construction, years
License period, years
/DURDL/, Lagged variables related to durables

Variable Description
DELAY Lagged value of DVALS
IDEL Number of delay periods in storage
NDEL Length of delay, number of time periods
/DURDS/, Decision and state variables related to durables

Variable Description
ACQT
ACQTR
ANAV
*DEPBAS
*DEPVAL Book value of durable, dollars
DGCAP
DVALD
DVALS
FOM
Acquisition time of present durable
Acquisition time of replacement durable
Annualized average of value of services, \$/year
Base for depreciation, dollars

Rate of change of value of services, \$/year

Rate of change of capacity of durable, megawatts/year
Rate of change of value of durable, \$/megawatt/year

Fixed operating and maintenance costs adjusted for inflation, \$/megawatt

GCAP Present capacity of durable, megawatts
PROD Production of energy, megawatt-hours/year
*PTIBAS Base for property tax and insurance, dollars
SALVAL Salvage value, \$/megawatt
*TAX Income tax, dollars

U

USCST
*UTH Increase in forced outage rate due to use, proportion of total time
*UTILC Average capacity utilization during operating time, proportion
*UTILT Time utilization, proportion
VALDUR Value of durable, \$/megawatt
VALSER Accumulated value of services, dollars
VIUSE Variable input use, units/year
VMAINT Variable maintenance, megawatts/year
/DUREX/, Exogenous variables related to durables
Variable Description
ACCST Acquisition cost, \$/megawatt
D
Depreciation rate, proportion/year
DCOST Decommissioning cost, \$/megawatt
FOMC Fixed operating and maintenance costs, \$/megawatt
*RCOST Relicensing cost, dollars
VICST Variable input cost, \$/megawatt
VMNTC Variable maintenance cost, \$/megawatt
/SYSEX/, Exogenous variables related to electric power system
Variable Description

CPR Customer price, \$/megawatt-hour
*FUELC Fuel constant, Btu's/unit
HPY Hours per year (8760).
PPR Price of Purchased energy, \$/megawatt-hour
*PRCOAL Price of coal, \$/ton
*PROIL Price of oil, \$/barre1
$R \quad$ Cost of capita1, proportion/year
RETINV Authorized return on capital base, proportion/year
RHO Discount rate, proportion/year
*TAXRAT Income tax rate, proportion
*XLOAD Energy demand, megawatt-hours/year
/SYSVR/, System variables
Variable Description

OBJ Objective function, dollars
*PINRG Purchased energy, megawatt-hours/year
*TPROD Total energy production, megawatt-hours/year
/LDMIC/, Parameters for load duration curve
Variable Description
*ALPK Minimum of load duration curve as a proportion of peak load
*CCPK Ratio of capacity to peak load

| *NPNT | Number of points in approximation of load duration curve. |
| :---: | :---: |
| *PPKADJ | Approximation of load duration curve, megawatts |
| *XLDPK | Peak load, megawatts |
| /EXODAT/, Data for e | exogenous variable projections |
| Variable | Description |
| *KLD | Number of data points in load growth probability distribution |
| *KLIC | Number of data points in license cost probability distribution |
| *KPRC | Number of data points in coal price probability distribution |
| *KPRO | Number of data points in oil price probability distribution |
| *XALD | Arguments of load growth probability distribution |
| *XALIC | Arguments of license cost probability distribution |
| *XAPRC | Arguments of coal price probability distribution |
| *XAPRO | Arguments of oil price probability distribution |
| *XVLD | Load growth probability distribution |
| *XVLIC | License cost probability distribution |
| *xVP只 | Coal price probability distribution |
| *XVPRO | Oil price probability distribution |
| /STODOM/, Variables | used in stochastic dominance analysis |
| Variable | Description |
| AMEAN | Mean values of strategies in efficient set |
| C | Temporary array used in stochastic dominance analysis |

CP Temporary array used in stochastic dominance analysis
IDSTRT Identification of strategies in efficient set
NEFF Number of strategies in efficient set
NPNT Pointer to next candidate for efficient set
R
Distributions of strategies in efficient set
STD Standard deviations of strategies in efficient set
$R \quad$ Distributions of strategies in efficient set
/STDPAR/, User-supplied parameters for stochastic dominance analysis Variable Description
RA
Boundary coefficients for risk analysis

