August 1993 No. 367

# SPATIAL HEDONIC PRICING AND TRADE

By

Jean-Paul Chavas
Thomas L. Cox
and
Edward V. Jesse
University of Wisconsin-Madison[1]

---

[1] Respectively professor, associate professor and professor of agricultural economics, University of Wisconsin-Madison. The seniority of authorship is shared. This research was funded in part by a Hatch grant from the College of Agriculture, University of Wisconsin-Madison.

# Spatial Hedonic Pricing and Trade

by

Jean-Paul Chavas

Thomas L. Cox

and

Edward Jesse[1/]

Abstract: A spatial trade model in a vertical sector is developed, allowing for an explicit analysis of nonmarket goods. The model helps bridge the gap between the Samuelson-Takayama-Judge approach to trade modeling, and Rosen's analysis of market allocation involving differentiated products. It provides a basis for analyzing the allocation and pricing of nonmarket goods in spatial and vertical markets. The usefulness of the model is illustrated in the context of resource allocation in the U.S. dairy sector.

J.E.L. Classification Number: C60, F10, Q13.

---

[1/] Respectively professor, associate professor, and professor of agricultural economics, University of Wisconsin-Madison. The seniority of authorship is equally shared. This research was funded in part by a Hatch grant from the College of Agriculture, University of Wisconsin-Madison.

## Spatial Hedonic Pricing and Trade

### I. Introduction.

The early work by Gorman, Becker and Lancaster on the allocation on nonmarket goods led to the development of household production theory, where the household obtains utility from nonmarket goods that are produced by combining market inputs with leisure time (e.g. Deaton and Muellbauer; Stigler and Becker). The shadow prices of these nonmarket goods have been the subject of much interest. The hedonic technique is motivated when the nonmarket goods are "characteristics" reflecting quality differences among goods. In estimating hedonic functions, the prices of market goods are typically regressed on the corresponding characteristics, the regression coefficients being interpreted as shadow price of these characteristics. (e.g. Griliches; Dhrymes). In a seminal paper, Rosen developed the analysis of the shadow price of characteristics in a competitive market equilibrium framework. He showed that, in general, the hedonic price function reflects both the distribution of the marginal rates of substitution over consumers, and the distribution of the marginal rate of transformation over producing firms. This has stimulated much research on the implicit pricing of nonmarket characteristics for differentiated products (e.g. Lucas; Ball and Kirwan; Palmquist; Epple).

At this point, the literature suggests that little research has been conducted on multimarket aspects of hedonic pricing. This is a situation of interest whenever nonmarket characteristics are allocated among several markets. These markets could be spatial markets for the same commodity, as well as vertical markets for successive stages in a marketing channel. There is considerable research developing commodity trade models under spatially dispersed competitive markets (e.g. Samuelson; Takayama and Judge). Also, the economic analysis of vertical market equilibrium in a marketing channel is now well established (e.g. Gardner), but such models have focused exclusively on the allocation of market goods. Thus, there is a need to extend this analysis to include the allocation of nonmarket characteristics in a multimarket framework. The objective of this paper is to develop a spatial trade model in a vertical sector, allowing for an explicit analysis of nonmarket characteristics. The intent is to help bridge the gap existing between the Samuelson-Takayama-Judge (STJ) approach to commodity trade modeling, and Rosen's analysis of market allocation involving differentiated products. This provides a basis for analyzing the allocation and pricing of nonmarket characteristics in vertical markets.

The paper is organized as follows. Section II presents an extension of the STJ model for spatial markets, allowing for explicit vertical market linkages. It considers a two-stage vertical sector, where primary commodities are used in the production of secondary commodities that are eventually consumed. Both primary and secondary commodities can be produced, consumed, and traded in spatial markets. This provides a basis for formulating a model of competitive spatial market equilibrium, reflecting the effects of production cost for the primary and secondary commodities, of transportation cost, and of the spatial distribution of consumer demands. In section III, this model is specialized to investigate the allocation of nonmarket characteristics across both spatial markets and successive stages of the vertical sector. This is done by relying on a Lancasterian-type model linking the market commodities with the nonmarket goods. It allows for an evaluation of the spatial shadow pricing of the nonmarket characteristics.

The usefulness of the model is illustrated in section IV which centers on resource allocation in the U.S. dairy sector. The investigation focuses on the allocation of milk production both spatially (among 14 producing regions) and vertically (through the production of 9 dairy products). The nonmarket characteristics are the basic components of milk (fat, protein, and carbohydrates) to be allocated among the 9 dairy products and the 14 regions. The model provides a basis for evaluating regional component pricing of fat, protein and carbohydrates. This is done under a competitive market scenario. Alternative scenarios incorporating in the model government price support and milk marketing orders are also evaluated. This allows for an investigation of the effects of dairy policy on resource allocation and regional welfare distribution in the U.S. dairy sector. Finally, concluding remarks are presented in section V.

## II. The Model.

Consider the spatial allocation of resources among J regions. The resources consist in a set of primary commodities and a set of secondary commodities. The primary commodities are not consumer goods: they are used exclusively as inputs in the production of the secondary commodities. Each region is a potential producer of the primary commodities, and a potential producer as well as potential consumer of the secondary commodities. Also, each region can trade both primary and secondary commodities with any other region . The question, then, is how to analyze the corresponding competitive spatial market equilibrium. This can be done by developing a market equilibrium model of resource allocation and trade over the J regions. In this section, we propose a simple formulation of spatial competitive market equilibrium and set up the notation for the rest of the paper.

Let N be the number of primary commodities, $w_{in}$ denoting the quantity produced of the n-th primary commodity in region i, and $x_{in}$ being the quantity of the n-th primary commodity used as an input in the production of the secondary commodities in region i, $n = 1, \ldots, N$, $i = 1, \ldots, J$. Let K be the number of secondary commodities. Denote by $y_{ik}$ the production level of the k-th secondary commodity in region i, $k = 1, \ldots, K$, $i = 1, \ldots, J$. And denote by $z_{ik}$ the consumption level of the k-th commodity in region i, $k = 1, \ldots, K$, $i = 1, \ldots, J$.

The production of the secondary commodities y will be influenced by the interregional trade in the primary commodities x and by the production technology transforming the primary commodities into the secondary commodities y. And the consumption of the secondary commodities z will be influenced by their corresponding production y and by the interregional trade in the secondary commodities. Denote by $T_{ijn} \geq 0$ the export of the n-th primary commodity from region i to region j, and by $T_{jin} \geq 0$ the import of the n-th primary commodity from region i to region j. Similarly, denote by $t_{ijk} \geq 0$ the export of the k-th secondary commodity from region i to region j, and by $t_{jik} \geq 0$ the import of the k-th secondary commodity from region i to region j. Using this notation, $T_{iin} \geq 0$ will be interpreted as the quantity of the n-th primary commodity that is both produced and used in the production of the secondary commodities within the i-th region (i.e. not exported to other regions). And $t_{iik} \geq 0$ will be interpreted as the quantity of the k-th secondary commodity that is both produced and consumed in the i-th region.

The production of the secondary commodities y involves two kinds of inputs: the primary commodities x, and other inputs denoted by the vector v. The technology involved in the transformation of the primary inputs x into the secondary inputs y in region i is given by the production possibility set $F_i$:

$$(v_i,\ x_i,\ y_i) \in F_i, \tag{1}$$

where $x_i = \{x_{in}: n = 1, \ldots, N\}$ is the vector of primary inputs, $y_i = \{y_{ik}: k = 1, \ldots, K\}$ is the vector of secondary outputs, and $v_i$ is the vector of other inputs (besides $x_i$) used in the production of $y_i$, i = 1, ..., J. Equation (1) simply establishes the technological relationship between inputs $(v_i, x_i)$ and feasible secondary outputs $y_i$ in each region. We assume that the production possibility set $F_i$ is nonempty, closed, and convex.

Under competition, let $r_i$ denote the vector of market prices for the inputs $v_i$, i = 1, ..., J. Then efficient use of the inputs $v_i$ requires that they are chosen in a cost minimizing way as follows:

$$G_i(x_i,\ y_i) = \min_{v_i}\{r_i' v_i: (v_i,\ x_i,\ y_i) \in F_i\}, \tag{2}$$

where $G_i(x_i, y_i)$ is a (restricted) cost function measuring the cost of optimal input use $v_i$, conditional on primary inputs $x_i$ and on output levels $y_i$, $i = 1, \ldots, J$. We will assume throughout the paper that the cost function $G_i(x_i, y_i)$ is a decreasing function of $x_i$, and an increasing function of $y_i$.

The trade flow constraints across regions take the form:

$$w_{in} \geq \sum_{j=1}^{J} T_{ijn}, \tag{3a}$$

$$\sum_{j=1}^{J} T_{jin} \geq x_{in}, \tag{3b}$$

$$y_{ik} \geq \sum_{j=1}^{J} t_{ijk}, \tag{3c}$$

$$\sum_{j=1}^{J} t_{jik} \geq z_{ik}. \tag{3d}$$

In any region, these equations guarantee that exports plus domestic use cannot be larger than domestic production, and that domestic consumption cannot exceed domestic production plus imports. This holds for primary commodities (equations (3a) and (3b)) as well as secondary commodities (equations (3c) and (3d)).

