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An Estimable Dynamic Model of Recreation Behavior

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Introduction

Over the past thirty years economists have made substantial theoretical and empirical advances in the measurement of the economic benefits associated with the consumption of nonmarket resources. For instance, the travel cost method (TCM) is now widely used to estimate the economic benefit of nonmarket resources for site-specific recreation activities. This method was first suggested by Hotelling in 1949, and later refined by Clawson and Knetsch (Clawson, Knetsch, Clawson and Knetsch). Recent applications of the method use nested logit (NL) estimation to reflect the essential truth that certain decisions associated with site-specific recreation are conditioned by previous decisions (Bockstael, Hanemann, and Kling; Bockstael, McConnell, and Strand; Milon; Morey, Shaw, and Rowe; Morey, Shaw, and Watson; Wegge, Carson, and Hanemann; Lyke).

Nested logit estimation has been only partially successful in modeling the sequential aspects of a recreation decision like the decision to fish. For instance, none of the recent studies formally considers that outcomes on the current trip affect the decision about when and where to take the next trip. Also absent is the possibly forward-looking nature of the recreation decision; *because* an angler knows early in the fishing season that plenty of opportunities to fish lie ahead, he may postpone a trip that he would not postpone late in the season. In other words, the *dynamic* nature of the decision process is absent from these models; they fail to consider that decisions today affect opportunities and information available tomorrow, and that consumers may behave in a manner consistent with this feedback relationship.

Structural estimation of the dynamic decision process of anglers would allow the analyst to attain an empirical esthetic not found in the usual static (NL) models; the analyst may be able to formulate a decision problem that most observers would agree looks more like "the real thing" than what is currently found in the literature. This pursuit of "the real thing" provides the opportunity not only to derive estimates of economic value as in the usual static models, but also to approximate the primitive parameters important to understanding the dynamics of angler effort and fish stocks. In their study of the Green Bay, Lake Michigan yellow perch fishery, Johnson et al. observe "The prospect exists for managing variability in harvest and stock size and for maximizing economic returns in the fishery, but more information is required, *primarily on sportfishing dynamics and angler preferences.* Stock-recruitment relations, density dependence of growth, *and dynamics of sportfishing effort* are the primary sources of uncertainty limiting the precision of our predictions" (Abstract, emphasis added). Johnson and Carpenter attempt to examine these dynamics using a simple predator-prey model, in which the angler has the role of predator. Swallow demonstrates the importance of intraseasonal management of a fishery to maximize angler welfare. Like Johnson et al. and Johnson and Carpenter, he concludes that additional research is required to understand angler behavior: "Evaluation of how different types of recreationists might switch days between subseasons in response to quality and regulations may prove critical. *The extant literature does not address the potential demand or equity implications for such intertemporal behavioral choices. Research on behavioral choices could enrich policy assessments based on recreational consumer's surplus*" [pg. 933, emphasis added].

The fundamentally dynamic structure of the trip decision is certainly understood by TCM practitioners; for instance, Morey, Shaw and Rowe observe, "In general, one might expect that the decision to fish at site *j* mode *m* in period *t* would affect the participation probability and site/mode probabilities in subsequent periods".[1] Yet such models have not yet been estimated, most likely for two reasons. First, such models require fairly detailed data. Second, developing a conceptually coherent dynamic model of behavior that is also tractable in estimation is considered a daunting task. Arguably the first reason is simply an effect of the second; if dynamic structural models were easily estimated, data to estimate them would be gathered.

In this paper we describe a dynamic structural model of the decision to visit a recreation site. The model is well-suited to the investigation of the dynamics of angler effort called for by Johnson et al. and Swallow, in particular the feedback relationship between catch and catch expectations. The fundamental estimation approach is actually the same as that first used by Rust (1987) in his study of the dynamics of the bus engine replacement decision, and Miranda and Schnitkey in their investigation of the dairy cow replacement problem. Provencher (1995a, 1995b) applied a conceptually similar model in his investigation of the harvest decisions of southern timber firms. The model is best described as a dynamic multinomial logit model. By virtue of its dynamic nature, the model avoids the problem of the independence of irrelevant alternatives that afflicts static multinomial logit models. On the other hand, because the model has the form of a multinomial logit model the calculation of likelihoods can be done relatively cheaply. The model is readily compared in a nested hypothesis test to a traditional (static) multinomial logit model in which each trip decision is independent of past and future decisions and events. Welfare effects of changes in site quality are easily calculated via dynamic programming.