## APPENDIX 2

## PROGRAM LIST

The complete program of AIDU as executed on the CDC computer at Michigan State University is listed on the following pages. This program generated the general test case described in the text and the outputs presented in Appendix 3.

```
    FROGRAM ATHU(INOLT, TUTFUT,TAFES=INEUT,TAFEO= CUTPLT,TAFFA*)
                            ANALYSIS OF INVESTNENT AND DISTNVESTMENT UNDER UNCERTAI'ITY
```



```
    CCMMCN /STDFAR/ RA(2)
    COEFFICIENTS FOR STOCHASTIC DOMINANCE
    OA-SOUNDARY COEFFICIENTS FOR PISK ANALYSIS
    DATA RA,' ZNUMEEP OF EFFICIENT STRATEGIES
    NFNT - POINTER TO CANDIDATE FOR EFFICIENT SET
    DATA NEFF,NFNT/1,1/
    READ SIMULATION PARAMETERS
    NUMBER OF STRATEGIESG NUMBER OF SAMPLE OESERVATIONS
    DEADTME INTERVAL,GEGINNING AND ENDING TIMES
    READ(S,1CC1) DT,GYIME,ETIME
    RFAD(S -SUMMARY 1-DETAILED
    READ(5,1 NCO) NDUQ
```

    SIMULATE NSTRAT DIFFERENT STRATEGIES
    DC 4 RCIISTRAT=1 NSTRAT
    \(\times\) RAN \(=\).
    STRAT FRODUCES A STRATEGY FOR EVALUATION EY THE MODEL
    CfLL STRAT
    LCOP FOR NSAMP DIFFERENT STATES CF NATUPE
    DC \(20 C\) ISAMP \(=1\), NSAMP
    SIMULATICN OF A STRATEGY GIVEN A RANOOM STATE OF NATURE
    WRITE(6,2000) ISAMP
    SIMULATE TVVESTMENT - DISINVESTMENT ANALYSIS
    CALL SIMIDA ( ORJ)
    RPITE (S, 2 OA1) ODJ
R(NPNTSAMP) $=$ OBJ / 1.EO
EVALUATE STRATEGY USING CRITERION OF STOCHASTIC DOMINANCE
PRINT EFFICIENT SET

WPITE (6,2SO2) ISTRATGICSTRT(ISTRAT), AMEAN (ISTRAT), STD (ISTRAT)

## CONTINUE

PRINT EFFICIENT SET

FCRMAT（2I5）
FCRNAT（F5．2，2F5．C）
FORMAT $6 H-I S A M F$ I 2 ）
FCRMAT（ 4 HOOQJ，



EVALUATE STRATEGY USING CRITERION OF STOCHASTIC DOMINANCE

```
        IF (TRNSAMP (NPNT) = ISTRAGO TO 400
        IOSTRT(NFNT) =
        CALL ASTCO(NSAMP),(R (NPNT,J),J=1,29)
```

```
    SUBPIUTIVE ACQUTR
```

        ACQUTSITICN OF DURABLES (USER SUPPLIED)
    CCMMON /CNTRL/
    + 
+ CCMMCN/DURCV/
+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 




## IF（TSTAT

## FUT NEWLY ACQUIRED DURABLES INTO OPERATIOV



```
WF(TGNG,NE, (ACNTR(IDUR) +TLEAD(IDUO))) GO TO 14%:
```



```
C ISTATSIDURSGOTSTTIOM CCST FOR USE AS FROPERTY TAX ANO INSURANCE
    PASE AND CCMPUTE BASE FOR DEPRECIATIOY
    PTIGAS(IDUQ)=ACCST(IDUR) * CAPA(IDUP)
    MEPEAS(INIDR)=
    GCAP(IDUR) = CAPC(IDUR)
    VALDUR(IDUR)=ACCST(IDUR)
    VALSER(IDUR) = ACOTR(IDUR)
        ACQT(IDUR) = ACQTR(IDUR)
        SALVAL(I\capUR)= YALDUR(IDUR)*GCAP(IDUR)-DCOST(IDUR)*CAPA(IDUR)
    UTH(IDUR)= =.
WRITE(6,GH
    WRITE(6,1Q13) GCAF(IDUR), VALSER(IDUR),ANAV(IDUR), OVALS(IDUR)
```

