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ON FAIRNESS AND WELFARE ANALYSIS UNDER UNCERTAINTY

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Abstract: Welfare analysis under uncertainty is developed by complementing the Pareto efficiency criterion with an equity criterion: the fairness-equivalence criterion. It focuses on the design and implementation of a public project, its method of financing, as well as the choice of the information available for public decision-making. Allocations that are both Pareto efficiency and fair-equivalent always exist. This provides a basis for conducting benefit-cost analysis under ordinal preferences and in the absence of interpersonal welfare comparison. We show that a fair-equivalent and Pareto efficient allocation leads to a maximin criterion defined in terms of individual ex-ante willingness-to-pay. The paper investigates in some detail the role of information in public decision-making in terms of its implications for both efficiency and fairness.

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On Fairness and Welfare Analysis Under Uncertainty

I- Introduction:

There is in economics a reluctance to go beyond the Pareto efficiency criterion when conducting normative analysis. The criterion is employed in any number of settings, an important instance being the problem of public goods. Formal understanding of Pareto efficiency in a public goods economy dates to Samuelson (1955), who studied the problem of public expenditure under certainty. More recently, Graham (1981, 1992) has presented a general framework for the identification of Pareto efficiency of a public project under uncertainty. Pareto is at once a leading tool in normative work and a severe limitation. It falls short of providing complete guidance for public decision making. While the set of Pareto efficient projects identifies a utility possibility frontier, it provides no guidance in choosing a point on this frontier. Some Pareto allocations can be quite inequitable from some intuitive distributional viewpoint. This suggests a need to complement the Pareto efficiency criterion with some equity criterion.

In the search for a normative criterion for public decision making, the concept of fairness (or the absence of envy) has been proposed as a reasonable equity criterion (Foley; Rawls; Kolm; Feldman and Kirman; Varian; Baumol). Intuitively, an outcome is fair if no agent wishes he/she were someone else. Fairness is appealing in that it treats agents symmetrically, is ordinal in nature and is free of interpersonal utility comparison. Unfortunately, as shown by Pazner and Schmeidler (1974), Pareto-efficient allocations are not always consistent with fairness. As a result, the issue of defining an adequate equity criterion in welfare analysis is still unsettled.

This paper presents an attempt to complement the Pareto efficiency criterion under uncertainty with an equity criterion. Our equity criterion, which selects one of the Pareto efficient allocations, is based on the concept of fairness-equivalence proposed by Pazner. An allocation is said to be fair-equivalent if its underlying welfare distribution could have been generated by a fair allocation in some hypothetical economy (Pazner, p. 463). Fairness-equivalence possesses the appealing symmetry property, is ordinal in nature and is free of interpersonal welfare comparison. Moreover, allocations that are both Pareto efficient and fair-equivalent always exist. Thus, the concept of fairness-equivalence appears to provide an attractive complement to the Pareto criterion in the welfare analysis of public choice.

The paper is organized as follows. Sections II and III present a general model of a public project under uncertainty, given an ordinal representation of individual preferences. The model builds on the work of Graham (1981, 1992) and extends his analysis by considering explicitly the possibility of learning over time. This is done by focusing on a simple two-period model where the amount of information available for public decision making increases between the two periods. Moreover, we treat the amount of learning as a subject of choice. Section III characterizes the set of Pareto-efficient allocations, including the project design, its financing and the optimal amount of information collected between period 1 and period 2. The introduction of fairness and fairness-equivalence in the analysis is presented in section IV. We show that a fair-equivalent and Pareto efficient allocation leads to a maximin criterion defined in terms of individual ex-ante willingness-to-pay. This maximin criterion suggests a simple mechanism for public decision making, as discussed in section V. Section VI investigates in some detail the role of information in public decision making in terms of its implications for both efficiency and fairness.

II- The model:

Consider a public project involving a set of n individuals $N = \{1, 2, \ldots, n\}$ facing the choice of public goods under uncertainty. The uncertainty is represented by a state of nature. Let e_j denote the j-th state of nature, and $E = \{e_1, e_2, \ldots, e_m\}$ be the finite set of all possible mutually exclusive states of nature, where m is the cardinality of E. It is typically not known ahead of time which state of nature will occur. However, it is possible to learn about this uncertainty.

The decisions involve public goods as well as inputs used in the production and implementation of these public goods. Part of these inputs involves the collection of relevant information to be used in the implementation of the public project. For our purpose, it will be convenient to assume that the public choice involves two time periods, $t = 1, 2$. Let x be the vector of decisions made at time $t = 1$, and let y be the vector of decisions made at time $t = 2$. In the first period, the group of n individuals may decide to collect information about the state of nature. Then, chosen at time $t = 1$, the vector x involves public goods, inputs used in the production of the public goods y, as well as information gathering activities.

The amount of public information available for choosing the public goods y at time $t = 2$ depends on the amount of learning generated by the information gathering activities chosen at time $t = 1$. The learning process is characterized by a partition P of the set E of states. A partition P is a collection of subsets of E whose intersection is empty and whose union is the set E itself. Let $\mathcal{P}$ denote the (finite) set of all possible partitions P of E. At time $t = 1$, nothing is known about which of the possible states will occur. One of the decisions made in the initial period is the information gathering decision which generates a partition $P \in \mathcal{P}$ at time $t = 2$.

The partitioning of the set E can be interpreted as the outcome of an experiment providing information on the state of nature. Let the result of the experiment be an observable signal or message $s \in S$, where S is the set of all possible signals. A message s may be sent by nature and provides information on the environment. It may be sent by any of the individuals $i \in N$, thus revealing information about their own preferences and/or behavior. A particular experiment generating public information is characterized by a function $s = g(e)$ mapping E into S. By associating each element of S with at least one element of E, the function g(e) partitions E into mutually disjoint sets $p(s) = \{e\colon s = g(e) \text{ for all } e \in E\} \in P$. Thus, p(s) is in the partition or information structure $P \in \mathcal{P}$ if and only if it is the set of states mapped by g(e) into the signal s. The mapping g is known at time t = 1 and reflects the nature of the underlying experiment. Observing a signal $s \in S$ at the beginning of period 2 means knowing in which member p(s) of the partition P the true state resides. Different experiments are associated with different functions g(e), and thus with different partitions P. The choice of the information gathering activities in x is interpreted here as a choice between different experiments, each producing different information structures $P \in \mathcal{P}$.

In general, there is an informational advantage to choosing an experiment that yields a fine partition, for the finer is P the less uncertainty will remain in period 2. This is formalized as follows.

Definition 1: A partition P is said to **at least as fine as** a partition P' if, for every $p \in P$ and $p' \in P'$, either $p \subseteq p'$ or $p \cap p' = \varnothing$. We denote this relationship by F, where $P \, F \, P'$ means that P is at least as fine P'.

"Perfect information" corresponds to the finest partition P of E in which every set p(s) includes a single state, i.e. where $P = \{e_1, e_2, \ldots, e_m\}$. At the other extreme, the coarsest partition consisting of only one member -- the set E itself with P = E -- represents "no information".