A market equilibrium must satisfy the technology constraints (1) and the trade flow constraints (3). It must also allocate resources in an efficient manner both across commodities and across space. One way of capturing this efficiency is to consider the following quasi-welfare function:

$$V(w, x, y, z) = \sum_{i=1}^{J} \{D_i(z_i) - S_i(w_i) - G_i(x_i, y_i)\}, \tag{4}$$

where $w = \{w_{in}: i = 1, ..., J, n = 1, ..., N\}$, $x = \{x_{in}: i = 1, ..., J, n = 1, ..., N\}$, $y = \{y_{ik}: i = 1, ..., J, k = 1, ..., K\}$, $z = \{z_{ik}: i = 1, ..., J, k = 1, ..., K\}$, and $G_i(x_i, y_i)$ is the cost function defined in equation (2). The quasi-welfare function V defined in (4) involves three sets of terms: D, S and G. The terms D is interpreted as a measure of the total benefits to the consumers purchasing the secondary goods z. And the terms S is interpreted as the cost of producing the primary commodities w. Given the cost function G defined in (2), it follows (S + G) is the total cost of production of the secondary goods z in the absence of trade. Then, the quasi-welfare function V in (4) is a measure of net social benefits (i.e. consumer benefits (D) minus total production cost (S + G)) in the absence of trade.

We will make the following assumption:

Assumption A: The function V(w, x, y, z) is differentiable and concave in (w, x, y, z), and satisfies:

$$\partial S_i/\partial w_{in} = p_{in}^s \geq 0, \quad n = 1, ..., N,$$

$$\partial D_i/\partial z_{ik} = p_{ik}^d \geq 0, \quad k = 1, ..., K,$$

where $p_{in}^s$ is the price received by the producers of the n-th primary commodity in region i, and $p_{ik}^d$ is the price paid by the consumers of the k-th secondary commodity in region i , i = 1, ..., J.

Assumption A states: that the quasi-welfare function is well-behaved; that the market prices of the primary commodities are equal to their marginal cost of production; and that the market prices of the secondary commodities are equal to their marginal consumer benefit. These conditions are consistent with competitive market equilibrium, where prices reflect the marginal valuation of the corresponding goods.

Let $C_{ijn} \geq 0$ be the unit cost of transportation of the n-th primary commodity from region i to region j. Similarly, let $c_{ijk} \geq 0$ be the unit cost of transportation of the k-th secondary commodity from region i to region j. We assume throughout the paper that $C_{iin} = 0$ and $c_{iik} = 0$, i.e. that transportation costs are zero in the absence of trade. Then, consider the following optimization model:

$$\max_{w,x,y,z,T,t} \{V(w,\ x\ ,y,\ z) - \sum_{i,j,n} T_{ijn} C_{ijn} - \sum_{i,j,k} t_{ijk} c_{ijk} : \qquad (5)$$
$$\text{equations (3)},\ w \geq 0,\ x \geq 0,\ y \geq 0,\ z \geq 0,\ T \geq 0,\ t \geq 0\}.$$

Equation (5) maximizes the quasi-welfare function V(w, x, y, z) net of transportation cost, subject to the trade flow constraints (3). Next, we show that, under assumption A, the optimization problem (5) generates the competitive spatial market equilibrium.

Under assumption A, problem (5) is a standard concave programming problem, subject to linear constraints. Provided that it has a bounded solution, it can be alternatively characterized as the saddle point of the following Lagrangean:

$$\begin{aligned} L = V(w,\ x,\ y,\ z) &- \sum_{i,j,n} T_{ijn} C_{ijn} - \sum_{i,j,k} t_{ijk} c_{ijk} \\ &+ \sum_{i,n} \alpha_{in}\ [w_{in} - \sum_{j} T_{ijn}] \\ &+ \sum_{i,n} \beta_{in}\ [\sum_{j} T_{jin} - x_{in}] \\ &+ \sum_{i,k} \gamma_{ik}\ [y_{ik} - \sum_{j} t_{ijk}] \\ &+ \sum_{i,k} \delta_{ik}\ [\sum_{j} t_{jik} - z_{ik}], \end{aligned}$$

where $\alpha \geq 0$, $\beta \geq 0$, $\gamma \geq 0$ and $\delta \geq 0$ are Lagrange multipliers corresponding to the constraints (2). Under assumption A, the Kuhn-Tucker conditions associated with the optimization problem (5) provide necessary and sufficient conditions for the solution to (5). They are:

$$\frac{\partial L}{\partial w_{in}} = -\frac{\partial S_i}{\partial w_{in}} + \alpha_{in} \leq 0, \; w_{in} = 0, \qquad = 0, \; w_{in} > 0, \tag{6a}$$

$$\frac{\partial L}{\partial x_{in}} = -\frac{\partial G_i}{\partial x_{in}} - \beta_{in} \leq 0, \; x_{in} = 0, \qquad = 0, \; x_{in} > 0, \tag{6b}$$

$$\frac{\partial L}{\partial y_{ik}} = -\frac{\partial G_i}{\partial y_{ik}} + \gamma_{ik} \leq 0, \; y_{ik} = 0 \qquad = 0, \; y_{ik} > 0, \tag{6c}$$

$$\frac{\partial L}{\partial z_{ik}} = \frac{\partial D_i}{\partial z_{ik}} - \delta_{ik} \leq 0, \; z_{ik} = 0, \qquad = 0, \; z_{ik} > 0, \tag{6d}$$

$$\frac{\partial L}{\partial T_{ijn}} = C_{ijn} + \beta_{jn} - \alpha_{in} \leq 0, \; T_{ijn} = 0, \qquad = 0, \; T_{ijn} > 0, \tag{6e}$$

$$\frac{\partial L}{\partial t_{ijk}} = c_{ijk} + \delta_{jk} - \gamma_{ik} \leq 0, \; t_{ijk} = 0, \qquad = 0, \; t_{ijk} > 0, \tag{6f}$$

$$\frac{\partial L}{\partial \alpha_{in}} = w_{in} - \sum_j T_{ijn} \geq 0, \; \alpha_{in} = 0 \qquad = 0, \; \alpha_{in} > 0, \tag{6g}$$

$$\frac{\partial L}{\partial \beta_{in}} = \sum_j T_{jin} - x_{in} \geq 0,\ \beta_{in} = 0 \qquad \text{(6h)}$$
$$= 0,\ \beta_{in} > 0,$$

$$\frac{\partial L}{\partial \gamma_{ik}} = y_{ik} - \sum_j t_{ijk} \geq 0,\ \gamma_{ik} = 0 \qquad \text{(6i)}$$
$$= 0,\ \gamma_{ik} > 0,$$

$$\frac{\partial L}{\partial \delta_{ik}} = \sum_j t_{jik} - z_{ik} \geq 0,\ \delta_{ik} = 0 \qquad \text{(6j)}$$
$$= 0,\ \delta_{ik} > 0.$$

From assumption A and equation (6a), it follows that $\alpha_{in}$ can be interpreted as the market price for the primary commodity $w_{in}$ in region i. Indeed, given $w_{in} > 0$, (6a) and assumption A imply that $\alpha_{in} = p_{in}^*$. Similarly, it follows from equation (6d) that $\delta_{ik}$ can be interpreted as the market price for the secondary commodities $z_{ik}$ in region i.[1/]

Equations (6e) and (6f) characterize the transportation arbitrage conditions expressed in terms of spatial prices. Note that, given $C_{iin} = c_{iik} = 0$, it follows from (6e) and (6f) that $\beta_{in} = \alpha_{in}$ (or $\gamma_{ik} = \delta_{ik}$) whenever $T_{iin} > 0$ ($t_{iik} > 0$). In this case, prices paid by producers as well as consumers for a particular commodity in a particular region are necessarily equal. This implies that $\beta_{in}$ can be interpreted as the market price of the primary commodity $x_{in}$. Similarly, $\gamma_{ik}$ can be interpreted as the market price of the secondary commodity $y_{ik}$. Equations (6e) and (6f) state that commodity prices between any two regions cannot differ by more than the corresponding unit transportation cost. And in the case where trade takes place (i.e., $T_{ijn} > 0$, $t_{ijk} > 0$, for $i \neq j$), then the spatial price difference between the importing region and the exporting region must be exactly equal to the unit transportation cost. Note that an implication of (6e) and (6f) is

$$[p^d_{jn} - p^d_{in} - c_{ijn}]\ t_{ijn} = 0,\ \forall i,\ \forall j,\ \forall n, \tag{7a}$$

$$[p^s_{jk} - p^s_{ik} - c_{ijk}]\ t_{ijk} = 0,\ \forall i,\ \forall j,\ \forall k. \tag{7b}$$

Equations (7a) and (7b) mean that the equilibrium conditions for trade necessarily imply zero profit from transportation activities. Thus, any departure from (6e) and (6f) cannot correspond to an equilibrium situation since it would provide incentives for transportation firms to alter trade patterns. In this sense, equations (6e) and (6f) characterize trade efficiency.