1012
4.13
140
DETURM
END

```
    OLOCK DATA OATGEM
        CATA FCR RANCCM VARIABLES (USER SUFFLIED)
    CCMMON/EXOOAT/ KLO,KLIC,KPRC,KPRO,OMRC(5),XAFRC(5),
    DATA KLD,KLIC,KFRC,KPRO, 5, 5, 3, 5 /
    DATA XALD / CO, -25,.55,.8.,.9%1
```



```
    DATA XALTC /, %O,.10,7%,%i.1:1.3/
    OATA XVORC / &&&, 10% %-12,/
```



```
    OATA XVFRO/.0};.0.9.12,.15,.18/
    FND
```


## TLOCK DATA EXDAT

$\quad$ CCMMON / CNTRL/
+
+
+
+
+
+

+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 
+ 





ISTAT(1CS, PROR(1, SALVAL(19), SU(1?),
VALDUR (1) ) DVALD (10), VALSER (1C), DVALS(10),
VIUSE (1), VMAINT (1,
$+$
$A C C S T(1 \cap)$
$F C M C(1, ~ R C O S T(1 O), V T C S T$


+ COMMCN /LOMICI
CCNSTANT FARAMETERS
DATA HPY 1876ニ.I

CATARATLATING TO FLANTS
DATA YDUR $\quad$ Q 1

END

```
    SUBRCUTINE EXOG(IDT)
    EXOGENOUS DATA (USER SUFFLIED)
```

        + COMMON/CNTRL/ BTIME, NTIME,NDT, DT,TIME,ISTRAT.
        + CCMMCN /DURCV/
    + 
+ 
+ 
+ 
+ 

$+C M M O N ~ / D U R D S / ~$



+ CCMMON/DURREX/

+ CCMMCN /SYSEX/
$+{ }^{+}$CCMMON /EXODAT/
+ 


+ CCMMCN /DURDS/
+
+
+
+

CFR, FUELC(5), HPY,
IFP (IDT $=E \dot{Q}$ 1) GC TO 5
IF (ID T EQQ $C$ (1) GC
XRAN STOCHASTIC VARIABLES COMMON TO THE SYSTEM
$\times R A N=R A N F(-1)$
FROIL $=$ FROIL
FROIL $=$ FROIL * (1.+STEP (XVPRO,KFRO, XAFRO,XRAI) *DT)
ORCDAL $=F R C D A L \star(1 .+S T E F(X V F R C, K D R C, X A P R C X X A N) * D T)$
XLDAD $=X L O A D$ * (1.+STEP
ALPHA $=$ STEF (XVLICGKLIC,XALTC XXAN)
CETERMIVISTIC VARIABLES COMMDN TO THE SYSTEM
CONTINUE
VARIAELES SPECIFIC TO PLANTS
OC V VARIAELES SPEC
$F C M C(I D U R)={ }^{\prime} F C M C O(I D U R) \star \operatorname{EXF}(.35 \star T I M E)$
© (IDUR) $=.025$
IF(TIME $\left.{ }^{\circ} E Q(I D U R)=(A C Q T R(I D U R)+T L E A D(I D U R))\right)$
$\operatorname{ACCST}(I O \cup R)=A C C S T(I C U R) * \operatorname{EXF}(. C 65 * T M E)$
DCDST (IOUR) $=1$ A*ACCST (IDUR)