We assume that the information structure available in the first period is E. This implies that the choice of x is made ex ante, i.e. before any information on the states in E becomes available. Learning can generate additional information before the public goods y are chosen at time t = 2. The amount of learning is influenced by the information gathering activities in x chosen at time t = 1. The resulting information structure P characterizes the public information available for the design and implementation of the public project at time t = 2. Since by definition P is a partition of E, it follows that $P \, F \, E$. This corresponds to expanding information over time: the information structure available at time $t = 2$ is as least as fine as the information structure available at time $t = 1$. We will limit our discussion to the situation where all public decisions made at time $t = 2$ are based on the same information, i.e. on the same partition P.[1/]

The n individuals face the following situation. They choose x and y, given the uncertainty reflected by the m states of nature $e = (e_1, e_2, ..., e_m)$. Let $y = (y_1, y_2, ..., y_m)$ where y_j denotes the second period decision in state j, $j = 1, 2, ..., m$. Throughout the paper, we will interpret the decision variables (x, y) as corresponding to a choice of the design of the public project.

Assume that the i-th individual's preferences over the first period (x) as well as second period (y) decisions can be represented by the ex ante ordinal utility function

$$u_i(-z_i, x, y, e), \quad i \in N,$$

where $z_i = (z_{i1}, z_{i2}, ..., z_{im})$, with z_{ij} denoting the monetary payment made by the i^{th} individual in state j, $j = 1, 2, ..., m$. The utility function $u_i(.)$ is ordinal, defined up to a monotonic increasing transformation. It is also ex ante, as it is evaluated in the initial period and depends on all possible states of nature $e = (e_1, e_2, ... ,e_m)$ which are not known at time $t = 1$.[2/] We will assume throughout the paper that $u_i(-z_i, x, y, e)$ is continuous in $(-z_i, x, y)$, and a strictly increasing function of $(-z_i)$ for all $i \in N$.[3/] This latter assumption corresponds to the "non-satiation assumption" commonly made in economics.

III- Efficient Resource Allocation:

The issue of interest here is the allocation of resources given by x, by $y = (y_1, y_2, ..., y_m)$ and by $z = \{z_{ij}: i \in N; j = 1, 2, ..., m\}$ among all n individuals, along with the information structure P. First, this allocation must be feasible. Here we distinguish between technical feasibility and fiscal feasibility. Technical feasibility is defined by the feasible set Ω: any $(x, y, P) \in \Omega$ is said to be technically feasible. The vector x chosen at time $t = 1$ can play three roles: 1/ it can include public goods that influence the welfare of the n individuals; 2/ it can include information gathering activities that condition the information structure P available at time $t = 2$; and 3/ it can include inputs which, in the presence of production lag, contribute to the production of public outputs y. The feasible set Ω thus represents the technology associated with the production of public goods at time $t = 1$ and $t = 2$, as well as with learning. In general, we allow for a joint production technology, where public outputs and information P are jointly produced (e.g. as would be the case under "learning by doing").[4] Throughout the paper, the set $\{(x, y): (x, y, P) \in \Omega\}$ is assumed to be convex and compact for every partition $P \in \mathcal{P}$. We also assume that $(0, 0, E) \in \Omega$, i.e. that doing nothing in the absence of learning is feasible.

Fiscal feasibility concerns the group budget constraint. Given a riskless investment available to all n individuals, we assume the existence of a single intertemporal budget constraint, where returns (or costs) at time $t = 2$ are discounted using the rate of return from the riskless asset (Graham, 1981). The group budget constraint involves the cost function $C(x, y_j, e_j)$ measuring the aggregate transaction costs associated with the choice of x and y in state j, $j = 1, 2, \ldots, m$. The function $C(x, y_j, e_j)$ is assumed continuous in (x, y_j). This cost function is interpreted as an intertemporal state dependent cost function, where costs incurred at time $t = 2$ are discounted using the riskless rate of return. The function $C(x, y_j, e_j)$ involves ex ante monetary costs incurred at time $t = 1$ (which depend on x) as well as discounted ex post costs incurred at time $t = 2$ (which depend on y_j and e_j when the actual state of nature is e_j). The ex ante costs include information costs (i.e. the cost of obtaining, processing and maintaining public information) as well as production costs of the public goods incurred at time $t = 1$. We assume that $C(0, 0, e_j) = 0$, i.e. that doing nothing collectively is costless for the group for any realization of $e_j \in E$. The feasibility definition we employ is as follows:

Definition 2: A resource allocation (z, x, y, P) is said to be feasible if it satisfies:

$$(x, y, P) \in \Omega, \tag{1a}$$

$$\Sigma_{i \in N} z_{ij} \geq C(x, y_j, e_j), j = 1, 2, \ldots, m, \text{ and} \tag{1b}$$

for every p in P, if e_j and $e_{j'}$ are both in p, then

$$y_j = y_{j'}, i \in N, \text{ and} \tag{1c}$$

$$z_{ij} = z_{ij'}, i \in N. \tag{1d}$$

Equation (1a) expresses technical feasibility. Equation (1b) is the group budget constraint, corresponding to the fiscal feasibility of the public project. It states that the sum of the payments made by all n individuals must cover the cost of group decision making in all possible states of nature. Thus, equation (1b) is a state dependent intertemporal budget restriction for public decision making. Equations (1c) and (1d) reflect the restrictions imposed on group behavior by the information structure at time $t = 2$. It shows how the choice of y and z depends on the information structure P available at time $t = 2$: if it is not possible to distinguish between the states in $p \in P$, then the second period decisions cannot depend on the e's in p.

The evaluation of the public choice (x, y) can rely on the ex ante willingness-to-pay of each of the n individuals. For the i-th individual, this is defined by z_i which satisfies:

$$U_i = u_i(-z_i, x, y, e), \quad i \in N, \tag{2}$$

where U_i is an ex ante reference utility level, and $z_i = (z_{i1}, z_{i2}, \ldots, z_{im})$ is the vector of state dependent payments for the i-th individual, $i \in N$. Equation (2) measures the schedule of state dependent payments z_i that make the i-th individual indifferent between facing (x, y) and a reference situation corresponding to ex ante utility level U_i.

The willingness-to-pay defined in equation (2) provides a basis for analyzing the efficiency of group decision making. This requires defining a net benefit criterion for public choice. First, following Graham (1981), consider the maximization of the sure net aggregate willingness-to-pay:

$$W(U) = \underset{z,x,y,P}{\text{Max}} \{\sum_{i\in N} z_{i1} - C(x,y_1,e_1)\text{: equ. (1a), (1c), (1d) and (2);} \\ \sum_{i\in N} z_{i1} - C(x,y_1,e_1) \leq \sum_{i\in N} z_{ij} - C(x,y_j,e_j),\ j=2,\ldots,m\}, \tag{3a}$$

where $U = (U_1, U_2, \ldots, U_n)$ and $z = (z_1, z_2, \ldots, z_n)$. In equation (3a), $[\Sigma_{i\in N} z_{ij}]$ is the gross aggregate willingness-to-pay (as defined in (2)) in state j, and $[\Sigma_{i\in N} z_{ij} - C(x, y_j, e_j)]$ is the net aggregate willingness-to-pay for the n individuals in state j. Note that all the inequality constraints $[\Sigma_{i\in N} z_{i1} - C(x, y_1, e_1)] \leq [\Sigma_{i\in N} z_{ij} - C(x, y_j, e_j)]$, $j = 2, \ldots, m$, are necessarily binding at the optimum. To see this, assume that one of these inequalities is not binding, say for $j = J$. Then, from the non-satiation assumption, a decrease in z_{iJ} in (2) can be associated with a feasible compensating increase in z_{i1}, keeping U constant. Such an increase would raise the value of the objective function in (3a), implying non-optimality. Thus, all the inequality constraints in (3a) could be written equivalently in terms of equality constraints, stating that the net aggregate willingness-to-pay is the same in all states of nature. This can be interpreted to mean that the net aggregate willingness-to-pay is "sure" in the sense that it does not depend on the state of nature. Equation (3a) then maximizes the net aggregate willingness-to-pay in state 1, subject to the feasibility restrictions (1a), (1c) and (1d) as well as the restriction that the net aggregate willingness-to-pay is the same in all states of nature.[5/]