The Lagrange multipliers $\beta$ and $\gamma$ measure the shadow price of the trade constraints (3b) and (3c). More specifically, $\beta_{in}$ measures the marginal social cost of one unit of the primary commodity $x_{in}$, $i = 1, \ldots, J$, $n = 1, \ldots, N$. Then, equation (6b) simply states that, at the optimum, the marginal cost of the commodity ($\beta_{in} \geq 0$) is equal to its marginal value ($-\partial G_i/\partial x_{in} \geq 0$) whenever $x_{in}$ is positive. But we have seen that $\beta_{in}$ can be interpreted as the market price of $x_{in}$ in region i. It follows that our model is consistent with a competitive market equilibrium, where market price is equal to the marginal value of each commodity at the optimum.

Similarly, $\gamma_{ik}$ measures the marginal social value of one unit of the secondary commodity $y_{ik}$, $i = 1, \ldots, J$, $k = 1, \ldots, K$. Then, equation (6c) states that, at the optimum, the marginal value of the commodity ($\gamma_{ik} \geq 0$) is equal to its marginal cost ($\partial G_i/\partial y_{ik} \geq 0$) whenever $y_{ik}$ is positive. We have seen that $\gamma_{ik}$ can be interpreted as the market price of $y_{ik}$. Thus, our model is consistent with a competitive market equilibrium, where market price is equal to the marginal cost of each commodity at the optimum.

Finally, equations (6g), (6h), (6i) and (6j), together with the complementary slackness conditions with respect to the corresponding Lagrange multipliers, are simply the trade flow constraints corresponding to (2). They represent the feasibility conditions for interregional trade.

These results indicate that the optimization problem (5) provides a representation of a competitive market equilibrium both across commodities and over space. By considering both trade and the transformation of primary commodities into secondary commodities, they provide an extension to the Samuelson-Judge-Takayama approach to spatial market equilibrium (see Samuelson; Takayama and Judge, pp. 107-121). As such, they should be useful in analyzing the spatial implications of resource allocation in a marketing channel.

## III. Spatial Hedonic Prices and Trade.

The model just developed can be refined when the production of the secondary commodities from the primary commodities involves well identified characteristics. In this case, the allocation of the primary characteristics both among secondary commodities and across space is of interest. This issue can be explored in the context of a Lancasterian-type model with trade. This section examines the implications of our analysis for spatial hedonic prices of characteristics under competitive markets and trade.

Assume that the N primary commodities involve S characteristics, the s-th characteristics being denoted by $r_s$, s = 1, ..., S. Each primary as well as secondary commodity in each region has a given composition in terms of its underlying characteristics. In region i, let $a_{ins} \geq 0$ denote the quantity of the n-th characteristic per unit of n-th primary commodity $x_{in}$. And let $b_{iks} \geq 0$ denote the quantity of the n-th characteristic per unit of the k-th secondary commodity. Assume that the characteristic composition of each commodity is constant, i.e. that $a_{ins}$ and $b_{iks}$ are constant. Under this assumption of constant proportions, consider that the production technology $F_i$ in region i (as given in equation (1)) takes the specific form:

$$y_{ik} = \min \{\sum_{n=1}^{N} x_{ink} \frac{a_{in1}}{b_{ik1}}, \ldots, \sum_{n=1}^{N} x_{ink} \frac{a_{inS}}{b_{ikS}}, f_{ik}(v_{ik}, x_{ik})\}, \forall i, \forall k, \tag{8}$$

where $x_{ink}$ is the quantity of the n-th primary input used in the production of the k-th secondary output in region i, which satisfies the identity: $x_{in} = \Sigma_k x_{ink}$. The production technology (8) assumes fixed proportions with respect to each of the characteristics used in the production of the secondary output $y_{ik}$, $\Sigma_n x_{ink} a_{ins}$, s = 1, ..., S. However, given the general function $f_{ik}(v_{ik}, x_{ik})$, it imposes no a priori restriction on the elasticities of substitution among the various inputs $(v_{ik}, x_{ik})$. Under the technology (8), the cost function given in equation (2) becomes:

$$g_i(x_i, y_i) = \min_v \{r_i' v_i : y_{ik} = f_{ik}(v_{ik}, x_{ik})\}, \ \forall i, \tag{9a}$$

subject to

$$\sum_{k=1}^{K} y_{ik} b_{iks} = \sum_{n=1}^{N} x_{in} a_{ins}, \tag{9b}$$

i = 1, ..., J, s = 1, ..., S. Equation (9b) ensures the balance in the allocation of component s in region i. It corresponds to a linear Lancasterian model where each commodity exhibits fixed proportions, but where the components are perfect substitutes in their allocation among commodities (see Lancaster).[2/] Then, the optimization problem (5) becomes:

$$\begin{aligned} \max_{w,x,y,z,T,t} \{ & \sum_i [D_i(z_i) - S_i(w_i) - g_i(x_i, y_i)] \\ & - \sum_{i,j,n} T_{ijn} C_{ijn} - \sum_{i,j,k} t_{ijk} c_{ijk} \\ & : \text{equations (3) and (9b)}, \\ & w \geq 0,\ x \geq 0,\ y \geq 0,\ z \geq 0,\ T \geq 0,\ t \geq 0 \}, \end{aligned} \tag{10}$$

with corresponding Lagrangean:

$$
\begin{aligned}
L = \sum_i \; & [D_i(z_i)-S_i(w_i)-g_i(x_i,\ y_i)] \\
& -\sum_{i,j,n} T_{ijn}C_{ijn} - \sum_{i,j,k} t_{ijk}c_{ijk} \\
& +\sum_{i,s} \lambda_{is}\ [\sum_n x_{in}\ a_{ins} - \sum_k y_{ik}\ b_{iks}] \\
& + \sum_{i,n} \alpha_{in}\ [w_{in} - \sum_j T_{ijn}] \\
& + \sum_{i,n} \beta_{in}\ [\sum_j T_{jin} - x_{in}] \\
& + \sum_{i,k} \gamma_{ik}\ [y_{ik} - \sum_j t_{ijk}] \\
& + \sum_{i,k} \delta_{ik}\ [\sum_j t_{jik} - z_{ik}],
\end{aligned}
$$

where $\lambda_{is} \geq 0$ is the Lagrange multiplier for the s-th characteristic constraint (9b) in region i. At the optimum, the $\lambda$'s provide a measure of the shadow price, or implicit market price, of the S components. This will give a convenient basis for evaluating component pricing in a spatial market equilibrium framework.

Except for (6b) and (6c), the Kuhn-Tucker conditions associated with the above Lagrangean satisfy equations (6). Equations (6b) and (6c) take the form:

$$
\frac{\partial L}{\partial x_{in}} = -\frac{\partial g_i}{\partial x_{in}} + \sum_{s=1}^{S} \lambda_{is} a_{ins} - \beta_{in} \leq 0, \ x_{in} = 0, \quad = 0, \ x_{in} > 0, \tag{11a}
$$

$$
\frac{\partial L}{\partial y_{ik}} = -\frac{\partial g_i}{\partial y_{ik}} - \sum_{s=1}^{S} \lambda_{is} b_{iks} + \gamma_{ik} \leq 0, \ y_{ik} = 0 \quad = 0, \ y_{ik} > 0. \tag{11b}
$$

Interpreting $\lambda_{is}$ as the shadow price of the s-th component in region i, expressions (11a) and (11b) indicate how the shadow valuation of components affects market equilibrium. Equation (11a) involves the marginal value of the n-th primary input $x_{in}$, which is equal to the marginal value associated with inputs $v_i$ ($-\partial g_i/\partial x_{in} \geq 0$), plus the marginal value of the S components ($\Sigma_s\ \lambda_{is}\ a_{ins} \geq 0$). This simply states that, at the optimum, marginal value is equal to the price of the primary input $\beta_{in}$, as found in a competitive market equilibrium. Equation (11b) involves the marginal cost of the k-the secondary product $y_{ik}$, which is equal to the marginal cost ($\partial g_i/\partial y_{ik} \geq 0$), plus the marginal cost of the S components ($\Sigma_k \lambda_{ik} b_{iks} \geq 0$). It shows that, at the optimum, marginal cost is equal to the market price $\gamma_{ik}$. Again, these results are consistent with resource allocation under competitive equilibrium.

Finally, the following additional Kuhn-Tucker condition must be satisfied:

$$\frac{\partial L}{\partial \lambda_{is}} = \sum_n x_{in} a_{ins} - \sum_k y_{ik} b_{iks} \geq 0, \; \lambda_{is} = 0 \qquad (11c)$$
$$= 0, \; \lambda_{is} > 0,$$

which simply represents the component balance constraint for component s in region i, i = 1, ..., J, s = 1, ..., S. This set of equations provides a convenient characterization of spatial competitive equilibrium of component allocation and their implicit pricing. The usefulness of these results is illustrated next in the context of resource allocation in the U.S. dairy industry.

## IV. Application to the U.S. Dairy Industry.

In this section, we first develop a competitive model of spatial allocation of resources in the U.S. dairy industry. The model, which focuses on annual allocation, is consistent with market conditions prevalent in 1990. We then incorporate in the model the milk price support program (implemented through federal government purchases) as well as milk marketing orders. This provides a basis for evaluating the regional impacts of U.S. dairy policy.