COST (IOUR) $=$ RCOSTO (IOUR) * ALPHA * EXP (.O5*TIME)
IF(INTP(IDUR) EQ. 2 ) VICST(IDUR) = PRCOAL
CONTINUE

```
    SUBRSUTIVE IVDAT
    CATA INITIALIZATICN (USER SUPFLIEN)
    + COMMON/CNTRL/ RTIMFQETIME,NDT&DT,TIME,ISTRAT,
    + NSTRAT,MSAMP,IPRT, XRAN,NDUR
```



```
    +CCMMCN /OURCV/
    +}
    +}
    +}
    +}
    +}
    INITIALIZATICN OF VARIABLES COMMON TO THE ENTIRE SYSTEM
    CPR=45.
    PROIL = 25
    PRCCAL = 3.0
    XLOAO = 6.5E6
    OC2 INITIALLIZATICN OF PLANTS
    ISTAT (IDUR)'= ?
    OC 4 TOUR=S,NDUR
    ISTAT(IDUR)'=-1
    OCIM IDUR=1,NDUR
    OELAYIT,GDUR
    \COST(IDUR)= = 1C** ACCST:(IDUR)
    ACQTR(IDUR)= ACQT(IDUR)
    GGAF(IDUR)=?.
    SALVAL(IDUR)="C.
    DVALS (IDUR)=*
    VMAINT(IOUR)=
    ANAV(IDUR)= =.
    OC 2NTTIALTZATITN OF ACTIVE PLANTS
    IF(ISTAT(IDUR) NE, 2) GO TO 2O
    DCOST (IDUR)=.1O**ACCSTQ(IDUR)
    VALDUR(IDUR) = 10** ACCSTO(IDUR)
    SCAF(IDUR) = CAP\cap(IDUP)
    VALSER(I\capUR)=-R*VALDUR(IDUR)
    OTIRAS(IDUR) = ACCSTA(IDUR) *CAFO(IDUQ)
    DEPEAS(IIUUR) = ACCSTO(IDUR) * CAP)
    DFPVAL(IDUR)= 5CN OEFEAS(IDUR)
    UTH(TOUR)= & . 
    2n CSCST(INUR) = Co.
    RETURN
```


17 $P F K A D J(I P N T)^{\prime}=A L P K+(1 .-A L P K) \star P P K N O R(I P N T)$
$A P E A=$
$\triangle C D C D T=1, ~ N F N T 1$
$A P E A=A R E A+(F F K A D J(I P N T)+P F K A D J(I P N T+1)) / 2$. *

+ (HPY/FEOAT (NPNT1)
$X L D F K=X L O A D / A R E A$
39
SUBRCUTINE LODUR
LODUR CONSTRUCTS A LOAD DURATION CURVE
ITS INPUTS INCLUDE THE LOAD AND BASE LOAD

```
    SUBRSUTIVE OEJECT ( CBJ)
    \capQJECT CDMPUTES THE OBJECTIVE FUNCTION
```

    + CMMMOY /CNTPL/ ETIMEEEINE NDT UTATIMESTSTRAT,
    

CTMMCN/OURDS/
$+$
+
+
+
+
+
+CCMMON / DUREX/
+CCMMON / SYSEX/

+ CCMMCN /SYSVR/
CCMMCN /SYSVR/ F
CIMENSICN DED (1: )

```
    OT 5% IDUR=1,NDUR
```


DEPRAT= (DEPLIF (IDUR)-(TIME-(ACQT (IDUR)+TLEAD(IDUR))-DT)) /
+ (DEFLIF(IDUR)*(DEPLIF (IDUR)+i.)/2.)
DEP (IDUR) = DEPAAS (IDUR) * DEFRAT

CEPVAL (IDUR) $=$ AMAX1 (DEFVAL(IDUR), $)$
CONTIVUE

## COMPUTE CUSTOMER FRICE, CFR

$$
\begin{aligned}
& \text { SUM } \\
& \text { DC } \\
& \text { IFCI } \\
& \text { SUM } \\
& +\quad+ \\
& +\quad+ \\
& \text { CRNT } \\
& \text { SUM } \\
& \text { CFR } \\
& \text { FFR } \\
& \text { RUC } \\
& \text { REV }
\end{aligned}
$$

    CFRD =
    OCIZCTDUR=1, NDUR
IFISTAT(IDUP) •LT. 2) GO TO 130
TAX (IDUR) $=(C F R \star H P Y \star U(I D U R) \star G C A F(I D U P)-V I C S T(I D U R) \star V I U S E(I D U F)$
+ - VNNTC(IDUR) *VMATN (IDUR) - FOMC (TOUR) *CAFC (TDUR)
+ - PTIRAT(IDUR) *PTIEAS(IDUR) - DEF (IDUR)) * TAXRAT
IF((TIME-ACQT (IDUR)) NE. TLIC(IDUR)) G2 TO 125
$R L C=P L C+R C D S T(I D U P)$