Expression (3a) is a nonlinear constrained optimization problem. Under some weak regularity conditions, it can be expressed equivalently as a saddle point problem of the corresponding Lagrangean (Takayama). Let $\lambda_j \geq 0$ denote the Lagrange multiplier associated with the constraint $[\Sigma_{i\in N} z_{i1} - C(x, y_1, e_1)] \leq [\Sigma_{i\in N} z_{ij} - C(x, y_j, e_j)]$, $j = 2, \ldots, m$. Also define $\lambda_1 = 1 - \Sigma_{j\geq 2} \lambda_j$. Then, a saddle point problem corresponding to problem (3a) takes the form:

$$\begin{aligned} W(U) &= \underset{\lambda\geq 0}{\text{Min}}\ \underset{z,x,y,P}{\text{Max}}\ \{\sum_{j=1}^{m}\lambda_j\ [\sum_{i\in N} z_{ij} - C(x,y_j,e_j)]: \\ &\qquad \text{equ. (1a), (1c), (1d) and (2)};\ \sum_{j=1}^{m}\lambda_j = 1\} \\ &= \underset{\lambda\geq 0}{\text{Min}}\ \{\underset{x,y,P}{\text{Max}}\ \{\sum_{i\in N}\underset{z_i}{\text{Max}}\ [\{\sum_{j=1}^{m}\lambda_j\ z_{ij}: \text{equ. (1d) and (2)}\} \\ &\qquad - \sum_{j=1}^{m}\lambda_j\ C(x,y_j,e_j)]: \text{equ. (1a) and (1c)}\}: \sum_{j=1}^{m}\lambda_j = 1\}, \end{aligned} \tag{3b}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$. It is well known that the existence of a saddle point in (3b) is sufficient for (3b) to provide a characterization of the solution to the constrained optimization problem (3a) (Sposito, p. 146). And the saddle point problem (3b) is always equivalent to problem (3a) if: 1/ Slater's constraint qualification is satisfied;[6/] and 2/ appropriate convexity conditions hold[7/] (Takayama; Sposito; Arrow and Enthoven).

Expression (3b) defines the net benefit criterion proposed by Graham (1992). Note that, from the complementary slackness condition implied by the saddle-point problem, the Lagrange multipliers are necessarily positive ($\lambda > 0$) since we have shown that under non-satiation the corresponding constraints are necessarily binding at the optimum. Each Lagrange multiplier λ_j has the useful interpretation of measuring the <u>shadow price</u> of the aggregate (state dependent) net benefit in state j, j = 1, ..., m. The properties and usefulness of expression (3b) are discussed in detail by Graham (1992). Some of these properties, to which the discussion now turns briefly, will prove useful.

First, note that W(U) is a strictly decreasing function of U. This follows from the non-satiation assumption stating that $u_i(-z_i, .)$ is a strictly increasing function of $(-z_i)$, $i \in N$. Second, denote by $z^*(U)$, $x^*(U)$, $y^*(U)$ and $P^*(U)$ the resource allocation obtained as the solution to the above optimization problem in (3a) or (3b). This solution involves the choice of public goods, the choice of information, as well as the financing of the project (as represented by the payments z^*). It is worth emphasizing that the payments z^* are actually made in (3). This is in contrast with the hypothetical compensation tests commonly used in applied welfare analysis (Kaldor; Hicks; Scitovsky; Samuelson, 1950). In other words, by treating the payments z^* as actual, we treat jointly the design of the public project (x^*, y^*) and its financing (z^*). Third, as shown by Graham (1992), expressions (3) provide a basis for evaluating the Pareto efficiency of resource allocation. This is formally stated in the following proposition.

Proposition 1: (Graham, 1992, p. 836). A necessary and sufficient condition for Pareto optimality is $W(U) = 0$. Then, for any U satisfying $W(U) = 0$, $z^*(U)$, $x^*(U)$, $y^*(U)$ and $P^*(U)$ provide a Pareto optimal allocation of resources among the n individuals.

Clearly, $W(U) \geq 0$ is required for the public choice to be fiscally feasible, i.e. to satisfy the group budget constraint (1b). But finding $W(U) > 0$ cannot be a Pareto optimal situation since it identifies the existence of a positive surplus which can always be redistributed to make at least one individual better off without making anyone worse off. As a result, $W(U) = 0$ is a necessary and sufficient condition for Pareto optimality. This is an important result since the Pareto optimality criterion is widely accepted by economists and provides a corner stone of theoretical and applied welfare economics. Moreover, interpreting C(.) as a measurement of transaction costs, then equations (3) and Proposition 1 show how to incorporate such costs in efficiency analysis.

Solving the equation $W(U) = 0$ for $U = (U_1, U_2, \ldots, U_n)$ generates an infinite number of solutions for U. These solutions identify the Pareto efficiency frontier expressing utility trade-offs under efficient allocation as welfare distribution varies across individuals (Samuelson; Graham, 1992). This raises the question of how movements along the utility frontier will affect the decisions $z^*(U)$,

$x^*(U)$, $y^*(U)$ and $P^*(U)$. This question is important for two reasons. First, if $x^*(U)$, $y^*(U)$ and $P^*(U)$ depend on U, then the Pareto optimality criterion (as stated in Proposition 1) falls short of providing precise guidance on which public choice should be made. In this case, Pareto optimality would suggest a class of possible collective decisions but with no basis for choosing among them. Second, even if $x^*(U)$, $y^*(U)$ and $P^*(U)$ are independent of U, particular individuals will not be indifferent between alternative choices of U on the utility frontier. This follows from interpreting $U = (U_1, U_2, \ldots, U_n)$ as the vector of utility levels actually obtained by the n individuals as a result of public decision making.

If preferences are quasi-linear, a familiar if somewhat restrictive condition, then the efficiency of resource allocation can be evaluated independently of distribution concerns. That is, $x^*(U)$, $y^*(U)$ and $P^*(U)$ are independent of U. The Coase theorem, which implicitly assumes the absence of income effects, is a well known example where efficiency analysis does not depend on distribution issues. Formally, under quasi-linear preferences, we have the following.

<u>Corollary 1</u>: If preferences are quasi-linear (where $u_i(k_i-z_{i1}, k_i-z_{i2}, \ldots, k_i-z_{im}, .) = f_i[k_i + h_i(-z_{i1}, -z_{i2}, \ldots, -z_{im}, .)]$ for some positive monotonic function f_i, $i \in N$), the optimal allocation (x^*, y^*, P^*) is independent of U.

Clearly, under quasi-linear preferences, any change in u_i can be generated by a corresponding change in income k_i without affecting the sub-utility index h_i, $i \in N$. This implies the absence of income effects, where a change in k_i such that $\Sigma_{i \in N}\, k_i = 0$ has no effect on optimal behavior. This also means that a move along the utility frontier can be done by a redistribution of income through the k_i's, without any influence on optimal choices. Then, efficient resource allocation is independent of the distribution of the k_i's across individuals. Moreover, the optimal public choice (x^*, y^*, P^*) in problem (3) is much simpler since it is independent of U. In this case, only the choice of z* is influenced by U. This can be interpreted to mean that the move along the utility frontier can be generated through a simple redistribution of income across individuals, redistribution that has no effect on the efficiency of resource allocation.