### A) The Competitive Model.

We consider the case of milk and its transformation into dairy products. Thus, we have a single primary commodity (N = 1): milk. We consider nine categories of dairy products (K = 9):

(1) fluid milk; (2) soft dairy products; (3) american cheese; (4) italian cheese; (5) other cheese; (6) butter; (7) frozen dairy products; (8) all other manufactured dairy products; and (9) nonfat dry milk. Also we divide the U.S. into 14 regions (J = 14).[3/] Finally, we consider three components-characteristics of milk (S = 3): (1) fat; (2) protein; and (3) carbohydrates. Farm milk is assumed to contain 3.66% fat, 3.20% protein, and 4.65% carbohydrates. The composition of fluid milk is: 2.20% fat, 3.32% protein, and 4.73% carbohydrates. The composition of all dairy products was estimated in a way consistent with their average composition in 1990.[4/] Our focus here is to investigate the spatial market equilibrium of the U.S. dairy sector that is consistent with the allocation and implicit pricing of these three components.[5/]

The objective function in (10) involves the consumer benefits D, the costs of milk production S, the other costs g, and transportation costs. Let $p_i^s(w_i)$ represent the price dependent supply function for the primary commodity (milk) in the i-th region, where $\partial p_i^s/\partial w_i > 0$, i = 1, ..., J. And let $p_{ik}^d(z_{ik})$ represent the price dependent demand function for the k-th dairy product consumed in the i-th region, where $\partial p_{ik}^d/\partial z_{ik} < 0$, i = 1, ..., J, k = 1, ..., K. We choose:

$$D_i = \sum_{k=1}^{K} \int_0^{z_{ik}} p_{ik}^d(q)\ dq, \tag{12a}$$

and

$$S_i = \int_0^{w_i} p_i^s(q)\ dq, \tag{12b}$$

i = 1, ..., J. Note that this particular choice satisfies assumption A.[6/] As argued above, model (10) therefore provides a representation of a spatial competitive model for dairy products..

We focus on the case where the demand function $p_{ik}^d(z_{ik})$ is the derived demand for the k-th dairy product at the farm gate. In other words, $p_{ik}^d$ is interpreted as the consumer price of the k-th dairy product <u>net of all marketing costs</u>, i = 1, ...., J. In this context, the various dairy products are considered to be obtained by simple recombination of the basic components of milk. This is equivalent to assuming that $g_i = 0$.

Equation (12a) is the sum of the total areas under the K derived demand curve in the i-th region. This area has sometimes been interpreted as a measure of benefits generated by the K commodities in the i-th region. Equation (12b) is the area under the supply curve. Since the supply curve is also the marginal cost of production under competition, it follows that (12b) can be interpreted as a measure of milk production cost in the i-th region. Then, given (12), the terms $[D_i - S_i]$ can be interpreted as a measure of welfare obtained by region i: the sum of producer plus consumer surplus. Note that consumer surplus (as measured from a Marshallian demand function) is only an approximate welfare measure in the presence of income effects (Willig). In that sense, the objective function in (5) or (10) cannot be interpreted as a true welfare measure. This motivates our characterization of this objective function as a "quasi-welfare function".[7/]

The empirical evaluation of the objective function in (10) requires estimates of the supply function for milk and the farm-level derived demand for the K dairy products in each region. The regional milk supply elasticities are taken from Buxton's analysis. The product demand elasticities are estimated as close to the farm gate as possible, using our "best judgement" from previous research results. For most products (except "fluid milk" and "other manufactured products"), the derived demand elasticities are obtained from Cox et al.'s research using wholesale level data. For the remaining products, the farm level demand elasticities are derived from retail demand elasticities (Huang; Haidacher et al.) adjusted by a price transmission elasticity (Kinnucan and Forker; McDowell et al.).[8/] In the absence of strong prior information on their functional form, the price dependent supply and demand functions in (12) are assumed to be linear. Their intercept and slope values are set consistent with dairy market conditions (i.e. price and quantity) prevalent in 1990.

The assumed transportation cost for farm milk and fluid milk is \$.35/cwt/100 miles. The transportation costs for other dairy products is estimated from actual transportation costs prevalent in 1990 for refrigerated products (soft dairy products, cheeses, butter, frozen products, and manufactured products) as well as nonrefrigerated products (nonfat dry milk). The use of actual transportation rates allows for asymmetric rates, where the unit transportation costs of a given commodity between two regions can differ for imports versus exports (e.g. because of backhauling opportunities).

The optimization model (10) is subject to constraints (3) and (9b). Imposing equation (3) in the dairy model is straightforward. Equation (9b) required some adjustments for components that never reach the consumers. For example, whey is a byproduct of cheese production. Although some whey is recovered and utilized in dairy products, a significant proportion of whey is typically discarded. Also, a small percentage of farm milk production is consumed on farm and therefore never reaches the market place. Appropriate adjustments in equation (9b) were made to reflect these characteristics of the dairy industry.

Finally, linear equation (9b) implicitly assumes that components are perfect substitutes in their allocation among different commodities. This may not be an appropriate assumption for some dairy commodities. In particular, there are technological constraints that prevent perfect substitution of components across commodities. Such constraints are typically associated with specialized plants that can use components only in the production of selected dairy commodities. First, because of the difference in fat composition, the production of fluid milk out of raw milk generates fat byproduct that is typically used only in the production of soft products, frozen products, or butter. Second, butter is a residual commodity using fat surpluses generated from two sources: 1/ the fat in whey associated with cheese production; and 2/ the fat surpluses due to induced production of butter and nonfat dry milk from "reserve fluid milk" that is needed to smooth seasonal fluctuations and uneven weekly bottling schedules in the fluid milk market. Two sets of constraints further restricting the allocation of components across commodities have been added to model (10) to incorporate these specific characteristics.

Model (10) is a well behaved nonlinear programming problem, with a strictly concave objective function and linear constraints. It was solved numerically using GAMS-MINOS software.

B) Government Price Support Scenario.

The competitive model discussed above was modified to account for federal government purchases of dairy commodities. Such purchases are part of the federal milk price support program, designed to stimulate aggregate demand for milk and maintain the price received by dairy farmers above a minimum level set by government. Those purchases are limited to the most storable dairy products. In 1990, government purchased 44 million lbs of american cheese, 404 million lbs of butter, and 100 million lbs of nonfat dry milk.

In an attempt to include government purchases in the model, an "additional region" was created to account for government demand. The quantity demanded by government was treated as exogenous and set at the 1990 level (as reported above). The model incorporating government purchases is also a well behaved nonlinear programming problem, with a strictly concave objective function and linear constraints.

C) Milk Marketing Orders Scenario.

In addition to government purchases, we also incorporated milk marketing orders, including the California milk marketing order as well as the federal milk marketing orders, in the model.

The federal milk marketing order program involves blend pricing of farm milk, as well as classified pricing rules based on the Minnesota-Wisconsin (MW) prices. We assumed that MW prices were the same as Wisconsin prices in our model. Blend pricing consists in paying dairy farmers the weighted average value of the dairy commodities produced from farm milk in each region. This allows for possible price discrimination across dairy markets, which can raise farm price and benefit farmers (see Helmberger, chapter 6). Blend price equations, defining the price received by farmers in each region, are added in the model.

The current federal milk marketing orders also place restrictions on the pricing of fluid milk. First, lower bounds on regional fluid milk prices are imposed, based on the fluid milk price in Wisconsin. More specifically, the fluid milk price in any region is restricted to be at least as large as the Wisconsin fluid milk price, plus a differential of $.21/cwt/100 miles distance from Wisconsin. These constraints are added to the model. Second, the federal milk marketing orders impose a base "class-I differential" between the price of fluid milk and the value of nonfluid uses of milk in Wisconsin. In 1990, this class-I differential was $1.25/cwt milk. This differential allows for price discrimination between the fluid milk market and the nonfluid dairy markets (Helmberger, p. 142-146). An additional constraint was included in the model to reflect this restriction.[9/] Finally, the blend price equation in California was specified to reflect the functioning of the California milk marketing order. In particular, it incorporates "make allowances" that reduce the value of cheese, butter, and nonfat dry milk used in the calculation of the price paid to California dairy farmers relative to other regions.

The addition of the above pricing constraints reflecting current milk marketing orders created a problem in our model. By influencing the first-order conditions, these constraints on prices imply that equations (6e) and (6f) are not necessarily satisfied at the optimum. Thus, the inclusion of additional price restrictions no longer guarantees the efficiency of trade across regions. As a result, equations (6e) and (6f) are explicitly added as constraints in the model in order to obtain an efficient trade allocation. After adding these non-linear, complementary slackness constraints, the resulting model is a nonlinear programming problem, with a strictly concave objective function and both linear and non-linear constraints.

D) Evaluation of the Results.