The Basic Model

Over the past ten years a literature concerning the estimation of stochastic dynamic behavioral models has emerged, and no doubt will continue to grow in the years ahead. Reviews of this literature are contained in Rust (1988), Eckstein and Wolpin, and Rust and Pakes. Here we cite a few examples to provide a sense of the breadth of the applications. Miranda and Schnitkey estimate the dairy cow replacement decision for a dairy farm in Ohio. In the model the farmer decides whether to keep or cull a dairy cow based on the cow's age (measured in lactation cycles) and current milk production. The model provides insights to how "good" farmers make their culling decisions. Fafchamps examines the seasonal labor decisions of farmers in Burkina Faso, West Africa. In this model, farmers make two decisions: the labor to use for planting and the labor to use for weeding. These decisions are based on the farmer's observations of crop growth and their understanding of the future variability of crop growth. In the only application of this methodology to the economics of natural resource use, Provencher (1995a,b) estimates a stochastic dynamic structural model of the timber harvest decision in southeast Georgia. Results

[1] We go a bit further; not only do past events and experiences affect the trip decision, but so too do expectations about *future* opportunities.

indicate that the expected gain from exploiting fluctuations in the price of timber are much smaller than predicted in the forest economics literature.

Because the estimation technique is fairly new, in the discussion here we present it in considerable detail. Before proceeding, a few comments about the general form of the problem are in order. As with all previous attempts to estimate stochastic decision problems, we assume preferences are time-separable and state-separable. This reflects adoption of the expected utility hypothesis. State separability implies that preferences in the current period depend only on the current state of nature, though this is not as restrictive as it might seem at first glance --it is a trivial matter in practice to include in the "current" state of nature past state values, or past outcomes. Of course, as emphasized below, the computational burden of estimating structural dynamic models places a premium on parsimony, both in terms of the set of state variables used in the analysis and the set of parameters requiring estimation, and so any representation of the effect of past behavior on current utility must be kept simple yet flexible. Rust and Pakes argue that the assumptions of time and state separability still leave the analyst with enough flexibility to develop a good model of behavior. They conclude, "While it is unlikely that human decision makers are literally solving time separable Markovian decision problems (either consciously or unconsciously), it turns out that this class is sufficiently rich and flexible to enable one to construct detailed models of most types of behavior" (pg. 7).

We present the model in the context of recreational fishing, though it is certainly applicable to other recreational activities with a clear dynamic component. Let T denote the total number of days in the fishing season, and let $t=1,...T$ denote an arbitrary day during the season. On day t an angler decides whether to fish, and what fishing alternative --as defined by the site visited and the species sought-- to choose. Ultimately the angler's decision depends on the state of nature at time t, defined by state variables such as the quality of fishing at various sites on day t, the time elapsed since the last day fished, wind speed, and so on. If the analyst knows all the variables that enter the angler's decision process, then in theory he can solve the appropriate dynamic decision problem to obtain an optimal "trip policy" that perfectly forecasts the fishing alternative chosen by the angler. Of course, knowing enough about the decision process to perfectly forecast trips is impossible, and so instead the analyst must concede the existence of random state variables entering the decision process that are observed contemporaneously by the angler but never observed by the analyst.

Suppose there are I trip alternatives, and let y_t denote the angler's trip decision on day t, with $y_t = i$ if on day t alternative i is chosen, $i=0...I$. $y_t = 0$ denotes the decision not to fish on day t. The vector of observable state variables affecting utility on day t is denoted by $\mathbf{x}_t$; by *observable* we mean that both the angler *and* the analyst observe the value of the state variable. Included in this vector are such determinants of the trip decision as weather variables, trip costs, and catch rates at the various fishing sites. Along with the observable state variables the model includes decision-specific unobservable state variables $\tilde{\varepsilon}_t = (\tilde{\varepsilon}_{0t}, \ldots \tilde{\varepsilon}_{It})$. These are random variables observed contemporaneously by the angler but *not* by the analyst.

