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Nonparametric Analysis of Production Efficiency

by

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NONPARAMETRIC ANALYSIS OF PRODUCTION EFFICIENCY

I- Introduction:

Much research has focused on the economic analysis of technical and allocative efficiency in production. The analysis has fallen into two broad categories: parametric and nonparametric. The parametric approach relies on a parametric specification of the production function, cost function or profit function (e.g., Forsund et al.; Bauer). It provides a consistent framework for investigating econometrically the technical and allocative efficiency of firms. However, it requires imposing parametric restrictions on the technology and the distribution of the inefficiency terms (Bauer). Alternatively, the nonparametric approach has been developed following the work of Afriat, Hanoch and Rothschild, Diewert and Parkan, and Varian. It has the advantage of imposing no a priori restriction on the underlying technology (e.g., Seiford and Thrall; Färe et al.).

Banker and Maindiratta proposed a nonparametric approach to the measurement of production efficiency. Their method is based on inner-bound and outer-bound representations of the underlying technology. For a given data set on production activities, the inner bound is obtained from data envelopment analysis (DEA) involving all the observations on inputs and outputs. The outer bound is obtained from a modification to the Afriat-Varian weak axiom of profit maximization (WAPM) applied to a subset of data points found to be consistent with profit maximizing behavior. These bounds can provide a basis for estimating technical and allocative efficiency indexes for each observation.

In this paper, we evaluate the Banker-Maindiratta approach and propose an alternative method for nonparametric production efficiency analysis. We argue that the Banker-Maindiratta approach suffers from a fundamental asymmetry: under production inefficiencies, the number of observations used to evaluate the upper-bound representation of the technology is smaller (and possibly much smaller) than the number of observations used to evaluate its inner-bound counterpart. This asymmetry can generate somewhat unrealistic representations of the technology. We propose an alternative nonparametric approach which does not suffer from this asymmetry problem. The method relies on the concept of "effective inputs and outputs", which can differ from actual inputs and outputs due to technical inefficiencies and/or technical change. We define an "effective technology" corresponding to these effective quantities and use it as the reference technology in the evaluation of technical and allocative efficiency. The approach is illustrated in an empirical application to US agriculture. Based on those data, the Banker-Maindiratta upper-bound estimate of the technology uses a single data point, generating a linear production frontier with infinite Allen elasticities of substitution. This appears to be a rather unrealistic representation of the real world. In contrast, our "effective technology" approach relies on all data points in the sample for both the inner-bound and outer-bound estimates. The empirical results from the two approaches are compared and contrasted. They both indicate the existence of a large gap between the upper-bound and lower-bound representations of the technology. This suggests that the data, together with production theory, may not be rich enough to provide a precise estimation of technology and production behavior.

II- Nonparametric Production Analysis:

This section reviews some key results on nonparametric analysis of production activities obtained by Afriat, Hanoch and Rothschild, Diewert and Parkan, and Varian. It also sets up the notation for the rest of the paper. Consider a competitive firm choosing (y, x) where $y = (y_1, \ldots,$

$y_m)' \geq 0$ is a $(m\times 1)$ vector of outputs, and $x = (x_1, ..., x_n)' \geq 0$ is a $(n\times 1)$ vector of inputs. We focus on a general multi-input multi-output joint technology represented by the feasible set $F \subset \mathbb{R}_+^m\times\mathbb{R}_-^n$, where $(y, -x) \in F$. We assume throughout the paper that the production set F is non-empty, convex and negative monotonic.[1/]

Assume that the firm behaves in a way consistent with the profit maximization hypothesis. Let $p = (p_1, ..., p_m)' > 0$ denote the $(m\times 1)$ vector of output prices, and $r = (r_1, ..., r_n)' > 0$ be the $(n\times 1)$ vector of input prices. Then, the firm production decisions are made as follows:

$$\pi(p, r) = \max_{y,x} \{p'y - r'x: (y, -x) \in F\}, \tag{1}$$

where $\pi(p, r)$ is the indirect profit function. The solution to (1) gives the profit maximizing output supplies and input demand correspondences denoted by $y^*(p, r)$ and $x^*(p, r)$.

Consider that the firm is observed making production decisions τ times. Let T be the set of these observations: $T = \{1, 2, ..., \tau\}$. The t-th observation on production decisions is denoted by (y_t, x_t), with corresponding prices (p_t, r_t), $t \in T$. We define economic rationality for production decisions in terms of profit maximizing behavior as stated in equation (1). We will say that a production set F rationalizes the data $\{(y_t, x_t; p_t, x_t): t \in T\}$ if $y_t = y^*(p_t, r_t)$ and $x_t = x^*(p_t, r_t)$, $t \in T$. A key linkage between observable behavior and production theory is given next.