However, the empirical evidence in favor of income effects is rather strong. When quasi-linearity cannot be assumed, the simple results from the Coase theorem or from Corollary 1 no longer apply. In general, it is no longer possible to separate efficiency analysis from distribution issues. The presence of income effects implies that x*(U), y*(U) and P*(U) are functions of U. This means that as U changes along the utility frontier, efficient public choices will also change.

Corollary 2: Unless x*(U) and y*(U) are independent of U, there are a multiple number of Pareto optimal projects.

The existence of multiple optimal projects is often troublesome. Unfortunately, given the empirical evidence in favor of income effects, such a situation can be expected to be the rule rather the exception. Policy-making typically requires the choice of a single alternative--the selection from among many alternatives of one "best" project. If economists cannot do more than trace out the (infinite) set of efficient candidate projects, it is understandable that policy-makers pay little heed to economic analysis. The need for a criterion that selects one efficient project is especially keen when alternative distributions affect efficient choices, i.e. when distribution and efficiency are joint issues that cannot be analyzed independently of each other. In the following section we develop our fairness criterion for use in public decision-making under uncertainty. It is an equity-based criterion that complements Pareto and that appears to satisfy a number of intuitively appealing distributional notions.

IV- <u>A Fairness Criterion</u>:

The literature on fairness in economics and welfare analysis is vast (Foley; Rawls; Kolm; Varian; Feldman and Kirman; Pazner; Pazner and Schmeidler, 1974, 1978; Crawford; Goldman and Sussangkarn; Thomson; Thomson and Varian; Baumol). Fairness has been defined as the absence of envy. A group is said to be characterized by the absence of envy if no individual would prefer what another has to what he/she has. The concept of fairness is appealing for several reasons: it provides an intuitive basis for analyzing distribution issues; it exhibits symmetry across individuals; it is consistent with on ordinal representation of individual preferences; and it is free of interpersonal comparison of utility. Indeed, fairness only requires each individual to evaluate others' bundle using their own (ordinal) preferences. Using fairness as a reasonable equity criterion has thus appeared attractive in the analysis of alternative welfare and resource distributions.

Some difficulties with the concept of fair allocation have been pointed out in the literature. First, as shown by Pazner and Schmeidler (1974) and Goldman and Sussangkarn, the fairness criterion is not always consistent with the Pareto efficiency criterion. In light of the general acceptance of the Pareto criterion, it would be desirable to have a concept of equity that never conflicts with Pareto efficiency. Second, as pointed out by Feldman and Kirman, a fair move away from a fair initial allocation can result in an unfair terminal allocation. This indicates the fragility of the fairness criterion in the evaluation of distribution issues. Finally, the definition of fairness found in the literature is based on the existence of private goods that can be transferred among individuals. This fails to capture an important feature of public goods which, by definition, are available to all individuals. It seems that an alternative definition of fairness applicable to public goods is needed.

Here, we propose a definition of fairness that has the following properties: 1/ it is consistent with an ordinal representation of individual preferences; 2/ it is free of interpersonal comparison of preferences; 3/ it is appropriate for the evaluation of public projects; and 4/ it is always consistent with the Pareto optimality criterion. It also preserves the essence of previous research on fairness: at an allocation satisfying our definition, no agent wishes he/she were in someone else's position.

Our approach to fairness involves a joint evaluation of the design of the public goods (x, y) and its method of financing (z). This is an intuitive requirement since, as discussed above, both the project design and its financing influence the distribution of welfare across individuals. This requirement is also attractive for another reason. It involves a private good, money (represented by z), that is transferable across individuals. The use of the fairness criterion requires at least one private transferrable good. Because money provides an universal means of exchange among individuals, it is employed here as a private good providing a basis for evaluating the fairness of a public project.

In the evaluation of a public project, we focus here on the fairness of the distribution of its net benefits among the n individuals. We wish to discover conditions under which the financing (z) of a particular public project (x, y) generates a fair distribution of its benefits in the group. For that purpose, we employ the concept of fairness-equivalence proposed by Pazner. Pazner (p. 463) defines an allocation to be fair-equivalent if there exists a fair allocation in some hypothetical economy in which each person enjoys the same welfare level as that enjoyed by him at the allocation under consideration. Our definition of fairness-equivalence is based on the monetary evaluation of the individual net benefits from the project. Given a project (x, y), its financing (z) and an information structure P, satisfying the feasibility conditions (1a), (1b), (1c) and (1d), the net ex ante benefit of the project to the i-th individual is measured by w_i satisfying:

$$u_i(-z_i, x, y, e) = u_i(w_i, 0, 0, e), \text{ subject to (1a), (1b), (1c) and (1d)}, \quad i \in N. \tag{4}$$

Let $w_i(z_i, x, y, P)$ denote the implicit solution of equation for w_i in (4), for each $i \in N$. It measures the ex ante selling price of (z, x, y, P), i.e. the smallest amount of money paid to the i-th individual that makes him willing to give up (z, x, y, P) and replace it with (0, 0, 0, E). Note that, in contrast to the actual payments z, the welfare measures w defined in (4) are hypothetical. Using w_i as a basis for evaluating the individual benefit of the project and its financing, we define fairness-equivalence as follows.

Definition 3: A project (z, x, y) under information structure P and satisfying (1a), (1b) and (1c) is said to be fair-equivalent if

$$w_1(z_1, x, y, P) = w_i(z_i, x, y, P), \quad i \in N, \tag{5}$$

where $w_i(z_i, x, y, P)$ is defined in (4).

Thus, a project (including its method of financing) is fair-equivalent if its net benefits are equally distributed among the n beneficiaries. This definition is intuitive. It is general in the sense that it is applicable to any feasible allocation, including allocations that are not Pareto efficient. It also indicates that any situation where $w_i(z_i, x, y, P) \neq w_h(z_h, x, y, P)$, $i \neq h \in N$, is not fair-equivalent: it generates net benefits that are not evenly distributed among the n individuals. This suggests the following index of fairness for the i-th individual:

$$I_i = w_i(z_i, x, y, P) - [\Sigma_{h\in N}\, w_h(z_h, x, y, P)]/n, \quad i \in N. \tag{6}$$

The index I_i is a simple measure of the excess-benefit obtained by the i-th individual as compared with the average benefit across all individuals within the group.[8] Using this index, an allocation is fair-equivalent if and only if $I_i = 0$ for all $i \in N$. And it is not fair-equivalent for the i-th individual if $I_i \neq 0$, i.e. if the i-th individual benefits are either less than the average ($I_i < 0$) or more than the average ($I_i > 0$). This provides useful information on the nature of the departure from fairness-equivalence. For example, finding $I_i > 0$ means that the allocation (z, x, y, P) is unfair and that the unfairness is in favor of the i-th individual. Alternatively, finding $I_i < 0$ means that the allocation (z, x, y, P) is unfair and that the unfairness is at the detriment of the i-th individual. Thus, the index I_i has some desirable characteristics. It can measure both the direction and the strength of unfairness or inequity. And it provides a continuous measure of envy or unfairness under ordinal preferences.[9] Thus, it seems to be an attractive measure of equity (or inequity) in the analysis of distribution issues related to public decision making.