For the purpose of understanding the angler's decision problem it is necessary to distinguish among various subvectors of $\mathbf{x}_t$. Let $\mathbf{x}_{at}$ denote the subvector of state variables with values observed by the angler at the time the fishing decision is made (where the subscript "a" denotes "anterior"), and let $\mathbf{x}_{pt}$ denote the subvector of state variables observed by the angler after the fishing decision is made (where the subscript "p" denotes "posterior"). In a model of recreational fishing the distinction between anterior and posterior state variables bears on the role of catch expectations in the trip decision; when the angler makes a decision he knows the conditional distribution of catch for each site, but not the actual catch, which is revealed only after the trip is taken. Those anterior state variables that evolve over time independently of the fishing decision are denoted by $\mathbf{x}_{rt}$. Those anterior state variables influencing the utility associated with alternative i are denoted by $\mathbf{x}_{ait}$. Finally, those posterior variables influencing the utility associated with alternative i are denoted by $\mathbf{x}_{pit}$.

Each day the angler solves a complex dynamic decision problem involving, among other things, the consumption of income and the fishing decision. Here we assume that the allocation of income for day t is independent of the decision to fish; this allows the separation of the income allocation and fishing decisions. Specifically, the consumption of income on day t is denoted by $b_t(B_t(\mathbf{x}_{rt}), \mathbf{x}_{rt})$, where B_t is the angler's budget for the fishing season on day t. The budget evolves according to

$$B_t = \begin{cases} \overline{B}(\mathbf{x}_{rt}) & \text{if} \quad t = 0 \\ B_{t-1} - b_{t-1} + \omega_t(\mathbf{x}_{rt}) & \text{if} \quad t = 1, \ldots T' \end{cases} \tag{1}$$

where $\omega_t(\mathbf{x}_{rt})$ is income added to the seasonal budget on day t. So, for instance, the model allows the possibility that the angler consumes more income on a warm, sunny weekend in July than on a cold rainy weekday in May, but it does not allow the possibility that more income is allocated to day t when the angler chooses fishing alternative I.

Let c_i denote the cost (price) of alternative i, i=$0,...I$. The cost of making no trip is zero, and so c_0=0. The utility associated with decision $y_t = i$ is generally denoted by

$$\widetilde{\mathrm{u}}_i\left(\mathbf{x}_{ait}, \mathbf{x}_{pit}, b_t(B_t(\mathbf{x}_{rt}), \mathbf{x}_{rt}) - c_i\right) + \widetilde{\varepsilon}_{it},$$

where the last argument in $\widetilde{\mathrm{u}}_i(\cdot)$ denotes consumption of the numeraire on day t.

In static random utility models of the recreation decision, utility is often representd as linear in the budget. This implies zero income effects, and so when the disturbance term $\widetilde{\varepsilon}_{it}$ has a certain structure, closed-form solutions for compensating and equivalent variation are possible. As Morey points out, in this regard the linear specification is therefore convenient but not necessary. Note, however, that this conclusion presumes that the budget (income) is observed. In our dynamic model the budget of interest is the *daily* budget, which is impossible to observe. As it turns out, a linear specification allows this empirical problem to be circumvented. In light of the small differences in the daily budget over time and across anglers, a linear specification may provide a reasonably good first-order approximation of the "true" model of indirect utility.

Suppose utility takes the linear form

$$\widetilde{\gamma}_{pi}\mathbf{x}_{pit} + \widetilde{\gamma}_{ai}\mathbf{x}_{ait} + \lambda(b_t - c_i) + \widetilde{\varepsilon}_{it}, \tag{2}$$

where $\tilde{\gamma}_{ai}$ and $\tilde{\gamma}_{pi}$ are conformable vectors of parameters and λ is the marginal utility of income. In applications where there is little price variation, any estimate of the marginal utility of income λ is likely to be fairly imprecise. This problem is readily avoided by dividing (2) by the marginal utility of income to obtain the money metric expression of utility u_i,

$$\begin{aligned} u_i\left(\mathbf{x}_{ait}, \mathbf{x}_{pit}, c_i, \varepsilon_{it}, b_t; \gamma_i\right) &= \frac{\tilde{\gamma}_{pi}\mathbf{x}_{pit}}{\lambda} + \frac{\tilde{\gamma}_{ai}\mathbf{x}_{ait}}{\lambda} + b_t - c_i + \frac{\tilde{\varepsilon}_{it}}{\lambda} \\ &= \gamma_{pi}\mathbf{x}_{pit} + \gamma_{ai}\mathbf{x}_{ait} + b_t - c_i + \varepsilon_{it} \end{aligned} \quad (3)$$