The results are summarized in tables 1-5. Five sets of data are presented: (1) the actual 1990 data; (2) the simulation results under both the price support program and the marketing orders; (3) the simulation results under the price support program alone; (4) the simulation results under the milk marketing orders (MMO) alone; and (5) the simulation results under a competitive market (i.e. without price support or milk marketing orders). The model results under these scenarios provide information on regional milk prices paid to farmers (table 1), regional milk production (table 2), regional implicit prices for milk components (table 3), market equilibrium for dairy products (table 4), and welfare distribution as measured by regional producer and consumer surpluses (table 5).[10]

Since both the price support program and the milk marketing orders were in place in 1990, comparing scenario (2) with the 1990 actual data provides a basis for validating the model. As shown in table 1, the percentage prediction error of the farm price of milk averages 2.5 percent across all 14 regions, with a maximum error of 10 percent. And from table 2, the percentage prediction error of milk production averages 0.6 percent, with a maximum error of 4.3 percent. These fairly low relative errors suggest that the model provides a reasonably good representation of the real world.

The effects of eliminating the milk marketing orders (both California and federal orders) can be assessed by comparing scenarios (2) and (3) in tables 1-5. These effects vary across regions. Removing the MMO's decreases farm price and milk production in all regions except Wisconsin, West North Central, North West and California (table 1). The largest reductions in milk production are in South Atlantic (-5.7 percent), the North East (-4.3 percent) and Central (-4.0 percent). In general, the MMO's raise the shadow price of fat (+12¢/lb on average) and lower the shadow price of proteins (-21¢/lb on average) (see table 3). As expected, the MMO's price discrimination scheme increases the price of fluid milk and depresses the price on nonfluid products (table 4). The results in table 5 indicate that eliminating the milk marketing orders would benefit consumers everywhere, and would make dairy producers better off in Wisconsin, West North Central, North West and California, but worse off in other regions.

The effects of removing the price support program can be evaluated by comparing scenarios (2) and (4). The results indicate that the price support program raises the price received by farmers by an average of 3.2 percent (see table 1) and stimulates U.S. milk production by 1.4 percent (table 2). Federal purchases are found to have a large positive influence of the shadow price of fat (+23¢/lb on average)[11] and a negative impact on the price of protein (-19¢/lb on average) (see table 3). They tend to increase significantly the price of butter, and depress the price of italian cheese and non fat dry milk (table 4). These results suggest that the 1990 federal purchases have important effects on the relative shadow prices of components, which in turn have significant influence on the relative prices of dairy commodities. As expected, eliminating the price support program would make milk producers worse off and consumers better off in every region (table 5).

Finally, the effects of eliminating both marketing orders and support price programs can be measured by comparing scenarios (2) and (5). The results indicate that these programs tend to redistribute welfare from consumers to producers, but with a differential impact on regional producer benefits. While dairy producers in Wisconsin would gain from eliminating such programs, producers in other regions would be made worse off by removing price support and milk marketing orders. This stresses the importance of a disaggregate analysis of the distributional impact of current U.S. dairy policy. From table 5, the deadweight loss to society associated with current dairy policy is estimated to be $350 million: $100 million due to the milk marketing orders, and $250 million due to the price support program.

In general, the results indicate the existence of important interaction effects between the price support program and milk marketing orders. For example, from table 3, the influence of marketing orders on the shadow price of components is fairly small in the absence of the price support program (scenarios (4) versus (5)), but becomes much larger under federal purchases (scenarios (2) versus (3)). This shows that the allocation and pricing of components play an important function in the determination of market equilibrium under alternative government interventions. It also suggests the general usefulness of our approach to better understand the role of nonmarket goods in spatially and vertically linked markets.

## V. Concluding Remarks.

This paper develops a multimarket competitive trade model that represents spatial resource allocation in a vertical sector including both market and nonmarket goods. This helps fill the gap between the Samuelson-Judge-Takayama's approach to commodity trade modeling, and Rosen's market model of hedonic functions associated with differentiated products. Using a Lancasterian-type approach, the model generates the spatial allocation of resources for the market goods, as well as the spatial distribution of hedonic prices for the nonmarket goods. An advantage of our approach is that corner solutions are easily handled. This is somewhat attractive given that corner solutions tend to make the empirical analysis of hedonic functions rather difficult (Deaton and Muellbauer, p. 252). The model generates a solution that is consistent with a competitive market equilibrium for both market and nonmarket goods. Moreover, our analysis shows how the model can be modified to incorporate some departures from perfect competition (i.e. the effects of government policy interventions).

The usefulness of the model is illustrated in the context of a regional analysis of the U.S. dairy sector. First, under the price support program and milk marketing orders, the model generates fairly accurate predictions of the actual 1990 situation. This indicates that our proposed modeling approach provides a reasonable approximation to the real world. Second, our model explicitly analyzes the allocation and shadow pricing of milk components in a way consistent with trade efficiency and market equilibrium. The empirical estimates of shadow prices provide useful information on the allocation of these nonmarket goods under alternative scenarios. Third, we evaluate the impact of current policy on resource allocation in the U.S. dairy industry. In particular, we assess the effects of U.S. dairy policy on the distribution of farmers' and consumers' welfare across regions. The empirical results suggest that our approach can provide valuable insights in the analysis of spatial and vertical resource allocation in the presence of nonmarket goods and government intervention. It is hoped that our paper will help stimulate further research on the economics of trade for differentiated products.

**Table 1. Milk Price Received by Farmers (Wholesale All-Milk Price, $/cwt).**

| REGION | (1) 1990 ACTUAL PRICES ($/cwt) | (2) PRICE SUPPORT AND MARKETING ORDERS PRICES | (2) % [(2)-(1)] | (3) PRICE SUPPORT PRICES | (3) % [(3)-(2)][1] | (4) MARKETING ORDERS PRICES | (4) % [(4)-(2)][2] | (5) COMPETITIVE MARKETS PRICES | (5) % [(5)-(2)][3] |
|---|---|---|---|---|---|---|---|---|---|
| North East | 14.62 | 15.18 | 3.8 | 14.5 | -4.3 | 13.88 | -8.6 | 14.03 | -7.6 |
| Mid Atlantic | 14.79 | 14.62 | -1.1 | 14.20 | -2.9 | 14.34 | -1.9 | 13.95 | -4.6 |
| South Atlantic | 14.91 | 15.44 | 3.5 | 14.56 | -5.7 | 13.86 | -10.2 | 13.97 | -9.5 |
| South East | 16.20 | 16.36 | 1.0 | 15.83 | -3.2 | 15.16 | -7.4 | 14.98 | -8.4 |
| Central | 14.50 | 15.13 | 4.4 | 14.53 | -4.0 | 14.54 | -3.9 | 14.32 | -5.4 |
| East South Central | 15.30 | 15.91 | 4.0 | 15.38 | -3.4 | 14.63 | -8.0 | 14.40 | -9.5 |
| West South Central | 14.40 | 14.94 | 3.7 | 14.53 | -2.7 | 14.02 | -6.2 | 13.93 | -6.7 |
| East North Central | 13.81 | 14.16 | 2.5 | 13.95 | -1.4 | 13.72 | -3.1 | 13.67 | -3.5 |
| Wisconsin | 13.47 | 13.27 | -1.5 | 13.72 | 3.4 | 13.22 | -0.4 | 13.43 | 1.2 |
| West North Central | 13.13 | 13.43 | 2.3 | 13.63 | 1.5 | 13.23 | -1.5 | 13.34 | -0.6 |
| West Central | 13.36 | 13.84 | 3.6 | 13.81 | -0.2 | 13.57 | -1.9 | 13.52 | -2.3 |
| North West | 12.92 | 13.42 | 3.8 | 13.44 | 0.2 | 13.05 | -2.7 | 13.09 | -2.4 |
| Mountain | 13.78 | 14.22 | 3.2 | 13.79 | -3.0 | 13.56 | -4.6 | 13.44 | -5.5 |
| California | 12.02 | 13.22 | 10.0 | 13.47 | 1.9 | 12.90 | -2.4 | 13.16 | -0.4 |
| U.S. Average | 13.73 | 14.08 | 2.5 | 13.99 | -0.6 | 13.63 | -3.2 | 13.63 | -3.2 |

Notes: 1/ These percentage changes reflect the impacts of removing the milk marketing orders.

2/ These percentage changes reflect the impact of removing the price support program.

3/ These percentage changes reflect the impact of removing both price support and milk marketing orders.