COMPUTE OBJECTIVE FUNCTION - FADT?
$O B J=O B J+E X F(-R H O * T I M E) *((R E V+(C F R D-P F R) * P I N R G) * D T-R L C)$
RETURN
EMO


[^0]


[^1]SUERCUTINE SIMIDA（ DEJ）


| CCMMCN／CNTRL／ |
| :--- |
| + |
| + CCMMCN／OURCV／ |
| + |
| + |
| + |
|  |
| CCMMON／OURDL／ |
|  |


INITIALIZATICN

```
CALL INDATME-BTINE)/DT + 1.OO1
TIME = BTIME - DT
OES = 0.
```

TIME LOOF
CALL OUTHD
CCS CC IDT＝1，NOT
ITME TIME＋DT
IT IDT ．EQ．1）GC TO 85
COMFUTE OBJECTIVE FUNCTION
CALL ORJECT（ OBJ）

```
    UPDATE STATE VARIABLFS
```

CALL STATES
EXOG DRTAINS VALUES OF EXOGENOUS VARIARLES FCR THIS OT
CALL EXOG（IDT）
IF IDT．EQ：1）GO TO 87
CALL SALYGE
CCNTINUE
CAL ACQUTR
COMPUTE UTILIZATION BY DURAELE AND DETERMIVE MAINTENANCE
CALL UTLMNT（ OBJ）
CALL OUTPUT（ OBJ）
WRITE（6，1C14）TINE

1915
1913
WRITC（6 1615 ）IDUR
FORMAT $8 \mathrm{X}, 5 \mathrm{HPLANT}, \mathrm{I} 3)$

600
＋CCNTINUE




```
    SURRCUTIME STATES
        compute state variables in the systen
        CCMMON /CNTRL/
        +}+\mathrm{ CCMMON /DURCV/
        +
        CCMMCN /OURDL\prime
        + CMMCN /OURDS/
        +
        DC 1:O IOUR=1;NDUR
    DC 1%O IOUR=1,NDUR 2 (ISTAT(IDUR) .LT. 2) GO TO 100
        UPDATE VALUE CF SERVICFS
    DVALS(IDUR)= CFR*HFY*U(IDUR)*GCAF (TDUR) - VICST (IDUR)*VIUSE(IDUR)
    + + (VALCUR(IOUR)-VMNTC(IDUR))*VMATNT(IDUR) - VR*SALVAL (ICUR)
    + + (VALCUR(IDUR)-VMNTC(IDUR))*VMAANT(IDUR) - VR*SALVAL (IDUUR)
+ - FOMC(IDUR)*CAPO(IDUR) - PTIRAT(IDUR)*PTIEAS(IDUR) - TAAX(IDUR)
    - FOMC(IDUR)*CAAPO(IDUR) - PTIRAT(IDUR)**PTIE
+ *EXP(-RHO*(TIME-ACOT(IDUR)))
    IDEL (IDUR) =MINC(NDEL,IDEL(TDUR)+1)
    OC 3G (I=2,NOEL (IDUQ)= DELLAY(IGIDUO)
            UFDATE CAPACITY
        OGCAF(IDUR)= VMAINT(IDUR) - UUSCST(IDUR)
            UFDATE MARKET VALUE CF DURABLE
        DVALD(IDUR) = -D(IDUR) * VALDUR(IDUR)
        DVALD(IDUR) = -D(IDUR) **VALDUR(IDUR)
        CONTINUF
            compute salvage value of flants
        DC 35OTIDUR=1, NDUR 2) GC TO 35A
```



```
        RETURV
        RETURV
```



```
S
```





## SUARCUTIVE STRAT

STRATEGY GENERATIC: (USER SUPPLIED)