At time t the angler's dynamic decision problem is to maximize the sum of expected current and future utility, where future utility is discounted by an "impatience" factor β. Formally, the angler's problem is,

$$\max_i E\sum_{s=t}^{T} \beta^{s-t}\left\{\gamma_{pi}\mathbf{x}_{pit} + \gamma_{ai}\mathbf{x}_{ait} + b_t - c_i + \varepsilon_{it}\right\}_{i=0}^{I}. \quad (4)$$

The expectation in (4) is taken over the three categories of random state variables $\mathbf{x}_{pt}$, $\mathbf{x}_{at}$, and ε_t. At the time of the harvest decision on day t, the values of the posterior state variables associated with alternative i are unknown and conditional on the state variables $\mathbf{x}_{ait}$ and the decision y_t. We denote by θ_{pi} the parameters associated with the distribution of $\mathbf{x}_{pit}$, and define $\theta_p = \left(\theta_{p1}, \theta_{p2}, \ldots \theta_{pI}\right)$. At the time of the harvest decision the values of the anterior state variables at time $t+1$ ($\mathbf{x}_{a,t+1}$) are unknown and conditional on $\mathbf{x}_{at}$ and y_t. We denote by θ_a the set of parameters associated with the distribution of $\mathbf{x}_{a,t+1}$. Finally, the unobservable state variables ε_t are independently and identically distributed over time according to a known multivariate probability distribution. We denote by μ the parameters of this distribution. It bears repeating that at the time the fishing decision is made, ε_t is known by the angler, but it is not observed by the analyst.

Now define $\Gamma = \left(\theta_a, \theta_p, \mu, \gamma\right)$, and define $c = \left(c_0, \ldots c_I\right)$. Also, let $v_{t+1}\left(\mathbf{x}_{a,t+1}, \varepsilon_{t+1}; c, \Gamma\right)$ denote the solution to (4) on day $t+1$. Then by Bellman's principle of optimality, the angler's problem can be restated,

$$v_t\left(\mathbf{x}_{at}, \varepsilon_t; c, \Gamma\right) = \max_i \; \gamma_{pi} E_{\mathbf{x}'_{pi}|\mathbf{x}_{ait},i}\mathbf{x}'_{pi} + \gamma_{ai}\mathbf{x}_{ait} - c_i + \varepsilon_{it} + \beta E_{\mathbf{x}'_a,\varepsilon'|\mathbf{x}_{at},i} v_{t+1}\left(\mathbf{x}'_a, \varepsilon'; c, \Gamma\right). \quad (5)$$

Here, $E_{\mathbf{x}'_{pi}|\mathbf{x}_{at},i}$ denotes the expectation over current (day t's) realizations of the state vector $\mathbf{x}_{pi}$, conditional on the current state $\mathbf{x}_{ait}$ and the current decision i; and $E_{\mathbf{x}'_{ai},\varepsilon|\mathbf{x}_{ai},i}$ denotes the expectation over *tomorrow's* realizations of the state vector $\mathbf{x}_{ai}$ and the utility shock ε, also conditional on $\mathbf{x}_{ait}$ and the current decision i. The daily budget b_t is eliminated from the decision problem because it is the same for all alternatives.

For expositional reasons define

$$V_{t+1}\left(\mathbf{x}_{at}, i; c, \Gamma\right) = E_{\mathbf{x}'_a,\varepsilon'|\mathbf{x}_{at},i} v_{t+1}\left(\mathbf{x}'_a, \varepsilon'; c, \Gamma\right), \quad (6)$$

and

$$U_i\left(\mathbf{x}_{ait}, i; \gamma, \theta_{pi}\right) = \gamma_{pi} E_{\mathbf{x}'_{pi}|\mathbf{x}_{ait}, i} \mathbf{x}'_{pi} + \gamma_{ai} \mathbf{x}_{ait}. \tag{7}$$

Equation (5) now can be restated as

$$\mathrm{v}_t\left(\mathbf{x}_{at}, \varepsilon_t; c, \Gamma\right) = \max_i \left\{U_i\left(\mathbf{x}_{ait}, i; \gamma, \theta_{pi}\right) - c_i + \varepsilon_{it} + \beta V_{t+1}\left(\mathbf{x}_{at}, i; c, \Gamma\right)\right\}. \tag{8}$$