Some general implications of our project fairness criterion are worth noting. First, an egalitarian allocation which treats every individual the same in the sense that $z_1 = z_i$, $i \in N$, is not necessarily fair-equivalent. This is in sharp contrast with the fairness of private goods found in the literature (e.g. Varian; Pazner and Schmeidler, 1978). The reason is that, in the evaluation of fairness-equivalence, we focus here on public goods that are available to all individuals and not subject to transfer across individuals. Indeed, under heterogenous preferences, the ability of different individuals to benefit from the public goods in (x, y) may vary greatly, implying that an egalitarian distribution of the individuals' payments z could be grossly unfair. In general, our fairness-equivalence criterion has the intuitive interpretation that individuals who benefit more from the public goods should pay a larger share of their cost. Second, if all individuals are identical, then an egalitarian distribution of the payments z (where $z_1 = z_i$, $i \in N$) would always be fair-equivalent. Indeed, in this case, each individual would obtain the same benefit from the public goods, implying that fairness-equivalence would hold only if their contributions z_i are also the same. Third, consider the situation where individuals are actually heterogenous but that there is no public information available to discriminate among them. In such a case, there would obviously be no basis for implementing a non-egalitarian payment scheme z. Under such an information structure, an egalitarian distribution of the payments z would necessarily be fair-equivalent.

Note that Definition 3 applies also to Pareto optimal allocations, i.e. to allocations (z*(U),

$x^*(U)$, $y^*(U)$, $P^*(U))$ where U satisfies $W(U) = 0$ (from Proposition 1). This implies that a project is Pareto efficient and fair-equivalent if and only if it satisfies

$$w_1^*(U) = w_i^*(U), \;\; i \in N,$$

and

$$W(U) = 0,$$

where $w_i^*(U) \equiv w_i(z_i^*(U), x^*(U), y^*(U), P^*(U))$, $i \in N$. Note that, given $C(0, 0, e_j) = 0$ for all j, it follows that any allocation that is Pareto optimal must satisfy $w_i^*(U) \equiv w_i(z_i^*(U), x^*(U), y^*(U), P^*(U)) \geq 0$, $i \in N$.

The next question concerns the existence and uniqueness of a project that is both efficient and fair-equivalent.

Proposition 2: There always exists a utility vector $U^* = (U_1^*, U_2^*, \ldots, U_n^*)$, unique up to positive monotonic transformations, that is on the Pareto utility frontier and corresponds to a fair-equivalent project, where

$$U^* = \{U: W(U) = 0; w_1^*(U) = w_i^*(U), \;\; i \in N\}.$$

The proof can be obtained by construction. It can be made in the w-space[10] or equivalently in the utility space. First, consider the feasible allocation $(z, x, y, P) = (0, 0, 0, E)$, i.e. "doing nothing". The corresponding individual net benefits are $w_i = 0$, $i \in N$, implying that this allocation is fair. Except in the trivial case where it happens to be Pareto optimal, this allocation will generate a utility vector $U = (U_1, U_2, \ldots, U_n)$ that is in the interior of the utility possibility set bounded by the utility frontier $W(U) = 0$. Under continuity, it is clearly possible to move from this allocation toward the utility frontier. Since this move can be made in any positive direction (in the utility space), it can generate any distribution of relative individual net gains. One of these positive directions corresponds to a fair allocation generating equal net benefits $w_i > 0$ (as defined in (4)) for all n individuals. If such a move is made all the way to the utility frontier, it necessarily generates a fair-equivalent and efficient allocation. This implies that a fair-equivalent and efficient allocation always exists. Moreover, it exists for a value of U that is unique up to positive monotonic transformations. This follows from continuity and the property that W(U) is a strictly decreasing function of U, while the move toward the utility frontier is increasing in U. In other words, there is a point $U^* = (U_1^*, U_2^*, \ldots, U_n^*)$, unique up to positive monotonic transformations, that satisfies both the efficiency criterion ($W(U) = 0$) and the fairness-equivalence criterion ($w_1^*(U) = w_i^*(U)$, $i \in N$).

Given U^*, the optimal and fair-equivalent allocation is then

$$z^+ = z^*(U^*)$$

$$x^+ = x^*(U^*)$$

$$y^+ = y^*(U^*)$$

$$P^+ = P^*(U^*).$$

Since U^* is well defined and unique, the optimal and fair-equivalent choice (z^+, x^+, y^+, P^+) is unique if and only if the optimization problem (3a) has a unique solution. In this case, our proposed framework provides a basis for improving the normative usefulness of economic analysis. It generates a unique recommendation about how a project should be designed in terms of both the public goods provided (x^+, y^+) and the method of financing them (z^+). Such a recommendation has the property of corresponding to an allocation that is always both Pareto efficient and fair-equivalent.

Since efficiency and our concept of fairness-equivalence can always be made consistent with each other, this suggests the possibility of combining them in economic analysis. More specifically, we look for a way to formulate an optimization problem that would incorporate fairness-equivalence with the net benefit criterion (3) used in the characterization of Pareto efficiency. The constructive proof of Proposition 3 suggests considering the following problem:

$$w^{+} = \underset{z,x,y,P}{\mathrm{Max}} \{\underset{i \in N}{\mathrm{Min}} \{w_i\}: u_i(-z_i,x,y,e) = u_i(w_i,0,0,e),\ i \in N;\ \text{equ. (1a), (1b) and (1c)}\}. \tag{7}$$

The solution of the optimization problem (7) always generates the efficient and fair-equivalent allocation. Indeed, by definition of a maximum, the solution is necessarily on the Pareto utility frontier. And it must be fair-equivalent and satisfy $w_1 = w_i$, $i \in N$, since any other efficient allocation could be subject to an efficiency-preserving income redistribution toward the individual with lowest w_i that would increase the value of the objective function. It follows that $w^+ = w_i^*(U^*)$, $i \in N$, where w^+ is the optimal objective function in (7). The optimization problem (7) thus provides a convenient basis for conducting a welfare analysis of public projects under the joint requirements of obtaining an efficient and fair-equivalent allocation.

The maximin characterization (7) exhibits a number of close similarities with Rawls' Theory of Justice. Indeed, Rawls proposed a similar maximin criterion as a measure of social welfare. Expression (7) shows that a maximin principle can lead to a Pareto-efficient allocation. It also suggests that the Rawlsian "original position" could be interpreted to be the one where (z, x, y, P) = (0, 0, 0, E). This is a fair egalitarian-equivalent (but not efficient) allocation where everyone is treated the same: it corresponds to "doing nothing" in the absence of learning. In this case, the "original position" would be the one before any public decisions on (x, y) have been made. And the absence of learning is consistent with Rawls' "veil of ignorance" when the states of nature include all the information required to discriminate among the n individuals. However, it should be emphasized that Rawls' maximin principle has been applied to personal utilities, which requires explicit interpersonal utility comparisons. In contrast, our maximin problem (7) is presented in a very different context: under ordinal individual preferences and in the absence of any interpersonal comparisons of utility. This suggests that, while a particular method of interpersonal utility comparisons can be consistent with the maximin criterion, such comparisons are not required to motivate the maximin principle in welfare analysis of public goods.