CATA FOR FROJECTED FLANT SIZES


CATA FDR STRATEGIES


UL $={ }^{80}$ STRAT ISTRAT -5
IF (KSTRAT GTGO) GO TO 10
$K S T R A T=$
$U L S T R A T$
CONTINUEO




SURRCUTTNE UTLMNT 60


CETERMINE FRCCUCTICN FOR EACH FLANT
INTTIALIZATICN
OC 1 I IDUR= TNOUR
U(IDUR) $=$ ?
UTILC (IDUR) $=0$.
UTILT(IDUR) $=0$.
PRDD(IDUR) $=6$.
VIUSE (IDUR) $=0$.
VIUSE (TDUR) =
USCST (TDUR)
IFRISTAT (IDUR)
F
FDRC (IDUR) = FCR(IDUR) + UTH (IDUR)
UTH $(I D U R)=1 .-\operatorname{SCHED}(I D U R)-F O R C(I D U R)$
AVL $) ~$
$X F T=1 \cdot 1$ FLOAT (NPT)
$A \cap G=-.5 * X P T$
COMPUTE VARIAQLE INPUT COST PER UNIT OF PRODUCTIOY
DC 12 IDUR=1, NDUR
IEFF 1 IDUR) $=$ IDUR
C(IDUR) $=$ (HEATQ (IDUR) /FUELC (TNTP (IDUR))) *VICST (IDUR)
SORT PLANTS RY OROER OF EFFICIENCY
ID 18 IDUR=1, NDUR1
IDUR1 = IDUR + 18 IDUR=IDUR1, NDUR
DC 18 IN

$x=C(I D U R)$
$C($ IDUR $)=C(J D U R)$
$C(J O U R)=X$
$I=I E F F$ (IDUR)
TEFF (IDUR) $=$ IEFF (JDUR)
IEFF (JDUR) $=$ I
CONT INEE
ONSIDER LOAC DURATION CURVE IN OPT SEGMEVTS

$A R G=A D G+X P T$
$X L=$ TABEL (PPKADJ, 21 CONSDER CLANTS IN C5, ARG)
DO 2 CKDUR=1, NDUR IS IN ORDER OF EFFICIEYCY


[^2]KOUR
OT SO LDUR=1, NDUR
IF (ISTAT (IDUR) OTLG?) GO TO
TMP(IDUR) $=$ TMF (IDUR) $+X$
XLO $X$ - $X$ *AVL (IDUR)
IF (XL IF . ) GC TO 55 + $x * A V L$ (IDUR)
$X S H R T=X S H R T+X L$
IF (ISTAT (IDUR) - LT TM) GO TO S? ${ }^{2}$ )

CrNT ACEXP (ICUR) *XT*AVL (IDUR)
AOJUST PROCUCTION FOR LINEAR ESTIMATION ERRORS
CCT $=$ IDUR=1, NDUR
TCT $=$ TOT + PRCD
TEROC $\overline{\text { OR }}$ IDUR $=1$, NDUR

U(IDUR) =FROO (ICUR), (GCAP (IDUR) *YPY)
UTILC (IDUR) $=$ U(IDUR) $/$ UTILT (IOUR)
VIUSE (IDUR) = HEATR(IQUR) * (1./FUELC(IVTP(IDUR))) * FROD (IOUR)

IC 1 O IDUR=1 NDUR
IF ISA (IDURS -LT. 2) GO TO 100
USER CNST
USCST (IDUR) = UCMAX (IDUR) * UTH(IDUR) * GCAP(IDUR)
CCMPUTE ENERGY PURCHASED OR SOLD
FINRG $=$ XLOAD - TFROD
CAPACITY/PEAK RATIO
$T C A F=?$.

CFPK = TCAF $\quad \times$ LDFK

```
FUNCTION STFF(VAL,K,ARG,XARG)
DTMENSION FUNCT(K), ARG(K)
OUM = XARG
STEF=V\hat{L}
    TE(CUM VITG ARG(I)) GO TO 2N
    STEP = V
    CCNTINUE
    RETURN
```