<u>Proposition 1</u>: (Afriat; Varian)

The following conditions are equivalent:

a) There exists a production set that rationalizes the data $\{(y_t, x_t; p_t, r_t): t \in T\}$ according to (1).

b) The data satisfy the Weak Axiom of Profit Maximization (WAPM):

$$p_t'y_t - r_t'x_t \geq p_t'y_s - r_t'x_s,\ \ t \in T,\ s \in T. \tag{2}$$

Given (2), there exists a family of convex, negative monotonic production sets F that:

- rationalizes the data in T according to (1),
- satisfies $F^i \subseteq F \subseteq F^o$, where

$$F^i = \{(y, -x): \textstyle\sum_{t\in T} y_t\theta_t \leq y;\ \sum_{t\in T} x_t\theta_t \geq x;\ \sum_{t\in T} \theta_t = 1;\ \theta_t \geq 0;\ x \geq 0,\ y \geq 0\}, \tag{3}$$

and

$$F^o = \{(y, -x): p_t'y - r_t'x \leq p_t'y_t - r_t'x_t,\ t \in T;\ x \geq 0;\ y \geq 0\}. \tag{4}$$

Proposition 1 establishes conditions for the existence of a production set that can rationalize observable production behavior. Equation (2) states that the t-th profit $(p_t'y_t - r_t'x_t)$ is at least as large as the profit that could have been obtained using any other observed production decision $(p_t'y_s - r_t'x_s)$, $s \in T$. It gives necessary and sufficient conditions for the data $\{(y_t, x_t; p_t, r_t): t \in T\}$ to be consistent with profit maximization (1). This is useful as a means of testing the relevance of production theory in particular situations. Perhaps more importantly, proposition 1 provides a basis for recovering some representations of the underlying production technology. More specifically, it identifies a whole family of production sets that are consistent with the data and the profit maximization hypothesis. This family is bounded by F^i in (3) and F^o in (4). Proposition 1 states that F^i in (3) gives the tightest inner bound while F^o in (4) is the tightest outer bound representation of the underlying technology. These representations are of considerable interest since they are empirically tractable and provide all the information necessary to conduct production efficiency analysis.

III- Production Efficiency:

In this section, we provide a brief review of production efficiency analysis. We also review the Banker-Maindiratta nonparametric approach to efficiency analysis when not all data points are consistent with production theory.

Technical Efficiency:

First, the concept of technical efficiency relates to the question of whether a firm uses the best available technology in its production process. Following the work of Debreu, Farrell, Farrell and Fieldhouse, and Färe et al., technical efficiency can be defined as the minimal proportion by which a vector of inputs x can be rescaled while still producing outputs y.[2/] For a firm choosing the output-input vector (y, x), this corresponds to the Farrell technical efficiency index, TE:

$$TE(y, x, F) = \inf_k \{k: (y, -kx) \in F, k \in \mathbb{R}_+\}. \tag{5}$$

In general, $0 < TE \leq 1$, where $TE = 1$ implies that the firm is producing on the production frontier and is said to be technically efficient. Alternatively, $TE < 1$ implies that the firm is not technically efficient. In this case, $(1 - TE)$ is the largest proportional reduction in inputs x that can be achieved in the production of outputs y. Alternatively, $(1 - TE)$ can be written as $[r'x - (TE)\, r'x]/(r'x)$, indicating that $(1 - TE)$ can be interpreted as the largest percentage cost saving that can be achieved by moving the firm toward the frontier isoquant through a radial rescaling of all inputs x.

Allocative Efficiency:

Following Farrell, and Farrell and Fieldhouse, the concept of allocative efficiency is related to the ability of the firm to choose inputs in a cost minimizing way.[3/] It reflects whether a technically efficient firm produces at the lowest possible cost. For an observed choice (y, x), this generates the Farrell index of allocative efficiency, AE:

$$AE(r,y,F) = \frac{C(r,y,F)}{(TE)r'x} = \frac{\mathrm{Min}_x\{r'x\colon (y,-x) \in F\}}{(TE)r'x}, \tag{6}$$

where C(r, y, F) is the cost function under technology F, and [(TE) x] is a technically efficient input vector from (5). In general, $0 < AE \leq 1$, where AE = 1 corresponds to cost minimizing behavior where the firm is said to be allocatively efficient. Alternatively, AE < 1 implies allocative inefficiency. In this case, (1 - AE) measures the maximal proportion of cost the firm can save by behaving in a cost minimizing way.

The two indexes TE in (5) and AE in (6) can be combined into an overall index, OE, as follows:

$$OE = TE \cdot AE = C(r, y, F)/(r'x), \tag{7}$$

where $0 < OE \leq 1$. Then OE = 1 implies that the firm is both technically and allocatively efficient. Alternatively, OE < 1 indicates that the firm is not efficient, (1 - OE) measuring the proportional reduction in cost that the firm can achieve by becoming both technically and allocatively efficient.

Banker and Maindiratta's approach:

The evaluation of the efficiency indexes just discussed require an empirically tractable representation of the underlying technology F. Nonparametric methods can be used for this purpose. However, the results presented in section II assume that all data points in T are consistent with the profit maximization hypothesis. When we allow for the possibility of production inefficiencies, this assumption may not be satisfied. Thus, there is a need to extend the nonparametric analysis reported in section II to generate a representation of the underlying technology F which does not assume that profit maximizing behavior is necessarily satisfied for all observations in T. Such an extension has been proposed by Banker and Maindiratta. In the situation where equation (2) is not satisfied for all s, $t \in T$, Banker and Maindiratta proposed a method relying on the subset of data points that are consistent with profit maximization. This subset is given by:

$$E = \{t: \Delta_t = 0; \Delta_t \equiv \mathrm{Max}_s\,[(p_t'y_s - r_t'x_s) - (p_t'y_t - r_t'x_t): s \in T]; t \in T\}. \tag{8}$$

Clearly, the criterion function Δ_t in (8) always satisfies $\Delta_t \geq 0$ for all $t \in T$. And $\Delta_t = 0$ only if there does not exist any data point $s \in T$ such that $p_t'y_t - r_t'x_t < p_t'y_s - r_t'x_s$, i.e. such that equation (2) is violated. As a result, any observation in $E \subseteq T$ is necessarily consistent with profit maximization with respect to all data points in T. For this reason, Banker and Maindiratta call E the "efficient subset" of T. Banker and Maindiratta obtained the following results.