**Table 2. Farm Level Milk Production (million pounds).**

| REGION | (1) 1990 ACTUAL QUANTITY | SHARE | (2) PRICE SUPPORT AND MARKETING ORDERS QUANTITY | %[(2)-(1)] | (3) PRICE SUPPORT QUANTITY | %[(3)-(2)][1] | (4) MARKETING ORDERS QUANTITY | %[(4)-(2)][2] | (5) COMPETITIVE MARKETS QUANTITY | %[(5)-(2)][3] |
|---|---|---|---|---|---|---|---|---|---|---|
| North East | 4,235 | 0.03 | 4,294 | 1.4 | 4,226 | -1.6 | 4,157 | -3.2 | 4,173 | -2.8 |
| Mid Atlantic | 21,090 | 0.14 | 20,648 | -2.1 | 20,572 | -0.4 | 20,695 | -0.2 | 20,350 | -1.4 |
| South Atlantic | 3,710 | 0.03 | 3,800 | 2.4 | 3,649 | -4.0 | 3,531 | -7.1 | 3,549 | -6.6 |
| South East | 5,853 | 0.04 | 5,897 | 0.8 | 5,753 | -2.4 | 5,568 | -5.6 | 5,521 | -6.4 |
| Central | 4,390 | 0.03 | 4,577 | 4.3 | 4,398 | -3.9 | 4,402 | -3.8 | 4,336 | -5.3 |
| East South Centra | 2,990 | 0.02 | 3,068 | 2.6 | 2,999 | -2.2 | 2,905 | -5.3 | 2,875 | -6.3 |
| West South Central | 8,192 | 0.06 | 8,435 | 3.0 | 8,252 | -2.2 | 8,018 | -4.9 | 7,979 | -5.4 |
| East North Central | 14,617 | 0.10 | 14,735 | 0.8 | 14,665 | -0.5 | 14,585 | -1.0 | 14,568 | -1.1 |
| Wisconsin | 24,059 | 0.16 | 23,792 | -1.1 | 24,395 | 2.5 | 23,727 | -0.3 | 24,009 | 0.9 |
| West North Central | 12,646 | 0.09 | 12,725 | 0.6 | 12,779 | 0.4 | 12,673 | -0.4 | 12,702 | -0.2 |
| West Central | 9,821 | 0.07 | 9,901 | 0.8 | 9,896 | -0.1 | 9,857 | -0.4 | 9,848 | -0.5 |
| North West | 8,833 | 0.06 | 8,964 | 1.5 | 8,972 | 0.1 | 8,869 | -1.1 | 8,878 | -1.0 |
| Mountain | 4,953 | 0.03 | 5,044 | 1.8 | 4,955 | -1.8 | 4,908 | -2.7 | 4,883 | -3.2 |
| California | 20,661 | 0.14 | 21,113 | 2.2 | 21,208 | 0.5 | 20,993 | -0.6 | 21,092 | -0.1 |
| U.S. TOTAL: | 146,049 | 1.00 | 146,994 | 0.6 | 146,719 | -0.2 | 144,887 | -1.4 | 144,762 | -1.5 |

Notes: 1/ These percentage changes reflect the impacts of removing milk marketing orders.

2/ These percentage changes reflect the impact of removing the price support program.

3/ These percentage changes reflect the impact of removing both price support and milk marketing orders.

**Table 3. Regional Implicit Component Prices ($/cwt) for Fat (in all products except butter, soft and frozen), Protein and Carbohydrates.**

| | (2) PRICE SUPPORT AND MARKETING ORDERS | | | (3) PRICE SUPPORT | | | (4) MARKETING ORDERS | | | (5) COMPETITIVE MARKET | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| REGION | FAT | PROTEIN | CARBO | FAT | PROTEIN | CARBO | FAT | PROTEIN | CARBO | FAT | PROTEIN | CARBO |
| North East | 174.61 | 220.15 | 20.40 | 162.89 | 239.45 | 19.48 | 148.38 | 236.70 | 23.43 | 146.66 | 237.74 | 22.71 |
| Mid Atlantic | 164.16 | 222.66 | 17.81 | 151.87 | 245.54 | 16.87 | 141.43 | 237.80 | 20.45 | 142.48 | 243.61 | 20.15 |
| South Atlantic | 192.57 | 236.47 | 12.43 | 163.69 | 247.39 | 13.95 | 147.17 | 243.48 | 18.98 | 145.20 | 244.08 | 18.25 |
| South East | 208.41 | 239.74 | 8.33 | 199.35 | 254.73 | 8.31 | 167.67 | 250.35 | 14.02 | 169.32 | 255.28 | 13.22 |
| Central | 142.68 | 197.91 | 10.23 | 147.50 | 278.83 | 4.46 | 131.92 | 280.43 | 5.05 | 132.05 | 291.54 | 3.33 |
| East South Central | 196.89 | 240.11 | 7.32 | 186.61 | 256.30 | 7.41 | 155.93 | 244.68 | 14.91 | 156.05 | 251.37 | 13.86 |
| West South Central | 170.43 | 226.11 | 14.76 | 163.01 | 248.54 | 13.15 | 143.70 | 242.04 | 17.89 | 144.00 | 245.64 | 17.19 |
| East North Central | 160.42 | 233.54 | 13.81 | 146.85 | 251.06 | 11.64 | 136.46 | 250.61 | 13.88 | 135.45 | 252.80 | 13.28 |
| Wisconsin | 151.46 | 238.13 | 8.13 | 134.43 | 268.99 | 4.15 | 122.94 | 265.48 | 6.45 | 123.10 | 270.40 | 5.88 |
| West North Central | 140.37 | 264.74 | 2.15 | 128.66 | 276.84 | 1.36 | 118.79 | 277.45 | 3.34 | 117.31 | 278.95 | 2.65 |
| West Central | 153.84 | 236.65 | 11.43 | 142.80 | 254.60 | 9.35 | 130.68 | 256.51 | 10.90 | 130.10 | 258.91 | 10.16 |
| North West | 135.76 | 262.34 | 0.00 | 124.83 | 277.32 | 0.00 | 112.81 | 279.69 | 0.00 | 112.30 | 280.60 | 0.00 |
| Mountain | 144.69 | 243.25 | 8.42 | 136.39 | 262.37 | 8.68 | 123.89 | 262.67 | 8.76 | 124.13 | 265.15 | 8.89 |
| California | 137.14 | 263.03 | 0.00 | 126.07 | 276.70 | 0.00 | 116.21 | 279.51 | 0.00 | 115.32 | 279.42 | 0.00 |
| U.S. Average | 150.79 | 240.03 | 10.02 | 138.26 | 261.50 | 8.69 | 127.49 | 259.55 | 11.18 | 121.70 | 253.58 | 11.74 |

## Table 4. Aggregate U.S. Wholesale Commodity Prices, Production and Consumption.

**AVERAGE U.S. WHOLESALE COMMODITY PRICES ($/cwt).**

| | (1) 1990 ACTUAL | (2) PRICE SUPPORT AND MARKETING ORDERS | | (3) PRICE SUPPORT | | (4) MARKETING ORDERS | | (5) COMPETITIVE MARKET | |
|---|---|---|---|---|---|---|---|---|---|
| COMMODITY | PRICE ($/cwt) | PRICE | %[(2)-(1)] | PRICE | %[(3)-(2)] | PRICE | %[(4)-(2)] | PRICE | %[(5)-(2)] |
| FLUID | 14.89 | 14.58 | 3.46 | 13.15 | -9.8 | 14.84 | 1.8 | 14.37 | -1.4 |
| SOFT | 29.34 | 29.00 | -1.15 | 29.59 | 2.0 | 28.32 | -2.4 | 28.47 | -1.8 |
| AMERICAN CHEESE | 110.00 | 124.22 | 12.93 | 128.78 | 3.7 | 125.15 | 0.8 | 128.51 | 3.5 |
| ITALIAN CHEESE | 120.00 | 101.53 | -15.39 | 107.54 | 5.9 | 112.49 | 10.8 | 113.11 | 11.4 |
| OTHER CHEESE | 125.00 | 109.53 | -12.38 | 108.97 | -0.5 | 102.63 | -6.3 | 103.83 | -5.2 |
| BUTTER | 82.89 | 83.23 | 0.41 | 92.17 | 10.7 | 1.14 | -98.6 | 1.22 | -98.5 |
| FROZEN | 24.72 | 23.54 | -4.76 | 23.95 | 1.7 | 22.60 | -4.0 | 22.86 | -2.9 |
| OTHER MFG | 40.71 | 41.23 | 1.28 | 42.87 | 4.0 | 42.60 | 3.3 | 42.94 | 4.1 |
| NONFAT DRY MILK | 85.00 | 90.56 | 6.54 | 98.74 | 9.0 | 99.15 | 9.5 | 100.04 | 10.5 |

**AGGREGATE U.S. WHOLESALE COMMODITY PRODUCTION (MILLION POUNDS).**

| | (1) 1990 ACTUAL | (2) PRICE SUPPORT AND MARKETING ORDERS | | (3) PRICE SUPPORT | | (4) MARKETING ORDERS | | (5) COMPETITIVE MARKET | |
|---|---|---|---|---|---|---|---|---|---|
| COMMODITY | QUANTITY | QUANTITY | (2)-(1)] | QUANTITY | %[(3)-(2)] | QUANTITY | %[(4)-(2)] | QUANTITY | %[(5)-(2)] |
| FLUID | 54,736 | 54,570 | -0.30 | 55,038 | 0.9 | 54,484 | -0.2 | 54,637 | 0.1 |
| SOFT | 3,760 | 3,777 | 0.46 | 3,745 | -0.8 | 3,814 | 1.0 | 3,805 | 0.7 |
| AMERICAN CHEESE | 2,891 | 2,868 | -0.80 | 2,850 | -0.6 | 2,820 | -1.7 | 2,807 | -2.1 |
| ITALIAN CHEESE | 2,209 | 2,299 | 4.09 | 2,271 | -1.2 | 2,248 | -2.2 | 2,245 | -2.4 |
| OTHER CHEESE | 961 | 987 | 2.71 | 988 | 0.1 | 997 | 1.0 | 995 | 0.8 |
| BUTTER | 1,302 | 1,306 | 0.30 | 1,296 | -0.7 | 1,221 | -6.5 | 1,223 | -6.3 |
| FROZEN | 7,188 | 7,251 | 0.88 | 7,212 | -0.5 | 7,340 | 1.2 | 7,316 | 0.9 |
| OTHER MFG | 3,652 | 3,609 | -1.18 | 3,550 | -1.6 | 3,561 | -1.3 | 3,549 | -1.7 |
| NONFAT DRY MILK | 877 | 877 | -0.02 | 846 | -3.5 | 745 | -15.0 | 742 | -15.4 |