$\frac{17}{25}$
$\begin{array}{ll}16 & 1 \\ 16 & 2 \\ 16 & 7 \\ 16 & 4 \\ 16 & 5 \\ 16 & 6 \\ 16 & 7 \\ 16 & 8 \\ 16 & 9 \\ 1611\end{array}$

```
    FUNCTION TABEL(VAL,K,SMALL,DIFF,XARG)
    OIMENSION YAL(K)
    DUM = XARG - SMALL
    IF(DUM.LT.O.) DUM = O.
    IF(DUM.GT.FLOAT(K-1)*DIFF) DUM = FLOAT (K-1)*OIFF
    IF=DUM. & CUM/DIFF
    YABEL\stackrel{EQ VAL(I)= K (VAL(I+1)-VAL(I))/DIFF) *}{=}
+ (DUM-FLOAT(I-1)*DIFF)
?ETUON
```

```
    SUBRCUTINF SSTCO(NSAMP)
    + NON/STOCOM/ NFF,VFVT,T(2O),ICSTRT(65),R(65,20),
    OIMENSION CE1(21),CE2(21:,LFLAEO(21)
    TFI = NSANF
    TFLAG=?
    20 \FLLAG(T)=1, \21
    \triangleMEAN(NDNT) =0.C
    OND=ND
    DCR1自 J=1:ND
    ANEAN (NPNT)=AMEAN(NPNT)+(1,/BND)*R(NFNT,J)
    VAR=VAR+(4,O/BND)*R(NFNT,J)*O(NPNT,J)
    T(J)=R(YPNT,J)
101 CONTINUE
    VAR=VAR-ANFAN (NFNT)*AMEAN (NPNT)
    STO(VFNT)=SQRT(VAR)
    WRITE (665O) IDSTRT (NPNT), AMEAN(NPNT), STD(NFNT)
    CALL RORDER (NSAMF)
    DC1GZ J=1 N(NO
102
    CONTINÚE
        81=0.0
        82=, = % k=1, 10
    R1=R1+(1./RND)*U1(R(NPNT,K))
    CONTINU!
1.5 CNNTINUE
        CE1(NPNT) =UI1(年1)
    IF(NFNT.FQ.1) GO TO 115
    I=NONT
    DC1 1O J=1,NEFF
        NF(CE1(I).GECCE1(J).AND.CEZ(I).GE.CEZ(J)) GO TO 6O
    NSD=C
    S0 CALL CUMCAL (JAINFSAMF)
    GO CALL CUMCAL (JAINFSAMF)
    IF(EU&GE.N'&)CGOTTO GZ,.CEZ(I).EQ.CEZ(J)) 60 TO 61
    NSD=C
    62 NSD=1
    61 CALL CUMCAL(II,J,NSAMP)
    61 CALL CUMCAL(II,JNNSAMP)
    IF(EU.GF.^.) GO TO A3
    NSD=C
    GC TO 1-7
    6 3
    NCD=-1
107 CONTINUE
    IF(NSDQEQ.C) GOO TO 196
    IF(IFLAGEVE.G
    CE2(J)=CE2(I)
    STD(J)=STD(T)
    IOSTRT(J)=IDSTRT(I)
    DC11OMM=1,ND
110 R (J,NM)=R(I;MM)
    FCAG-1
120 IFLAGEIFLAG)=J
106 CONTINUE
    CCNTINUE
    -NFFF-TFLAG+1
    IF(NEFF.GT.64)NEFF=64
1 1 5
    IF(IFLAG.LE.1.OR.LFLAG(1).GT.NEFF) GO TO 13?
    IF(TFLAG-LE %
    LE=LFLAG(1)
    OC
CONTINUE
        . ./RND)*UZ(R(NFNT,K))
    GCRTO 107
    A2 GO TO 1.7
    CD=-1
    GO TO 1.76
```

170
171


```
    SURRCUTTNE CUMCALIM1,MZNSAMP)
    +
    ND = NSAMP
    k l}=
    C(1)=0?
    CP(1)=
    ENO=ND
    NOZ=ND+?+1
    IF(K1.GT=ND.AND.K2.GTGND) 60 TO 550
    TF(K1.GT:ND) GO TC 553
551 IF(O(M1,K1)-R(M2,K2)) 551,552,553
552 C(I) =C(I-1)
    CF(I)=R(Y^,K1)
        k}1=k 1+
        K2=k = +0+1
553
    C(I)=C(I-1)-(1.0/BND)
        k2=k2+1
    550 C(I) =0.0,
500 CNNTINUE
            RETURN
```

```
        SURRCUTINE RORDER (NSAMF)
```

```
        SURRCUTINE RORDER (NSAMF)
```




```
        NO = NSAMP
```