Proposition 2: (Banker and Maindiratta)

Assuming that E is non-empty, the following conditions are equivalent:

a) There exists a production set that rationalizes the data $\{(y_t, x_t; p_t, r_t): t \in E\}$ according to (1), and satisfies $(y_t, -x_t) \in F$ for all $t \in T$.

b) The data satisfy the Weak Axiom of Profit Maximization (WAPM):

$$p_t'y_t - r_t'x_t \geq p_t'y_s - r_t'x_s,\ \ t \in E,\ s \in T. \tag{9}$$

Given (9), there exists a family of convex, negative monotonic production sets F that:

- satisfies $(y_t, -x_t) \in F$ for all $t \in T$,
- rationalizes the data in E according to (1),
- satisfies $F^i \subseteq F \subseteq F_E^o$, where F^i is given in (3) and

$$F_E^o = \{(y, -x): p_t'y - r_t'x \leq p_t'y_t - r_t'x_t,\ t \in E;\ x \geq 0;\ y \geq 0\}. \tag{10}$$

Note that proposition 2 reduces to proposition 1 when E = T. Since we are interested in production inefficiencies, we focus here on the case where $\emptyset \subset E \subset T$, i.e. where E is a non-empty proper subset of T. Clearly, the observations in E are consistent with profit maximizing behavior, implying that their efficiency cannot be refuted by the data. In contrast, the observations that are in T but not in E are inconsistent with profit maximization. Moreover, F^i in (3) and F_E^o in (10) can be used as inner bounds and outer bounds representations of the underlying technology F. In turn, such representations can be used to evaluate production efficiency indexes and provide useful insights on the nature and magnitude of the inefficiencies.

This well defined approach proposed by Banker and Maindiratta has one drawback. Under production inefficiencies, the efficient subset E can be smaller than the set T. This means that the number of observations in E used to evaluate the outer-bound representation F_E^o can be smaller (and possibly much smaller) than the number of observations in T used to evaluate its inner-bound counterpart. This implies an asymmetry in the evaluation of the bounds on technology. This asymmetry could be fairly undesirable. For example, if E were to consist of only a few data points, the associated technology F_E^o would have few kinks, implying a relatively flat production frontier. Although not inconsistent with production theory, such a representation of the real world may be somewhat unrealistic. This point will be further illustrated in an empirical application presented in section V below. This suggests a need to explore some alternative approach to nonparametric representations of technology under production inefficiencies. This is the topic of the next section.

IV- An Alternative Approach:

In this section, we propose an alternative approach to the estimation of bounds on technology. In contrast with the Banker-Maindiratta approach, our method relies on all sample observations for both the upper bound and the lower bound representations. As such, our approach does not suffer from the asymmetry problem just discussed.

We propose a distinction between actual quantities (y_t, x_t) and "effective quantities" denoted by (Y_t, X_t). This can be done through an "augmentation hypothesis". Following Chavas and Cox, or Cox and Chavas, assume that actual and effective quantities are related through the functional relationships:

$$Y_{jt} = Y(y_{jt}, A_{jt});\ j = 1, \ldots, m, \text{ and } X_{it} = X(x_{it}, B_{it});\ i = 1, \ldots, n;\ t \in T, \tag{11}$$

where $Y_j(y_j, .)$, $j = 1, \ldots, m$, and $X_i(x_i, .)$, $i = 1, \ldots, n$, are one-to-one increasing functions, and A_{jt} and B_{it} are technology indexes associated with the t-th observation. This states, intuitively, that the technology indexes A and B can "augment" the actual quantities into effective quantities. Using (11), assume that problem (1) takes the form:

$$\pi(p_t, r_t, A_t, B_t) = \max_{y,x} [p_t'y - r_t'x: (Y(y, A_t), -X(x, B_t)) \in F^e], t \in T, \tag{12}$$

where $A_t = (A_{1t}, ..., A_{mt})'$ is a $(m\times 1)$ parameter vector and $B_t = (B_{1t}, ..., B_{nt})'$ is a $(n\times 1)$ parameter vector. The production technology F^e in (12) is an "effective technology" expressed in terms of effective quantities: $(Y_t, -X_t) \in F^e$, $Y_t = (Y_{1t}, ..., Y_{mt})'$ and $X_t = (X_{1t}, ..., X_{nt})'$ being vectors of effective quantities for the t-th observation with $Y_{jt} \equiv Y(y_{jt}, A_{jt})$ and $X_{it} \equiv X(x_{it}, B_{it})$.