If the parameter vector Γ is known, and the state variable vector ε_t is observed, then the decision problem (8) can be solved via backwards recursion (see Bellman, or Bertsakas). In general this is not a trivial problem. For instance, deriving $\mathrm{V}_{t+1}\left(\mathbf{x}_{at}, i; \Gamma\right)$ involves I-dimensional integration over the state vector ε_{t+1} at each stage of the recursion. The problem is greatly simplified, however, by assuming that the state variables ε_t are mutually independent Gumbel distributed random variables with location parameters $\mu_0 \ldots \mu_I$ and scale parameter σ; in our notation, $\mu = \left(\mu_0, \ldots \mu_I, \sigma\right)$. From the standard properties of Gumbel-distributed random variables (see Ben-Akiva and Lerman), we know that integrating both sides of (8) with respect to ε' for day $t+1$ yields,

$$E_{\varepsilon'} v_{t+1}\left(\mathbf{x}_{a,t+1}, \varepsilon'; c, \Gamma\right) = \frac{1}{\sigma} \ln\left\{\sum_{j=0}^{I} \exp \sigma\left[U_j\left(\mathbf{x}_{aj,t+1}, j; \gamma_j, \theta_{pj}\right) - c_i + \mu_{jt} + \beta V_{t+2}\left(\mathbf{x}_{a,t+1}, j; c, \Gamma\right)\right]\right\}. \tag{9}$$

Substituting (9) into (6) yields

$$V_{t+1}\left(\mathbf{x}_{at}, i; c, \Gamma\right) = E_{\mathbf{x}'_a|\mathbf{x}_{at}, i} \frac{1}{\sigma} \ln\left\{\sum_{j=0}^{I} \exp \sigma\left[U_j\left(\mathbf{x}'_{aj}, j; \gamma_j, \theta_{pj}\right) - c_i + \mu_{jt} + \beta V_{t+2}\left(\mathbf{x}'_a, j; c, \Gamma\right)\right]\right\}, \tag{10}$$

and so with $\mathrm{V}_{t+2}(\cdot)$ known from the previous stage in the recursion, calculation of $\mathrm{V}_{t+1}(\cdot)$ is a relatively simple affair involving integration over the random elements in the observable state vector.

Solution of the DP problem (8)-(10) is essential to estimating the parameter set Γ. In particular, we know from the properties of the Gumbel distribution that the probability of observing decision i given the observable state vector $\mathbf{x}_{at}$ and the parameter vector Γ has the same structure as its counterpart for static multinomial logit models (see Ben-Akiva and Lerman). Formally,

$$\Pr\left(i|\mathbf{x}_{at}, c, \Gamma\right) = \frac{\exp \sigma\left[U_i\left(\mathbf{x}_{ait}, i; \gamma_i, \theta_{pi}\right) - c_i + \mu_i + \beta \mathrm{V}_{t+1}\left(\mathbf{x}_{at}, i; c, \Gamma\right)\right]}{\sum_{j=0}^{I} \exp \sigma\left[U_j\left(\mathbf{x}_{ajt}, j; \gamma_j, \theta_{pj}\right) - c_j + \mu_j + \beta \mathrm{V}_{t+1}\left(\mathbf{x}_{at}, j; c, \Gamma\right)\right]} \tag{11}$$

Now suppose that the trip behavior of N anglers is observed for all T days of the fishing season. Letting y_{nt} denote the decision of angler n on day t, the likelihood of the sample is

$$\mathrm{L}(\Gamma) = \prod_{N} \prod_{T} \Pr\left(\mathbf{x}_{a,nt}, y_{nt} \mid c, \Gamma\right). \tag{12}$$

Maximum likelihood estimation thus involves a nested inner algorithm in which a dynamic programming (DP) algorithm derives $\mathrm{V}_t(\cdot), t = 1,...T$, for the specified vector Γ, and an outer hill-climbing algorithm that searches for the value of Γ yielding the highest value of $\mathrm{L}(\Gamma)$. This nested structure places a premium on parsimony, both in the state vector $\mathbf{x}_t$ and in the parameter vector Γ; each time a new value of Γ is examined in the maximization routine, a DP problem must be solved.