        NO = NSAMP
        ND1=ND-1
        ND1=ND-1
        \(A_{M}=T(I)\)
        \(A_{M}=T(I)\)
        \(\mathrm{N}=\mathrm{I}\)
        \(\mathrm{N}=\mathrm{I}\)
        \(I I=I+1\)
        \(I I=I+1\)
        1 CC
    $\mathrm{IF}\left(A^{N}-L E=T(j)\right) \quad G O$ TO 401
1 CC
$\mathrm{IF}\left(A^{N}-L E=T(j)\right) \quad G O$ TO 401
$\mathrm{N}=\mathrm{J}$
$\mathrm{N}=\mathrm{J}$
$A M=T(J)$
$A M=T(J)$
401 CONTIMUE
$T(I)=A M$
$T(I)=A M$
LCO CONTINUE
RETURVUE
END

```
    RETURVUE
END
```




```
    SUERCUTINE SD (MU,NSAMP)
    CCMMCN ISTODOM/ NEFFMP) NNT, T(20), IOSTRT(65) , R (65, 20),
+ REAL MU
    \(N D=N S A M P\)
    \(M U=0\).
\(N L=2 \star N D\)
    \(\because D 2=2 * N D\)
```



```
    \(I F(C(N L)\)
\(M L=V L-1\)
\(M U=C D\)
    \(M U=C i^{\circ}\).
\(\begin{array}{ll}\mathrm{MU}=\mathrm{C} \\ 653 & \mathrm{G} C \\ \mathrm{MU}=\mathrm{MU}+(U 2(C P(N L+1))-U 2(C P(N L))) \star C(N L)\end{array}\)
\(653 \begin{array}{ll}M U=M U+(U 2(C P(N L+1))-U 2(C P(N L))) \star C(N L)\end{array}\)
    \(N L=V L-1\)
```



```
    TU \(=(U 2(C P(N L+1),-U 2(C P(N L))) * C(N L)\)
\(I F(M U+T U) 654.652,655\)
655
601
654
    \(\begin{array}{cc}61 & C O N T I N U E \\ 54 & T T=N U / C(N L)+U 2(C F(N L+1))\end{array}\)
```



```
        \(C P(N L+1)=Q\)
```



```
    \(N L=N L-1 K\)
    IF (NLLLE 1 , GO TO 65 ?
    \(T U=(11(C F(N L+1))=U 1(C F(A L))) * C(N L)\)
\(I F(M U+T U) 656,652,65 ?\)
\(656 \quad\) ~U \(=\mathrm{MU}+\mathrm{TU} \quad 656,652,65\) ?
    \(M U=M U+T U\)
\(N L=N L-1\)
\(C O N T L U F\)
\(\begin{array}{ll}6 C 2 & \text { NL=NL-1 } \\ 657 & \text { TCNTINUF } \\ 651 C( \end{array}\)
    \(T S=N U / C(N L)+U 1(C F(N L+1))\)
\(=U T Y(T S)\)
    \(Q=U T 1\) (TS )
    \(C F(N i+1)=6\)
0.0
\(n 0\)
\(n 0\)
CONTLNUE \(=\)
    \(I F(M U+T U\)
\(M U=M U+T U\)
\(V L=V L-1\)
    \(M U=M U+T U\)
VE =NL-1
CONTINUE
GO TO 6 )0
CONTINUE
RETURNE
END
    REAL MU
                            GO TO 65 ,
    \(M y=0.6\)
\(N L=N L-1\)
```




```
FUNCTICM, UIA(X) RA(2)
UT1=EXF(x)-Q\Delta(1)
RETURN
END
```

2200
$\rightarrow 510$
$22-\frac{2}{2}$

## FUNCTION UTZ(X) RA(2) CCMMCN/STDPAR/ RA(2) <br> UIZ $=E X F(X)-R A(Z)$ <br> EETU

## APPENDIX 3

## SAMPLE PRINT-OUTS

Results of the general test case printed from AIDU are shown on the following pages to illustrate the output formats programmed in the model package. These are presented in tables as follows:

Table A: Standard output for Strategy 1, samples 1 and 20 with mean and standard deviation for that strategy.

Table B: Standard output for the efficient set of strategies, with, for each strategy in the efficient set, objective function mean and standard deviation and values for each state of nature. In this case, the efficient set consisted only of Strategy 1.

Table C: Detailed output for Strategy 1, Sample 1, time 1.00.


Table B


Table C



[^0]:    GUNAFAFAAAAAN

[^1]:    

[^2]:    1402
    1495
    1496
    1497
    $1499^{\circ}$
    1499
    15
    
    