**AGGREGATE U.S. WHOLESALE LEVEL COMMODITY CONSUMPTION (MILLION POUNDS).**

| | (1) 1990 ACTUAL | (2) PRICE SUPPORT AND MARKETING ORDERS | | (3) PRICE SUPPORT | | (4) MARKETING ORDERS | | (5) COMPETITIVE MARKET | |
|---|---|---|---|---|---|---|---|---|---|
| COMMODITY | QUANTITY | QUANTITY | %[(2)-(1)] | QUANTITY | %[(3)-(2)] | QUANTITY | %[(4)-(2)] | QUANTITY | %[(5)-(2)] |
| FLUID | 54,338 | 54,177 | -0.30 | 54,643 | 0.9 | 54,090 | -0.2 | 54,243 | 0.1 |
| SOFT | 3,735 | 3,752 | 0.46 | 3,720 | -0.9 | 3,789 | 1.0 | 3,780 | 0.8 |
| AMERICAN CHEESE | 2,741 | 2,684 | -2.07 | 2,666 | -0.7 | 2,681 | -0.1 | 2,667 | -0.6 |
| ITALIAN CHEESE | 2,231 | 2,317 | 3.86 | 2,289 | -1.2 | 2,266 | -2.2 | 2,263 | -2.3 |
| OTHER CHEESE | 1,129 | 1,151 | 1.92 | 1,151 | 0.1 | 1,160 | 0.8 | 1,159 | 0.7 |
| BUTTER | 906 | 906 | -0.04 | 897 | -1.0 | 989 | 9.2 | 989 | 9.2 |
| FROZEN | 7,137 | 7,246 | 1.53 | 7,208 | -0.5 | 7,335 | 1.2 | 7,311 | 0.9 |
| OTHER MFG | 3,536 | 3,516 | -0.56 | 3,458 | -1.7 | 3,467 | -1.4 | 3,455 | -1.7 |
| NONFAT DRY MILK | 706 | 685 | -2.94 | 655 | -4.5 | 653 | -4.7 | 650 | -5.2 |

## Table 5. Regional Surplus Measures under Alternative Scenarios.

**PRICE SUPPORT AND MARKETING ORDERS (2):**

| REGION | PRODUCER SURPLUS million $ | PRODUCER SURPLUS SHARE | CONSUMER SURPLUS[1] million $ | CONSUMER SURPLUS[1] SHARE | PROD + CONS SURPLUS[2] million $ | PROD + CONS SURPLUS[2] SHARE |
|---|---|---|---|---|---|---|
| North East | 876.65 | 0.03 | 3,975.20 | 0.05 | 4,851.86 | 0.05 |
| Mid Atlantic | 2,500.71 | 0.10 | 11,273.97 | 0.15 | 13,774.68 | 0.14 |
| South Atlantic | 422.37 | 0.02 | 4,319.41 | 0.06 | 4,741.78 | 0.05 |
| South East | 637.45 | 0.03 | 8,536.12 | 0.11 | 9,173.57 | 0.09 |
| Central | 353.76 | 0.01 | 2,557.12 | 0.03 | 2,910.87 | 0.03 |
| East South Central | 368.21 | 0.01 | 3,850.82 | 0.05 | 4,219.04 | 0.04 |
| West South Central | 783.69 | 0.03 | 6,633.51 | 0.09 | 7,417.20 | 0.07 |
| East North Central | 3,185.20 | 0.13 | 10,034.63 | 0.13 | 13,219.84 | 0.13 |
| Wisconsin | 2,118.45 | 0.08 | 2,720.36 | 0.04 | 4,838.80 | 0.05 |
| West North Central | 3,056.97 | 0.12 | 1,726.88 | 0.02 | 4,783.86 | 0.05 |
| West Central | 2,924.51 | 0.12 | 3,530.94 | 0.05 | 6,455.45 | 0.06 |
| North West | 1514.72 | 0.06 | 2,669.89 | 0.04 | 4,184.62 | 0.04 |
| Mountain | 612.31 | 0.02 | 3,445.94 | 0.05 | 4,058.26 | 0.04 |
| California | 5,893.95 | 0.23 | 9,500.72 | 0.13 | 15,394.66 | 0.15 |
| U.S. TOTAL | 25,248.95 | 1.00 | 74,775.51 | 1.00 | 10,0024.49 | 1.00 |

**PRICE SUPPORT (3):**

| REGION | PRODUCER SURPLUS million $ | PRODUCER SURPLUS %[(3)-(2)] | CONSUMER SURPLUS million $ | CONSUMER SURPLUS %[(3)-(2)] | PROD + CONS SURPLUS[3] million $ | PROD + CONS SURPLUS[3] %[(3)-(2)] |
|---|---|---|---|---|---|---|
| North East | 849.04 | -3.1 | 4,002.49 | 0.7 | 4,851.53 | -0.0 |
| Mid Atlantic | 2,412.98 | -3.5 | 11,346.10 | 0.6 | 13,759.08 | -0.1 |
| South Atlantic | 389.52 | -7.8 | 4339.37 | 0.5 | 4,728.89 | -0.3 |
| South East | 606.73 | -4.8 | 8,542.95 | 0.1 | 9,149.68 | -0.3 |
| Central | 326.66 | -7.7 | 2,558.72 | 0.1 | 2,885.38 | -0.9 |
| East South Central | 351.99 | -4.4 | 3,856.93 | 0.2 | 4,208.91 | -0.2 |
| West South Central | 749.92 | -4.3 | 6,656.32 | 0.3 | 7,406.24 | -0.1 |
| East North Central | 3,155.00 | -0.9 | 10,030.13 | -0.0 | 1,3185.14 | -0.3 |
| Wisconsin | 2,227.10 | 5.1 | 2,742.83 | 0.8 | 4,969.93 | 2.7 |
| West North Central | 3,082.67 | 0.8 | 1,732.39 | 0.3 | 4,815.06 | 0.7 |
| West Central | 2,921.58 | -0.1 | 3,548.73 | 0.5 | 6,470.32 | 0.2 |
| North West | 1517.23 | 0.2 | 2,680.65 | 0.4 | 4,197.88 | 0.3 |
| Mountain | 590.96 | -3.5 | 3,460.86 | 0.4 | 4,051.83 | -0.2 |
| California | 5,947.34 | 0.9 | 9,553.97 | 0.6 | 15,501.30 | 0.7 |
| U.S. TOTAL | 25,128.72 | -0.5 | 75,052.44 | 0.4 | 10,0181.17 | 0.2 |

**MARKETING ORDERS (4):**

| REGION | PRODUCER SURPLUS million $ | PRODUCER SURPLUS %[(4)-(2)] | CONSUMER SURPLUS million $ | CONSUMER SURPLUS %[(4)-(2)] | PROD + CONS SURPLUS million $ | PROD + CONS SURPLUS %[(4)-(2)] |
|---|---|---|---|---|---|---|
| North East | 821.78 | -6.3 | 4,004.77 | 0.7 | 4,826.54 | -0.5 |
| Mid Atlantic | 2,441.84 | -2.4 | 11,354.18 | 0.7 | 13,796.03 | 0.2 |
| South Atlantic | 364.68 | -13.7 | 4,344.87 | 0.6 | 4,709.55 | -0.7 |
| South East | 568.35 | -10.8 | 8,571.65 | 0.4 | 9,140.01 | -0.4 |
| Central | 327.16 | -7.5 | 2,567.83 | 0.4 | 2,894.99 | -0.5 |
| East North Central | 330.07 | -10.4 | 3,871.03 | 0.5 | 4,201.10 | -0.4 |
| West South Central | 708.08 | -9.6 | 6,656.30 | 0.3 | 7,364.37 | -0.7 |
| East North Central | 3,120.90 | -2.0 | 10,095.86 | 0.6 | 13,216.76 | -0.0 |
| Wisconsin | 2,106.91 | -0.5 | 2,732.79 | 0.5 | 4,839.69 | 0.0 |
| West North Central | 3,031.75 | -0.8 | 1,736.43 | 0.6 | 4,768.17 | -0.3 |
| West Central | 2,898.27 | -0.9 | 3,559.18 | 0.8 | 6,457.45 | 0.0 |
| North West | 1,482.53 | -2.1 | 2,689.90 | 0.7 | 4,172.42 | -0.3 |
| Mountain | 579.76 | -5.3 | 3,470.95 | 0.7 | 4,050.72 | -0.2 |
| California | 5,827.20 | -1.1 | 9,547.97 | 0.5 | 15,375.17 | -0.1 |
| U.S. TOTAL | 24,609.28 | -2.5 | 75,203.71 | 0.6 | 99,812.97 | -0.2 |