The functions in (11), being one-to-one, can be inverted and expressed equivalently as: $y_{jt} = y(Y_{jt}, A_{jt})$, $j = 1, ..., m$, and $x_{it} = x(X_{it}, B_{it})$; $i = 1, ..., n$; $t \in T$. Then, equation (12) can be alternatively written as:

$$\pi(p_t, r_t, A_t, B_t) = \max_{Y,X} [p_t'y(Y, A_t) - r_t'x(X, B_t): (Y, -X) \in F^e], t \in T. \tag{12'}$$

Many specifications for the functions y(Y, A) and $x(X_i, B_i)$ are possible. Two of these specifications appear particularly appealing:[4] the scaling hypothesis corresponding to the multiplicative specification $y_j = Y_j/A_j$ and $x_i = X_i B_i$; and the translating hypothesis corresponding to the additive specification $y_j = Y_j - A_j$ and $x_i = X_i + B_i$. For simplicity, we will focus here on the translating hypothesis.[5] Under translating, equation (12') becomes:

$$\begin{aligned}\pi(p_t,r_t,A_t,B_t) &= \max_{Y,X} [p_t'(Y-A_t) - r_t'(X+B_t): (Y, -X) \in F^e] \\ &= - p_t'A_t - r_t'B_t + \max_{Y,X} [p_t'Y - r_t'X: (Y, -X) \in F^e],\end{aligned} \tag{13}$$

for $t \in T$. Equation (13) is a standard profit maximizing problem similar to (1), except that it involves the effective quantities (Y, X). The associated "augmented" Weak Axiom of Profit Maximization (corresponding to (2)) is:

$$p_t'Y_t - r_t'X_t \geq p_t'Y_s - r_t'X_s, \quad s, t \in T, \tag{14}$$

or

$$p_t'[y_t+A_t] - r_t'[x_t-B_t] \geq p_t'[y_s+A_s] - r_t'[x_s-B_s], \quad s, t \in T. \tag{14'}$$

Next, consider the following optimization problem:

$$\min_{A,B} [\textstyle\sum_{t\in T} (\sum_j \alpha_{jt} A_{jt} + \sum_i \beta_{it} B_{it}): \text{equation (14')}; A \geq 0, B \geq 0; Y_t \geq 0, X_t \geq 0, t \in T], \tag{15}$$

where the α and β are positive parameters.[6] Note that the solution to (15) for the A's and B's is necessarily consistent with the WAPM conditions (14) or (14'). Obtaining this solution is straightforward since (15) is a standard linear programming problem. Using the solution from (15) for the A's and B's, we can obtain the corresponding effective quantities: $Y_t = y_t + A_t$ and $X_t = x_t - B_t$, $t \in T$. Since these effective quantities necessarily satisfy the WAPM condition (14) for <u>all</u> s, $t \in T$, it follows that all the results presented in proposition 1 apply with respect to the effective quantities $(Y, -X) \in F^e$. And with $A \geq 0$ and $B \geq 0$ from (15), it follows that $y_t \leq Y_t$ and $x_t \geq X_t$, implying that $(y_t, -x_t) \in F^e$ for all $t \in T$. The "effective technology" F^e can thus be interpreted as the technology that is "as close to the data as possible" while satisfying WAPM in (14) for <u>all</u> data points. Substituting (Y, X) for (y, x), equation (3) gives an inner bound representation of the effective technology, F^{ei}, while equation (4) gives its outer bound representation, F^{eo}. Note that, in contrast to the Banker-Maindiratta approach, the evaluation of the outer-bound representation F^{eo} uses all the data points in T. These inner and outer bounds can then be used to estimate production efficiency indexes.

V- An Application:

In this section, we illustrate the usefulness of the results presented above in the context of time series data on U.S. agriculture. The analysis uses the data developed by Capalbo and Vo on U.S. agriculture. It involves annual data covering the period 1948-1983 on the prices and quantities of agricultural outputs and inputs. Outputs consist of six categories: (1) small grains; (2) coarse grains; (3) field crops; (4) fruits; (5) vegetables; and (6) animal products. Inputs consist of ten categories: (1) family labor; (2) hired labor; (3) land; (4) structures; (5) other capital; (6) energy; (7) fertilizer; (8) pesticides; (9) feed and seed; and (10) miscellaneous. The quantity indexes are all equal to 1 in 1977. The price indexes are implicit prices defined such that price multiplied by quantity equals expenditure.

The nonparametric methods discussed above are applied to this data set. The WAPM condition (2) is evaluated for all data points. A number of data points are found to violate WAPM in (2). In this situation, we want to evaluate the bounds on technology. As discussed in section III and IV, we consider two options: either working with the "efficient subset" E in (5), as proposed by Banker and Maindiratta; or using the "effective quantity" approach.

In the context of the data set, we evaluate the efficient subset E given in (5) to find that E = {1982}, i.e. that the 1982 data point is the only point which does not violate WAPM in (2). This is due in large part to the high rate of technical progress in U.S. agriculture (e.g., Capalbo and Antle; Cox and Chavas). Indeed, technical progress expands the production set, which makes "older" data points appear technically inefficient and thus inconsistent with profit maximization based on a stable technology. In our case, the fact that the efficient set E consists of a single data point has strong implications for Banker and Maindiratta's approach. It implies that the boundary of the production set $F_E{}^o$ given in (10) is in fact linear in x and y. This means for example that the elasticities of substitution among any two inputs are infinite. This also means that the solution to the profit maximization problem (1) based on $F_E{}^o$ either is zero or it is unbounded.[7/] This suggests that the outer bound production set $F_E{}^o$ in (10) is "too flat" and does not give a realistic representation of U.S. agricultural technology.