Finally, our results can be compared and contrasted with bargaining models that have been proposed in the search for normative criteria evaluating the distribution of resources (e.g. Nash; Harsanyi; Zeuthen; Kalai and Smorodinsky; Riddell). First, such models have typically been developed in the context of cardinal utility.[11/] This is in contrast with our approach that only requires personal ordinal ranking of alternative allocations. Second, the idea of a "threat point" in bargaining models is similar to our reference situation (0, 0, 0, E) used to evaluate the fairness of the project: it is the outcome that would be obtained in the absence of any agreement among the n individuals. Third, Nash bargaining models have been developed both from an axiomatic viewpoint (Nash) and from a procedural viewpoint (Zeuthen; Harsanyi). The procedural approach to bargaining has clearly both intuitive and practical appeal in economic analysis. This raises the question of searching for some procedural scheme that would achieve our proposed public choices, a question to which we now turn.

V- A Mechanism:

Having defined the efficient and fair-equivalent allocation $z^+ = z^*(U^*)$, $x^+ = x^*(U^*)$, $y^+ = y^*(U^*)$ and $P^+ = P^*(U^*)$, we now devise a simple mechanism that achieves this allocation. The mechanism we focus on is suggested by the optimization problem (7), and consists of a succession of iterative proposals. Formally, the mechanism specifies the nature of the offer made at each iteration and a termination rule. This process eventually reaches the efficient and fair-equivalent allocation (z^+, x^+, y^+, P^+). It is possible to imagine a sequence of offers arising from two different sources. They may be made by the members of the group, or by individuals outside the group -- perhaps by an outside arbitrator.

Suppose that the iterative procedure begins at the reference situation (0, 0, 0, E), and let the offer made at the k-th iteration be denoted by (z^k, x^k, y^k, P^k). Given an offer, the welfare of each individual $i \in N$ can be evaluated using the ex ante welfare measure w_i^k defined in equation (4):

$$u_i(-z_i^k, x^k, y^k, e) = u_i(w_i^k, 0, 0, e), \text{ given } P^k, i \in N.$$

Intuitively, the process starts at the reference allocation representing inaction, and drives the outcome toward the efficient and fair-equivalent allocation. The process ends when it is not possible to improve the welfare of the worst-off person. It is possible for this simple process to fail to converge to (z^+, x^+, y^+, P^+). The sequence of offers could approach some other allocation asymptotically, but at each stage the worst-off member is still strictly better off than was the worst-off member from the previous stage. In order to ensure that the process does not become "stuck" in this fashion, we assume that there is a scalar $\delta > 0$ such that the improvement in the worst-off welfare level from one stage to the next is at least δ.[12] Throughout, we employ the following assumptions:

Assumption 1: Each offer must satisfy the feasibility conditions (1a), (1b), (1c), and (1d).

Assumption 2: There exists a finite number $\delta > 0$ such that at each stage $k = 1, 2, 3, \ldots,$

$$\text{Min } \{w_i^k: i \in N\} - \text{Min } \{w_i^{k-1}: i \in N\} \geq \delta.$$

As is true of the process proposed by Zeuthen and described by Harsanyi (p. 152), at the final stage these two assumptions may be in conflict. In this case, assumption 1 takes precedence.

Formally, we propose the following rules:

Rule 1 - (first iteration): An offer (z^1, x^1, y^1, P^1) is made such that Assumption 1 is satisfied, and $w_i^1 \geq 0$, $i \in N$.

Rule 2 - (k-th iteration, $k = 2, 3, \ldots$): An offer (z^k, x^k, y^k, P^k) is made such that Assumptions 1 and 2 are satisfied.

Rule 3 - (convergence rule): The iterative process converges when there does not exist any new offer that satisfies rule 2.

We now argue that this process must generate the efficient and fair-equivalent allocation $z^*(U^*)$, $x^*(U^*)$, $y^*(U^*)$ and $P^*(U^*)$. Indeed, from Rule 2 and equation (7), since Min $\{w_i^k: i \in N\}$ necessarily increases from one iteration to the next, it can reach convergence (as defined in rule 3) only at the upper bound given by w^+ in (7). And by Assumption 2, it must necessarily converge in a finite number of steps. Thus, upon convergence, the process yields the fair-equivalent and efficient allocation. In other words, this simple scheme can provide a mechanism for the group to make collective decisions.

The role played by our Assumption 2 is crucial. How reasonable is it or, put another way, when might it be violated? It might be violated if the individual who is treated the most unfairly (i.e. with the lowest w_i) would see arbitrarily small improvements in his net benefit through the process. Or it could correspond to a situation where successive offers change the identity of the individual receiving the lowest benefit w_i without generating a significant increase in Min$\{w_i: i \in N\}$. In either case, the proposed iterative process would fail to generate a fair-equivalent resource allocation.[13/]

The treatment given above can place offer-making authority in the hands of members themselves. In this case, the rule governing which individual within the group makes an offer at a particular iteration appears to be important. If the individual treated the most unfairly (i.e with the lowest w_i) is the one making the offer, he/she has some incentive to speed up the move toward a fair-equivalent outcome. This can help obtaining a fast convergence of the mechanism to an efficient and fair-equivalent allocation. On the other hand, if the individuals treated unfairly are not involved in making proposals, then there is no private incentive to move toward a fair-equivalent allocation. This could clearly happen if the successive offers are made exclusively by the individuals benefiting from the unfair allocations. In such a situation, Assumption 2 may not be satisfied through the iterative process and the mechanism may fail to converge to a fair-equivalent allocation.

It is also interesting to consider the case in which offers are made by individuals outside the group. One obvious example would be an arbitration setting, where an outside arbitrator is brought into the negotiation process when the parties involved fail to reach an agreement acceptable within the group. Another example would be the involvement of the Courts following a disagreement about resource distribution that cannot be settled between the parties involved. To the extent that arbitrators or the Courts are concerned with fairness, our proposed mechanism may provide useful insights into their role and influence on the efficiency and distribution of resource allocation in public decision making.

VI- The Role of Information:

Recall that the decisions analyzed here include both the choice of information and the choice of public goods. In this section, we explore the role of information in public decision making.[14] This is motivated by a need to assess the informational efficiency of a particular public decision making process, and by the fact that collective choices are often made in a situation where interest groups try to lobby and influence the information publicly available. In this section, we explore the implications of different information structure on the efficiency and fairness-equivalence of the outcome.

Consider a decomposition of problem (3) into two stages: first choose (z, x, y) given a feasible information structure P_o; and second choose the optimal information structure. A feasible partition P_o is defined as an information structure which satisfies $P_o \in \{P: \exists\ (x, y)$ such that $(x, y, P) \in \Omega;\ P \in \mathcal{P}\}$ and can be obtained at a finite cost $C(x, y_j, e_j)$ for all j. Given some feasible P_o, the first stage problem takes the form

$$V(U,P_o) = \underset{z,x,y}{\mathrm{Max}}\{\sum_{i\in N} z_{i1}-C(x,y_1,e_1):\ U_i=u_i(-z_i,x,y,e),\ i\in N;\ \text{equ. (1a), (1c), (1d) and (2)};$$
$$\sum_{i\in N} z_{i1}-C(x,y_1,e_1) \leq \sum_{i\in N} z_{ij}-C(x,y_j,e_j),\ j=2,\ldots,m;\ P=P_o\}, \tag{8}$$

which has as a solution the conditional choices $z^c(U,P_o)$, $x^c(U,P_o)$ and $y^c(U,P_o)$, i.e. the choices (z, x, y) that are conditional on the information structure P_o. The second stage is

$$W(U) = \mathrm{Max}_P \{V(U,P): \text{for all feasible partitions } P\}.$$

Denote by $P^*(U)$ the optimal solution of stage two. Combining the two stages gives the allocation discussed above: $z^* = z^c(U,P^*(U))$, $x^*(U) = x^c(U,P^*(U))$ and $y^*(U) = y^c(U,P^*(U))$.