**COMPETITIVE MARKET (5):**

| REGION | PRODUCER SURPLUS million $ | PRODUCER SURPLUS %[(5)-(2)] | CONSUMER SURPLUS million $ | CONSUMER SURPLUS %[(5)-(2)] | PROD + CONS SURPLUS million $ | PROD + CONS SURPLUS %[(5)-(2)] |
|---|---|---|---|---|---|---|
| North East | 828.10 | -5.5 | 4,018.26 | 1.1 | 4,846.36 | -0.1 |
| Mid Atlantic | 2,361.24 | -5.6 | 11,379.90 | 0.9 | 13,741.15 | -0.2 |
| South Atlantic | 368.55 | -12.7 | 4,357.89 | 0.9 | 4,726.44 | -0.3 |
| South East | 558.62 | -12.4 | 8,568.03 | 0.4 | 9,126.65 | -0.5 |
| Central | 317.45 | -10.3 | 2,567.27 | 0.4 | 2,884.72 | -0.9 |
| East North Central | 323.32 | -12.2 | 3,872.04 | 0.6 | 4,195.36 | -0.6 |
| West South Central | 701.12 | -10.5 | 6,659.35 | 0.4 | 7,360.47 | -0.8 |
| East North Central | 3,113.30 | -2.3 | 10,087.37 | 0.5 | 13,200.66 | -0.1 |
| Wisconsin | 2,157.17 | 1.8 | 2,730.66 | 0.4 | 4,887.83 | 1.0 |
| West North Central | 3,045.95 | -0.4 | 1,738.44 | 0.7 | 4,784.39 | 0.0 |
| West Central | 2,893.04 | -1.1 | 3,562.19 | 0.9 | 6,455.23 | -0.0 |
| North West | 1,485.64 | -1.9 | 2,692.21 | 0.8 | 4,177.85 | -0.2 |
| Mountain | 573.75 | -6.3 | 3,475.35 | 0.9 | 4,049.10 | -0.2 |
| California | 5,882.52 | -0.2 | 9,586.54 | 0.9 | 15469.06 | 0.5 |
| U.S. TOTAL | 24,609.77 | -2.5 | 75,295.50 | 0.7 | 99,905.27 | -0.1 |

Notes:
1/ Consumer surplus does not include the cost to taxpayers.
2/ In scenario (2), the cost to U.S. taxpayers is $477 million, generating a total net surplus of $99,547 million.
3/ In scenario (3), the cost to U.S. taxpayers is $523 million, generating a total net surplus of $99,658 million.

References

Ball, M.J. and R.M. Kirwan "Urban Housing Demand: Some Evidence from Cross-Sectional Data" Applied Economics 9(1977):343-366.

Becker, G. S., "A Theory of the Allocation of Time." Economic Journal 75(1965):493-517.

Buxton, B. M., Factors Influencing U.S. Milk Production. USDA, ERS, Agricultural Economic Report No. 527, March 1985.

Cox, T.L., D. Lewis and R. Selinski "Estimation of Wholesale Demand for U.S. Dairy Products" University of Wisconsin-Madison, Department of Agricultural Economics Staff Paper, 1992.

Deaton, A. and J. Muellbauer, Economics and Consumer Behavior. Cambridge University Press, Cambridge, 1980.

Dhrymes, Phoebus J. "Price and Quality Changes in Consumer Capital Goods: An Empirical Study" in Price Indexes and Quality Change Zvi Griliches, ed., Cambridge University Press, 1971.

Epple, Dennis "Hedonic Prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products" Journal of Political Economy 95(1987):59-80.

Gardner, Bruce L. "The Farm-Retail Price Spread in a Competitive Food Industry" American Journal of Agricultural Economics 57(1975):399-409.

Gorman, W.M. "A Possible Procedure for Analyzing Quality Differentials in the Egg Market" Mimeo, Iowa State College, Ames, 1956.

Griliches, Z. "Hedonic Price Indexes for Automobiles: an Econometric Analysis of Quality Change" in Price Indexes and Quality Change: Studies in New Methods of Measurement Z. Griliches, ed., Harvard University Press, Cambridge, Mass., 1971.

Haidacher, Richard C., J. Blaylock, and L.H. Myers Consumer Demand for Dairy Products U.S. Department of Agriculture, Economic Research Service, Agricultural Economic Report No. 586, 1988.

Helmberger, Peter G. Economic Analysis of Farm Programs McGraw-Hill, Inc., New York, 1991.

Huang, Kuo S. U.S. Demand for Food: A Complete System of Price and Income effects U.S. Department of Agriculture, Economic Research Service, Technical Bulletin No. 1714, 1985.

Kinnucan, H.W. and O.D. Forker "Asymmetry in Farm-Retail Price Transmission of Major Dairy Products" American Journal of Agricultural Economics 69(1987):285-292.

Lancaster, K. J., "A New Approach to Consumer Theory." Journal of Political Economy 74(1966):132-57.

Lancaster, K. J., Consumer Demand: A New Approach. Columbia University Press, New York, 1971.

Lucas, Robert E.B. "Hedonic Price Functions" Economic Inquiry 13(1975):157-178.

McDowell, H., A. M. Fleming and R. F. Fallert, Federal Milk Marketing Orders: An Analysis of Alternative Policies. USDA, ERS, Agricultural Economic Report No. 598, September 1988.

McDowell, H., A. M. Fleming and F. Spinelli, U.S. Milk Markets Under Alternative Federal Order Pricing Policies. USDA, ERS, Staff Report No. AGES 9068, November 1990.

Palmquist, Raymond B. "Estimating the Demand for the Characteristics of Housing" Review of Economics and Statistics 66(1984):394-404.

Rosen, S., "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition." Journal of Political Economy 82(1974):34-55.

Samuelson, P. A., "Spatial Price Equilibrium and Linear Programming." American Economic Review 42(1952):283-303.

Stigler, G. J., and G. S. Becker, "De Gustibus non est Disputandum." American Economic Review 65(1975):283-300.

Takayama, Y. and G. G. Judge, Spatial and Temporal Price and Allocation Models. North Holland Publ. Co., Amsterdam, 1971.

Willig, Robert D. "Consumer's Surplus without Apology" American Economic Review 66(1976):589-597.

Footnotes

1. To see this, note that given $z_{ik} > 0$, (6d) and assumption A imply that $\delta_{ik} = p_{ik}^{d}$.

2. Note that this assumption of perfect substitutability among commodities can be relaxed by including appropriate constraints in addition to (9b). This will be illustrated in section 4 below.

3. The 14 regions are: 1/ New England (Maine, New Hampshire, Vermont, Massachusetts, Connecticut, Rhode Island); 2/ Middle Atlantic (New York, Pennsylvania, New Jersey); 3/ South Atlantic (Delaware, Maryland, West Virginia, Virginia); 4/ South East (North Carolina, South Carolina, Georgia, Florida); 5/ Central (Kentucky, Tennessee); 6/ East South Central (Alabama, Mississippi, Arkansas, Louisiana); 7/ West South Central (Oklahoma, Texas, New Mexico); 8/ East North Central (Ohio, Indiana, Illinois, Michigan); 9/ Wisconsin; 10/ West North Central (Minnesota, South Dakota, North Dakota); 11/ West Central (Missouri, Kansas, Iowa, Nebraska); 12/ North West (Idaho, Oregon, Washington); 13/ Mountain (Arizona, Colorado, Utah, Nevada, Wyoming, Montana); and 14/ California.

4. We neglect other components of milk (i.e, water and minerals). We implicitly assume that these other components have a zero shadow price and are disposable at no cost.

5. In contrast with previous research (e.g. McDowell et al.), our model uses a disaggregate analysis of the demand for "non-fluid milk". Perhaps more importantly, it specifies an allocation of milk among the various dairy products that is always consistent with the milk component balance.

6. Note that the specification (12a) neglects possible cross-price demand effects across dairy commodities. This simplification is motivated by the current absence of reliable information on the nature and magnitude of these cross-price effects at the farm gate (see below). If such information became available, it could be easily incorporated in the model.

7. However, these approximations do not affect the validity of the arguments presented earlier that our model generates a competitive market equilibrium.

8. The price transmission elasticity reported by Kinnucan and Forker and used by McDowell et al. is .5. On that basis, our estimate of the farm-level derived demand elasticity is half of the corresponding retail demand elasticity for "fluid milk" (as reported by Huang) and for "other manufactured products" (as reported by Haidacher et al.).

9. The class-I differential for Wisconsin was specified to incorporate an additional wedge of \$.75/cwt (beyond the \$1.25/cwt specified for the federal marketing orders) between milk used in fluid and all other uses. This additional wedge reflects the intense competition by Wisconsin cheese processors to bid milk away from fluid uses.

10. Note that the consumer surplus reported in table 5 does not include the cost to the taxpayers. If the cost to the taxpayers were included, then the price support program would always generate a deadweight loss to society (as compared to a competitive situation).

11. Note that the shadow price for fat reported in table 3 does not concerns fat in butter, soft, and frozen products. The price of fat in butter,soft, and frozen products tends to be lower than the one reported in table 3 because of additional constraints imposed in the model reflecting the limited substitutability of fat between these products and other dairy products.