We evaluate indexes of technical efficiency (TE), allocative efficiency (AE) and overall efficiency (OE) at each data point. First, this is done following the Banker-Maindiratta representations of the technology.[8/] The estimate of TE at time t based on the inner-bound representation F^i is obtained as $TE^i = TE(y_t, x_t, F^i)$ from (5), where F^i is given in (3). The estimate of OE at time t based on the inner-bound representation F^i is denoted by OE^i, It is obtained from (7) after solving numerically for $C(r_t, y_t, F^i) = \min_x [\ r_t'x: (y_t, -x) \in F^i]$, $t \in T$. The corresponding allocative efficiency index AE^i is then calculated as $AE^i = OE^i/TE^i$ from (7).[9/] Estimates of technical, overall, and allocative efficiency indexes are similarly obtained based on the Banker-Maindiratta outer-bound representation of the technology $F_E{}^o$ given in (10). These indexes are denoted by TE^o, OE^o, and AE^o, respectively.[10/] The results are reported in Table 1.

Next, the efficiency indexes TE, AE and OE are estimated following the "effective quantity" approach proposed above. Under translating, the technology parameters A and B are obtained by solving equation (15) with $\alpha_{jt} = 1$ and $\beta_{it} = 1$.[11/] Using the solution for A and B from (15), the corresponding effective quantities are computed as: $Y_t = y_t + A_t$ and $X_t = x_t - B_t$. The A's and B's measure the difference between actual quantities (y, x) and effective quantities (Y, X) and account for technical change and/or technical inefficiencies in U.S. agriculture over the last decades (Cox and Chavas). After substituting (Y, X) for (y, x), the inner bound representation of the effective technology, F^{ei}, is obtained from (3), while its outer bound representation, F^{eo} is obtained from (4). The associated technical, allocative and overall efficiency indexes are again estimated from (5), (6) and (7). They are denoted by TE^{ei}, AE^{ei}, and OE^{ei} when based on the inner-bound representation F^{i}, and by TE^{eo}, AE^{eo}, and OE^{eo} when based on the outer-bound representation F^{eo}.[12/] The results are reported in Table 2.

In comparing Table 1 with Table 2, the estimates of production efficiency indexes based on <u>inner-bound representations</u> of technology are fairly similar. The results based on F^{i} (using the Banker-Maindiratta approach reported in Table 1) or based on F^{ei} (using the "effective technology" approach reported in Table 2) indicate that most observations are either efficient or very close to being efficient. The lowest index of overall efficiency OE^{i} in Table 1 is 0.967 for 1951 and 1959. And the lowest index of overall efficiency OE^{ei} in Table 2 is 0.957 for 1978. Recall that these indexes can be interpreted as upper-bound estimates of overall efficiency indexes OE in (7). It follows that, using either approach, it is possible to interpret the data in such a way that the U.S. agricultural sector is subject to little technical and allocative inefficiencies during the sample period.

This contrasts sharply with the production efficiency indexes based on <u>outer-bound representations</u> of technology. Indeed, the corresponding efficiency indexes take values that can be much smaller than one (see Tables 1 and 2). For example, the technical efficiency index TE^{o} (based on the Banker-Maindiratta outer-bound representation F_E^{o}) is as low as 0.656 in 1950 (see

Table 1). And the technical efficiency index TE^{eo} (based on the outer-bound representation of the effective technology F^{eo}) is as low as 0.628 in 1949 (see Table 2). Recall that these indexes can be interpreted as lower-bound estimates of technical efficiency. This suggests that it is possible to interpret the data in such a way that the U.S. agricultural sector is subject to important technical inefficiencies and/or technical change during the sample period. This would be consistent with previous parametric evidence reporting a high rate of technical progress in U.S. agriculture over the last few decades (e.g., Antle and Capalbo). Note that, in the absence of technical inefficiency, our index TE in (5) becomes an input-based productivity index which uses F as the reference technology (see Caves et al.). Thus, if we were willing to assume away technical inefficiencies, the TE indexes reported in Tables 1 and 2 could be interpreted as productivity indexes measuring the rate of technical progress in U.S. agriculture. With this interpretation in mind, the indexes TE^{o} in Table 1 as well as TE^{eo} in Table 2 would suggest important and fairly steady technical progress in U.S. agriculture from 1948 to 1983. However, TE^{eo} tends to increase a little faster over time than TE^{o}. In other words, on average, the Banker-Maindiratta approach (reported in Table 1) would identify <u>less</u> technical inefficiencies and/or technical progress compared to the "effective technology" approach (reported in Table 2).