The first stage in (8) provides a basis for evaluating the role of information in public decision making. In order to assess the efficiency of a particular information structure P_o, consider constrained-Pareto optimality defined as Pareto optimality under the restriction that $P = P_o$, i.e. conditional on P_o. Then, combining (8) with Proposition 1, constrained-Pareto optimality corresponds to any U satisfying $V(U, P_o) = 0$. Moreover, the following result sheds some light on the implications of learning.

Proposition 3: Given two feasible partitions of E, P and P', if $P\ F\ P'$ and $\{(x, y): (x, y, P) \in \Omega\} \supseteq \{(x, y): (x, y, P') \in \Omega\}$, then $V(P, U) \geq V(P', U)$.

Proposition 3 compares two information structures P and P', where P is at least as fine as P', $P\ F\ P'$, and where the feasible set for (x, y) under P is at least as large as the one under P', $\{(x, y): (x, y, P) \in \Omega\} \supseteq \{(x, y): (x, y, P') \in \Omega\}$. From (1c) and (1d), a finer information structure tends to expand the feasible region for (y, z). Then, under the stated assumptions, the feasible region for (z, x, y) in (8) is at least as large under P as compared to P'. The result in Proposition 3 follows from the fact that the optimal value of the objective function in a maximization problem cannot decrease with a larger feasible region.

Proposition 3 states that a finer information structure that does not increase the cost C(.) tends to shift up the constrained-Pareto efficiency frontier. In other words, better information that can be obtained without a higher cost necessarily constitutes a Pareto improving move in public decision making. This can be interpreted to mean that the value of costless information is always non-negative in public decision-making. This result is consistent with the literature on moral hazard or adverse selection, where a lack of information has adverse effects on the efficiency of resource allocation.

However, it should be kept in mind that the result stated in Proposition 3 does not hold in general if obtaining additional information is costly, or if the two information structures considered cannot be ranked according to *F*. To see that, consider any two feasible information structure P_a and P_b. Assume that we want to compare the relative efficiency of P_a and P_b, using P_a as the reference situation. We have seen that constrained-Pareto optimality in the reference case is given by any U_a satisfying $V(P_a, U_a) = 0$. Then, a measure of the difference in aggregate net benefit generated by P_a versus P_b is: $V(P_b, U_b) - V(P_a, U_a) = V(P_b, U_a)$. Thus, $V(P_b, U_a)$ can be interpreted as the aggregate welfare gain (or loss if negative) associated with replacing information structure P_a by P_b, using U_a as reference utility levels. Finding $V(P_b, U_a) > 0$ would identify a Pareto improving move from P_a, implying that the information structure P_a is not Pareto optimal. Alternatively, finding $V(P_b, U_a) < 0$ means that the information structure P_b is not Pareto efficient.

We turn now to the implications of information for fairness. In equation (5), we defined fairness-equivalence as a situation where $w_1(z_1, x, y, P) = w_i(z_i, x, y, P)$, where the willingness-to-pay w_i is given in equation (4), $i \in N$. It follows that, for a given feasible information structure P_o, an allocation is both fair and constrained-Pareto efficient if it satisfies $V(P_o, U) = 0$ and $w_1(z_1, x, y, P_o) = w_i(z_i, x, y, P_o)$, $i \in N$. As in Proposition 2, it can be shown that such an allocation always exists. Let U_o^* denote the utility vector (uniquely defined up to positive monotonic transformations) corresponding to a fair-equivalent and constrained-Pareto optimal allocation. Conditional on the information structure P_o, this fair and efficient allocation is given by $z_o^c = z^c(U_o^*, P_o)$, $x_o^c = x^c(U_o^*, P_o)$ and $y_o^c = y^c(U_o^*, P_o)$. In general, all the results obtained above apply to the choice of (z, x, y), conditional on the information structure P_o. For example, under a fair-equivalent and constrained-efficient allocation given P_o, the fairness index (6) for the i-th individual is $I_i(U_o^*, P_o) \equiv w_i(z_{oi}^c, x_o^c, y_o^c, P_o) - [\Sigma_{h \in N} w_h(z_{oh}^c, x_o^c, y_o^c, P_o)]/n = 0,\ i \in N$.

What happens to fairness (e.g. as measured by our fairness index (6)) when the information structure P_o changes? A special case of interest is when all individuals are identical. In such a situation, treating all individuals the same would always be fair under any information structure. However, whenever all individuals are not identical and at least some information can be obtained on their individual characteristics, then any change in P_o can affect fairness in any possible direction: holding U_o^* constant, $I_i(U_o^*, P_o)$ can increase, remain constant or decrease with a change in P_o for any $i \in N$. This follows from the fact that learning (or alternatively the lack of information) can differentially affect individual welfare, thus influencing the distribution of the individual benefits w_i in possibly non-equitable ways. This is true whether the resulting allocation is constrained-Pareto optimal or not. Also, this is true whether the new information is costly or not. Thus, obtaining more costless information is not always "better" (as implied by the efficiency results in Proposition 3) once fairness is taken into consideration. This reflects in part the possibility that new information can affect the public perception of individual benefits: a particular distribution of individual benefits may be deemed fair under some information structure, but unfair under another.

Thus, the control of information can play a significant role in assessing the fairness of public decision making when all individuals are not identical and at least some information can be obtained on their individual characteristics. In this case, information will in general affect <u>both</u> the efficiency and the fairness of the outcome. This can provide incentives for individuals to influence public information to their own personal advantage. It suggests a need for a more refined investigation of the role of information in public decision making, in welfare analysis under uncertainty, and in the investigation of equity issues.

VII- Concluding Remarks:

This paper has presented an approach to welfare analysis under uncertainty by complementing the Pareto efficiency criterion with an equity criterion: the fairness-equivalence criterion. It focuses on the design and implementation of a public project, its method of financing, as well as the choice of the information available for public decision-making. Allocations that are both Pareto efficient and fair-equivalent always exist. This provides a basis for conducting benefit-cost analysis under ordinal preferences and in the absence of interpersonal welfare comparison. We have shown that a fair-equivalent and Pareto efficient allocation leads to a maximin criterion defined in terms of individual ex-ante willingness-to-pay. We have also investigated in some detail the role of information in public decision making, especially the effect of information on efficiency and fairness.

By incorporating equity issues in welfare analysis, our proposed approach explicitly addresses both efficiency and distribution issues in the analysis of public decision-making. To the extent that fairness is a relevant criterion, this should help economists to be more effective in the evaluation, design and implementation of public projects and their method of financing. It is hoped that this paper will help stimulate additional research on this important topic.

References

Arrow, K.J. Social Choice and Individual Values second ed., Yale University Press, New Haven, 1963.

Arrow, K.J. and A.C. Enthoven "Quasi-Concave Programming" Econometrica 29(1961):779-800.

Baumol, W.J. Superfairness The MIT Press, Cambridge, Mass., 1986.