The Issue of the Independence of Irrelevant Alternatives

A well known weakness of static multinomial logit models is the property of *independence of irrelevant alternatives* (IIA): the odds of choosing one alternative over another depends only on the attributes of the two alternatives. So, for instance, in the example offered by Bockstael, the odds of visiting a saltwater beach instead of a freshwater beach does not depend on whether a third beach is itself a saltwater or freshwater site. The IIA property does not extend to our dynamic model. The log odds ratio can be stated

$$\begin{aligned}\log\left\{\frac{\Pr(y_t = i \mid \mathbf{x}_{at})}{\Pr(y_t = j \mid \mathbf{x}_{at})}\right\} &= \left[U_i\left(\mathbf{x}_{ait}, i; \gamma_i, \theta_{pi}\right) - c_i + \beta \mathrm{V}_{t+1}\left(\mathbf{x}_{at}, i; c, \Gamma\right)\right] \\ &\quad - \left[U_j\left(\mathbf{x}_{ajt}, j; \gamma_j, \theta_{pj}\right) - c_j + \beta \mathrm{V}_{t+1}\left(\mathbf{x}_{at}, j; c, \Gamma\right)\right]\end{aligned} \tag{13}$$

and so by virtue of the presence of $\mathbf{x}_{at}$ in $\mathrm{V}_{t+1}(\cdot)$, the odds of choosing to fish at site i instead of site j depends on the attributes (state of nature) at all possible sites.

Testing a Static Multinomial Logit Model against the Dynamic Model

A nested test of a static multinomial logit model against the dynamic model presented above can take one of two forms. The first involves a test for $\beta = 0$. To see this, note that with $\beta = 0$, (8) reduces to

$$\Pr(i \mid \mathbf{x}_{at}, \Gamma) = \frac{\exp \sigma\left[U_i\left(\mathbf{x}_{ait}, i; \gamma_i, \theta_{pi}\right) - c_i + \mu_i\right]}{\sum_{j=0}^{I} \exp \sigma\left[U_j\left(\mathbf{x}_{ajt}, j; \gamma_j, \theta_{pj}\right) - c_j + \mu_j\right]}, \tag{14}$$

which is the classical presentation of the probability of choosing alternative i in the static multinomial logit models found in the travel cost literature (see, for instance, Bockstael). The alternative is to test the null hypothesis of temporal independence in the anterior state variables (no linkage between $\mathbf{x}_{at}$ and $\mathbf{x}_{a,t+1}$). Inspection of (5) reveals that given independence, the angler merely solves a sequence of static decisions.

Of these two approaches, only the latter is consistent with the welfare analysis found in the travel cost literature. In particular, welfare estimates for a season are usually calculated as the sum of welfare estimates for each choice occasion. But this approach implies no discounting ($\beta = 1$), which contradicts a test of a static model based on $\beta = 0$. More generally, it appears to be inconsistent --as a representation of behavior-- to construct static models in which the state of nature is not independent over time. Such models imply either that $\beta = 0$, or that the economic agent is irrational; neither assumption is particularly attractive or readily defended. In theory the impatience factor β is equal to the discount rate for money; in a daily model of behavior, this implies $\beta \approx 1$. Generally a sensible approach is to simply set $\beta = 1$, and to test the static model of behavior by testing for temporal independence in the anterior state variables .

Welfare Analysis Using the Model

On the last day of the season, day T, the angler's dynamic decision problem reduces to a static one. Consider now the welfare effect of a change in c and Γ on the last day of the season. Let c^0 denote the set of original prices of the various decision alternatives (the set derived in estimation), and let c^1 denote an alternative set. Similarly, let Γ^0 denote the original (estimated) set of parameter values, and let Γ^1 denote an alternative set. From (3)-(5) it is apparent that v_T+b_T is a money measure of conditional indirect utility on the last day of the season, and so assuming nonsatiation ($b_T=B_T$), compensating variation (CV_T) and equivalent variation (EV_T) are defined by

$$CV_T = EV_t = v_T\left(\mathbf{x}_{aT}, \varepsilon_T; c_T^1, \Gamma_T^1\right) - v_T\left(\mathbf{x}_{aT}, \varepsilon_T; c_T^0, \Gamma_T^0\right) + B_T^1 - B_T^0;$$

with B_T unchanged,

$$CV = EV = v_T\left(\mathbf{x}_{aT}, \varepsilon_T; c_T^1, \Gamma_T^1\right) - v_T\left(\mathbf{x}_{aT}, \varepsilon_T; c_T^0, \Gamma_T^0\right).$$