The allocative efficiency (AE) and overall efficiency (OE) indexes based on outer bound representations of technology can also be much smaller than one. For example, the smallest value of OE^{o} (based on the Banker-Maindiratta outer-bound representation F_E^{o}) is 0.083 for 1950 (see Table 1). And the smallest value of OE^{eo} (based on the outer-bound representation of the effective technology F^{eo}) is 0.118 for 1950 (see Table 2). Recall that these estimates can be interpreted as lower-bound estimates of overall efficiency. They suggest that it is possible to interpret the data in such a way that the U.S. agricultural sector has been subject to very large inefficiencies during the sample period. These measures show that large inefficiencies characterize the early part of the sample, and that such inefficiencies have been greatly reduced in the latter part of the sample (see Tables 1 and 2). They indicate that a large part of the overall inefficiencies are in fact allocative inefficiencies. The allocative efficiency indexes AE^o in Table 1 and AE^{eo} in Table 2 tend to rise over time. They are very low in the late 1940's and early 1950's and become close to one in the early 1980's.[13/] Also, the AE^o index in Table 1 starts lower and rises faster than AE^{eo} in Table 2. This indicates that the Banker-Maindiratta approach tends to find higher levels of allocative inefficiencies that the "effective technology" approach.

These results have some important implications. First, they indicate how the Banker-Maindiratta approach can differ from our proposed "effective technology" approach. Tables 1 and 2 suggest that, compared the Banker-Maindiratta approach, our approach tends to give lower technical efficiency indexes, higher allocative efficiency indexes, and higher overall efficiency indexes. Second, these results indicate that, using either approach, the gap between the inner-bound and outer-bound representations of technology is <u>quite large</u>. This gap is illustrated by finding very little inefficiencies using the inner-bound representations of technology, while uncovering very large inefficiencies using the outer-bound representations of technology. This shows that these data (along with the theory) are unable to provide tight estimates of the underlying technology. In other words, there seems to be a fairly wide range of technologies that are consistent with the data and production theory. This suggests that, for a given data set, the production technology is largely <u>underidentified</u>: there are possibly many different representations of the underlying technology that are consistent with both the data and the theory. In this context, empirical searches (e.g. through the parametric testing of alternative functional forms) for a "true technology" may be futile. The nonparametric bounds discussed here could help better assess the range of identification (or underidentification) of the underlying technology, and better evaluate the strength of the information that can be recovered from a particular data set.

VI- <u>Conclusion</u>:

After reviewing the Banker-Maindiratta approach to production efficiency analysis, this paper proposes an alternative nonparametric method to estimate inner bounds and outer bounds of a technology and their associated technical, allocative, and overall efficiency indexes. The proposed method relies on the characterization of an "effective technology". Compared the Banker-Maindiratta approach, it has the advantage of using all the data points in the estimation of both bounds. This was illustrated in an empirical application for which the Banker-Maindiratta approach generated an outer-bound representation that was deemed somewhat unrealistic. The empirical results from the two approaches were compared and contrasted. Compared to the Banker-Maindiratta approach, our "effective technology" approach tends to generate lower technical efficiency indexes, but higher allocative and overall efficiency indexes. Using either approach, the gap between the inner-bound and outer-bound representations of the technology is found to be quite large. This indicates an underidentification of the underlying technology, i.e. many possible representations of the production technology can be found to be consistent with both the data and the theory. Two important implications follow. First, based on those data, parametric attempts to find a single "true representation" of the technology are likely to be futile. Second, it would be helpful to assess the range of identification of a technology that is possible from a particular production data set. The nonparametric bounds discussed in this paper should help economists better assess this range, allowing them to become more aware of the strengths as well as limitations of their data, and to better evaluate their informational content.

Table 1- Production Efficiency Indexes: Banker-Maindiratta's Approach

	Technical Efficiency		Allocative Efficiency		Overall Efficiency	
	TE^i	TE^o	AE^i	AE^o	OE^i	OE^o
1948	1.000	0.669	1.000	0.127	1.000	0.085
1949	1.000	0.661	0.994	0.131	0.994	0.087
1950	1.000	0.656	0.974	0.126	0.974	0.083
1951	1.000	0.671	0.967	0.127	0.967	0.086
1952	1.000	0.687	0.978	0.135	0.978	0.093
1953	1.000	0.703	0.999	0.162	0.999	0.114
1954	1.000	0.705	0.985	0.157	0.985	0.111
1955	1.000	0.708	0.975	0.176	0.975	0.125
1956	1.000	0.751	1.000	0.197	1.000	0.148
1957	1.000	0.733	1.000	0.237	1.000	0.174
1958	1.000	0.752	1.000	0.231	1.000	0.174
1959	1.000	0.740	0.967	0.257	0.967	0.190
1960	1.000	0.755	0.982	0.287	0.982	0.217
1961	1.000	0.765	0.977	0.287	0.977	0.220
1962	1.000	0.778	0.986	0.288	0.986	0.224
1963	1.000	0.790	0.993	0.298	0.993	0.236
1964	1.000	0.804	1.000	0.314	1.000	0.252
1965	1.000	0.801	0.992	0.308	0.992	0.246
1966	1.000	0.806	1.000	0.358	1.000	0.288
1967	1.000	0.821	1.000	0.403	1.000	0.331
1968	1.000	0.825	1.000	0.444	1.000	0.366
1969	1.000	0.832	1.000	0.480	1.000	0.399
1970	1.000	0.818	0.992	0.515	0.992	0.421
1971	1.000	0.851	1.000	0.501	1.000	0.426
1972	1.000	0.857	0.997	0.479	0.997	0.411
1973	1.000	0.867	0.981	0.480	0.981	0.416
1974	1.000	0.885	1.000	0.565	1.000	0.500
1975	1.000	0.892	1.000	0.602	1.000	0.537
1976	1.000	0.898	1.000	0.640	1.000	0.575
1977	1.000	0.923	1.000	0.693	1.000	0.640
1978	1.000	0.905	0.980	0.740	0.980	0.670
1979	1.000	0.942	0.983	0.786	0.983	0.741
1980	1.000	0.940	1.000	0.886	1.000	0.833
1981	1.000	0.991	1.000	0.904	1.000	0.896
1982	1.000	1.000	1.000	1.000	1.000	1.000
1983	1.000	0.935	1.000	0.872	1.000	0.815