Bergson, A. "A Reformulation of Certain Aspects of Welfare Economics" Quarterly Journal of Economics 52(1938):310-334.

Coase, R.H. "The Problem of Social Cost" Journal of Law and Economics 3(1960):1-44.

Crawford, V.P. "A Game of Fair Division" Review of Economic Studies 44(1977):235-247.

Feldman, A. and A. Kirman "Fairness and Envy" American Economic Review 64(1974):995-1005.

Foley, D.K. "Resource Allocation in the Public Sector" Yale Economic Essays 7(1967):73-76.

Goldman, S.M. and C. Sussangkarn "On the Concept of Fairness" Journal of Economic Theory 19(1978):210-216.

Graham, D. "Cost Benefit Analysis under Uncertainty" American Economic Review 71(1981):715-725.

Graham, D. "Public Expenditure under Uncertainty: the Net Benefit Criterion" American Economic Review 82(1992):822-846.

Harsanyi, J.C. Rational Behavior and Bargaining in Games and Social Situations Cambridge University Press, Cambridge, 1977.

Hicks, J.R. "The Foundations of Welfare Economics" Economic Journal 49(1939):696-700, 711-712.

Kalai, E. and M. Smorodinski "Other Solutions to Nash's Bargaining Problem" Econometrica 43(1975):513-518.

Kaldor, N. "Welfare Propositions of Economics and Interpersonal Comparisons of Utility" Economic Journal 49(1939):549-552.

Kolm, S.C. Justice et Égalité Éditions du Centre National de la Recherche Scientifique, Paris, 1972.

Nash, J.F. "The Bargaining Problem" Econometrica 43(1950):155-162.

Pazner, E.A. "Pitfalls in the Theory of Fairness" Journal of Economic Theory 14(1977):458-466.

Pazner, E.A. and D. Schmeidler "A Difficulty in the Concept of Fairness" Review of Economic Studies 41(1974):441-443.

Pazner, E.A. and D. Schmeidler "Egalitarian Equivalent Allocations: A New Concept of Economic Equity" Quarterly Journal of Economics 92(1978):671-687.

Rawls, J. A Theory of Justice Harvard University Press, Cambridge, Mass., 1971.

Riddell, W.C. "Bargaining under Uncertainty" American Economic Review 71(1981):579-590.

Rubinstein, A., Z. Safra, and W. Thomson "On the Interpretation of the Nash Bargaining Solution and its Extension to Non-Expected Utility Preferences" Econometrica 60(1992):1171-1186.

Samuelson, P.A. "Evaluation of Real National Income" Oxford Economic Papers 2(1950):1-29.

Samuelson, P.A. "The Pure Theory of Public Expenditure" Review of Economics and Statistics 36(1955):387-389.

Scitovsky, T. "A Note on Welfare Propositions in Economics" Review of Economic Studies 9(1941):77-88.

Sposito, V.A. Linear and Nonlinear Programming Iowa University Press, Ames, 1975.

Takayama, A. Mathematical Economics Cambridge University Press, Cambridge, 1986.

Thomson, W. "Problems of Fair Division and the Egalitarian Solution" Journal of Economic Theory 31(1983):211-226.

Thomson, W. and H.R. Varian "Theories of Justice Based on Symmetry" in L. Hurwicz, D. Schmeidler and H. Sonnenschein (eds.) Social Goals and Social Organization: Essays in Memory of Elisha Pazner Cambridge University Press, Cambridge, 1985.

Varian, H.R. "Equity, Envy and Efficiency" Journal of Economic Theory 9(1974):63-91.

Zeuthen, F. Problems of Monopoly and Economic Warfare Routledge and Kegan Paul, London, 1930.

Footnotes

1. The extension of our analysis from a two-period model to a multi-period model would be fairly straightforward. It would require defining successively finer partitions of E, each one characterizing the information used to make decisions at time t = 2, 3, ... In that sense, our simple two-period model seems to capture the essence of a dynamic decision making process with learning.

2. As a special case, the utility function $u_i(.)$ could be the expected utility function

$$u_i(.) = \Sigma_j \pi_j v_i(-z_i, x, y, e_j),$$

where π_j is the probability of state j, and $v_i(.)$ is the i-th individual's state-dependent von Neuman-Morgenstern utility function, $i \in N$. However, this particular utility function would be cardinal, an assumption that is not needed for the analysis which follows. In its most general form, the utility function $u_i(-z_i, y, x, e_j)$ does not have to be linear in the probabilities. It does not even require that probabilities exist. It only requires that individuals have a consistent subjective ex-ante evaluation of the uncertain states.

3. We use a broad interpretation of the public goods in (x, y): they can be either "goods" (having a positive influence on individual welfare) or "bads" (having a negative effect on $u_i(.)$).

4. This allows for the special case where the learning process is independent of the production of the public goods. The production technology would then be non-joint: the choice of information gathering activities in x would have no influence on the production of public goods; and the production process for public goods would have no effect on the amount of learning.

5. The relationship between expression (3a) and the criteria proposed in welfare analysis under uncertainty is discussed in detail by Graham (1981).

6. Slater's condition is (Takayama, p. 73):

there exists a point (z^0, x^0, y^0, P^0) satisfying the feasibility conditions (1a), (1c) and (1d) along with the condition $\Sigma_{i\in N} z_{ij}{}^0 - C(x^0, y_j{}^0, e_j) - [\Sigma_{i\in N} z_{i1}{}^0 - C(x^0, y_1{}^0, e_1)] > 0$, $j = 2, 3, \ldots, m$.

Note that Slater's condition can be dispensed with in special cases (e.g. when the cost function $C(x, y_j, .)$ is linear in (x, y_j); see Takayama, p. 75).

7. The convexity condition is that the set K is convex (see Sposito, p. 146), where

$$K = \{(\alpha, \beta_2, \ldots, \beta_m): \Sigma_{i\in N} z_{i1}\text{-}C(x,y_1,e_1) \geq \alpha; \Sigma_{i\in N} z_{ij}\text{-}C(x,y_j,e_j) - [\Sigma_{i\in N} z_{i1}\text{-}C(x,y_1,e_1)] \geq \beta_j,\ j = 2, \ldots, m; (\alpha, \beta_2, \ldots, \beta_m) \in \mathbf{R}^m; \text{for some feasible point } (z, x, y, P) \text{ satisfying (1a), (1c) and (1d)}\}.$$

See also Takayama, and Arrow and Enthoven in the context of quasi-concave optimization.

8. An alternative index with similar properties would be obtained by substituting the median of w for its mean in equation (6).

9. This can be contrasted with the indexes of envy proposed by Feldman and Kirman: those indexes are either discrete or require cardinal utility functions (e.g. Feldman and Kirman, p. 997). As such, the fairness index (6) may be an improvement over previous indexes found in the literature.

10. A proof in the w-space can be obtained following Pazner and Schmeidler (1978).

11. However, Rubinstein et al. have recently proposed a Nash bargaining model under ordinal preferences.

12. The procedure we describe bears some resemblance to Zeuthen's approach to the bargaining process. Harsanyi (p. 152) establishes the mathematical equivalence between Zeuthen's procedure and the Nash bargaining game. Harsanyi's "minimum concession" assumption is akin to our Assumption 2.

13. The process may also fail to generate a Pareto optimal allocation. This would correspond to a situation of rent seeking behavior generating an efficiency loss.

14. A good discussion of the efficiency implications of alternative choices for (z, x, y) is provided in Graham (1992).