Backwards induction reveals that v_t+B_t is a money measure of welfare for the remainder of the season on day t, yet clearly welfare analysis associated with a change in prices or parameters is notably richer than on the last day of the season, because now such change involves a temporal aspect; the issue is not simply *whether* a parameter is changed, but when and for how long. Let c_t^1 and Γ_t^1 denote the values of c and Γ on day t under the alternative setting. In this more general setting we amend (5) slightly to define

$$v_t\left(\mathbf{x}_{at}, \varepsilon_t; c_t, \Gamma_t\right) = \max_i \; \gamma_{pi} E_{\mathbf{x}'_{pi}|\mathbf{x}_{at},i} \mathbf{x}'_{pi} + \gamma_{ai} \mathbf{x}_{ait} - c_{it} + \varepsilon_{it} + \beta E_{\mathbf{x}'_a, \varepsilon'|\mathbf{x}_{at},i} v_{t+1}\left(\mathbf{x}'_a, \varepsilon'; c_{t+1}, \Gamma_{t+1}\right).$$

Then for $B_t^0 = B_t^1$, compensating and equivalent variation are defined by

$$CV_t = EV_t = v_t\left(\mathbf{x}_{at}, \varepsilon_t; c_t^1, \Gamma_t^1\right) - v_t\left(\mathbf{x}_{at}, \varepsilon_t; c^0, \Gamma^0\right). \qquad (15)$$

Here both compensating and equivalent variation equal the one-time payment required on day t to make the angler indifferent to the proposed change in prices and parameter values. This value is readily obtained by solving two DP problems. In principle the estimated model is amenable to a rich variety of welfare analyses of intraseasonal management programs, such as those which shift the fish catch from one part of the season to another.

The definitions of compensating and equivalent variation offered above are *conditional* on the state of nature $(\mathbf{x}_{at}, \varepsilon_t)$. Insofar as ε_t is not observable by the analyst, we can define an alternative notion of compensating variation as the expected value of CV_t, where the expectation is taken over ε_t:

$$\overline{CV_t} = E_{\varepsilon'} v_t\left(\mathbf{x}_{at}, \varepsilon'; c_t^1, \Gamma_t^1\right) - E_{\varepsilon'} v_t\left(\mathbf{x}_{at}, \varepsilon'; c^0, \Gamma^0\right).$$

For policy analysis there remains the issue of presenting welfare measures as conditional on the observable state variables $\mathbf{x}_{at}$. Two points about this are noteworthy. First, for policy analysis interest most likely lies in seasonal welfare measures, and with $\beta = 1$ such welfare measures are unlikely to be much affected by the initial state $\mathbf{x}_{a0}$. This proposition can be examined via sensitivity analysis. Second, in the event $\mathbf{x}_{a0}$ does have a substantial impact on the value of $\overline{CV_0}$, historical data may be used to estimate the (unconditional) distribution of $\mathbf{x}_{a0}$, and a measure of compensating variation for the season is then

$$\overline{\overline{CV_0}} = E_{\varepsilon', \mathbf{x}_a'} v_0\left(\mathbf{x}_a', \varepsilon'; c_0^1, \Gamma_0^1\right) - E_{\varepsilon', \mathbf{x}_a'} v_0\left(\mathbf{x}_a', \varepsilon'; c^0, \Gamma^0\right).$$

Summary

This paper presents an estimable dynamic model of recreation behavior that avoids a number of consistency issues arising in static random utility models, while permitting standard welfare analysis. By virtue of its explicit statement of the "primitives" of the decision problem, the model is well-suited to the analysis of the dynamic response in recreational behavior to various management strategies. Nonetheless, estimation of the model requires surmounting several notable obstacles. Obtaining the requisite data is generally quite expensive, as it involves fairly detailed panel data. Moreover, the estimation procedure involves a considerable amount of computer programming. In particular, the solution algorithm is comprised of an "inner" dynamic programming algorithm nested within an "outer" algorithm used to find maximum likelihood estimates. Whether this modeling approach is worth the expense and effort is an empirical question that can be answered only after it is applied and compared to the usual static models in a number of different situations.

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