Table 2- Production Efficiency Indexes: "Effective Quantity" Approach

	Technical Efficiency		Allocative Efficiency		Overall Efficiency	
	TE^{ei}	TE^{eo}	AE^{ei}	AE^{eo}	OE^{ei}	OE^{eo}
1948	1.000	0.636	0.996	0.206	0.996	0.131
1949	1.000	0.628	0.994	0.200	0.994	0.126
1950	1.000	0.623	0.974	0.189	0.974	0.118
1951	1.000	0.638	0.966	0.212	0.966	0.135
1952	0.997	0.654	0.995	0.248	0.993	0.162
1953	1.000	0.669	0.980	0.307	0.980	0.206
1954	1.000	0.671	0.979	0.293	0.979	0.197
1955	0.996	0.674	0.977	0.333	0.973	0.225
1956	1.000	0.716	0.997	0.432	0.997	0.309
1957	1.000	0.698	1.000	0.459	1.000	0.320
1958	1.000	0.717	1.000	0.486	1.000	0.348
1959	1.000	0.706	0.994	0.545	0.994	0.385
1960	1.000	0.721	0.974	0.578	0.974	0.417
1961	1.000	0.731	0.973	0.610	0.973	0.446
1962	1.000	0.744	0.980	0.617	0.980	0.459
1963	1.000	0.755	0.980	0.646	0.980	0.488
1964	1.000	0.769	1.000	0.663	1.000	0.510
1965	1.000	0.767	1.000	0.731	1.000	0.561
1966	1.000	0.772	0.987	0.690	0.987	0.533
1967	1.000	0.786	0.994	0.739	0.994	0.581
1968	1.000	0.790	0.999	0.764	0.999	0.604
1969	1.000	0.797	0.998	0.780	0.998	.0622
1970	1.000	0.784	0.978	0.795	0.978	0.623
1971	1.000	0.816	0.987	0.838	0.987	0.684
1972	0.998	0.823	0.968	0.842	0.966	0.694
1973	0.986	0.833	0.972	0.818	0.959	0.682
1974	1.000	0.851	0.994	0.871	0.994	0.741
1975	1.000	0.858	0.997	0.946	0.997	0.812
1976	1.000	0.864	0.989	0.931	0.989	0.805
1977	1.000	0.890	0.995	0.966	0.995	0.859
1978	0.990	0.872	0.967	0.951	0.957	0.830
1979	0.989	0.909	0.985	0.990	0.974	0.900
1980	1.000	0.907	1.000	1.000	1.000	0.907
1981	1.000	1.000	1.000	1.000	1.000	1.000
1982	1.000	1.000	1.000	1.000	1.000	1.000
1983	1.000	0.902	0.999	0.991	0.999	0.893

REFERENCES

Afriat, S.N. "Efficiency Estimation of Production Functions." International Economic Review 13(October 1972):568-598.

Banker, R.D. and A. Maindiratta "Nonparametric Analysis of Technical and Allocative Efficiencies in Production" Econometrica 56(November 1988):1315-1332.

Bauer, P.W. "Recent Developments in the Econometric Estimation of Frontiers" Journal of Econometrics 46(1990):39-56.

Binswanger, H.P. "The Measurement of Technical Change Biases with Many Factors of Production" American Economic Review 64(1974):964-976.

Capalbo, S.M. and J.M. Antle Agricultural Productivity: Measurement and Explanation Resources for the Future, Inc., Washington, D.C., 1988.

Capalbo, S.M. and T.V. Vo "A Review of the Evidence on Agricultural Productivity and Aggregate Technology" in Agricultural Productivity: Measurement and Explanation S.M. Capalbo and J.M. Antle, eds., Resources for the Future, Inc., Washington, D.C., 1988.

Caves, D.W., L.R. Christensen, and W.E. Diewert "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity" Econometrica 50(1982):1393-1414.

Chavas, J.P. and T.L. Cox "A Nonparametric Analysis of Productivity: The Case of U.S. and Japanese Manufacturing" American Economic Review 80(June 1990):450-464.

Cox, T.L. and J.P. Chavas "A Nonparametric Analysis of Productivity: the Case of U.S. Agriculture" European Review of Agricultural Economics 17(1990):449-464.

Diewert, W.E. and C. Parkan "Linear Programming Tests of Regularity Conditions for Production Functions" in Quantitative Studies on Production and Prices W. Eichhorn, R. Henn, K. Neumann, and R.W. Shephard, eds., Physica Verlag, Wurzburb, 1983.

Debreu, G. "The Coefficient of Resource Utilization" Econometrica 19(1951):273-292.

Färe, R., S. Grosskopf, and C.A.K. Lovell The Measurement of Efficiency of Production Kluwer-Nijhoff Publishers, Boston, 1985.

Farrell, M.J. "The Measurement of Productive efficiency" Journal of the Royal Statistical Society, Series A, 120(1957):253-290.

Farrell, M.J. and M. Fieldhouse "Estimating Efficient Production under Increasing Return to Scale" Journal of the Royal Statistical Society, Series A, 125(1962):252-267.

Forsund, F.R., C.A.K. Lovell, and P. Schmidt "A Survey of Frontier Production Functions and their Relationship to Efficiency Measurement" Journal of Econometrics 13(1980):5-25.

Hanoch, G. and M. Rothschild "Testing the Assumptions of Production Theory: A Nonparametric Approach" Journal of Political Economy 80(1972):256-275.

Pollak, R.A. and T.J. Wales "Demographic Variables in Demand Analysis" Econometrica 49(1981):1533-1551.

Seiford, L.M. and R.M. Thrall "Recent Developments in DEA: The Mathematical Programming Approach to Frontier Analysis" Journal of Econometrics 46(1990):7-38.

Varian, H. "The Nonparametric Approach to Production Analysis." Econometrica 52(May 1984):579-597.

Footnotes

1.The set F is said to be negative monotonic if $z \in F$ and $z' \leq z$ implies that $z' \in F$. This is basically a "free disposal" assumption.

2. Alternative measures of technical efficiency have been proposed in the literature. For example, an index of technical efficiency can be obtained by rescaling outputs instead of inputs (see Färe et al., chapter 4). Although input-based and output-based indexes of technical efficiency are identical under constant return to scale, they differ under variable return to scale (Färe et al., p. 132). More specifically, the input-based index of technical efficiency is lower (higher) than the corresponding output-based index under decreasing (increasing) returns to scale (Färe et al., p. 133). Also, Färe et al. have proposed analyzing technical efficiency without the "negative monotonicity" assumption (where our "strong disposability" assumption is replaced by a "weak disposability" assumption). Finally, non-radial measures of technical efficiency have also been proposed (e.g., Färe et al., chapter 7).

3. An alternative approach is to evaluate allocative efficiency in terms of profit maximizing behavior. This is the approach followed by Banker and Maindiratta, who proposed to define an allocative efficiency index in terms of a profit ratio (instead of a cost ratio as in equation (6)). Note that, in order to be meaningful, such an index requires profit to be positive, a condition that may not always be satisfied empirically.

4. For example, see Pollak and Wales for the use of scaling and translating hypotheses in the context of consumer demand.

5. In the context of scaling, equation (12') becomes:

$$\pi(p_t, r_t, A_t, B_t) = \max_{Y,X} [p_t'(Y/A_t) - r_t'(X\, B_t): (Y, -X) \in F^c]$$

for $t \in T$. The associated Weak Axiom of Profit Maximization is:

$$p_t'y_t - r_t'x_t \geq p_t'[y_s\, A_s/A_t] - r_t'[x_s\, B_t/B_s], \quad s, t \in T.$$

Note that, in contrast with (14'), the above expression is nonlinear in A and B. This nonlinearity makes the scaling hypothesis a little more difficult to use empirically (compared to the translating hypothesis). This is the main reason why we focus our attention here on the translating hypothesis.

6. The issue of the choice of the parameters α and β will discussed in the next section.

7. Banker and Maindiratta proposed restricting the feasible set to fall within the range of the data. While this eliminates the possibility of finding unbounded solutions, it simply transforms the solution to problem (1) into a "bang-bang solution" which lies at the boundary of the data range. Note that such restrictions also tend to increase the estimated value of the efficiency indexes TE and OE.

8. In contrast with Banker and Maindiratta (who used profit ratios as efficiency indexes), we measure efficiency in terms of cost ratios as given in (5), (6) and (7). The reason is that, in our data, not all profits are positive, thus making profit ratios rather unattractive measures of efficiency.

9. Note that TE^i and OE^i (both based on the inner bound representation F^i given in (3)) are upper-bound estimates of, respectively, technical efficiency and overall efficiency indexes. However, it does not follow that $AE^i = OEi/TE^i$ is necessarily an upper-bound estimate of the allocative efficiency index. This is because the ratio of maxima is not necessarily the maximum of the ratio (see Banker and Maindiratta).

10. Note that TE^o and OE^o (based on the outer-bound representation of the technology F_E^o)can be interpreted as lower-bound estimates of technical and overall efficiency indexes, respectively.

11. The choice for α and β was made on the basis that all output and input quantity indexes are equal to 1 for the year 1977. In other words, for that year, all agricultural inputs and outputs have the same index, thus making the corresponding A's and B's comparable.

12. Again, TE^{ei} and OE^{ei} (based on the inner-bound representation of the effective technology F^{ei}) can be interpreted as upper-bound estimates of technical and overall efficiency indexes, respectively. And TE^{eo} and OE^{eo} (based on the outer-bound representation of the effective technology F^{eo}) can be interpreted as lower-bound estimates of technical and overall efficiency indexes, respectively.

13. Note that, in the context of technical progress, the rise in the allocative efficiency index over time is consistent with biased technical change in U.S. agriculture (e.g., see Binswanger). Under biased technical change, the shift in the production frontier takes place in a non-homothetic way, which alters optimal factor proportions. Our estimated index of allocative efficiency could then simply reflect the changing optimal factor proportions over time, the index being higher (lower) for data points that are close to (far from) the reference technology.